

Financial Anarchy. A new measure of financial fragility

Dimitris Voliotis
Dpt. of Banking and Financial Management,
University of Piraeus, 18758, Piraeus, Greece

Abstract

This paper presents a simple banking system in a game-theoretic framework where banks operate as self-interested agents, maximizing their leverage at the expense of the overall financial stability. This strategic inefficiency raises substantive concerns about the way banks manage the “financial stability” good, which is appropriated in a “tragedy of commons” situation. The inefficiency is conceptualized using the “price of anarchy” (Koutsoupias & Papadimitriou, 2009). The game admits a best response potential function (Voorneveld, 2000) that makes the “price of anarchy” measure being bounded from above. In this framework, we seek the optimal regulatory framework that minimizes the “price of anarchy” or equivalently the level of financial fragility.

1 Introduction

Financial stability has always been a critical element for the smooth functioning of economic activity. When the financial system becomes unstable, the everyday business can be disrupted, making the real economy more fragile and vulnerable to adverse shocks. For, the monetary authorities have been engaged in the endeavor to keep financial conditions sound, especially for the backbone of the financial system, which is the banking sector. Insofar, the main toolkit for addressing financial stability has been emphasized on a macro-prudential framework, as more suitable to tackle with systemic financial risks. Here, we suggest that we should pay some attention at the micro-level, as well.

No doubt, the regulation of financial markets comes along with significant costs. Banks, conforming to the regulatory framework, adjust their balance sheets to keep their overall risk and their leverage under certain bounds, deliberately narrowing down their opportunities to enhance their business. For instance, the imposition of a cash reserve ratio inhibits banks to allocate their high-powered money to higher return investments. Alike, capital adequacy ratio makes banks more resilient to economic downturns, but also bounds their effective leverage. Occasionally, bank managers have identified the adverse role of regulation to banks’ profitability and have figured out the ways to downsize regulatory “costs”. The collection of those new financial instruments and practices, outside the traditional banking, is known in the industry as “shadow banking”.

Shadow banking is described as the credit intermediation involving entities or activities (fully or partially) outside the regular banking system Financial Stability Board (2013). Occasionally, banks securitize mortgage-backed loans to increase their leverage or transfer deposits to sponsored money market funds to bypass the cash reserve ratio. As result, they manage to increase their leverage and pass-through their individualistic risks at the cost of increasing contagion risks.

Is this an optimal behavior by the banks, especially when “shadow practices” come along with an increase in systemic risks? It seems that the expense of financial stability for the increase of short-term profits is no different from the appropriation of common pool resource in the technology of financial services. Indeed, financial stability can be seen as an “open access” good of limited supply, where banks and other financial intermediaries appropriate for their profit. The over-consumption of the financial stability “good” engenders a tragedy of commons situation, in which all stakeholders will suffer the losses of a financial distress.

This paper presents a simple banking system in a game-theoretic framework where banks operate as self-interested agents, maximizing their profits at the expense of the overall financial stability. This “tragedy of commons” situation results inefficient strategic outcomes. For the measurement of inefficiency we use the concept of “price of anarchy” by (Koutsoupias & Papadimitriou, 2009). To my knowledge, the concept has been previously used in economics in Moulin (2007) and Juarez (2006). The game is classified in the broader class of congestion games, which admit an ordinal potential function Monderer & Shapley (1996). The optimization of the potential function guarantees the equilibrium existence. A next step is to measure the maximum inefficiency that can incur in the equilibrium and use this information for measuring financial fragility. Moreover, I investigate the bounds of strategic inefficiency when the latter is quantified in terms of “price of anarchy” (Vetta (2002), and Roughgarden (2006, 2012)). Calculating the upper bound can give us a good measure of how detrimental can be the opportunistic behavior of banks.

Section 2 presents the model and introduces an appropriate measure of strategic inefficiency. In addition, it associates the boundedness properties of inefficiency level to the existing literature for generic cost-minimization games. Section 3 extends the basic model for the existence of a group of bankers with benefit to destabilize the financial system. Finally, Section 4 concludes.

2 The model

Suppose an one-shot game played by $I = \{1, \dots, n, n+1\}$ players, $n \geq 2$ banks and the financial regulatory authority (FRA). Each bank has a simple balance sheet where on the liabilities side we have the Deposits (D) and the Capital (K). For convenience, we set the deposit rate to be zero ($r^D = 0$). On the assets side, we have the cash balances and the position in a single asset A with positive return $r^m > 0$. Someone can think of r^m as the return on assets (ROA) and without loss of generality we assume that it is identical across banks. The FRA will have a decisive role in the game. It will opt for the *capital adequacy ratio* ψ , i.e. the capital that is required to be held as a percentage of bank’s assets, and in addition, for the *reserve requirements ratio* θ , i.e. the minimum reserves of the deposits that ought to be held the in cash. Eventually, the banks incur

a “regulatory tax” amounting to the opportunity cost of holding reserves and binding capital that otherwise could invest in the market portfolio with positive return. The evaded profits for the bank i due to the regulatory tax are estimated to be

$$RT_i = \psi \cdot A_i \cdot r^m + \theta \cdot D_i \cdot r^m = r^m(\psi \cdot A_i + \theta \cdot D_i).$$

In absence of regulation the bank could invest these resources, both reserved deposits ($\theta \cdot D_i$) and reserved capital ($\psi \cdot A_i$), to the asset and enjoy with certainty a positive return r^m .

