Endogenous Skill-Biased Innovation, Growth, and Inequality

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Abstract

Based on a three-factor, two-level constant elasticity of substitution production function, this paper develops an endogenous skill-biased innovation model using the neoclassical growth model to analyze the dynamics of income inequalities. Stiglitz (2014) argued that the formulation of the induced skill-biased innovation is a promising research approach for analyzing the various inequalities in OECD countries, but did not extend the model analysis. Extending the innovation possibility frontier that has a skill-biased innovation type, we investigate the dynamic properties of income inequality and labor shares in the growth model. We show that capital–skill complementarity production technology and the possible shift in the induced innovation possibility frontier incorporating the externality of capital accumulation play significant roles in the stability of steady state and the dynamics of labor shares and inequality. The implications of the heterogeneity of population growth on the equilibrium skill-biased innovation and dynamics of inequality are also investigated.

JEL: D33, E13, E25, O33

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1. Introduction

Parallel to the widespread inequality in advanced countries, there has been a growing literature on skill-biased technology and inequality.[1] Among these studies, having analyzed the dynamics of technical unemployment in the framework of the induced factor-biased of innovation, Stiglitz (2014) argued that the formulation of the induced skill-biased innovation is a promising research approach for analyzing the various inequalities in OECD countries. One of the implications of the induced innovation framework in line with Kennedy (1964) and Samuelson (1965) is that relatively increasing the factor share can induce firms to introduce their own factor-augmenting technical progress in the maximization of the instantaneous cost reduction rate of change on the concavity of the innovation frontier. In this setting, an increase in the capital share of income relative to the labor share of income, for instance, leads to a bias toward capital-augmenting technical progress. If the elasticity of substitution between capital and labor is smaller than unity, the steady state is stable, which reduces the capital share and leads to lower capital-augmenting innovation. However, if the elasticity of substitution is larger than unity, the long-run equilibrium is unstable, and thus this capital-augmenting technical progress further increases the capital share without bounds. Stiglitz (2014) conjectured that the same logic can apply to the case of the innovation of augmenting skilled labor and unskilled labor. In other words, an increase in the income of skilled labor relative to unskilled labor can lead to skill-biased innovation. If the elasticity of substitution between skilled and unskilled labor is larger than unity, skill-biased technology further increases the skilled labor income, which induces even more skill-biased innovation.

However, to consider formally how this conjecture affects the induced skill-biased innovation in a growth economy, at least two modifications are needed. The first is a three-factor framework and the associated elasticity of substitution in the three-factor case. In a growth model, capital accumulation plays a role in the dynamics and associated income inequalities. Therefore, the analysis of induced skill-biased innovation needs a three-factor framework including capital stock. In such a framework, the substitutability and complementarity among skilled, unskilled labor and capital play significant roles in the stability of the steady state and formulation of endogenous biased innovation and inequality. Under three-factor production structures, the economy has the widely estimated relevant capital-skill complementarity structure that implies an elasticity of substitution between capital and skilled labor smaller than unity but that of substitution between

[1] See Acemoglu (2002) and Hornstein et al. (2005). Recent studies have examined capital-augmenting technical progress as an improvement in automation and artificial intelligence technologies. See Acemoglu and Restrepo (2017a, 2017b), Kotlikoff and Sachs (2012), Graetz and Michaels (2015), and Korinek and Stiglitz (2017). In this paper, as an exogenous parameter, we analyze the impact of capital-augmenting technology on skill-biased innovation and income inequality.

capital and unskilled labor larger than unity.[2] Hence, we investigate the stability condition of the steady state and examine how the induced skill-biased innovation and associated income inequality behave in the dynamic system.

The second modification is the innovation possibility frontier.[3] The weakness of the innovation possibility frontier is that it does not have the possibility of shifting.[4] This weakness makes the determination of the growth rate unrealistic as well as ensures the constancy of biased technologies and of the factor income ratio at the steady state. In particular, the latter case is produced by the unique relationship between skill-augmenting technical progress and the factor income ratio, which is derived from the maximization of the instantaneous cost reduction rate of change on the concavity of the stationary innovation frontier. Therefore, in this stationary innovation frontier framework, skill-biased innovation and income inequality do not change unless the frontier curve shifts or the growth economy is in a transitional state.

To address these two respects, based on a three-factor framework with a two-level constant elasticity of substitution (CES) production function that can capture relevant capital–skill complementarity technology,[5] we develop endogenous skill-biased innovation using Solow's (1956) standard neoclassical growth model to analyze the dynamics of income inequalities. As noted, Stiglitz (2014) conjectured about the skill-biased innovation, but did not extend the model analysis. Developing the possible shift in the innovation possibility frontier in line with Kennedy (1964), Samuelson (1965), and Drandakis and Phelps (1966) as well as assuming a tradeoff relationship between skill-augmenting and unskilled-augmenting technical progress,[6] we investigate the characteristics of the induced skill-biased innovation and dynamic properties of income inequality and labor shares in the three-factor growth model.

Further, according to the formulation of Adachi et al. (2019), we develop the induced innovation frontier incorporating the externality of capital accumulation that can expand outward at the innovation frontier.[7] R&D activity can be embodied in the new capital stock, and thus we implicitly take account of this property as the possible shift in the innovation frontier. Developing these modifications, our model can then analyze growth and inequality in the induced innovation framework.

Our three main results are as follows. First, capital–skill complementarity technology and the shift in the innovation possibility frontier play significant roles in the stability of the steady state

[2] See Krusell et al. (2000) and Duffy et al. (2004).

[3] See Acemoglu (2010) and Korinek and Stiglitz (2017).

[4] See Nordhaus (1973) and Acemoglu (2015).

[5] For pioneering works, see Griliches (1969) and Sato (1967).

[6] Caselli and Coleman (2006) considered the same type of frontier in capital–skill complementarity. However, they did not analyze the induced biased innovation.

[7] Samuelson (1965) suggested that the capital share of income can be one of the shift parameters of the innovation possibility frontier.

and dynamics of labor shares and wage inequality. The stability is fulfilled with some empirically relevant capital–skill complementarity and this steady state is oscillatory. This finding implies that even if the elasticity of substitution between skilled and unskilled labor is larger than unity, which is the empirically relevant case,[8] when the elasticity of substitution between capital and skilled labor is smaller than unity, the steady state becomes stable. Hence, there is no biased innovation at this steady state. However, the induced skill-biased innovation can occur in the transitional state.

Second, when the innovation frontier does not shift, the induced skill-biased innovation and skilled /unskilled labor income ratio, which means labor income inequality, do not change at the steady state. However, when the innovation frontier shifts with capital accumulation, a rising population has a favorable influence on growth and the income distribution, as the population growth can expand the innovation possibility frontier outward because of its positive influence on capital accumulation. Thus, in some conditions, the increase in population growth can produce economic growth and decreases income inequality. However, the opposite case applies. In other words, a population decline, which is the relevant case in most advanced countries, is likely to lead to a fall in the growth rate and an increase in income inequality.

Third, our analysis provides an implication for how the heterogeneity of population growth in both labor types affects the induced bias of innovation, growth, and distribution. If population growth in unskilled labor is larger than that in skilled labor, which means a scarcity of skilled labor in the growth process, skill-biased innovation can be introduced even at the steady state, which can then provide more growth. However, this is likely to produce greater income inequality.

The structure of the remainder of this paper is as follows. Section 2 presents the basic model. Section 3 analyzes the dynamic system and the implications for the induced skill-biased innovation and income inequality. Section 4 concludes.

2. Basic Model

2.1 Three-factor production function

We consider a three-factor production function that is twice differentiable and homogeneous of degree one:

$$Y = F(A_1 L_1, A_2 L_2, BK), \tag{1}$$

where Y is output, L_1 is skilled labor, L_2 is unskilled labor, K is capital stock, A_1 is skilled labor efficiency, A_2 is unskilled labor efficiency and B is capital efficiency.

