Cheap Talk with Transparent and Monotone Motives to an Informed Receiver

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Abstract

We develop a model of cheap talk with transparent and monotone motives to an informed receiver. By transparent and monotone motives, we mean that the sender's utility function does not depend on the state of the world, and it is increasing in the choice of the receiver regardless of the state of the world. We first show that if the receiver is not informed at all, only the babbling equilibrium is possible. Then, we obtain our main result that even if the receiver has the slightest information about the state of the world, full revelation can be supported by the cross-checking strategy of the receiver if the sender's utility function is strictly concave, unless the receiver has too much information. Paradoxically, no information and too much information of the receiver both dispense with the fully revealing equilibrium. We also get a counterintuitive result that the sender prefers a more informed receiver than a less informed receiver.

Keywords: cheap talk; cross-checking strategy; fully revealing equilibrium; informed receiver; monotone motive; permissible deviation; transparent motive

JEL Classification Code: C7, D8

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1 Introduction

People often get advice from experts. Lawyers give legal advice to clients. Patients obtain medical advice from doctors. Mechanics recommend some repair services to customers. Academic advisers give their opinions on theses of students. In those cases, advisees are often not completely ignorant of the subject, although it is true that they are less informed than the experts.

In literature on cheap talk games that follow Crawford and Sobel (1982),[1] however, the feature that the receiver (she) is also partially informed has been largely neglected.[2] It seems obvious that if the receiver is informed in a degree, the message of the sender (he) will be affected by the informativeness of the receiver. Then, a natural question will be whether the informativeness of the receiver can discipline the sender's message. How different a mechanic's recommendations to a complete novice and to a customer with some expertise could be?[3] Does he make more honest recommendation to knowledgeable customers? Is he more fraudulent to novices? Does an academic adviser write a recommendation letter of his student differently to someone who knows of him a little than to someone who has no prior information about the student at all? Will a professor who has some information about the student interpret the same recommendation letter differently? Are lawyers more honest to professional clients in their legal service? In general, does an advisee's partial information always make the recommendation of the expert credible? If so, how much?

Let us take some examples. Suppose a consumer considers purchasing a computer, either a Mac or a PC, and they are not fully compatible. It is well known that there is a network externality in adopting technology which is not compatible with different technology.[4] She will ask a Mac-user about the quality of Mac, because a computer is an experience good the

[1] Crawford and Sobel (1982) will be abbreviated as CS hereafter.

[2] Exceptions include Seidmann (1990), Watson (1996), Olszewski (2004), Lai (2014), Ishida and Shimizu (2016) etc. to name a few.

[3] The example of a mechanic is often used in literature on experts starting from Pitchick and Schotter (1987). They introduced an insightful model of consumer fraud in which an informed expert recommends a certain kind of repair to a consumer, and investigated how honest the expert is in equilibrium. As a disciplining device of fraud, they considered second opinions obtained by searches and Wolinsky (1993) considered the expert's reputation, but neither considered the consumer's expert as a disciplining device. Also, in their model, an expert's recommendation is a binding option like the repair price, not a cheap talk.

[4] See, for example, Farrell and Saloner (1985).

quality of which is learned to a consumer only after he uses one. Due to network externality, however, the user will have an incentive to exaggerate the quality of Mac, because his utility is increased if the referrer decides to buy a Mac. Therefore, the message of the user will not be taken as credible. However, if the consumer has some experience of using a Mac even if he is not an expert, the Mac-user may not be able to exaggerate too much.

As another example which will be used as the main scenario in our paper, consider a salesperson who sells a product.[5] If he gets paid based on his sales performance, he will not have a proper incentive to be honest about the quality of the product.[6] Such an incentive to exaggerate the quality may be disciplined, though, if the consumer is partially informed about the quality herself. Actually, many firms hire marketing experts to sell the products they want to sell using deceitful marketing practices, and consumers's lack of information often tricks them to purchase products which turn out to be unsatisfactory. If a consumer does not have all of the needed information about a product, she may mistakenly purchase that product, but her proper information about the product may protect her from being deceived into purchasing an unnecessary amount of goods.

In both examples, the utility of the sender increases with respect to the quantity to be purchased by the receiver regardless of the quality of the product. In other words, the sender has a transparent motive in the sense that his preference does not depend on his information of the quality, and he has a monotone motive in the sense that he always prefers the receiver purchasing more.[7] There was a widely held conjecture that if the sender has transparent motives, credible communication is not possible and only the babbling equilibrium exists, since the interests of the sender and the receiver are too much conflicting. But the falsity of the conjecture was shown by Seidmann (1990) when the receiver has some private information and her action is one-dimensional,[8] and by Charkraborty and Harbaugh (2010) when the private information of the sender is multi-dimensional, and by Jung and Kim (2019) when

[5]This example is borrowed from Charkraborty and Harbaugh (2010).

[6]The seller's strategy in this model is distinguished from his strategy of revealing the product quality completely or partially in persuasion games by Milgrom (1981) or Lipman and Seppi (1995), because the seller's strategy in our model is an unverifiable cheap talk.

[7]Seidmann (1990) used the terms (IA) and (CI) for a transparent motive and a monotone motive respectively.

[8]He also demonstrates that the conjecture is incorrect when the receiver has no information and her action is two-dimensional.

there are multiple senders.

This paper addresses the general question whether more informativeness of the receiver helps the sender communicate truthfully when the sender has transparent and monotone motives. Seidmann (1990) argued that cheap talk could influence the receiver's equilibrium actions even if the sender has transparent and monotone motives if the receiver is herself privately informed, but he did not answer the question of whether more information of the receiver makes honest communication of the sender easier or more difficult.

In this paper, we argue that there exists a truthfully revealing communicative equilibrium in a fairly general setting of the unbounded state space. To support the truth-revealing outcome as an equilibrium, we will use a specific form of strategy of the receiver, what will be called "cross-checking strategies". By a crosschecking strategy of the receiver, we mean a strategy whereby the receiver believes the sender's message if his message is congruent enough with her information in the sense that the message falls within the normal (confidence) range, i.e., it is not too far above from the receiver's information, and punish him by believing that he was fibbing, if it falls in the punishment range, i.e., it is much higher than the receiver's information. We will call the distance between the receiver's information and the punishment range the *permissible deviation*.

We first show that unless the sender's information and the receiver's information are both noiseless, the cross-checking strategy cannot induce full revelation if the utility function of the sender is linear in the receiver's choice. The difficulty in this case arises mainly because the assumption of the unbounded state space makes off-the-equilibrium messages vanish completely under the normal distribution of the noise. Even if the message is too high or too far from the receiver's information, it is a possible event, although the likelihood is very low. So, the receiver cannot believe that it is a consequence of the sender's lying. This makes it difficult to penalize strongly enough the sender who sends a higher message than the true value.

We next show that if the utility function of the sender is strictly concave, full honesty of the sender is possible with the cross-checking strategy even if the receiver has the slightest information about the state of the world, unless her information is too precise. In this case, strict concavity of the sender's utility function can make the penalty from inflating the message exceed the reward from it, so that it can discipline the sender who is tempted to lie. However, if the receiver is too well informed, such a truthful communication may not be possible. Rough intuition behind this result goes as follows. For any small permissible deviation, as the receiver gets better informed in the sense that the variance of her information approaches zero, it is more likely that a slightly inflated message of the sender falls within the confidence range, and, as a result, the punishment probability converges to zero, implying that a sender would have an incentive to inflate his message if the receiver is too well informed. However, in fact, the permissible deviation gets larger as quite a well informed receiver gets much better informed, so the sender will clearly inflate his message. This surprising result that more information of the receiver may hinder effective communication and consequently hurt her counters the widely held perception that better information of the receiver will pay off by disciplining the sender's message. We also show that the sender having a CARA (constant absolute risk aversion) utility function is the necessary and sufficient condition for the existence of a fully revealing equilibrium with the cross-checking strategy.