However, in case of financial distress we assume that there is going to be a horizontal event and all banks will incur a haircut of ω percent. Hence, the objective of bank i is to minimize the total cost that includes the regulatory tax and the expected cost of financial distress. The total cost of bank i for a proper subset of banks $S \subseteq I \setminus \{n + 1\}$ is given by the equation

$$C_i = \alpha_i[r^m(\psi \cdot A_i + \theta \cdot D_i)] + (1 - \alpha_i) \frac{\sum_{j=1}^{\#S} (1 - \alpha_j)}{n} \omega \cdot A_i,$$

which for $\alpha_i \in (0, 1)$ and substituting RT_i becomes

$$C_i = \alpha_i RT_i + (1 - \alpha_i)^2 \frac{\omega A_i}{n} + (1 - \alpha_i) \frac{\sum_{j=1}^{\#S \setminus \{i\}} (1 - \alpha_j)}{n} \omega \cdot A_i. \quad (1)$$

Thoroughly, the bank i decides to evade a percentage $(1 - \alpha_i)$ of the regulatory tax by committing shadow banking practices, whereas the remaining percentage α_i will be “remitted” regularly. Hence, the parameter α_i is the strategic variable of bank i and determines how much of the regulatory tax wants to evade by conducting shadow banking. As follows, the first term in the right hand side is the cost originated by the regulatory tax. Respectively, the second and third term of the right hand side are going to be the expected loss from a financial distress. If $S = \{j \in I \setminus \{n + 1\} | s.t. \alpha_j < 1\}$ is the subset of banks that evade some or all of their regulatory tax then the probability of financial distress is going to be $\frac{\sum_{j=1}^{\#S} (1 - \alpha_j)}{n}$ and the haircut cost for the individual bank i will be ωA_i . It is evident that as the cardinal of evading banks increases, i.e. $\#S \rightarrow n$ and together the regulatory tax evasion increases ($\alpha_i \rightarrow 0$), the probability of the financial distress tends to one.

From the FRA’s perspective the *social optimum cost* is all banks to opt for $\alpha_i = 1$ that makes the overall cost equal to $\bar{\mathbf{C}} = \sum_i C_i((\psi, \theta, \alpha = \mathbf{1}))$. Clearly, the *social optimum cost* would be the overall regulatory tax, i.e.

$$SOC = \bar{\mathbf{C}} = \sum_i RT_i = RT.$$

In the strategic equilibrium, the overall (social) cost will be denoted by $\mathbf{C}^* = \sum_i C_i((\psi^*, \theta^*), \alpha^*)$. One way to measure departures from the social optimum is to employ a *coordination ratio*, which is known in the game-theoretic literature as the *price of anarchy* (Koutsoupias & Papadimitriou, 2009). Since we emphasize on the financial fragility of the banking system, we more appropriately name it for the occasion as the *price of financial anarchy*.

Definition 1 *The Price of Financial Anarchy (PFA) is defined as the deviation from the social optimum cost for the worst case equilibrium (in the equilibrium set), i.e. the ratio*

$$PFA = \max_{\alpha^* \in NE} \frac{\mathbf{C}^*}{\bar{\mathbf{C}}}. \quad (2)$$

Similar to the PFA is the Price of Financial Stability (PFS) which measures the deviation from the best case equilibrium. Of course in case of a unique equilibrium the two measures coincide.

The FRA, having a stabilizing role in the financial system, aims to minimize the objective $C_{n+1} = |PFA - 1|$ by appropriately choosing the policy parameters (ψ, θ) . Overall, a strategy profile of the game is a pair $((\psi, \theta), \alpha)$ with $(\psi, \theta) \in [0, 1]^2$, the policy mix of the FRA, and $\alpha \in [0, 1]^n$ the strategy profile of the banks.

Definition 2 *The Financial Regulation game is defined by the tuple*

$$\Gamma = \{I, [0, 1]^{n+2}, \{C_i\}_{i \in I}\}.$$

Remark. Someone can think the same game in a sequential form, where the FRA decides first the policy parameters (ψ, θ) and then the simultaneous decision of all banks follows.