We assume that this three-factor production function is a weakly separable sub-

[8] See Ciccone and Peri (2005). In the two-factor case, if the elasticity of substitution between skilled and unskilled labor is larger than unity, the steady state may always be unstable and thus the induced skill-biased innovation can raise the income of skilled labor over time.

aggregate production function. For the production function, we specify a nested two-level CES production function that has two elasticity parameters: the elasticity of substitution between capital and skilled labor σ_1 and the elasticity of substitution between capital and unskilled labor σ_2:

$$Y = F(L_1, L_2, K)$$
$$= \left[[\delta_2\{\delta_1 L_1^{(1-\sigma 1)/\sigma 1} + (1-\delta_1)K\}^{(1-\sigma 1)/\sigma 1}]^{(1-\sigma 2)\sigma 1/(1-\sigma 1)\sigma 2} + (1-\delta_2)L_2^{(1-\sigma 2)/\sigma 2} \right]^{\sigma 2/(1-\sigma 2)} \quad (2)$$

In this specification, $\sigma_2 > \sigma_1$ provides a capital–skill complementarity technology[9] that has been widely estimated (Krusell et al., 2000; Hornstein et al., 2005), and we deal with this ongoing technical progress. In particular, our analysis focuses on inequalities in empirically relevant capital–skill complementarity $\sigma_2 > 1 > \sigma_1$.[10]

Under the assumption of constant returns to scale, we can rewrite the production function as follows:

$$Y = BKf(A_1L_1/BK, A_2L_2/BK) \quad (3)$$

Since we consider the long-run perfect competitive economy that has full employment, we simply denote L_1, L_2 as follows:

$$L_1 = uL, L_2 = (1-u)L \quad (4)$$

where $u(\equiv L_1/L)$ is a proportion of skilled labor supply in total labor supply L. From (3), output per capital $y(\equiv Y/K)$ is then described as

$$y = Bf[ucx/B, (1-u)x/B] \quad (5)$$

where $c(\equiv A_1/A_2)$ is the skilled/unskilled labor efficiency ratio and $x(\equiv A_2L/K)$ is the effective labor/capital ratio evaluated by unskilled labor efficiency. Over time, the movement of the induced skill-biased innovation represents the dynamics of c and capital accumulation represents the dynamics of x. Later, we examine the dynamics of these two variables.

In the long-run economy, the skilled wage w_1 and unskilled wage w_2 are competitively determined as their own marginal products. In this setting, the skilled wage rate w_1 and unskilled wage rate w_2 are given by $w_1 = \partial F/\partial L_1 = A_1 f_1$ and $w_2 = \partial F/\partial L_2 = A_2 f_2$,

[9] Defining $c_{ij} \equiv F_{ij}F/F_iF_j$ as the partial elasticity of the complementarity between i and j, capital–skill complementarity is described as an inequality in which the elasticity of complementarity between capital and skilled labor is larger than that between capital and unskilled labor c_{1K} (= $F_{1K}F/F_1F_K$) > c_{2K} (= $F_{2K}F/F_2F_K$). In our two-level CES production technology, $\sigma_2 > \sigma_1$ implies $c_{1K} > c_{2K}$ since $c_{1K} - c_{2K} = (\sigma_1^{-1} - \sigma_2^{-1})/(1 - F_2L_2/F)$. Thus, $\sigma_2 > \sigma_1$ implies capital–skill complementarity. In addition, in our three-factor case, the elasticity of substitution is not always equal to the inverse of the elasticity of complementarity. Specifically, $c_{1K} = (\sigma_1^{-1} - \sigma_2^{-1})/(1 - F_2L_2/F) + \sigma_2^{-1} \neq \sigma_1^{-1}$ although $c_{2K} = \sigma_2^{-1}$.

[10] See Duffy et al. (2004) and Hornstein et al. (2005).

respectively. Therefore, wage inequality $\omega(\equiv w_1 / w_2)$ and labor income inequality $a / b(\equiv w_1 L_1 / w_2 L_2)$, implying the labor share ratio, are described respectively as follows:

$$\omega = f_1 c / f_2 , \quad a / b = w_1 L_1 / w_2 L_2 = f_1 c / f_2 \{u / (1-u)\} \tag{6}$$

where a is the skilled labor share and b is the unskilled labor share. The movements in wage inequality and income inequality synchronize as long as the proportion of labor is constant; in other words, no labor mobility across sectors occurs.

2.2 Induced innovation frontier

Consider the induced biased technologies in line with the Kennedy (1964) and Samuelson (1965) type. We focus on two augmenting technologies, namely skill-augmenting technology $\alpha(\equiv \dot{A}_1 / A_1)$ and unskilled-augmenting technology $\beta(\equiv \dot{A}_2 / A_2)$, where the dot denotes dx/dt. Their rates of technical changes are given by the following innovation possibility frontier $q(\alpha, \beta, g) = 0$, which is rewritten as

$$\beta = \beta(\alpha, g), \qquad \beta_\alpha < 0,\ \beta_{\alpha\alpha} < 0,\ \beta_g > 0,\ \beta_{\alpha g} > 0 \tag{7}$$

Here, $\beta_\alpha < 0$ and $\beta_{\alpha\alpha} < 0$ exhibit the concavity of the innovation frontier that implies the resource constraints devoted to these factor-biased technologies. Moreover, $g(\equiv \dot{K} / K)$ expresses capital accumulation and $\beta_g > 0$ represents a shift in the innovation possibility frontier.[11] Following Adachi et al. (2019), we assume that this comes from the R&D activity embodied in the new capital stock, which can produce more possible biased technologies. We formulate this as the external effect of capital accumulation on the innovation frontier. Thus, $\beta_g > 0$ expresses the expansion of the innovation frontier. Furthermore, we assume $\beta_{\alpha g} > 0$, implying that the equilibrium skill-augmenting technology is an increasing function of capital accumulation. Then, representative firms facing the innovation frontier aim to maximize the instantaneous cost reduction rate of change $a\alpha + b\beta(\alpha, g)$ with respect to α, where a is the skilled labor share and b is the unskilled labor share. Solving this maximization problem yields $-\beta_\alpha(\alpha, g) = a / b$. This means that the tangency of the innovation possibility curve equivalent to the skilled/unskilled labor share ratio, implying income inequality, determines the equilibrium skill-biased innovation. This is explicitly shown as

$$-\beta_\alpha(\alpha, g) = \frac{f_1[ucx / B, (1-u)x / B]cu}{f_2[ucx / B, (1-u)x / B](1-u)} \tag{8}$$

where

[11] As noted before, Samuelson (1965) suggested that the capital share of income is one of the shift parameters at the innovation possibility frontier that has a tradeoff between capital-augmenting and labor-augmenting technical progress.

$$a = w_1 L_1 / Y = f_1 cux / Bf = \frac{f_1[ucx/B,(1-u)c/B]ucx/B}{f[ucx/B,(1-u)x/B]} \quad (9)$$

$$b = w_2 L_2 / Y = f_2(1-u)x / Bf = \frac{f_2[ucx/B,(1-u)x/B](1-u)x/B}{f[ucx/B,(1-u)x/B]} \quad (10)$$

Equation (8) is solved for each equilibrium biased innovation α^* and β^* as follows.