If the receiver has no information or a completely noisy information in the sense that the variance is infinity, it is clear that only the babbling equilibrium in which no meaningful message is possible. It is interesting that the sender's message cannot be fully revealing in two opposite extreme cases, i.e., either if the receiver has no information or if she has very precise information. In the former case, the sender says anything because the receiver has no way to check the honesty of his message due to lack of information or useless information. In the latter case, the sender has no reason to be honest because there is little possibility that the receiver will get an unacceptable message from the sender thereby penalizing him due to the high precision of her information.

There is some literature on cheap talk with transparent motives. Chakraborty & Harbaugh (2010) and Lipnowski & Ravid (2018) are most notable examples. The term of transparent motives is used to mean that the informed sender does not care about the state but only about the receiver's action. The authors of both papers show that cheap talk can be informative even if the sender has a transparent motive. The main difference of our model from theirs is that the receiver is also (partially) informed in our model. In Charkraborty & Harbaugh (2010), informativeness of cheap talk relies on the multi-dimensionality of the state variable which implies that the receiver cares about multiple issues rather than one issue, which is not assumed in our model. Also, while we assume that the state space and the receiver's action space are unbounded, Lipnowski & Ravid (2018) uses a different assumption that the state space and the action space are compact.

The possibility of an informed receiver in a cheap talk game has been considered by several authors. However, this paper provides a quite different setup and a new insight. Above all things, we assume that the support of each information is unbounded, whereas all other papers assume a finite or bounded support.[9] This difference is important because there is no off-the-equilibrium message under the assumption of the unbounded support. That is, it is impossible to detect a false message for sure, because any message is possible in equilibrium as far as noises have a normal distribution we assume. Therefore, we cannot use Lai's punishment strategy based on a proper off-the-equilibrium belief due to inconsistent supports between the sender's message and the receiver's information, or have no reason to resort to Watson's matching strategy because the sender's message and the receiver's information do not match with probability one even if the sender is honest, as far as the information of both players have noises with unbounded supports. Since no off-the-equilibrium message exists in our model, the receiver cannot penalize a sender who is presumed to have sent a false message, but instead he can use what we call a crosschecking strategy whereby the receiver chooses the action favorable to the sender if the sender's message is not too far from above from her own information, and chooses the action unfavorable to him if it is too high.

Does the receiver believe that the sender lied if the sender's message is too high from her own information? It is not the case. She does believe that the sender sent the truthful message (because she knows that the sender chose the equilibrium strategy of reporting truthfully), but does not take it seriously by ignoring the message and taking only her own information which is lower. Otherwise, the sender would inflate his message. In other words, the receiver overrules the sender's message, not because she believes that the message is likely to be wrong, but because she needs to give him an incentive to report correctly. This feature of the cross-checking strategy is somewhat similar to the trigger strategy by Green and Porter (1984) in that the player who receives some signal penalizes the opponent even if she knows that the opponent did not cheat, because he would cheat otherwise.

The main results of Lai (2014) and Ishida and Shimizu (2016) have a common feature

[9] These papers include Seidmann (1990), Watson (1996), Olszewski (2004), Chen (2009), Moreno de Barreda (2012), Galeotti *et al.* (2013), Lai (2014) and Ishida and Shimizu (2016). The support of the receiver's information is assumed to be binary in Chen (2009), Galeotti (2013) and Lai (2014), finite in Seidmann (1990), Watson (1996), Olszewski (2004), Ishida and Shimizu (2016), and bounded in Moreno de Barreda (2012).

with our result in that the incentive for truthful communication diminishes as the receiver is better informed. However, their main insight that the receiver's response becomes less sensitive to the sender's message as she is better informed as a result of Bayesian belief updating is not valid in our model. In our model, as the receiver's expertise becomes more precise, the possibility of truthful communication disappears discontinuously at some level of her expertise. In their model, the receiver's belief is a weighted average of her information and the sender's information. So, as the receiver's information is more accurate, the weight on her own information should be larger, and thus she must rationally respond to the sender's message less sensitively. In our model, the receiver's belief is not a weighted average of her information and the sender's information, but either her information or the sender's information. Depending on whether the two pieces of information lie within the confidence range or not, the receiver responds discontinuously to the sender's message. Both features are unique in our model.

Our results are sharply contrasted with Lai in several respects. In Lai (2014), the receiver who gets informed could benefit only if the receiver's information is sufficiently different from the sender's, i.e., it is very useful information. In our model, reliable information could be conveyed from the sender only when the receiver's information is sufficiently close to the sender's information. Besides, in our model, a receiver who is informed is always better off than a receiver who is uninformed, whereas a completely uninformed receiver may be better off than an informed receiver in his model, because the sender would provide an informed receiver less informative advice that yields the receiver a lower payoff. Also, the results by Lai (2014) and Ishida and Shimizu (2016) hold only when preferences of the sender and the receiver are sufficiently congruent, whereas our result of perfectly truthful communication holds even if their preferences are conflicting enough in the sense that the sender's favorite is independent of the receiver's favorite. Another important difference is that, in Lai (2014), the receiver's information does not elicit full revelation, which is not the case in our model.

Seidmann (1990) is pioneering in games of cheap talk to an informed receiver, but his model is quite restrictive in the sense that the supports of the players' information are finite. In our model in which any message is possible in equilibrium, the crosschecking strategy can work well to discipline the sender's incentive to inflate his information. The difference between the two pieces of information can give the receiver some information about whether the sender lied or not because of the correlation between the sender's information and the receiver's information, although it cannot give perfect information about it. Therefore, the cross-checking strategy is to exploit the correlation structure between the sender's information and the receiver's information. Correlation between them is essential to driving the outcome of full revelation. This distinguishes our paper from Seidmann.

The paper is organized as follows. In Section 2, we set up a cheap talk model of an informed receiver. In Section 3, we characterize the fully revealing communicative equilibrium with cross-checking strategies in the cheap talk game. In Section 4, we provide some comparative statics to examine the effect of a change in the receiver's informativeness. Concluding remarks and some ramifications follow in Section 5. Proofs are provided in Appendix.

2 Model

There are a sender S, and a receiver R. The state of nature θ is a random variable which is distributed over $\mathbb{R}$. For example, θ could be the quality of a product that a salesperson sells to a consumer. For simplicity, we assume that θ is uniformly distributed over $\mathbb{R}$,[10] i.e., players have no information about θ *a priori*. Although neither the sender nor the receiver knows the accurate value of θ, both of them receive a noisy signal on the state of nature $v_i \in V = \mathbb{R}$ for $i = S$ and R where $v_i = \theta + \epsilon_i$, ϵ_i is stochastically independent with θ, and ϵ_i's are independent. It is important to note that R is also partially informed. We assume that ϵ_i follows a normal distribution with its mean zero and the variance σ_i^2, $i = S, R$, where $\sigma_S^2 < \sigma_R^2$. The assumption of the inequality in the variances reflects the feature that the sender has higher expertise about θ than the receiver.

The game proceeds as follows. First, the state of nature θ is realized and then a sender and a receiver receive a private signal v_S and v_R respectively without knowing θ. After observing private information v_S, S sends a payoff-irrelevant message (cheap talk) $m \in M = \mathbb{R}$ to R.[11] Then, receiving a message $m \in M$, R updates her posterior belief about v_S, $\hat{b}(m)$, and then

[10] Note that we are assuming an improper prior distribution.

[11] Since the cheap talk message of the sender, m, is payoff-irrelevant by the definition of cheap talk, the payoffs of the players (U^S and U^R) which are described below should not depend on m. This is distinguished from Pitchick and Schotter. In their model, an expert makes a binding recommendation, for example, about the price, so it is not cheap talk, whereas we consider an unbinding recommendation of an expert (for example, about the quality) thereby making the payoff of the receiver not directly depend on the recommendation m.

forms her belief about θ, $b(m, v_R)$, by using m and v_R, where $b : M \times V \to \mathbb{R}$,[12] based on which she chooses an action $a \in A(=\mathbb{R})$. A strategy of the receiver determines the sender's payoff as well as her own payoff.