Definition 3 *The strategic equilibrium is a strategy profile $((\psi^*, \theta^*), \alpha^*)$ such that for all i banks and the FRA*

1. $C_i((\psi^*, \theta^*), \alpha^*) \leq C_i((\psi^*, \theta^*), (\alpha_i, \alpha_{-i}^*))$ for all α_i .
2. $C_{n+1}(\alpha^*, (\psi^*, \theta^*)) \leq C_{n+1}(\alpha^*, (\psi, \theta))$ for all (ψ, θ) .

Remark. The equilibrium always exists. Indeed, both the strategy sets and cost functions are convex, hence a minimum always exists.

The Financial Regulation game admits a *best response potential*. The *best-response potential games* is a special class of potential games that guarantee the existence of Nash equilibrium when the cost function of players is non-linear. The game $\Gamma = \{I, [0, 1]^{n+2}, \{C_i\}_{i \in I}\}$ admits a *best-response potential* $P : [0, 1]^{n+2} \mapsto R$ such that

$$\arg \max_{\alpha_i} C_i(\alpha) = \arg \max_{\alpha_i} P(\alpha)$$

Lemma 1 *The best-response potential function of the Financial Regulation game is*

$$P(\alpha) = \sum_i (1 - a_i)^2 \frac{\omega A_i}{n}.$$

Proof. See Appendix A.

Proposition 1 *The Financial Regulation game has a unique strategic equilibrium.*

Proof. By Lemma 1 and Proposition 2.2 in Voorneveld (2000) the game has a strategic equilibrium. \square

For exposition, think the equivalent sequential version of the game where the FRA sets out the policy parameters (ψ, θ) and then bankers simultaneously decide their α 's. A straightforward case is when bankers share the same characteristics.

Example. Suppose the full symmetric case where all banks share the same characteristics. Then it is all banks that choose the same α_i , for all i , to minimize the cost function

$$C_i = \alpha_i RT_i + (1 - \alpha_i)^2 \omega A_i.$$

The necessary first order conditions give

$$\alpha_i^* = 1 - \frac{RT_i}{2\omega A_i} = 1 - \frac{r^m(\psi A_i + \theta D_i)}{2\omega A_i}.$$

What someone can notice is that the higher the level of haircut (ω) is the higher α_i will be, which means that the cost of distress prevents bankers not to comply to financial regulation. On the other hand, the more costly the regulatory framework is, i.e. higher ψ or θ the more enticed the bankers are to evade the regulatory tax.

Next, we illustrate that if the cost of financial distress is relatively low then bankers will always have incentive to deviate from full compliance to financial regulation.

Corollary 1 *When all banks but i fully conform to financial regulation. i.e. $\alpha_j = 1$ and $RT_i \geq \omega A_i$, then bank i always evades some of the regulatory tax, i.e. $\alpha_i < 1$.*

Proof. If this is the case, the cost function of i reduces to $C_i = \alpha_i RT_i + \frac{(1-\alpha_i)^2}{n} \omega A_i$, which evidently takes its minimum value for $\alpha_i = 1 - \frac{nRT_i}{2\omega A_i} < 1$, whenever $RT_i > 0$. \square

Broadly, the finiteness of the game makes the equilibrium outcome Pareto inefficient. The level of inefficiency remains to be found. For this task, we provide some further definitions.

We denote $\alpha_{-i} \geq \alpha'_{-i}$ whenever component-wise for all $j \neq i$ it is $\alpha_j \geq \alpha'_j$. With some abuse of notation let also $\alpha_{-i} = \alpha_j$.

Definition 4 *We say that the cost function exhibits decreasing differences if for $a_i \geq a'_i$ and $a_j \geq a'_j$ it is*

$$C_i(\alpha_i, \alpha_j) - C_i(\alpha_i, \alpha'_j) \leq C_i(\alpha'_i, \alpha_j) - C_i(\alpha'_i, \alpha'_j), \quad \forall i. \quad (3)$$

A game with cost functions that exhibit decreasing differences will be called *submodular*. Next, we show that the cost function of bankers exhibits decreasing differences.

Lemma 2 *The cost functions of banks for the Financial Regulation game exhibit decreasing differences, i.e. for $\alpha_i \geq \alpha'_i$ and $\alpha_j \geq \alpha'_j$ it is*

$$C_i(\alpha_i, \alpha_j) - C_i(\alpha_i, \alpha'_j) \leq C_i(\alpha'_i, \alpha_j) - C_i(\alpha'_i, \alpha'_j), \quad \forall i.$$

Proof. see Appendix B.

Also, important role in this analysis plays the individual ability of a bank to affect the social cost. We define the concept of *pivotal cost* of banker i at α_i as the social cost when all bankers but i comply with financial regulation. We denote the pivotal cost by $PC_i((\psi, \theta), \alpha_i < 1, \alpha_j = 1)$ and the total pivotal cost $TPC = \sum_i PC_i$, accordingly.