$$\alpha^* = \alpha(x,c,g,B,u) \quad (11a)$$

$$\beta^* = \beta[\alpha(x,c,g,B,u),g] = \beta(x,c,g,B,u) \quad (11b)$$

2.3 Dynamics

We consider a standard Solow type neoclassical growth model. Under this model, aggregate savings determine investment and thus can provide capital accumulation. Then, the rate of change of the effective labor/capital ratio evaluated at unskilled labor efficiency $\dot{x}/x$ and that of the skilled/unskilled labor efficiency ratio $\dot{c}/c$ are given by

$$\dot{x}/x = \dot{A}_2/A_2 + \dot{L}/L - \dot{K}/K = \beta + n - sBf[ucx/B,(1-u)x/B] \quad (12)$$

$$\dot{c}/c = \dot{A}_2/A_2 - \dot{A}_1/A_1 = \beta - \alpha \quad (13)$$

where *s* represents the saving rate assumed to be simply constant and *n* is the rate of change in total labor supply implying population growth. Then, equations (12) and (13) give the following dynamic system:

$$\dot{x}/x = \beta(x,c;g,B,u) + n - sBf[cux/B,(1-u)x/B] \quad (14)$$

$$\dot{c}/c = \alpha(x,c;g,B,u) - \beta(x,c;g,B,u) \quad (15)$$

$$g = sBf[cux/B,(1-u)x/B] \quad (16)$$

The steady state is given by the solution to

$$\beta[\alpha(x^*,c^*;g^*,B,u),g^*] + n = sBf[uc^*x^*/B,(1-u)x^*/B] \quad (17)$$

$$\alpha(x^*,c^*;g^*,B,u) = \beta[\alpha(x^*,c^*;g^*,B,u),g^*] \quad (18)$$

$$g^* = sBf[c^*ux^*/B,(1-u)x^*/B] \quad (19)$$

At the steady state, there are the non-bias innovations: therefore, in our model, the skill-biased innovation appears only in the transitional case. However, even at the steady state, we can also have skill-biased innovation in the case of the heterogeneity of population growth, particularly when the population growth of unskilled labor is larger than that of skilled labor. In the next section, we analyze the properties of the dynamics and effects on the induced bias and inequalities.

3. Analysis

We first consider the stability of the dynamics and then analyze the comparative statics of income inequality, the induced bias innovation, and growth at the steady state. Finally, we investigate the implication of the heterogeneity of population growth.

3.1 Stability

Before analyzing the dynamic system, we start by examining the properties of the equilibrium biased technologies. In equation (8), totally differentiating with respect to *α*, *x*, *c*, *g*, and *B* yields the following equation.

$$(\beta_{\alpha\alpha}\alpha / \beta_{\alpha})\hat{\alpha} = (f_{11}l_1 / f_1 + f_{12}l_2 / f_1 - f_{21}l_1 / f_2 - f_{22}l_2 / f_2)(\hat{x} - \hat{B}) + (f_{11}l_1 / f_1 - f_{21}l_1 / f_1 + 1)\hat{c}$$

$$-(\beta_{\alpha g} g / \beta_{\alpha})\hat{g} \qquad (20)$$

where $\hat{x}(\equiv dx / x)$ denotes the percentage change in *x* and $l_1 \equiv A_1 L_1 / BK, l_2 \equiv A_2 L_2 / BK$. Based on our two-level CES production function, this equation is specified as follows:[12]

$$(\beta_{\alpha\alpha}\alpha / \beta_{\alpha})\hat{\alpha} = \frac{\kappa}{1-b}(\sigma_2^{-1} - \sigma_1^{-1})(\hat{x} - \hat{B}) + \frac{1}{1-b}(-a\sigma_2^{-1} - \kappa\sigma_1^{-1} + a + \kappa)\hat{c} - (\beta_{\alpha g} g / \beta_{\alpha})\hat{g}$$

$$(21)$$

where $\kappa(=1-a-b)$ represents the capital share. Therefore, assuming capital–skill complementarity technology ($\sigma_2 > \sigma_1$), we find the effect of these parameters on the equilibrium skill-biased innovation in the elasticity form as follows:

$$\alpha_x x / \alpha = \frac{\beta_{\alpha}}{\beta_{\alpha\alpha}\alpha}\frac{\kappa}{1-b}(\sigma_2^{-1} - \sigma_1^{-1}) < 0 \qquad (22a)$$

$$\alpha_c c / \alpha = \frac{\beta_{\alpha}}{\beta_{\alpha\alpha}\alpha}\frac{1}{1-b}(-a\sigma_2^{-1} - \kappa\sigma_1^{-1} + a + \kappa) \qquad (22b)$$

$$\alpha_g g / \alpha = \frac{-\beta_{\alpha g}}{\beta_{\alpha\alpha}\alpha} g > 0 \qquad (22c)$$

$$\alpha_B B / \alpha = \frac{-\beta_{\alpha}}{\beta_{\alpha\alpha}\alpha}\frac{\kappa}{1-b}(\sigma_2^{-1} - \sigma_1^{-1}) > 0 \qquad (22d)$$

Similarly, we find the effect of each parameter on the equilibrium unskilled-biased innovation in the elasticity form as follows:

$$\beta_x x / \beta = \beta_{\alpha}\alpha_x x / \beta > 0 \qquad (23a)$$

$$\beta_c c / \beta = \beta_{\alpha}\alpha_c c / \beta \qquad (23b)$$

$$\tilde{\beta}_g g / \beta = (\beta_{\alpha}\alpha_g + \beta_g) g / \beta \qquad (23c)$$

$$\beta_B B / \beta = \beta_{\alpha}\alpha_B B / \beta < 0 \qquad (23d)$$

The results of *x*, *c*, and *B* come from the movement in the tradeoff relationship between skill-augmenting and unskilled-augmenting technical progress at the innovation frontier. Thus, we

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[12] In our weakly separable two-level CES production technology, $f_{ij}l_i / f_j (i, j = 1,2)$ are specified as follows: $f_{11}l_1 / f_1 = -(\kappa\sigma_1^{-1} + ab\sigma_2^{-1}) / (1-b) < 0$, $f_{12}l_2 / f_1 = b\sigma_2^{-1} > 0$, $f_{21}l_1 / f_2 = a\sigma_2^{-1} > 0$, and $f_{22}l_2 / f_2 = -(1-b)\sigma_2^{-1} < 0$. Thus, these specifications lead to equation (21).

have the converse effects on α and β. However, the results of g come from the shift in the innovation frontier. This provides the possibility of increasing both α and β. In our later analysis of inequality, we see that these differences provide different outcomes.

Here, an increase in x leads to a decrease in α, but an increase in β. However, increases in g and B lead to an increase in α but a decrease in β. A similar result applies to the effect of c, although this effect is ambiguous. However, the increase in c can sufficiently lead to a decrease in α and therefore an increase in β if the stability condition in (27) is satisfied. This stability condition corresponds to an elasticity of substitution less than unity in the two-factor case. In the three-factor, two-level CES case, some of the capital–skill complementarity in $\sigma_2 > 1 > \sigma_1$ and $1 > \sigma_2 > \sigma_1$ provide the stability. In this case, the relative increase in skill-biased technology decreases the skill-biased innovation, which can produce stability. However, the effect of capital accumulation g produces different outcomes. We have a positive effect of g on both α and β if $\beta_{\alpha g} > 0$ and $\tilde{\beta}_g = \beta_\alpha \alpha_g + \beta_g > 0$ are provided. These expansion effects of capital accumulation play significant roles in the comparative statics of inequalities at the steady state.