The payoff to S is given by a continuously differentiable function $U^S : A \to \mathbb{R}$ and the payoff to R is given by twice continuously differentiable function $U^R : A \times \Theta \to \mathbb{R}$. Throughout the paper, we will assume that (1) $U^S(a) = u(a)$ where $u' > 0$, $u'' \leq 0$, i.e., $u(a)$ is increasing and concave in a, and (2) $U^R(a, \theta) = -(a-\theta)^2$. The receiver's utility function implies that it has a unique maximum in a for all θ and the maximizer of U^R, denoted by $a^R(\theta)$, is strictly increasing in θ. Independence of the sender's utility function on θ means that S has transparent motives, and the utility which is increasing in a means that S has monotone motives. The monotonic increase of $a^R(\theta)$ in θ means that the receiver will want to buy more units of high θ which can be interpreted as quality. A typical example that corresponds to these assumptions is a situation in which a salesperson gets paid based on the quantity he sells, so that the salesperson's utility is increasing with respect to the consumer's purchasing choice regardless of θ.

A strategy for S specifies a signaling rule given by a measurable function $s : V \to M$. A strategy for R is an action rule given by a function $\alpha : M \times V \to A$. The equilibrium concept that we will employ is that of weak Perfect Bayesian equilibrium (wPBE). An equilibrium of this game consists of a signaling rule for S, an action rule of R and a system of beliefs $(s^*(v_s), \alpha^*(m, v_R), (\hat{b}(m), b(m, v_R)))$ such that

(2-I) $s^*(v_S) \in \arg\max_m \int_{-\infty}^{\infty} U^S(\alpha^*(m, v_R))h(v_R \mid v_S)dv_R$, where $h(v_R \mid v_S)$ is the conditional density function of v_R given v_S,

(2-II) $\alpha^*(m) \in \arg\max_a U^R(a, b(m, v_R))$.

(2-III) R's posterior belief $\hat{b}(m)$ is consistent with the Bayes' rule on the equilibrium path and $b(m, v_R)$ is an unbiased estimator of θ based on the observations m and v_R.[13]

[12] The belief $\hat{b}$ could be defined by a function μ that corresponds a probability distribution to each message m, but we prefer our notation mainly because of its simplicity and intuitiveness. So, $\mu(\theta = v \mid m) = 1$ in the standard notation can be denoted simply by $\hat{b}(m) = v$ in our notation.

[13] Since R is not perfectly informed of two values, v_S and θ, she must form both of the beliefs. However, the belief that she can infer from the weak consistency requirement of wPBE is only about v_S, not about θ because the value of θ is not known to S, either. Instead, R can obtain an estimator for θ from the two observations, m and v_R. But since θ is *not a type* of S (because S does not know the value), the

Henceforth, we will simply use the notation of $h(v_R)$ for the density function conditional on v_S and $H(v_R)$ for the corresponding distribution function by suppressing v_S. Before we characterize equilibria, we will adapt some standard definitions often used in literature.

Definition 1. *A message m induces an action a of the receiver with v_R iff $a = \alpha^*(m; v_R)$.*

Definition 2. *An equilibrium is communicative iff there exist two different observations v_S, v'_S such that $s^*(v_S) \neq s^*(v'_S)$ and $\alpha^*(m; v_R) \neq \alpha^*(m'; v_R)$ for some v_R where $m = s^*(v_S)$, $m' = s^*(v'_S)$. An equilibrium is uncommunicative (or babbling) otherwise.*

In other words, if two different messages sent by two different types of the sender induce two different actions of the receiver with some information v_R, the equilibrium is communicative in the sense that some meaningful message that can affect the receiver's choice is conveyed by cheap talk communication in equilibrium.

Definition 3. *A communicative equilibrium is fully-revealing iff $s^*(v_S) \neq s^*(v'_S)$ for any v_S, v'_S such that $v_S \neq v'_S$. In particular, if $s^*(v_S) = v_S$, a fully-revealing equilibrium is called a truth-revealing equilibrium.*[14]

Observe that, insofar as the receiver has no information, the message sent by the sender cannot be credible at all in this model with transparent and monotone motives. In the CS model, the payoff function of S as well as that of R is single-peaked, so that, given θ, the favorite actions to S and R do not differ very much even if they do differ. This implies that, for some low value of θ, both S and R prefer one action to another, while the reverse is true for some other high value of θ. In other words, there is room for coordination between S and R and in effect cheap talk enables such coordination to occur by conveying the message whether θ is high or low. In our model, however, the assumption of single-peaked preferences

definition of wPBE does not impose any requirement for the estimator. It could be any weighted average of the information inferred from the message and her own information, $\lambda\hat{b}(m) + (1-\lambda)v_R$ where $\lambda \in [0, 1]$, in particular, $\hat{b}(m)$ or v_R by ignoring one information, if one requires the unbiasedness of the estimator at the very least. All of them are perfect unbiased estimators of θ. Our $b(m, v_R)$ is a summary statistic that can be obtained after two separate processes, the inference process and the estimation process based on the inference.

[14] Since even fully-revealing strategies which are $v_S \neq s^*(v_S)$ reveal the truth in equilibrium, those strategies are literally truth-revealing, So, in fact, the terms "fully-revealing" and "truth-revealing" could be exchangeable.

is violated and all the types of S prefer a higher level of the receiver's action a. Thus, S would like to pretend to have observed as highest v_S as possible to induce R's highest action possible, regardless of his type.

We now summarize with

Proposition 1. *If R has no information about θ, there exists no communicative equilibrium.*

It is easy to prove it. Suppose, for some distinct v_S and v'_S, $m \neq m'$ and $a \neq a'$ where $m = s^*(v_S)$, $m' = s^*(v'_S)$, $a = \alpha^*(m)$ and $a' = \alpha^*(m')$ in a communicative equilibrium. If $a < a'$, S with information v_S would prefer sending m' to m, since $u(a) < u(a')$. If $a' < a$, S with information v'_S would choose m instead of m', since $u(a') < u(a)$. This violates the definition of an equilibrium.

However, if R has some information about θ, the above argument breaks down. Suppose m induces a and m' induces a' with $a < a'$ for some v_R. Nonetheless, we cannot conclude that S will prefer sending m' to m, because m' could induce a lower level of action for some other v'_R.

In the next section, we will make a formal analysis of cheap talk to an informed receiver.

3 Informed Receiver

For our purpose, let us concentrate on the following specific form of strategy profile;

(3-I) S with v_S announces $m = v_S$.

(3-II) R believes $b(m, v_R) = m$ if $m - v_R \leq \rho$ and believes $b(m, v_R) = v_R$ if $m - v_R > \rho$ for some $\rho > 0$.[15]

(3-III) R chooses $\alpha(m, v_R) = b$.

R's action rule given by (3-III) will be called a "crosschecking strategy",[16] and ρ will be called permissible deviation between m and v_R. Note that there is no off-the-equilibrium

[15] As we put in Footnote 12, these on-the-equilibrium beliefs are perfectly legitimate beliefs about θ, since the weak consistency condition of wPBE requires $\hat{b}(m)$ to be consistent with the equilibrium messaging strategy of the sender ($\hat{b}(m) = m$) but does not impose any requirement on the estimator for θ based on $\hat{b}(m)$ and v_R.

[16] Navin Kartik commented on the monotonicity of our strategies. We could say that the receiver's strategy is monotonic if $\partial\alpha(m, v_R)/\partial m \geq 0$. It is not difficult to see that the cross-checking strategy is not monotonic.

message, because any message can occur even if the sender tells the truth, as long as ϵ_i follows a normal distribution over $(-\infty, \infty)$.