Proposition 2 *The PFA is bounded from above by the $\frac{TPC}{SOC}$.*

Proof. See Appendix C.

From Proposition 2 someone infers that the higher the strategic power of bankers to affect the social cost, attributed by the level of total pivotal cost, the higher the supremum of the inefficiency in the system. Suppose now that the FRA seeks a regulatory policy (θ, ψ) that would make everyone to fully comply and opt for $\alpha = \mathbf{1}$. If this is the case, then an appropriate incentive compatibility constraint would be

$$RT_i \leq PC_i((\psi, \theta), \alpha_i < 1, \alpha_j = 1),$$

that is the cost incurred to the arbitrary banker i by complying to the regulatory policy $C_i(\alpha = \mathbf{1}) = RT_i$ is less or equal to his pivotal cost, the cost that succeeds by unilaterally deviating. An optimum behavior by the FRA would be to set the (θ, ψ) such that for all bankers $RT_i = PC_i$. Then, by proposition 2 it would be that PFA would take always its optimal value, i.e. it would be bounded by 1,

$$PFA \leq \frac{\sum_i PC_i}{\sum_i RT_i} = 1.$$

However, since bankers have different strategic influence to affect the social cost, and no discretionary policy is allowed such that the FRA to assign bank specific policy parameters, i.e. (ψ_i, θ_i) , a more appropriate compatibility constraint in regulatory policy formation would be,

$$\max_i RT_i \leq \min_i \{PC_i((\psi, \theta), \alpha_i < 1, \alpha_j = 1)\}.$$

A special class of cost minimization games with sum objective like financial regulation games is known as smooth games. These games satisfy a certain boundedness property that guarantess an optimal worst case upper bound on the price of anarchy. In the next subsection it is provided the linkage of smooth games with our model.

2.1 PFA robustness for smooth Financial Regulation games

Key concept in defining sufficient conditions for upper bounds of PFA is the smoothness property of cost minimization games, introduced by Roughgarden (2009). Smoothness provides a boundedness property of cost functions, estimated at the equilibrium. In detail,

Definition 5 (Roughgarden (2009)) *A Financial Regulation game Γ is (λ, μ) -smooth with respect to a strategy profile α and a strategic equilibrium $((\psi^*, \theta^*), \alpha^*)$ if*

$$C((\psi^*, \theta^*), \alpha_i, \alpha_{-i}^*) \leq \lambda \cdot C((\psi^*, \theta^*), \alpha) + \mu \cdot C((\psi^*, \theta^*), \alpha^*),$$

with the convention $C = \sum_{i=1}^n C_i$.

For the strategy profile of full compliance $\alpha = \mathbf{1}$, the associated social optimum cost is denoted by \bar{C} . If the Financial Regulation game is (λ, μ) -smooth, it is

$$C((\psi^*, \theta^*), \alpha_i, \alpha_{-i}^*) \leq \lambda \cdot \bar{C} + \mu \cdot C^*.$$

One important result appearing in Roughgarden (2009, 2012) is that the price of anarchy for cost minimization games is bounded from above by $\lambda/(1 - \mu)$. The following result verifies that the upper bound we estimated in Proposition 2 coincides to this bound whenever the game is (λ, μ) -smooth.

Proposition 3 *If the Financial Regulation game Γ is (λ, μ) -smooth then it is $\frac{TPC}{SOC} = \frac{\lambda}{1-\mu}$.*

Proof. see Appendix D.

3 The game with “Byzantine” bankers

In an economic system, there might be a group of agents having a destabilizing role. For instance, in financial markets speculative traders can manipulate an abrupt fall in security prices by taking appropriate short positions; or bonds traders might put pressures to some issuer aiming at the triggering of credit default swaps that they hold in advance. These trades are not always traceable. The recent years the volume of trading via “dark pools” is keep increasing, accounting to an almost 14% of all US stock trading volume, according to Businessweek. Dark pools are trading platforms that allow institutional investors to mask their trading activity from other market participants. In fact, they manage to by-pass the official markets, fragment the market, and as result to disrupt the market information.

Once there is masking trading activity in financial markets, some bankers as financial actors may pursue the destabilization of the financial system. Let us call these traders as Byzantine bankers,

or “malicious” bankers, borrowing the term from the Byzantine Generals’ problem encountered in computer networks literature. Accordingly, we define the game *with Byzantine Bankers* where bankers are divided into two classes, the proper subset of profit maximizing bankers I^p , and the malicious bankers I^m , with $I = I^p \cup I^m \cup \{FRA\}$. Specifically, we assume that each banker in I^m aims at the destabilization of the system, pursuing to maximize the difference $|PFA - 1|$ or equivalently to minimize the $C_i = -|PFA - 1|$. Eventually, we anticipate that no Byzantine banker will opt for a positive α_i , whatever the incurring cost to their balance sheet.