Now, let us consider the stability condition of the dynamics. Linearizing in equations (17) and (18) that incorporates (19) at the steady state and rearranging, we have the following dynamic matrix equation:

$$\begin{pmatrix} \delta(\dot{x}/x)/\delta x \\ \delta(\dot{c}/c)/\delta c \end{pmatrix} = \begin{pmatrix} \dfrac{\beta_\alpha \alpha}{\beta+n}(\alpha_x x/\alpha) + A(a+b) & \dfrac{\beta_\alpha \alpha}{\beta+n}(\alpha_c c/\alpha) + Aa \\ \dfrac{\alpha}{\alpha+n}(1-\beta_\alpha)(\alpha_x x/\alpha) + B(a+b) & \dfrac{\alpha}{\alpha+n}(1-\beta_\alpha)(\alpha_c c/\alpha) + Ba \end{pmatrix} \begin{pmatrix} dx/x \\ dc/c \end{pmatrix} \tag{24}$$

where

$$A = \beta_\alpha \alpha_g + \beta_g - 1 = \tilde{\beta}_g - 1, \tag{25a}$$

$$B = (1-\beta_\alpha)\alpha_g - \beta_g = \alpha_g - \tilde{\beta}_g. \tag{25b}$$

Describing the matrix of the partial derivatives of the differential equations as J, the stability of the steady state is then locally satisfied when the trace of the matrix J is negative and the determinant of the matrix J is positive. With some calculation, the trace and determinant of the matrix are respectively given by

$$trJ = \frac{\alpha}{\alpha+n}\{\beta_\alpha \alpha_x x/\alpha + (1-\beta_\alpha)\alpha_c c/\alpha\} + A(a+b) + Ba$$

$$= -(1-\beta_\alpha-\beta_g)\left[\frac{\alpha}{(\alpha+n)^2}\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\frac{1}{b(1-b)}(a\sigma_2^{-1} + b\kappa\sigma_1^{-1} - a - b\kappa) + b\right] < 0 \tag{26}$$

$$\det J \equiv \Delta = \frac{\alpha}{\alpha+n}\{\beta_\alpha B - (1-\beta_\alpha)A\}\{a\alpha_x x/\alpha - (a+b)\alpha_c c/\alpha\}$$

$$= \frac{1}{\alpha+n}\frac{\beta_\alpha}{\beta_{\alpha\alpha}}(1-\beta_\alpha-\beta_g)\frac{1}{1-b}(a\sigma_2{}^{-1}+b\kappa\sigma_1{}^{-1}-a-b\kappa)>0 \qquad (27)$$

From these two inequalities, we have the following proposition about stability at the steady state.

Proposition 1

With endogenous factor-biased innovation that may shift because of capital accumulation in the three-factor, two-level CES production economy, the steady state is sufficiently stable if the expansion effect of the innovation frontier is weak and there is some capital–skill complementarity. Specifically, the steady state is sufficiently stable and oscillatory if

$$1-\beta_\alpha-\beta_g>0\,,\quad a\sigma_2{}^{-1}+b\kappa\sigma_1{}^{-1}-a-b\kappa>0 \qquad (28)$$

[Insert Figures 1 and 2]

Figures 1 and 2 show Proposition 1 in the case of $1-\beta_\alpha-\beta_g>0$. We make three remarks. First, the two stability conditions imply that dynamic stability is produced by the stable stationary innovation frontier and some factor complementarity production technology. The latter case corresponds to an elasticity of substitution smaller than unity in the two-factor case. However, the former case is a new one. The implication is that the steady state becomes unstable even for some factor complementarity if β_g is so large such that the R&D associated with capital accumulation can produce more expansion at the frontier. However, if this is not likely to be the case over time, it is plausible to focus on the case of $1-\beta_\alpha-\beta_g>0$. In this case, Figure 1 shows Proposition 1 in two kinds of elasticity of parameters spaces $\sigma_1{}^{-1}-\sigma_2{}^{-1}$.

Second, in the three-factor case, the steady state is stable in capital–skill complementarity $1>\sigma_2>\sigma_1$ as well as in some empirically relevant capita–skill complementarity $\sigma_2>1>\sigma_1$. The latter implies that even if the elasticity of substitution between skilled and unskilled labor is larger than unity, implying $\sigma_2>1$, which is the empirically relevant case,[13] the smaller elasticity of substitution between capital and skilled labor makes the steady state stable. Figure 2 shows the stability case in which convergence is oscillatory. However, if $a\sigma_2{}^{-1}+b\kappa\sigma_1{}^{-1}-a-b\kappa<0$, implying some factor substitutability, the steady state is unstable and the saddle point shown in Figure 3. This is the same as the elasticity of substitution being lager than unity in the two-factor case. In other words, in this case, skill-biased technology produces more skilled labor income and thus more skill-biased innovation. Hence, this induced innovation produces more income inequality if $a\sigma_2{}^{-1}+b\kappa\sigma_1{}^{-1}-a-b\kappa<0$, implying the larger substitutability between

[13] See Ciccone and Peri (2005). As noted in footnote 8, if the elasticity of substitution between skilled and unskilled labor is larger than unity, the steady state may always be unstable in the two-factor case.

unskilled labor and capital even for some relevant capital–skill complementarity.

[Insert Figure 3]

Finally, this stability condition in unrelated to the aggregate elasticity of substitution between capital and overall labor $\sigma^A (= (a+b\kappa)/(a\sigma_1^{-1} + b\kappa\sigma_2^{-1}))$.[14] Indeed, even if the aggregate elasticity of substitution is smaller than unity, instability is likely to occur.[15] This is because the innovation frontier does not contain capital-augmenting technology and thus the maximization of the instantaneous cost reduction rate of change does not focus on capital-biased innovation. Thus, if the innovation frontier has capital-augmenting technology and labor-augmenting technology in the three-factor case, an aggregate elasticity of substitution smaller than unity may play a crucial role in the stability condition.

3.2 Comparative statics

Analyzing the comparative statics at the steady state, we see that the factor substitutability and the expansion effect of the innovation frontier have a significant effect on the results of the analysis. First, we consider the case of the effective labor/capital ratio and skilled/unskilled efficiency ratio. Total differentiation in the steady state produces the following matrix:

$$\begin{pmatrix} \frac{\beta_\alpha \alpha}{\beta+n}(\alpha_x x/\alpha) + A(a+b) & \frac{\beta_\alpha \alpha}{\beta+n}(\alpha_c c/\alpha) + Aa \\ \frac{\alpha}{\alpha+n}(1-\beta_\alpha)(\alpha_x x/\alpha) + B(a+b) & \frac{\alpha}{\alpha+n}(1-\beta_\alpha)(\alpha_c c/\alpha) + Ba \end{pmatrix} \begin{pmatrix} dx/x \\ dc/c \end{pmatrix} = \begin{pmatrix} -A \\ -B \end{pmatrix} ds/s$$

$$+ \begin{pmatrix} -\frac{\beta_\alpha \alpha}{\beta+n}(\alpha_B B/\alpha) - Ac \\ -\frac{\alpha}{\alpha+n}(1-\beta_\alpha)(\alpha_B B/\alpha) - Bc \end{pmatrix} dB/B + \begin{pmatrix} -\frac{n}{\beta+n} \\ 0 \end{pmatrix} dn/n \tag{29}$$

We assume the stability condition $\Delta > 0$ is satisfied. In this case, from this matrix, we obtain the following results for capital–skill complementarity technology:

$$\hat{x}/\hat{s} = \frac{1}{\Delta}\frac{1}{\alpha+n}(1-\beta_\alpha-\beta_g)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\frac{1}{1-b}(-a\sigma_2^{-1} - \kappa\sigma_1^{-1} + a + \kappa) < 0 \tag{30a}$$

$$\hat{x}/\hat{B} = \frac{1}{\Delta}\frac{1}{\alpha+n}(1-\beta_\alpha-\beta_g)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\kappa(1-\sigma_1^{-1}) < 0 \tag{30b}$$

[14] σ^A is derived in the appendix.
[15] See Figure 1.

$$\hat{c} / \hat{s} = \frac{1}{\Delta}\frac{1}{\alpha+n}(1-\beta_\alpha-\beta_g)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\frac{-\kappa}{1-b}({\sigma_2}^{-1}-{\sigma_1}^{-1}) > 0 \tag{31a}$$

$$\hat{c} / \hat{B} = \frac{1}{\Delta}\frac{1}{\alpha+n}(1-\beta_\alpha-\beta_g)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\frac{-\kappa}{1-b}({\sigma_2}^{-1}-{\sigma_1}^{-1}) > 0 \tag{31b}$$

$$\hat{x} / \hat{n} = \frac{1}{\Delta}\frac{-n}{\beta+n}\left[\frac{1}{\alpha+n}(1-\beta_\alpha)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\frac{1}{1-b}(-a{\sigma_2}^{-1}-\kappa{\sigma_1}^{-1}+a+\kappa)+(\alpha_g-\tilde{\beta}_g)a\right] \tag{32a}$$

$$\hat{c} / \hat{n} = \frac{1}{\Delta}\frac{n}{\beta+n}\left[\frac{1}{\alpha+n}(1-\beta_\alpha)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\frac{\kappa}{1-b}({\sigma_2}^{-1}-{\sigma_1}^{-1})+(\alpha_g-\tilde{\beta}_g)a\right] \tag{32b}$$

Thus, both an increase in the saving rate and further capital-augmenting technical progress lead to a decrease in the effective labor/capital ratio evaluated at unskilled efficiency, but an increase in the skill/unskilled efficiency ratio. Alternatively, the effects of population growth on these two equilibrium variables are ambiguous. However, if the expansion effect of the frontier is small or the effects of capital accumulation on both α and β are similar, then the decline in population growth has the same influence on *x* and *c*. Specifically, the decline in population growth decreases the effective labor/capital ratio evaluated at unskilled efficiency, but increases the skill/unskilled efficiency ratio. Hence, skill-biased innovation occurs only on the transition path toward the new steady state (Figure 4). Therefore, even in the case of population growth, there is non-biased technical progress at the steady state.