Since it is obvious that (3-III) is R's optimal decision, it is enough to focus on the optimal decision of the sender. S will maximize

$$U^S(m; v_S) = \int_{-\infty}^{m-\rho} u(v_R)h(v_R)dv_R + \int_{m-\rho}^{\infty} u(m)h(v_R)dv_R. \tag{1}$$

The economic reasoning behind this formula goes as follows. The first term represents the punishment that the sender would get when v_R is very low ($v_R < m - \rho$). The second term indicates his utility when v_R falls into a normal confidence region ($v_R \geq m - \rho$). Thus, the effect of inflating the message on the sender's utility is

$$\begin{aligned}\frac{\partial U^S}{\partial m} &= u(m-\rho)h(m-\rho) + u'(m)\int_{m-\rho}^{\infty} h(v_R)dv_R - u(m)h(m-\rho) \\ &= u'(m)\int_{m-\rho}^{\infty} h(v_R)dv_R - (u(m) - u(m-\rho))h(m-\rho). \end{aligned} \tag{2}$$

The first term is the effect of utility increases in normal cases due to the inflated announcement and the second term is the loss that he is expected to bear from being punished by increasing his announcement marginally.

Truthful revelation requires $\frac{\partial U^S}{\partial m}|_{m=v_S} = 0$, which implies that

$$u'(v_S)\int_{v_S-\rho}^{\infty} h(v_R)dv_R = (u(v_S) - u(v_S-\rho))h(v_S-\rho). \tag{3}$$

The left hand side is the marginal benefit of inflating m above v_S, and the right hand side is the marginal cost due to an increase in the expected penalty.

If $u(a)$ is linear, $u'(v_S) = \frac{u(v_S)-u(v_S-\rho)}{\rho}$. It is easy to see that there is no ρ which satisfies (3). The proof exploits $u(v_S) - u(v_S-\rho) = \rho u'(v_S)$ due to the linearity of the utility function. Then, because of the equality, it is clear that the direct utility effect measured in a large range of $v_R \in [v_S + \epsilon - \rho, \infty)$, which is $u'(v_S)\int_{v_S-\rho}^{\infty} h(v_R)dv_R$, exceeds the penalty effect measured in a small range of $v_R \in [v_S - \rho, v_S + \epsilon - \rho]$ for a small $\epsilon > 0$, which is roughly $\rho u'(v_S)h(v_S-\rho)$.

If we impose monotonicity of R's strategy, there would be no equilibrium other than the babbling equilibrium in this game.

Proposition 2. *If the utility function $u(a)$ is linear,*[17] *there is no truth-revealing equilibrium with the crosschecking strategy.*

If $u''(\cdot) < 0$, however, $u(v_S) - u(v_S - \rho) > \rho u'(v_S)$, i.e., the penalty effect of exaggerating v_S becomes more severe, so equation (3) may have a solution for ρ which is independent of v_S.[18] We will denote the solution by ρ^*.

To confirm the existence of the equilibrium permissible deviation ρ^*, take $u(a) = 1 - e^{-a}$. Note that this utility function satisfies the assumptions we made, because $u'(a) = e^{-a} > 0$ and $u''(a) = -e^{-a} < 0$. Equation (3) can be simplified into

$$1 - H(v_S - \rho) = (e^\rho - 1)h(v_S - \rho), \tag{4}$$

or equivalently,

$$\frac{1}{e^\rho - 1} = G(v_S - \rho), \tag{5}$$

where $G(v_S - \rho) = \frac{h(v_S - \rho)}{\int_{v_S-\rho}^{\infty} h(v_R) dv_R}$ is the hazard rate. This equation determines the equilibrium permissible deviation ρ^*. Moreover, since $v_R (= v_S + \epsilon_R - \epsilon_S)$ has the distribution of $N(v_S, \sigma_S^2 + \sigma_R^2)$, we have

$$\begin{aligned} h(v_S - \rho^*) &= \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{\rho^*}{\sigma}\right)^2}, && (6) \\ H(v_S - \rho^*) &= \text{Prob}(v_R \leq v_S - \rho^*) \\ &= \Phi\left(\frac{v_S - \rho^* - v_S}{\sigma}\right) \\ &= \Phi\left(-\frac{\rho^*}{\sigma}\right) \\ &= 1 - \Phi\left(\frac{\rho^*}{\sigma}\right), && (7) \end{aligned}$$

where $\sigma \equiv \sqrt{\sigma_S^2 + \sigma_R^2}$ and $\Phi(x) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{x}{\sqrt{2}}\right)\right]$. Note that neither depends on v_S, so ρ^* does not, either. By substituting (6) and (7) into (4), we obtain

$$\frac{1}{2}\left[1 + \text{erf}\left(\frac{\rho^*}{\sqrt{2}\sigma}\right)\right] = \frac{e^{\rho^*} - 1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{\rho^*}{\sigma}\right)^2}. \tag{8}$$

[17] If $u''(a) > 0$, it is obvious that there is no truth-revealing equilibrium with the crosschecking strategy, because $u''(a) > 0$ implies $u'(v_S) > \frac{u(v_S) - u(v_S - \rho)}{\rho}$, so that a gain from exaggeration exceeds the loss.

[18] If the equilibrium value of ρ depends on v_S, it is of no practical meaning, because R's optimal strategy relies on ρ^* which in turn depends on v_S the value of which she does not know.

Figure 1 shows for this particular utility function that (i) there exists $\bar{\sigma}(> 0)$ such that the first order condition given by (3) is satisfied for some $\rho^*(\sigma)$ whenever $\sigma \geq \bar{\sigma}$ and (ii) there does not exist $\rho^*(\sigma)$ for low values of $\sigma(< \bar{\sigma})$. It also shows that there are two solutions of $\rho^*(\sigma)$ except for $\sigma = \bar{\sigma}$. Note that higher values of $\rho^*(\sigma)$ change more elastically with respect to a change in σ than lower values. If we define the elasticity of ρ^* with respect to σ by $\epsilon_\rho = \frac{d\rho^*}{d\sigma}\frac{\sigma}{\rho^*}$, we can notice that $\epsilon_{\rho_1} > 1 > \epsilon_{\rho_2}$ where $\rho_1(\sigma) > \epsilon_2(\sigma)$. Figure 2 illustrates $\bar{\sigma} > 0$ which is the minimum σ for the existence of a fully revealing equilibrium that consists of strategies and beliefs given in (3-I) – (3-III), and above which there are two equilibrium values for $\rho^*(\sigma)$, roughly speaking, an elastic one and an inelastic one. The appendix shows that this solution satisfies the second order condition and global optimality as well.

Proposition 3. *There exists $\bar{\sigma} > 0$ such that for any $\sigma \geq \bar{\sigma}$, there exists a truth-revealing equilibrium for some $\rho^* > 0$ which is independent of v_S and v_R, if the utility function u is any negative affine transformation of e^{-a}, i.e., $u(a) = \gamma - \beta e^{-a}$ where $\beta > 0$.*

This proposition says that the utility function $u(a) = \gamma - \beta e^{-a}$ satisfies the differential equation given by (3) for some ρ which is independent of v_S and v_R, implying that under this utility function, there is a possibility that there exists ρ^* characterizing the equilibrium cross-checking strategy and moreover, it does not depend on v_S and v_R. This utility function enables the sender to reveal truth by making the punishment larger than the direct gain when he inflates his information.

Is it still possible that there exists a truth-revealing equilibrium for a different form of utility function? The following proposition suggests that it is not possible.

Proposition 4. *If there exists a truth-telling equilibrium for small $\rho > 0$ which is independent of v_S and v_R, then the utility function must have the form of $u(a) = \gamma - \beta e^{-ca}$ where $\beta > 0$ and $c = \frac{2}{\rho}\left(\frac{1}{G(v_S-\rho)\rho} - 1\right) > 0$.*

It says that an affine transformation of e^{-ca} for some $c > 0$, i.e., $u(a) = \gamma - \beta e^{-ca}$, is indeed a necessary condition as well as a sufficient condition for the existence of a fully revealing equilibrium. So, we can conclude that when the receiver is partially informed, it is possible to fully reveal private information of the sender with the cross-checking strategy only if the sender has a utility function of this form.

This result could be interpreted as a possibility theorem in the sense that truth-telling is possible in equilibrium if the sender has this CARA utility function, or interpreted as an impossibility theorem in the sense that truth-telling is possible only if the sender has the CARA utility function. Considering the fact that the CARA function is a reasonable approximation to the real but unknown utility function,[19] we believe that this result is reassuring.