Definition 6 *The Byzantine Financial Regulation (BFR) game is defined by the cost minimization game*

$$\Gamma = \{I, [0, 1]^{n+2}, \langle \{C_i\}_{i \in I^p}, \{C_i\}_{i \in I^m}, \{C_{n+1}\} \rangle\}.$$

In this specification, the overall social cost includes only the cost incurred to the profit maximizing bankers, $i \in I^p$. It is legitimate to exclude Byzantine bankers from the social cost since we assumed that they role is undermining for the social benefit. Thus, $\bar{\mathbf{C}} = \sum_{i \in I^p} C_i((\psi, \theta, \alpha = \mathbf{1}|I^m))$. Accordingly, we define the equilibrium of the game with Byzantine players, $((\psi^*, \theta^*), \alpha^*)$, which we call the *Byzantine strategic equilibrium* as well as the overall cost of the profit maximizing bankers at the equilibrium, $\mathbf{C}^* = \sum_{i \in I^p} C_i((\psi^*, \theta^*), \alpha^*|I^m)$.

Bankers are aware of the presence of the Byzantine bankers, they know their number but they cannot identify them. For, we modify the price of financial anarchy to accomodate the presence of Byzantine bankers.

Definition 7 (Price of Byzantine Financial Anarchy) *The Price of Financial Anarchy (PBFA) is defined as the deviation from the social optimum cost for the worst case equilibrium (in the equilibrium set), i.e. the ratio*

$$PBFA(I^p; I^m) = \max_{\alpha^*} \frac{\mathbf{C}^*(I^p; I^m)}{\bar{\mathbf{C}}(I^p)}. \quad (4)$$

As claimed, we do not calculate the cost of Byzantine bankers to the overall social cost. However, we take into account their strategic influence to the cost of the remaining (profit maximizing) bankers. Moreover, following Moscibroda *et al.* (2006) we define a measure called *Price of Malice* that conceptualizes the relative inefficiency with respect to the original game.

Definition 8 *The Price of Malice measures the inefficiency in the system caused by the presence of Byzantine bankers and is given by the ratio*

$$PoM(I^m) = \frac{PBFA(I^p; I^m)}{PFA(I^p)}. \quad (5)$$

Let us now see how much detrimental can be the role of Byzantine bankers in the financial system.

Proposition 4 *The PBFA is bounded from above by the ratio*

$$PFA + \frac{m}{SOC} \frac{\sum_{i \in I^p} (1 - a_i^*) \omega A_i}{n}.$$

Proof. See Appendix E.

What appears interesting in this result is that the PBFA can be expressed as the decomposition of the original PFA attributed to the strategic inefficiency of the profit maximizing bankers and the component of financial risks that comes from the Byzantine bankers. This decomposition might be helpful for the assessment of different regulatory policy schemes. For each scheme, we can calculate how much discouraging the regulation could be for the profit maximizing bankers and how much immune the financial system becomes from the presence of Byzantine bankers. When policies are targeting primarily against the destabilizing role of Byzantine bankers, then it might be more appropriate to use as indicator the Price of Malice.

Corollary 2 *The Price of Malice of the Byzantine Financial Regulation game is*

$$PoM(I^m) = \frac{m \frac{\sum_{i \in I^p} (1 - a_i^*) \omega A_i}{n}}{TPC}$$

Proof. It follows trivially by the definition. □

3.1 The BFR game with incomplete information

The assumption of known number of malicious bankers seems to be too restrictive and a more general framework is of interest for our goals. One way to proceed is to assume that bankers are unaware of the number of malicious bankers but possess beliefs for the pursuits of their competitors. Assuming Bayesian bankers, the BFR game with incomplete information can be depicted as follows.

Assume two types of bankers as before, the regular profit maximizing bankers (t_p) and the (malicious) Byzantine bankers (t_m). Assume that a type profile $t \in \{t_p, t_m\}^n$ is drawn according to a distribution P that is common knowledge. Given this distribution, one can calculate its expected cost, i.e. $EC_i = \sum p(t) C_i((\psi, \theta), \alpha; t)$. In Bayesian terms, the equilibrium is defined as follows.