[Insert Figure 4]

Next, let us consider the effects of the parameters on the labor shares, inequality, and biased technologies at the steady state. Table 1 summarizes the results. Total differentiation with respect to the skilled labor share a, unskilled labor share b, and aggregate labor share $s_L(=a+b)$ in our specified production function provides the following equations:

$$\hat{a} = (f_{11}l_1/f_1 + f_{12}l_2/f_1 + 1 - f_1l_1/f - f_2l_2/f)(\hat{x}-\hat{B}) + (1 + f_{11}l_1/f_1 - f_1l_1/f)\hat{c}$$
$$= \frac{\kappa}{1-b}(b{\sigma_2}^{-1}-{\sigma_1}^{-1}+1-b)(\hat{x}-\hat{B}) + \frac{1}{1-b}(-\kappa{\sigma_1}^{-1}-ab{\sigma_2}^{-1}+\kappa+ab)\hat{c} \tag{33}$$

$$\hat{b} = (f_{21}l_1/f_2 + f_{22}l_2/f_1 + 1 - f_1l_1/f - f_2l_2/f)(\hat{x}-\hat{B}) + (f_{21}l_1/f_2 - f_1l_1/f)\hat{c}$$
$$= \kappa(1-{\sigma_2}^{-1})(\hat{x}-\hat{B}) + a({\sigma_2}^{-1}-1)\hat{c} \tag{34}$$

$$\hat{s}_L = \frac{a}{a+b}\hat{a} + \frac{b}{a+b}\hat{b} \tag{35}$$

Taking account of (30) – (32) in the above equations, we have the following consequences in some empirically relevant capital–skill complementarity $(\sigma_2 > 1 > \sigma_1)$.

Concerning the saving rate and capital-augmenting technical progress, we find the same results because the frontier does not shift at the steady state:

$$\hat{a}/\hat{s} = \hat{b}/\hat{s} = \hat{s}_L/\hat{s} = \frac{1}{\Delta}\frac{1}{\alpha+n}(1-\beta_\alpha-\beta_g)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\kappa(1-\sigma_2^{-1})(1-\sigma_1^{-1}) < 0 \tag{36}$$

$$\hat{a}/\hat{B} = \hat{b}/\hat{B} = \hat{s}_L/\hat{B} = \frac{1}{\Delta}\frac{1}{\alpha+n}(1-\beta_\alpha-\beta_g)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\kappa(1-\sigma_2^{-1})(1-\sigma_1^{-1}) < 0 \tag{37}$$

$$\hat{a}/\hat{s} - \hat{b}/\hat{s} = 0, \quad \hat{a}/\hat{B} - \hat{b}/\hat{B} = 0 \tag{38}$$

$$\hat{\alpha}/\hat{s} = \hat{\beta}/\hat{s} = 0, \quad \hat{\alpha}/\hat{B} = \hat{\beta}/\hat{B} = 0 \tag{39}$$

In other words, both an increase in the saving rates and the advancement of capital-augmenting technical progress decrease the skilled labor shares, unskilled labor share, and aggregate labor share at the same rates. Thus, the capital shares rise, but labor income inequality a/b does not occur. These results come from the fixity of the innovation possibility frontier at the steady state at which not shifting the innovation frontier provides the constancy of the tangency of frontier curve, which is equivalent to the labor/income ratio. Hence, the movement of the skilled labor shares can synchronize with that of the unskilled labor share and unbiased innovation occurs in at steady state.

However, the consequences of population growth provide different outcomes because the innovation may expand because of capital accumulation. As a result, the shift in the innovation frontier has a positive influence on growth, which may affect income inequality. The results are given as follows:

$$\hat{a}/\hat{n} = \frac{1}{\Delta}\frac{-n}{\beta+n}\left[\frac{1}{\alpha+n}\frac{\beta_\alpha}{\beta_{\alpha\alpha}}(1-\beta_\alpha)\kappa(1-\sigma_2^{-1})(1-\sigma_1^{-1}) + (\alpha_g-\tilde{\beta}_g)\frac{b}{1-b}(a\sigma_2^{-1}+\kappa\sigma_1^{-1}-a-\kappa)\right] \tag{40}$$

$$\hat{b}/\hat{n} = \frac{1}{\Delta}\frac{-n}{\beta+n}(1-\sigma_2^{-1})\left[\frac{1}{\alpha+n}\frac{\beta_\alpha}{\beta_{\alpha\alpha}}(1-\beta_\alpha)\kappa(1-\sigma_1^{-1}) + (\alpha_g-\tilde{\beta}_g)a\right], \tag{41}$$

$$\hat{s}_L/\hat{n} = \frac{1}{\Delta}\frac{-n}{\beta+n}\kappa\left[\frac{1}{\alpha+n}\frac{\beta_\alpha}{\beta_{\alpha\alpha}}(1-\beta_\alpha)(1-\sigma_2^{-1})(1-\sigma_1^{-1}) + (\alpha_g-\tilde{\beta}_g)\frac{b}{1-b}\frac{a}{1-\kappa}(\sigma_1^{-1}-\sigma_2^{-1})\right] \tag{42}$$

$$\hat{a}/\hat{n} - \hat{b}/\hat{n} = \frac{-1}{1-\beta_\alpha-\beta_g}\frac{\beta_{\alpha\alpha}}{\beta_\alpha}(\alpha_g-\tilde{\beta}_g)n, \tag{43}$$

$$\hat{\alpha}/\hat{n} = \hat{\beta}/\hat{n} = \frac{\beta_g}{1-\beta_\alpha-\beta_g}\frac{n}{\beta} > 0. \tag{44}$$

Specifically, in some relevant capita–skill complementarity, although the effects of population growth on each labor share become ambiguous because of the expansion effect, labor income inequality declines if its influence on skill-biased innovation is greater than that on unskilled-biased innovation. Moreover, if the influence of the expansion frontier on each biased innovation is similar, population growth leads to decreases in the skilled labor share, unskilled labor share, and aggregate labor share. Further, we find a positive influence on both skill-biased and unskilled-biased innovations at the same rate, and thus a positive effect on growth. Conversely, in this case, a population decline provides the opposite outcomes. Summarizing the outcomes, we have the following proposition.