4 Comparative Statics

Is the receiver more ready to believe the sender's message as she is less informed? We will investigate the effect of an increase in σ_R on the equilibrium permissible deviation ρ^*. Consider the first order condition of the sender's incentive compatibility given by (5) as follows;

$$\frac{1}{e^{\rho}-1} = G(v_S - \rho).$$

The left hand side is the ratio of marginal benefit to marginal cost of inflating the message slightly in terms of utility. Roughly, $G(v_S - \rho)$ can be interpreted as the relative ratio of the punishment probability to the no-punishment probability. The punishment probability is affected by both ρ and σ_R. If σ_R increases, the probability is increased because the tail probability $\int_{-\infty}^{v_S-\rho} h(v_R)dv_R$ is increased. If ρ increases, the probability decreases. Therefore, if the left hand side of (5) does not change very much for large values of ρ, a larger σ_R should be complemented by a larger ρ to maintain the constant probability that a slightly inflated message is penalized, which means that a larger value of σ is associated with a larger value of ρ. This is why the receiver uses a more lenient strategy which allows a deviation to be more permissible, as information is less accurate, for most of the values of σ_R (except for small values of σ_R and ρ). Of course, if σ_R and ρ are very small, this monotonicity may not hold.

Now, on the sender's side, is it more likely that the sender tells the truth as R is more informed, i.e., σ_R^2 gets smaller? Proposition 3 suggests that it is not true, because there may not exist a truth-revealing equilibrium for a very low value of $\sigma^2 \equiv \sigma_S^2 + \sigma_R^2$. To see the intuition, consider the limiting case of σ_S^2 and σ_R^2. Given any fixed ρ, if σ_S^2 and σ_R^2 (with maintaining $\sigma_S^2 < \sigma_R^2$) keep falling, the penalty probability approaches zero, and thus, the expected net loss from inflating m converges to zero, implying that the sender will have

[19] See Zuhair *et al.* (1992).

an incentive to inflate his message. However, as σ approaches zero (so that σ is too small), $\rho^*(\sigma)$ gets larger and thus the penalty probability gets even smaller; hence, no communicative equilibrium for low values of σ_S^2 and σ_R^2.[20]

We will briefly compare the utilities of players when the receiver is completely uninformed and when she is partially informed. It is clear that the receiver cannot be worse off by obtaining some noisy information v_R with any finite variance $\sigma_R^2 < \infty$. Conditional on information m and v_R,[21] R's belief about θ is $b(m, v_R) = \hat{b}(m) = v_S$ if $m \leq v_R + \rho$ and v_R if $m > v_R + \rho$, implying that the conditional distribution of θ follows the normal distribution $N(v_S, \sigma_S^2)$ or $N(v_R, \sigma_R^2)$, depending on $m \leq v_R + \rho$ or $m > v_R + \rho$. So, assuming that the sender reveals v_S truthfully, the loss function of the receiver, $\text{MSE}(a \mid v_i) = E[(a - \theta)^2 \mid v_i]$ is minimized when R chooses $a = E(\theta \mid v_i) = v_i$ for $i = S$ or R. Since $\min \text{MSE}(a \mid v_i) E[(v_i - \theta)^2 \mid v_i] = E(\epsilon_i^2) = \sigma_i^2$, it is strictly lower than $\text{MSE}(a = 0) = \infty$, which is the R's loss when she is uninformed. It is also clear that the sender has no reason to prefer an informed receiver, since an informed receiver may choose $a < 0$, if $v_R < 0$, while an uninformed receiver will always choose $a = E(\theta) = 0$.

It is more intriguing to compare utilities when the receiver is more informed and less informed. Increasing σ_R has two effects on the sender's utility, its direct effect and its indirect effect through ρ^*.[22] The direct effect is negative, because the probability that v_R is low enough to penalize S is higher. On the other hand, the indirect effect is positive, because the sender is less likely to be penalized with larger ρ^*, implying that R chooses $a = v_S$ than v_R more often, as far as the sender reveals truthfully. The next proposition shows that the direct nagative effect dominates the indirect positive effect, so U^S decreases as σ_R^2 increases for any v_S. This is rather contrary to the widely held belief that the sender prefers a less informed receiver. The correct intuition behind this counterintuitive result is that wrong information of a less informed receiver may screw up the deal between the sender and the receiver in the sense that she chooses a very low quantity based on her own preposterous information.

[20] If $\sigma_S^2 = \sigma_R^2 = 0$, a fully revealing equilibrium can be attained by the receiver's extreme form of cross-checking strategy ($\rho = 0$), $b(m, v_R) = m$ if $m \leq v_R$ and $b(m, v_R) = v_R$ if $m > v_R$. However, our assumption that $\sigma_S < \sigma_R$ excludes this case.

[21] Strictly speaking, v_S is inferred from the message m, but we will not distinguish the term between information and an inference here.

[22] The formal derivation of this decomposition is provided in Appendix.

Proposition 5. *In equilibrium, the sender's utility decreases as $\sigma(<\infty)$ increases for any v_S, if $\epsilon_\rho \equiv \frac{d\rho}{d\sigma}\frac{\sigma}{\rho} \in (0,1)$.*

Intuitively, if the permissible deviation schedule is inelastic with respect to a change in σ, the positive indirect effect through ρ^* is outweighed by the negative direct effect, so the result immediately follows.

The sender's preference for an informed receiver clearly relies on his information v_S. It is obvious that the sender's utility in a communicative equilibrium increases as v_S increases, because a higher value of v_S just shifts the conditional distribution of v_R to the right and it has the direct positive effect of $u'(v_S) > 0$. This leads to the following proposition.

Proposition 6. *For any $\sigma_R < \infty$, there exists large $\bar{v}_S$ such that for any $v_S \geq \bar{v}_S$, the sender with v_S prefers an informed receiver with expertise σ_R to an uninformed receiver.*

This proposition suggests that monotonicity of U^S with respect to σ_R for any v_S is not extended to $\sigma_R = \infty$.

Now, consider the receiver's preference. Does a receiver prefer the communicative equilibrium to the babbling equilibrium as she is more informed? Increasing σ_R has two effects on the receiver. It has the direct effect and the indirect effect through ρ^*. It is clear that the direct effect is negative, because there will be more loss from wrong penalties due to inaccurate information as the expertise of the receiver is lower. The indirect effect through a change in ρ^* is positive in the sense that R prefers high ρ^* resulting from higher σ. The intuition goes as follows. The receiver prefers to estimate θ based on v_S rather than v_R, because v_S has a lower variance. Note that v_S is symmetrically distributed around v_R, but R penalizes by ignoring v_S only if v_S is far from v_R from above. This asymmetry makes R believe that $\theta = v_S$ with higher probability than $\theta = v_R$, based on the realization of v_S, because $\text{Prob}(v_S \leq v_R + \rho^*) = \Phi(\frac{\rho^*}{\sigma}) > \frac{1}{2}$. As ρ^* becomes larger, the probability that R penalizes S by ignoring v_S gets even lower. This is why R prefers a larger ρ^*.

Proposition 7. *Suppose that the sender reports truthfully. Then, the receiver prefers the crosschecking strategy with a larger permissible deviation.*

This proposition has an important implication on the case that $\sigma_R(<\infty)$ is also private information of the receiver. If σ_R is private information of R, does the receiver always have an incentive to exaggerate her own expertise? Does the expert (sender) have a good reason to believe what the receiver says about her expertise? Interestingly, this proposition says that the receiver would rather pretend to be less informed, because she prefers a larger ρ^*, insofar as it does not affect the sender's incentive to be honest, so there would be no truth-revealing equilibrium in which the receiver sends an honest message about her expertise to the sender.