Definition 9 *The Bayesian strategic equilibrium is a strategy profile $((\psi^*, \theta^*), \alpha^*)$ for a cumulative distribution P over type profiles such that for all i banks and the FRA*

1. $EC_i((\psi^*, \theta^*), \alpha^*; t) \leq EC_i((\psi^*, \theta^*), (\alpha_i, \alpha_{-i}^*))$ for all α_i .

$$2. C_{n+1}(\alpha^*, (\psi^*, \theta^*); t) \leq C_{n+1}(\alpha^*, (\psi, \theta); t) \text{ for all } (\psi, \theta).$$

In this Bayesian setup, the price of Byzantine financial anarchy depends on distribution P . When the distribution is degenerate, this measure of inefficiency reduces to the previous case, where the number of Byzantine bankers is known. If this is not the case, for different distributions the Byzantine financial anarchy receives different values. One way to circumvent the problem is to define a distribution-free PBFA, in the spirit of independent Price of Anarchy introduced by Roughgarden (2012). Specifically, assume the universe of distributions \mathcal{P} over type specifications. The independent price of Byzantine financial anarchy is the worst case scenario for all strategy and type profiles.

Definition 10 For the universe of distributions \mathcal{P} , the independent price of Byzantine Financial Anarchy is defined to be

$$iPBFA = \max_{\substack{\alpha^* \in NE \\ P \in \mathcal{P}}} \frac{\mathbf{C}^*}{\bar{\mathbf{C}}}.$$

The measurement of an upper bound to iPBFA turns to be a very hard task. The strategic outcome is sensitive to private information, which evidently is not available knowledge to FRA. Once the consistency of beliefs is ensured, multiple equilibria may emerge. Still, it might be useful for the FRA to use its perception for the information diffusion into the financial system and roughly approximate this upper bound. For instance, for efficient markets the strategic power of bankers is quite small, almost negligible, and Byzantine bankers cannot be very harmful. The iPBFA bound will approximate the PBFA bound, which makes efficient markets uninteresting. It is the shallow markets that are vulnerable to malicious behavior, and deviations might be significant. Then, the iPBFA serves to determine how much detrimental the strategic power of bankers is, across priors and information diffusion schemes.

4 Discussion

The recent financial crisis experienced by the world economy brought to our attention the necessity of an early warning method that would alert financial regulatory authorities for taking the necessary pre-emptive measures. Insofar, to my perception there are no financial indicators that securely can play this role. The macro-prudential approach uses a series of financial indicators that may identify the symptoms of financial fragility, still seem to be inadequate to detect the real causes, that may be attributed to thin market institutions or perverse motives of financial actors. This paper attempts to address the problem at micro-level, emphasizing the incentives of profit maximizing bankers to circumvent market regulations and make extra profits at the cost of systemic stability. Current early warning systems lack the ability to detect financial practices like "shadow banking" and ignore the considerable role of financial innovation for developing rent opportunities. On top of that, there are backward looking and cannot conceive the structural characteristics of the market.

The financial regulatory game developed here, is an abstract still powerful framework to address the strategic considerations of banks as financial actors. Banks have the opportunity to increase incognito their leverage and make extra profits, As long as these practices bear no cost to bankers and many financial instruments are highly unregulated, these perverse motives will remain alive. Motivated by the concept of "price of anarchy", it is introduced an new early warning indicator that attempts to capture the social inefficiency caused by the profit maximizing bankers. The metric of the price of anarchy is used extensively in congestion and network games and can fulfill effectively the role of a financial indicator as well.

Financial regulatory authorities can use the price of financial anarchy in two different respects. First to assess different market rules and regulations with respect to their ability to correct the vulnerability of financial system; and second to calculate the critical values of the price of financial anarchy that makes the financial system fragile and pursue to regulate market for keeping it well below these threshold values. In a broad sense, financial fragility conceptualizes the idea of triggering a financial crisis by an external (small) financial or economic shock. Here, we emphasized the fact that the more unregulated the financial system is the higher the risk for a crisis to trigger. To put this in a different way, financial actors will always pursue to circumvent financial regulation so as to fulfill their profit maximizing motives. No doubt, speculative profits emerge not only in upturns but at the downturns as well. This is basically the essence of the role of Byzantine bankers, which represent actors that will make money out of a financial turmoil.

APPENDICES

A Proof of Lemma 1

Proof. For arbitrary i 's cost function eq. 1, the first derivative gives

$$\frac{\partial C_i}{\partial \alpha_i} = RT_i - 2(1 - \alpha_i) \frac{\omega A_i}{n} - \sum_{j=1}^{\#S \setminus \{i\}} (1 - \alpha_j) \frac{\omega \cdot A_i}{n}.$$

Now define the function $\hat{C}_i = (1 - \alpha_i)^2 \frac{\omega A_i}{n}$ which the first derivative is $\frac{\partial \hat{C}_i}{\partial \alpha_i} = -2(1 - \alpha_i) \frac{\omega A_i}{n}$

Since the domain of α_i is a convex subset of reals and both C_i, \hat{C}_i are quadratic, the first order conditions are also sufficient, hence both achieve a minimum for some α_i . In fact, it is easy to note that both functions achieve minimum for the same α_i , i.e.

$$\arg \min C_i(\alpha_i; \alpha_{-i}) = \arg \min \hat{C}_i(\alpha_i; \alpha_{-i}).$$