Proposition 2

With endogenous factor-biased innovation that may shift because of capital accumulation in the three-factor case, with some relevant capital–skill complementarity technology, the following holds:

1. *Because, at the stable steady state, the change in the saving rate and advancement of capital-augmenting technology cannot produce a shift in the innovation possibility frontier, neither skill-biased nor unskilled-biased innovation changes. Thus, labor income inequality does not occur, although the increase in the saving rate and advancement of capital-augmenting technical progress can lead to decreases in the skilled labor share, unskilled labor share, and aggregate labor share at the same rates.*
2. *However, even at the stable steady state, because population growth may shift the innovation frontier, both skill-biased and unskilled-biased innovation can proportionally change. Therefore, population decline is likely to move the frontier inward, leading to decreases in the skilled labor share, unskilled labor share, and aggregate labor shares if the expansion effect of both skill-biased and unskilled-biased innovation is weak. Furthermore, an increase in income inequality can occur if the expansion effect of skill-biased innovation is larger than that of unskilled-biased innovation.*

Table 1. Effects of the parameters on labor shares, inequality, and biased innovation

	χ	c	a	b	s_L	a/b	α	β	g
s	$-$	$+$	$-$	$-$	$-$	0	0	0	0
B	$-$	$+$	$-$	$-$	$-$	0	0	0	0
n	$+^d$	$-^e$	$+^d$	$+^d$	$+^d$	$-^f$	$+$	$+$	$+$

Note: $\sigma_2 > 1 > \sigma_1$, $\Delta = a\sigma_2^{-1} + b\kappa\sigma_1^{-1} - a - b\kappa > 0$

$d > 0, e < 0$ if $\alpha_g \cong \tilde{\beta}_g$, $f < 0$ if $\alpha_g > \tilde{\beta}_g$.

In the stability case of the steady state, our outcomes for the labor shares at the steady state depend crucially on the property of some substitutability between capital and unskilled labor; in other words, some substitutability between skilled and unskilled labor among some relevant capital–

skill complementarity technology $\sigma_2 > 1 > \sigma_1$. Conversely, even at the steady state, there is some complementarity between capital and unskilled labor among capital–skill complementarity $1 > \sigma_2 > \sigma_1$, and thus some outcomes can be reversed.

3.3 Implications of the heterogeneity of population growth

Finally, we consider the case of the heterogeneity of population growth. One of the weaknesses of our analysis thus far is that the steady state has no biased innovation, implying that skill-biased innovation is the equivalent to unskilled-biased innovation. However, even at the steady state in our framework, the equilibrium skill-biased innovation can be introduced if the population growth of unskilled labor is larger than that of skilled labor. This is likely to occur if the newly arrived technologies lead to the obsolescence of many types of skills, which implies an increasing number of unskilled workers (i.e., the greater scarcity of skilled labor in the growth process).

To analyze this case, we modify our framework. Let n_1 and n_2 denote the population growth of skilled labor and unskilled labor, respectively, and we assume that the population growth of unskilled labor is larger than that of skilled labor $n_2 > n_1$. In this setting, rewriting output per capital leads to

$$y = Bf(\tilde{c}\tilde{x}/B, \tilde{x}/B) \tag{45}$$

where $\tilde{c}(\equiv A_1L_1/A_2L_2)$ is the effective skilled/unskilled labor ratio and $\tilde{x}(\equiv A_2L_2/K)$ is the effective unskilled labor/capital ratio. Then, the dynamics of $\tilde{x}$ and $\tilde{c}$ are given as follows:

$$\dot{\tilde{x}}/\tilde{x} = \beta + n_2 - sBf(\tilde{c}\tilde{x}/B, \tilde{x}/B), \qquad \dot{\tilde{c}}/\tilde{c} = \beta - \alpha + n_2 - n_1 \tag{46}$$

Since the formulation of factor-biased innovation is the same, the solution to the maximization of the instantaneous cost reduction rate of change on the concavity of the innovation possibility frontier yields $-\beta_\alpha(\alpha, g) = a/b$, which is shown as $-\beta_\alpha(\alpha, g) = \dfrac{f_1(\tilde{c}\tilde{x}/B, \tilde{x}/B)\tilde{c}}{f_2(\tilde{c}\tilde{x}/B, \tilde{x}/B)}$. Thus, we have $\alpha^* = \alpha(\tilde{x}, \tilde{c}, g, B)$, $\beta^* = \beta[\alpha(\tilde{x}, \tilde{c}, g, B), g] = \beta(\tilde{x}, \tilde{c}, g, B)$. Therefore, the dynamics are explicitly given by

$$\dot{\tilde{x}}/\tilde{x} = \beta(\tilde{x}, \tilde{c}, g, B) + n_2 - sBf(\tilde{c}\tilde{x}/B, \tilde{x}/B), \tag{47a}$$

$$\dot{\tilde{c}}/\tilde{c} = \alpha(\tilde{x}, \tilde{c}, g, B) - \beta(\tilde{x}, \tilde{c}, g, B) + n_2 - n_1. \tag{47b}$$

$$g = sBf(\tilde{c}\tilde{x}/B, \tilde{x}/B) \tag{47c}$$

Hence, the steady state is given by the solution to

$$\beta(\tilde{x}^*, \tilde{c}^*, g^*, B) + n_2 = sBf[\tilde{c}^*\tilde{x}^*/B, \tilde{x}^*/B] \tag{48}$$

$$\alpha(\tilde{x}^*, \tilde{c}^*, g^*, B) + n_1 = \beta(\tilde{x}^*, \tilde{c}^*, g^*, B) + n_2 \tag{49}$$

$$g^* = sBf(\tilde{c}^*\tilde{x}^*/B, \tilde{x}^*/B). \tag{50}$$

[Insert Figure 5]

Figure 5 illustrates this steady state. As far as $n_2 > n_1$, we have $\alpha(\tilde{x}^*, \tilde{c}^*, g^*, B) > \beta(\tilde{x}^*, \tilde{c}^*, g^*, B)$. In other words, in this case, more skill-biased innovation is introduced because the scarcity of skilled labor induces firms to promote skill-augmenting technology. Moreover, purely skill-biased innovation can appear if the population growth of unskilled labor is high such that the upward line satisfying $\alpha + n_1 = \beta + n_2$ intersects with point α_0 on the innovation frontier in Figure 5.

In addition, we can see the consequences of the population growth in unskilled labor at the steady state. Calculating the comparative statics provides the following results:[16]

$$\hat{\tilde{x}} / \hat{n}_2 = \frac{1}{\Delta} \frac{n_2}{\beta + n_2} \left[\frac{-1}{\alpha + n_1} \frac{\beta_\alpha}{\beta_{\alpha\alpha}} \frac{1}{1-b} (-a\sigma_2^{-1} - \kappa\sigma_1^{-1} + a + \kappa) + (1-\alpha_g)a \right] \tag{51a}$$

$$\hat{\tilde{c}} / \hat{n}_2 = \frac{1}{\Delta} \frac{n_2}{\beta + n_2} \left[\frac{1}{\alpha + n_1} \frac{\beta_\alpha}{\beta_{\alpha\alpha}} \frac{\kappa}{1-b} (\sigma_2^{-1} - \sigma_1^{-1}) - (1-\alpha_g)a \right] \tag{51b}$$

$$\hat{a} / \hat{n}_2 = \frac{1}{\Delta} \frac{n_2}{\beta + n_2} \left[\frac{-1}{\alpha + n_1} \frac{\beta_\alpha}{\beta_{\alpha\alpha}} \kappa(1-\sigma_2^{-1})(1-\sigma_1^{-1}) + (1-\alpha_g) \frac{b}{1-b} (a\sigma_2^{-1} + \kappa\sigma_1^{-1} - a - \kappa) \right] \tag{52}$$

$$\hat{b} / \hat{n}_2 = \frac{1}{\Delta} \frac{n_2}{\beta + n_2} (1-\sigma_2^{-1}) \left[\frac{-1}{\alpha + n_1} \frac{\beta_\alpha}{\beta_{\alpha\alpha}} \kappa(1-\sigma_1^{-1}) + (1-\alpha_g)a \right], \tag{53}$$

$$\hat{s}_L / \hat{n}_2 = \frac{1}{\Delta} \frac{n_2}{\beta + n_2} \kappa \left[\frac{-1}{\alpha + n_1} \frac{\beta_\alpha}{\beta_{\alpha\alpha}} (1-\sigma_2^{-1})(1-\sigma_1^{-1}) + (1-\alpha_g) \frac{b}{1-b} \frac{a}{1-\kappa} (\sigma_1^{-1} - \sigma_2^{-1}) \right] \tag{54}$$

$$\hat{a} / \hat{n}_2 - \hat{b} / \hat{n}_2 = \frac{1}{1-\beta_\alpha - \beta_g} \frac{\beta_{\alpha\alpha}}{\beta_\alpha} (1-\alpha_g) n_2 \tag{55},$$

$$\hat{\alpha} / \hat{n}_2 = \frac{1}{1-\beta_\alpha - \beta_g} \frac{n_2}{\alpha} > 0. \tag{56}$$

$$\hat{\beta} / \hat{n}_2 = \frac{\beta_\alpha + \beta_g}{1-\beta_\alpha - \beta_g} \frac{n_2}{\beta} \tag{57}$$

$$\hat{\alpha} / \hat{n}_2 - \hat{\beta} / \hat{n}_2 = \frac{n_2}{\alpha\beta(1-\beta_\alpha - \beta_g)} (\beta - \beta_\alpha \alpha - \beta_g \alpha) \tag{58}$$

From these, we have Proposition 3.