To see why, suppose that the receiver always tells the truth about σ_R and the sender believes whatever she says. Given that the sender believes any message of the receiver, he will send the true message $m = v_S$, based on his presumption that the receiver will use the permissible deviation $\rho^*(\sigma_R)$. Given that, the receiver will have an incentive to pretend to be less informed ($\sigma'_R > \sigma_R$) to change the permissible deviation to $\rho^*(\sigma'_R) > \rho^*(\sigma_R)$. Even if she misreports that her standard deviation is $\sigma'_R(> \sigma_R)$, the sender will believe it and send the truthful message on the belief that the permissible deviation is $\rho^*(\sigma'_R)$, and accordingly, as far as the sender sends the truthful message, the receiver prefers $\rho^*(\sigma'_R) > \rho^*(\sigma_R)$ so has an incentive to misreport her variance. Therefore, it cannot an equilibrium for her to use the crosschecking strategy with $\rho^*(\sigma_R)$. Of course, if the receiver is completely uninformed ($\sigma_R = \infty$), the uninformed receiver ($\sigma_R = \infty$) may have an incentive to pretend to be an informed receiver ($\sigma_R < \infty$), since she knows that only the babbling equilibrium is possible if it is known that she is completely uninformed. However, knowing this incentive, the sender will not believe the receiver's cheap talk message saying that she is informed. In this case, truthful communication about the receiver's expertise would not be possible, either.

5 Conclusion

We have shown that a sender can be disciplined by an informed receiver so that he will reveal his information truthfully because he would be more likely to be penalized by conveying false information. In reality, persuasion by a salesperson can be credible if consumers have some relevant information about the product. This is also true for referral processes of the quality of a newly introduced experience good. Information diffusion by word-of-mouth communication can be credible if consumers who seek opinions have some expertise.

Even though the arguments in this paper have been made within a limited context of a salesperson, the general insight can be carried over to enormous economic situations. For instance, university professors may want more students of his own to get an admission or a job in be decent graduate schools. If a professor does not care about his reputation at all – this is usually the case for a professor from abroad –, he will always write the most favorable recommendation letters as he can. This is the reason why most graduate schools do not believe references from foreign countries. Of course, this is one equilibrium (babbling equilibrium). However, apart from the reputational consideration, a professor sometimes writes a very sincere and fair letter especially for an applicant in the job market, if the recipient has some information about the applicant, because he is afraid that his evaluation is too much different from the recipient's own opinion.

Appendix

Proof of Proposition 2: We have

$$\begin{aligned} u'(v_S)\int_{v_S-\rho}^{\infty} h(v_R)dv_R &= u'(v_S)\left[\int_{v_S-\rho}^{v_S} h(v_R)dv_R + \int_{v_S}^{\infty} h(v_R)dv_R\right] \\ &> \rho u'(v_S)h(v_S-\rho) \\ &= (u(v_S)-u(v_S-\rho))h(v_S-\rho). \end{aligned}$$

The inequality follows from $\int_{v_S-\rho}^{v_S} h(v_R)dv_R > \rho h(v_S-\rho)$ and $\int_{v_S}^{\infty} h(v_R)dv_R > 0$, and the last equality follows from $u(v_S)-u(v_S-\rho) = \rho u'(v_S)$ due to the linearity of $u(a)$. Therefore, the first order condition given by (3) cannot be satisfied for any ρ. ■

Proof of Proposition 3:

Lemma 1. *If $u(a) = 1-e^{-a}$, the first order condition of the sender's optimization given by (8) implies the second order condition.*

Proof. We have

$$\begin{aligned} \frac{\partial^2 U^S}{\partial m^2} &= u''(m)\int_{m-\rho}^{m} h(v_R)dv_R - u'(m)h(m-\rho) \\ &\quad -(u'(m)-u'(m-\rho))h(m-\rho) - (u(m)-u(m-\rho)h'(m-\rho). \end{aligned} \tag{9}$$

Therefore, we have

$$\begin{aligned}\left.\frac{\partial^2 U^S}{\partial m^2}\right|_{m=v_S} &= -e^{-v_S}\left[\int_{v_S-\rho}^{\infty} h(v_R)dv_R + h(v_S-\rho) + (e^{\rho}-1)(h'(v_S-\rho) - h(v_S-\rho))\right] \\ &= -e^{-v_S}\left[h(v_S-\rho) + (e^{\rho}-1)h'(v_S-\rho)\right] \qquad (10) \\ &< 0,\end{aligned}$$

since $\rho > 0$, $h'(v_S - \rho) > 0$ and $1 - H(v_S - \rho) = (e^{\rho} - 1)h(v_S - \rho)$ from the first order condition given by (4). ■

Lemma 2. *If $u(a) = 1 - e^{-a}$, local optimality of the sender implies global optimality.*

Proof. Since it is clear that a sender will not deviate to $m < v_S$, we will check only the incentive to deviate to $m > v_S$.

Since $u(a) = 1 - e^{-a}$, equation (2) can be rearranged into

$$\frac{\partial U^S}{\partial m} = e^{-m}[1 - H(m-\rho) - (e^{\rho}-1)h(m-\rho)]. \qquad (11)$$

We will show $\psi(m) \equiv 1 - H(m-\rho) - (e^{\rho}-1)h(m-\rho) < 0$, $\forall m > v_S$.

Without loss of generality, assume that $v_S = 0$. Since $m = 0$ is a local maximum, $m = 0$ must satisfy the first order condition,

$$1 - H(m-\rho) = (e^{\rho}-1)h(m-\rho), \qquad (12)$$

or equivalently,

$$\frac{1}{e^{\rho}-1} = G(m-\rho), \qquad (13)$$

where $G(x)$ is a hazard rate function. It is well known that a normal distribution function satisfies log-concavity so its hazard rate function $G(x)$ is monotonic increasing, i.e., $G'(x) > 0$. (See Bagnoli and Bergstrm (2005).) Also, we have $\lim_{x\to-\infty} G(x) = 0$ and $\lim_{x\to\infty} G(x) = \infty$. Therefore, the equation (13) has the unique solution, which is $m^* = 0$ and $\psi(m) < 0$ $\forall m > 0$. This completes the proof. ■

Now, we can rewrite $u(a)$ as following:

$$u(a) = \gamma - \beta e^{-a} = \gamma(1 - \frac{\beta}{\gamma}e^{-a}) = \gamma(1 - e^{-(a - \ln \beta/\gamma)}). \qquad (14)$$

The graph of $u(a)$ is obtained simply by scaling of the vertical axis and transition of the a-axis. This does not change the first order condition and second order condition. ■

Proof of Proposition 4: By Taylor expansion, we have

$$u(v_S-\rho)=u(v_S)-u'(v_S)\rho+u''(v_S)\frac{\rho^2}{2}+O(\rho^3). \tag{15}$$

So, for small $\rho>0$, the first order condition given by equation (3) can be rewritten as

$$u'(v_S)\int_{v_S-\rho}^{\infty}h(v_R)dv_R=(u'(v_S)\rho-u''(v_S)\frac{\rho^2}{2})h(v_S-\rho). \tag{16}$$

By using $G(v_S-\rho)\equiv\frac{h(v_S-\rho)}{\int_{v_S-\rho}^{\infty}h(t)dt}$, one can reduce equation (16) into

$$u''(v_S)=-\frac{2}{\rho}\left(\frac{1}{G(v_S-\rho)\rho}-1\right)u'(v_S). \tag{17}$$

Note that the solution for the differential equation given by (17) must be of the form $u(x)=\gamma-\beta e^{-cx}$ where $c=\frac{2}{\rho}\left(\frac{1}{G(v_S-\rho)\rho}-1\right)=\frac{2}{\rho}\left(\frac{e^{\rho}-1}{\rho}-1\right)>0$ for $\rho>0$. Also, $u'>0$ implies that $\beta>0$. ■

Derivation of $\frac{dU^S}{d\sigma}$:

The sender's utility in the fully revealing equilibrium is given by

$$U^S(v_S;v_S)=\int_{-\infty}^{v_S-\rho^*(\sigma)}u(v_R)h(v_R)dv_R+\int_{v_S-\rho^*(\sigma)}^{\infty}u(v_S)h(v_R)dv_R, \tag{18}$$

where $u(x)=1-e^{-x}$ and $h(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{\rho}{\sigma}\right)^2}$. We will use the following Leibniz rule.