Now define the function,

$$P(\alpha) = \sum_i \hat{C}_i = \sum_i (1 - \alpha_i)^2 \frac{\omega \cdot A_i}{n}$$

The function P is twice differentiable with respect to α_i and strictly convex. Differentiating,

$$\frac{\partial P}{\partial \alpha_i} = -2(1 - \alpha_i) \frac{\omega A_i}{n} = \frac{\partial C_i}{\partial \alpha_i}. \quad (6)$$

It follows that

$$\arg \min C_i(\alpha_i; \alpha_{-i}) = \arg \min \hat{C}_i(\alpha_i; \alpha_{-i}) = \arg \min P(\alpha_i; \alpha_{-i}),$$

which satisfies the condition for the best-response potential. \square

B Proof of Lemma 2

Proof. By substituting (1) into the definition of decreasing differences (3), we have for the left hand side

$$C_i(\alpha_i, \alpha_j) - C_i(\alpha_i, \alpha'_j) = \frac{(1 - \alpha_i)\omega A_i}{n} \left(\sum_{S \setminus \{i\}} (1 - \alpha_j) - \sum_{S \setminus \{i\}} (1 - \alpha'_j) \right).$$

Similarly, for the right hand side,

$$C_i(\alpha'_i, \alpha_j) - C_i(\alpha'_i, \alpha'_j) = \frac{(1 - \alpha'_i)\omega A_i}{n} \left(\sum_{S \setminus \{i\}} (1 - \alpha_j) - \sum_{S \setminus \{i\}} (1 - \alpha'_j) \right).$$

For $\alpha_i \geq \alpha'_i$ it is always

$$\frac{(1 - \alpha_i)\omega A_i}{n} \leq \frac{(1 - \alpha'_i)\omega A_i}{n},$$

hence $C_i(\alpha_i, \alpha_j) - C_i(\alpha_i, \alpha'_j) \leq C_i(\alpha'_i, \alpha_j) - C_i(\alpha'_i, \alpha'_j)$. \square

C Proof of Proposition 2

Proof. By Topkis (1998)(lemma 2.6.1. p49), we know that the sum of submodular functions is submodular. Hence, the collective cost $\mathbf{C} = \sum_i C_i$ is a submodular function. Consider the following profiles: $\bar{\alpha} = (1, \mathbf{1})$ and $\alpha^* = (\alpha_i^*, \alpha_j^*)$ with $\bar{\alpha}$ the profile where all bankers fully comply to financial regulation and corresponds to the optimal solution from the stand of FRA, and α^* the strategic equilibrium. By submodularity we have,

$$\begin{aligned} \mathbf{C}(\bar{\alpha}_i, \bar{\alpha}_j) - \mathbf{C}(\bar{\alpha}_i, \alpha_j^*) &\leq \mathbf{C}(\alpha_i^*, \bar{\alpha}_j) - \mathbf{C}(\alpha_i^*, \alpha_j^*) \\ \frac{\mathbf{C}(\alpha_i^*, \alpha_j^*)}{\mathbf{C}(\bar{\alpha}_i, \bar{\alpha}_j)} + 1 &\leq \frac{\mathbf{C}(\alpha_i^*, \bar{\alpha}_j) + \mathbf{C}(\bar{\alpha}_i, \alpha_j^*)}{\mathbf{C}(\bar{\alpha}_i, \bar{\alpha}_j)} \\ \frac{\mathbf{C}(\alpha_i^*, \alpha_j^*)}{\mathbf{C}(\bar{\alpha}_i, \bar{\alpha}_j)} &\leq \frac{\mathbf{C}(\alpha_i^*, \bar{\alpha}_j) + \mathbf{C}(\bar{\alpha}_i, \alpha_j^*)}{\mathbf{C}(\bar{\alpha}_i, \bar{\alpha}_j)} - 1 \end{aligned}$$

It follows that it should be

$$\max_{\alpha} \frac{\mathbf{C}(\alpha_i^*, \alpha_j^*)}{\mathbf{C}(\bar{\alpha}_i, \bar{\alpha}_j)} = PFA \leq \max_{\alpha} \left[\frac{\mathbf{C}(\alpha_i^*, \bar{\alpha}_j) + \mathbf{C}(\bar{\alpha}_i, \alpha_j^*)}{\mathbf{C}(\bar{\alpha}_i, \bar{\alpha}_j)} - 1 \right]$$

It is easy to verify that

$$\begin{aligned} \mathbf{C}(\bar{\alpha}_i, \bar{\alpha}_j) &= \mathbf{C}(\bar{\alpha}_i, \alpha_j^*) = \sum_i RT_i \\ \mathbf{C}(\alpha_i^*, \bar{\alpha}_j) &= \sum_i \alpha_i^* RT_i + \sum_i (1 - \alpha_i^*)^2 \frac{\omega A_i}{n} \end{aligned}$$