Proposition 3

With endogenous factor-biased innovation that may shift because of capital accumulation in the three-factor case, with some relevant capital–skill complementarity technology as well as relatively large population growth in unskilled labor, the following holds:

[16] The details are available on request.

If the population growth of unskilled labor is larger than that of skilled labor, skill-biased innovation can be introduced at the steady state. In the stable steady state, the increase in the population growth of unskilled labor can lead to expanding the frontier, resulting in more growth and skill-biased innovation if the expansion effect of unskilled-biased innovation is small. However, larger population growth of unskilled labor can lead to a greater increase in the skilled labor share than in the unskilled labor share, and thus may result in more labor income inequality if the expansion effect of skill-biased innovation is small.

We obtain our results when the population of skilled labor is relatively scarce and the innovation possibility frontier is rather stationary. Therefore, if skilled labor supply is increasing, and the expansion effect of the innovation frontier is large, some outcomes may be reversed. However, if capital-augmenting technology such as automation and artificial intelligence technologies can play the same roles for skilled workers and influence the expansion of the innovation possibility frontier significantly as well as change the capital–skill complementarity structure, the consequences of our analysis may lead to different outcomes.[17] Then, an alternative framework such as formulating endogenous capital-augmenting technical progress may be needed.

4. Concluding remarks

In this paper, based on a three-factor, two-level CES production function that has capital–skill complementarity, we developed an endogenous skill-biased innovation model based on the neoclassical growth model to analyze the dynamics of income inequalities. Extending the innovation possibility frontier that has a skill-biased innovation type, we showed that some relevant capital–skill complementarity technology and the possible shift in the innovation frontier because of capital accumulation play significant roles in the stability of the steady state and the behaviors of labor shares and inequalities. We also showed that relatively large population growth of unskilled labor can introduce more skill-biased innovation and thus raise income inequality. However, to investigate the implications of automation and artificial intelligence technologies on growth, inequality, and unemployment, an alternative formulation of endogenous capital-augmenting technical progress and its analysis are needed. These issues will be addressed in future research.

[17] See Acemoglu and Restrepo (2018) and Berg et al. (2018).

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Appendix

Aggregate elasticity of substitution

We can derive the endogenous elasticity of substitution as follows. w/r is given by the following equation:

$$w/r = \{uw_1 + (1-u)\,w_2\} \,/\, \{f\,(l_1, l_2) - f_1(l_1, l_2)\, l_1 - f_2(l_1, l_2)\, l_2\} \quad \text{(A1)}$$

$$= v\,(l_1, l_2, u)$$

In this case, $l_1 = u/k$ and $l_2 = (1-u)/k$, where $k \equiv K/L$. Denoting $\theta_{l1k} \equiv (k/l_1)(\mathrm{d}/l_1/\mathrm{d}k)$, $\theta_{l2k} \equiv (k/l_2)(\mathrm{d}/l_2/\mathrm{d}k)$, we have the following equations:

$$\theta_{l1k} = -1 \text{ and } \theta_{l2k} = -1 \quad \text{(A2)}$$

Moreover, defining $\eta_1 \equiv (l_1/v)\,\partial v/\partial l_1$, $\eta_2 \equiv (l_2/v)\,\partial v/\partial l_2$, the aggregate elasticity of substitution σ^A is then written as

$$\sigma^A \equiv [(w/r)/k]\mathrm{d}k/\mathrm{d}(w/r) \quad \text{(A3)}$$

$$= \{(k/v)\,\mathrm{d}v/\mathrm{d}k\}^{-1}$$

$$= (\eta_1\theta_{l1k} + \eta_2\theta_{l2k})^{-1}$$

$$= (-\eta_1 - \eta_2)^{-1}$$

Calculating $\eta_1 \equiv (l_1/v)\,\partial v/\partial l_1$ and $\eta_2 \equiv (l_2/v)\,\partial v/\partial l_2$, and specifying these in our two-level CES production function yield

$$\eta_1 = (b\sigma_2^{-1} - \sigma_1^{-1})\, a/(a + b\kappa),\ \eta_2 = -\,\sigma_2^{-1} b/(a + b) < 0. \quad \text{(A4)}$$

Thus, the aggregate elasticity of substitution is given by

$$\sigma^A = (-\eta_1 - \eta_2)^{-1}$$

$$= (a + b\kappa)/(a\sigma_1^{-1} + b\kappa\sigma_2^{-1}) \quad \text{(A5)}$$

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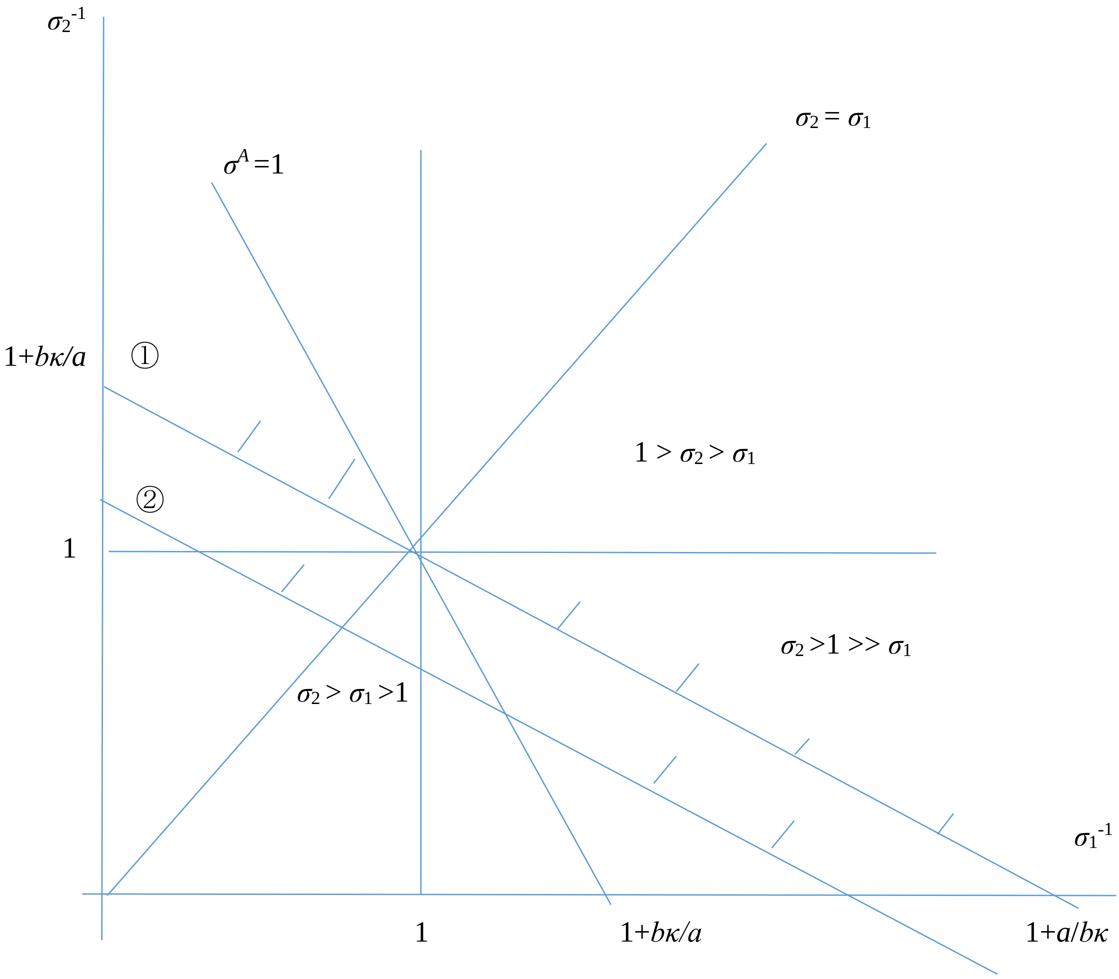