$$\frac{d}{dx}\int_{a(x)}^{b(x)}f(x,t)dt=f(x,b(x))\frac{db}{dx}-f(x,a(x))\frac{da}{dx}+\int_{a(x)}^{b(x)}\frac{\partial}{\partial x}f(x,t).$$

Then, we obtain

$$\begin{aligned}\frac{\partial U^S(v_S;v_S)}{\partial\sigma} &= u(v_S-\rho^*(\sigma))h(v_S-\rho^*(\sigma))\Big(-\frac{\partial\rho^*}{\partial\sigma}\Big)+\int_{-\infty}^{v_S-\rho^*(\sigma)}u(v_R)\frac{\partial}{\partial\sigma}h(v_R)dv_R\\ &\quad +u(v_S)\Big(h(v_s-\rho^*(\sigma))\frac{\partial\rho^*}{\partial\sigma}+\int_{v_S-\rho^*(\sigma)}^{\infty}\frac{\partial}{\partial\sigma}h(v_R)dv_R\Big).\end{aligned}$$

Using $h(v_S-\rho^*(\sigma))=h(v_S+\rho^*(\sigma))$, $\frac{\partial}{\partial\sigma}h(v_R)=h(v_R)\Big(\frac{(v_R-v_S)^2}{\sigma^3}-\frac{1}{\sigma}\Big)$ and $\int_{v_S-\rho^*(\sigma)}^{v_S}\frac{\partial}{\partial\sigma}h(v_R)dv_R=\int_{v_S}^{v_S+\rho^*(\sigma)}\frac{\partial}{\partial\sigma}h(v_R)dv_R$, we have

$$\frac{\partial U^S(v_S; v_S)}{\partial \sigma} = \Big(u(v_S) - u(v_S - \rho^*(\sigma))\Big) h(v_S - \rho^*(\sigma)) \frac{\partial \rho^*}{\partial \sigma} + \int_{-\infty}^{v_S - \rho^*(\sigma)} u(v_R) h_\sigma(v_R) dv_R + \int_{v_S - \rho^*(\sigma)}^{\infty} u(v_S) h_\sigma(v_R) dv_R,$$

where $h_\sigma \equiv \frac{\partial}{\partial \sigma} h(v_R)$. The first term is the indirect effect, and the second and third terms are the direct effects.

Proof of Proposition 5: Let $U^{S*}(v_S, \sigma) \equiv U^S(v_S; v_S, \sigma)$ be the equilibrium utility of a sender whose information has standard deviation σ. We will prove that $\frac{\partial U^{S*}(v_S,\sigma)}{\partial \sigma} < 0$ for any v_S. To do that, we will show that (i) $\frac{\partial U^{S*}(v_S,\sigma)}{\partial \sigma}$ is increasing in v_S, and (ii) $\lim_{v_S \to \infty} \frac{\partial U^{S*}(v_S,\sigma)}{\partial \sigma} < 0$.

(i) Let $x = \frac{v_R - v_S}{\sigma}$. Then, x follows a standard normal distribution. Let the density function and the distribution function of the standard normal distribution be $f(x)$ and $F(x)$ respectively. Then, U^{S*} can be rewritten as

$$U^{S*} = \sigma \left[\int_{-\infty}^{-\frac{\rho}{\sigma}} u(v_S + \sigma x) f(x) dx + \int_{-\frac{\rho}{\sigma}}^{\infty} u(v_S) f(x) dx \right]. \tag{19}$$

Due to Young's Theorem, it suffices to show that $\frac{\partial^2 U^{S*}}{\partial \sigma \partial v_S} = \frac{\partial^2 U^{S*}}{\partial v_S \partial \sigma} > 0$.

By using $u(v_S) = 1 - e^{-v_S}$ and $u(v_S + \sigma x) = 1 - e^{-v_S - \sigma x}$, we have

$$\frac{\partial U^{S*}}{\partial v_S} = e^{-v_S} \Psi(\sigma), \tag{20}$$

where $\Psi(\sigma) = \sigma \left[\int_{-\infty}^{-\frac{\rho}{\sigma}} e^{-\sigma x} f(x) dx + \int_{-\frac{\rho}{\sigma}}^{\infty} f(x) dx \right]$. By using Leibniz rule, we have

$$\begin{aligned} \frac{d\Psi(\sigma)}{d\sigma} &= \int_{-\infty}^{-\frac{\rho}{\sigma}} e^{-\sigma x} f(x) dx + \int_{-\frac{\rho}{\sigma}}^{\infty} f(x) dx \\ &\quad + \sigma \left[\left(\frac{\rho}{\sigma^2} \right) e^{\rho} f(-\frac{\rho}{\sigma}) - \int_{-\infty}^{-\frac{\rho}{\sigma}} x e^{-\sigma x} f(x) dx - \frac{\rho}{\sigma^2} f(-\frac{\rho}{\sigma}) \right] + \Delta \\ &= \int_{-\infty}^{-\frac{\rho}{\sigma}} (1 - \sigma x) e^{-\sigma x} f(x) dx + \int_{-\frac{\rho}{\sigma}}^{\infty} f(x) dx \\ &\quad + \frac{\rho}{\sigma} (e^{\rho} - 1) f(-\frac{\rho}{\sigma}) + \Delta, \end{aligned} \tag{21}$$

where $\Delta = \frac{d\Psi(\sigma)}{d\rho}\frac{d\rho^*}{d\sigma} = f(-\frac{\rho}{\sigma})(1-e^{\rho})\frac{d\rho^*}{d\sigma}$. Therefore, we obtain

$$\frac{d\Psi(\sigma)}{d\sigma} = \int_{-\infty}^{-\frac{\rho}{\sigma}} (1-\sigma x)e^{-\sigma x} f(x)dx + \int_{-\frac{\rho}{\sigma}}^{\infty} f(x)dx + (e^{\rho}-1)f(-\frac{\rho}{\sigma})\left(\frac{\rho}{\sigma} - \frac{d\rho^*}{d\sigma}\right) > 0, \quad (22)$$

if $\epsilon_\rho < 1$, where $\epsilon_\rho = \frac{d\rho^*}{d\sigma}\frac{\sigma}{\rho}$ is the elasticity of ρ^* with respect to σ.

(ii) $\lim_{v_S\to\infty} \frac{\partial U^{S*}(v_S,\sigma)}{\partial\sigma} = \int_{-\infty}^{\infty} u(v_R)h_\sigma dv_R$, since $\lim_{v_S\to\infty} u(v_S) - u(v_S-\rho) = 0$, $\lim_{v_S\to\infty} h(v_S-\rho) = 0$ and $\frac{\partial\rho^*}{\partial\sigma}$ is independent of v_S. We want to show that $\lim_{v_S\to\infty}\frac{\partial U^{S*}(v_S,\sigma)}{\partial\sigma} < 0$.

We have

$$\begin{aligned}
\int_{-\infty}^{\infty} u(v_R)h_\sigma dv_R &= \int_{-\infty}^{\infty}(1-e^{-v_R})h_\sigma dv_R \\
&< \int_{-\infty}^{\infty} h_\sigma dv_R \\
&= \int_{-\infty}^{\infty} \frac{h(v_R)}{\sigma}\left(\left(\frac{v_R - v_S}{\sigma^2}\right)^2 - 1\right) dv_R \\
&= \int_{-\infty}^{\infty} f(x)(x^2-1)dx \\
&= 0,
\end{aligned}$$

since $1 - e^{-v_R} < 1$. ∎

Proof of Proposition 6: Since the sender's utility in a babbling equilibrium is $U^S = u(0) = 1 - e^0 = 0$, it suffices to show that there exists large $\bar{v}_S$ such that for all $v_S \geq \bar{v}_S$, $U^S(v_S, v_S) > 0$, for all $\sigma < \infty$.