But $\mathbf{C}(\alpha_i^*, \bar{\alpha}_j)$ is the *TPC* Substituting in the above inequality,

$$\begin{aligned} PFA &\leq \max_{\alpha^*} \left[\frac{TPC}{\sum_i RT_i} + \frac{\sum_i RT_i}{\sum_i RT_i} - 1 \right] \\ &\leq \max_{\alpha^*} \frac{TPC}{\sum_i RT_i} = \max_{\alpha^*} \frac{TPC}{SOC}. \end{aligned}$$

□

D Proof of Proposition 3

Proof. It is evident that $PC_i < C_i((\psi^*, \theta^*), \alpha_i, \alpha_{-i}^*)$. Summing all over n , it is $TPC < C((\psi^*, \theta^*), \alpha_i, \alpha_{-i}^*)$. From the (λ, μ) -smooth game we have

$$TPC < C((\psi^*, \theta^*), \alpha_i, \alpha_{-i}^*) \leq \lambda \cdot \bar{\mathbf{C}} + \mu \cdot \mathbf{C}^*.$$

Then it follows easily that

$$\begin{aligned} TPC &< \lambda \cdot \bar{\mathbf{C}} + \mu \cdot \mathbf{C}^* \\ \frac{TPC}{\bar{\mathbf{C}}} &< \lambda + \mu \cdot \frac{\mathbf{C}^*}{\bar{\mathbf{C}}} = \lambda + \mu \cdot PFA \end{aligned}$$

By Proposition 2, when the PFA takes its highest value it is $PFA = \frac{TPC}{\bar{\mathbf{C}}}$. Substituting above we get

$$\begin{aligned} PFA &< \lambda + \mu \cdot PFA \\ &< \frac{\lambda}{1 - \mu}. \end{aligned}$$

□

E Proof of Proposition 4

Proof. Similar to Proposition 2 we define the strategy profiles $\bar{\alpha} = (1, \mathbf{1}; \alpha_m = 0)$ and $\alpha^* = (\alpha_i^*, \alpha_j^*; \alpha_m = 0)$ with α^m is the m -dimensional vector of zeros standing for the strategy profile of malicious bankers. As previously $\bar{\alpha}$ is the profile where all bankers fully comply to financial regulation and corresponds to the optimal solution from the stand of FRA, and α^* the strategic equilibrium.

It is easy to verify (by mere substitutions) that

$$\mathbf{C}(\bar{\alpha}_i, \bar{\alpha}_j; \alpha_m = 0) = \mathbf{C}(\bar{\alpha}_i, \alpha_j^*; \alpha_m = 0) = \sum_i RT_i = SOC$$

But,

$$\mathbf{C}(\alpha_i^*, \bar{\alpha}_j; \alpha_m = 0) = \sum_{i \in I^p} \alpha_i^* RT_i + \sum_{i \in I^p} (1 - \alpha_i^*)^2 \frac{\omega A_i}{n} + \sum_{i \in I^p} (1 - \alpha_i^*) \frac{\sum_{j \in I^m} \mathbf{1} \omega A_i}{n},$$

with $\mathbf{1} = \begin{cases} 1 & j \in I^m \\ 0 & j \notin I^m \end{cases}$. The previous equations can be rewritten as

$$\begin{aligned} \mathbf{C}(\alpha_i^*, \bar{\alpha}_j; \alpha_m = 0) &= \sum_{i \in I^p} \alpha_i^* RT_i + \sum_{i \in I^p} (1 - \alpha_i^*)^2 \frac{\omega A_i}{n} + \sum_{i \in I^p} (1 - \alpha_i^*) \frac{m \omega A_i}{n} \\ &= TPC + \sum_{i \in I^p} (1 - \alpha_i^*) \frac{m \omega A_i}{n} \end{aligned}$$

Then, it is

$$\begin{aligned} PBF A &\leq \frac{\mathbf{C}(\alpha_i^*, \bar{\alpha}_j; \alpha_m = 0) + \mathbf{C}(\bar{\alpha}_i, \alpha_j^*; \alpha_m = 0)}{\mathbf{C}(\bar{\alpha}_i, \bar{\alpha}_j; \alpha_m = 0)} - 1] \\ &\leq \frac{TPC + \sum_{i \in I^p} (1 - \alpha_i^*) \frac{m \omega A_i}{n} + SOC}{SOC} - 1 \\ &\leq \frac{TPC}{SOC} + \frac{m}{SOC} \sum_{i \in I^p} (1 - \alpha_i^*) \frac{\omega A_i}{n} \\ &\leq PFA + \frac{m}{SOC} \sum_{i \in I^p} (1 - \alpha_i^*) \frac{\omega A_i}{n}. \end{aligned}$$

□

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