Lemma 3. *Let $g(v_R) \equiv u(v_R)h(v_R)$. Then, $g(v_R) \to 0$ as $v_R \to -\infty$.*

Proof. We have

$$g(v_R) = (1-e^{-v_R})\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{v_R - v_S}{\sigma}\right)^2}. \quad (23)$$

Let $x \equiv \frac{v_R - v_S}{\sigma}$. Then, (23) is reduced to

$$\begin{aligned}
g(v_R) &= (1-e^{-(\sigma x + v_S)})\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}x^2} \\
&= \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}x^2} - \frac{1}{\sqrt{2\pi}\sigma}e^{-(\sigma x + v_S + \frac{1}{2}x^2)} \\
&\equiv \hat{g}(x).
\end{aligned}$$

Therefore, $\lim_{v_R\to-\infty} g(v_R) = \lim_{x\to\infty}\hat{g}(x) = 0$. ∎

Lemma 4. *For any $\epsilon > 0$, there exists $\bar{v}_S$ such that for any $v_S \geq \bar{v}_S(\epsilon)$, $\int_{-\infty}^{v_S-\rho^*} g(v_R)dv_R > -\epsilon$.*

Proof. We have

$$\int_{-\infty}^{v_S-\rho^*} g(v_R)dv_R = \int_{-\infty}^{-\frac{\rho^*}{\sigma}} \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}x^2}dx - \int_{-\infty}^{-\frac{\rho^*}{\sigma}} \frac{1}{\sqrt{2\pi}\sigma}e^{-(\sigma x+v_S+\frac{1}{2}x^2)}dx.$$

If $\sigma > 1$, we have

$$\int_{-\infty}^{-\frac{\rho^*}{\sigma}} \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}x^2}dx = \frac{1}{\sigma}\Phi(-\frac{\rho^*}{\sigma}) \in (0, \frac{1}{2}).$$

Note that $\int_{-\infty}^{-\frac{\rho^*}{\sigma}} \frac{1}{\sqrt{2\pi}\sigma}e^{-(\sigma x+v_S+\frac{1}{2}x^2)}dx = \int_{-\infty}^{-\frac{\rho^*}{\sigma}} \frac{1}{\sqrt{2\pi}\sigma}e^{-(x+\sigma)^2}e^{-v_S+\frac{1}{2}\sigma^2}dx$. Since $\lim_{v_S\to\infty} e^{-v_S} = 0$ and $\int_{-\infty}^{-\frac{\rho^*}{\sigma}} \frac{1}{\sqrt{2\pi}\sigma}e^{-(x+\sigma)^2}dx$ is finite, we have

$$\lim_{v_S\to\infty}\int_{-\infty}^{-\frac{\rho^*}{\sigma}} \frac{1}{\sqrt{2\pi}\sigma}e^{-(\sigma x+v_S+\frac{1}{2}x^2)}dx = 0.$$

Therefore, for any $\epsilon > 0$, there exists $\bar{v_S}(\epsilon)$ such that for any $v_S \geq \bar{v_S}(\epsilon)$,

$$\left|\int_{-\infty}^{-\frac{\rho^*}{\sigma}} \frac{1}{\sqrt{2\pi}\sigma}e^{-(\sigma x+v_S+\frac{1}{2}x^2)}dx\right| < \epsilon.$$

This implies that

$$-\epsilon < \int_{-\infty}^{v_S-\rho^*} g(v_R)dv_R < \frac{1}{2} + \epsilon.$$

■

For any given $\sigma_R(< \infty)$ and the corresponding $\rho^*(\sigma_R) < \infty$, (18) implies

$$U^S = \int_{-\infty}^{v_S-\rho^*} g(v_R)dv_R + \int_{v_S-\rho^*}^{\infty} u(v_S)h(v_R)dv_R.$$

Since $\lim_{v_S\to\infty} u(v_S) = 1$ and $\int_{v_S-\rho^*}^{\infty} h(v_R)dv_R > \frac{1}{2}$, it follows from Lemma 4 that $U^S > -\epsilon + \frac{1}{2} > 0$ for any $v_S \geq \bar{v_S}(\epsilon)$. ■

Proof of Proposition 7: Since $b(v_S, v_R) = m$ if $m \leq v_R+\rho^*$ and $b(v_S, v_R) = v_R$ if $m > v_R+\rho^*$, the receiver's interim utility in the fully revealing equilibrium is given by

$$|U^R(m = v_S)| = \int_{-\infty}^{v_R+\rho}\int_{-\infty}^{\infty}(v_S-\theta)^2 f(\theta \mid v_S)d\theta h(v_S)dv_S + \int_{v_R+\rho}^{\infty}\int_{-\infty}^{\infty}(v_R-\theta)^2 f(\theta \mid v_R)d\theta h(v_S)dv_S,$$

where $\theta \mid_{v_i} \sim N(v_i, \sigma_i^2)$. Let $L(v_i, \sigma_i^2) = \int_{-\infty}^{\infty}(v_i - \theta)^2 f(\theta \mid v_i)d\theta$, $i = S, R$. Then, it can be reduced to

$$|U^R| = \int_{-\infty}^{v_R+\rho} L(v_S, \sigma_S^2)h(v_S)dv_S + \int_{v_R+\rho}^{\infty} L(v_R, \sigma_R^2)h(v_S)dv_S.$$

Therefore, we have

$$\frac{d|U^R|}{d\rho} = L(v_R + \rho, \sigma_S^2)h(v_R + \rho) - L(v_R, \sigma_R^2)h(v_R + \rho) < 0.$$

This completes the proof. ■

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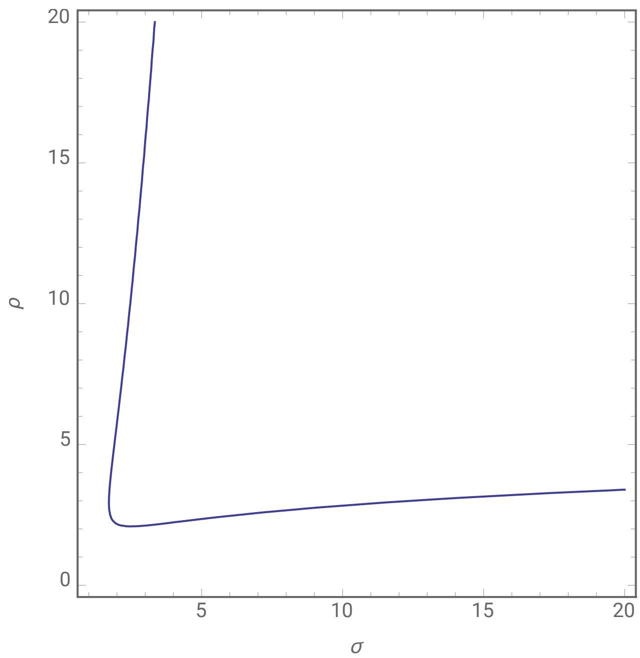

Figure 1: Optimal ρ* for various values of σ

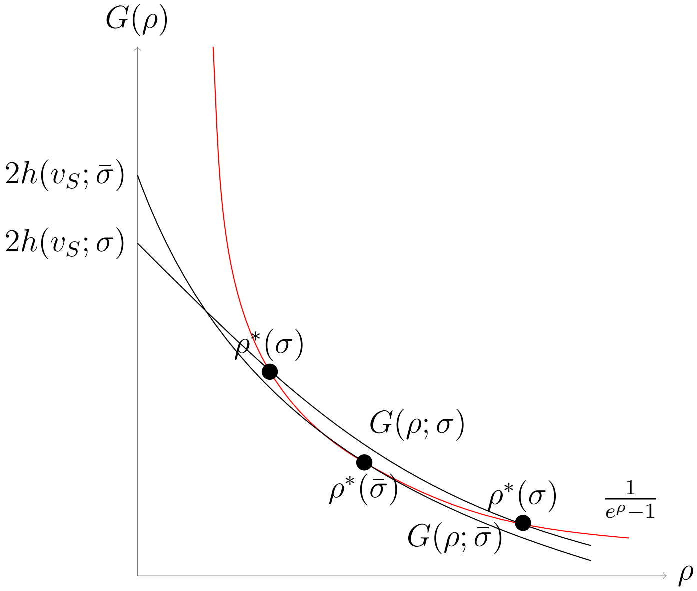

Figure 2. Multiple Solutions for $\rho^*(\sigma)$ when $\sigma > \bar{\sigma}$