Figure 1 Stability condition

① det $J > 0$ if $a\sigma_2^{-1} + b\kappa\sigma_1^{-1} > a + b\kappa$

② tr$J < 0$

$\sigma^A = (a + b\kappa)/(a\sigma_1^{-1} + b\kappa\sigma_2^{-1})$

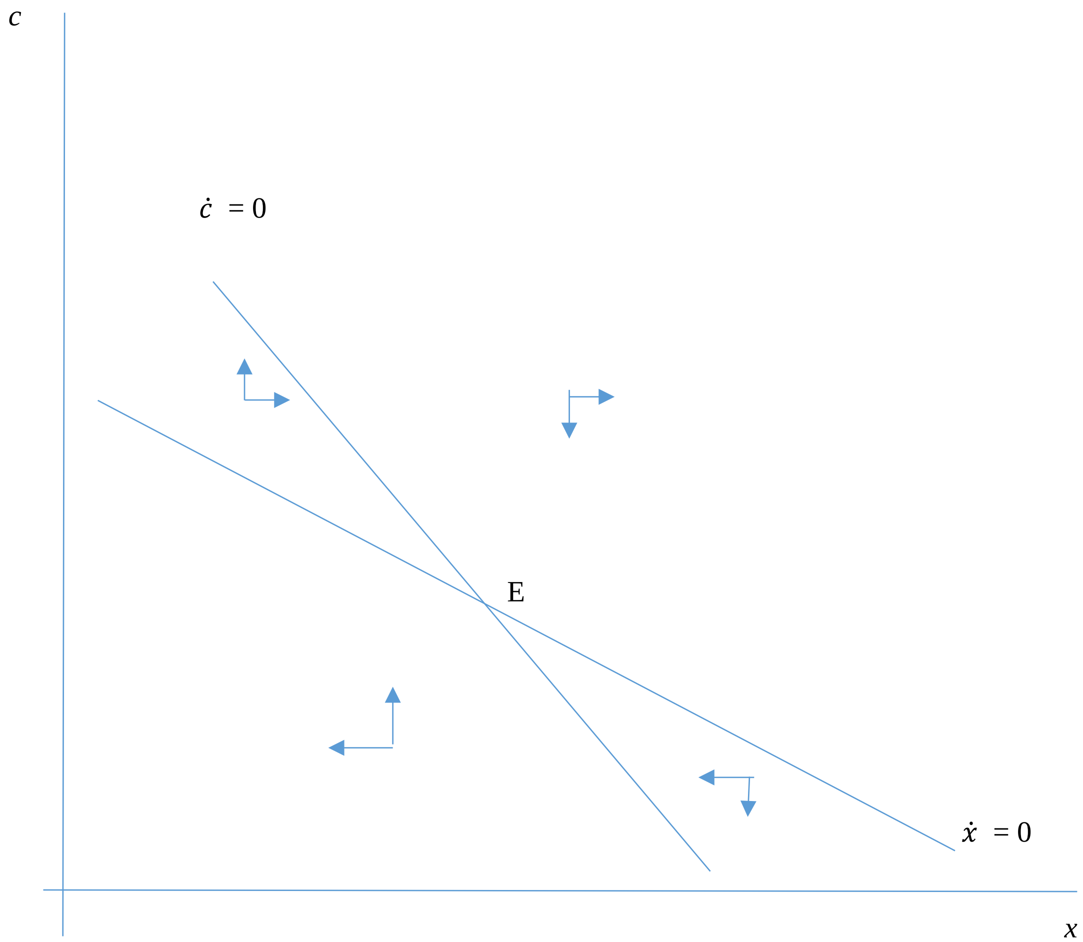

Figure 2 Stable case $a\sigma_2^{-1} + b\kappa\sigma_1^{-1} > a + b\kappa$

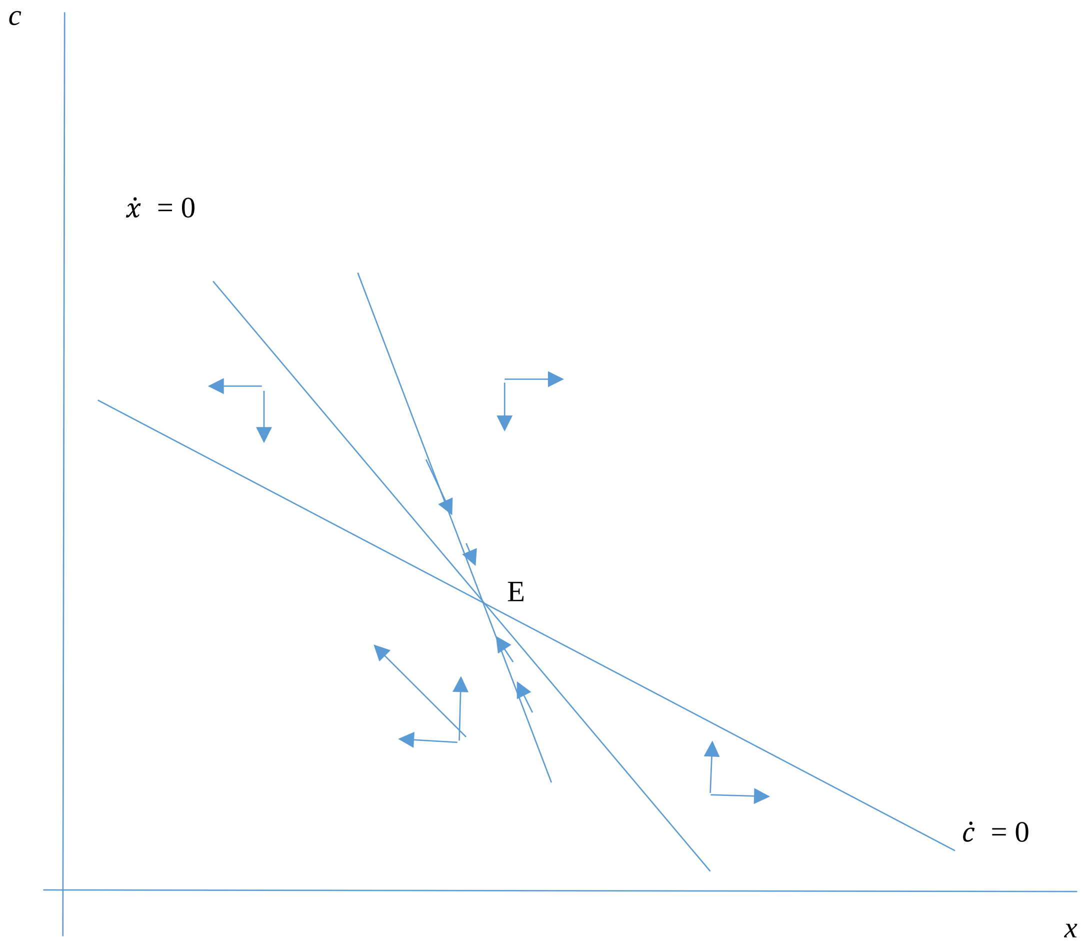

Figure 3 Unstable case $a\sigma_2^{-1} + b\kappa\sigma_1^{-1} < a + b\kappa$

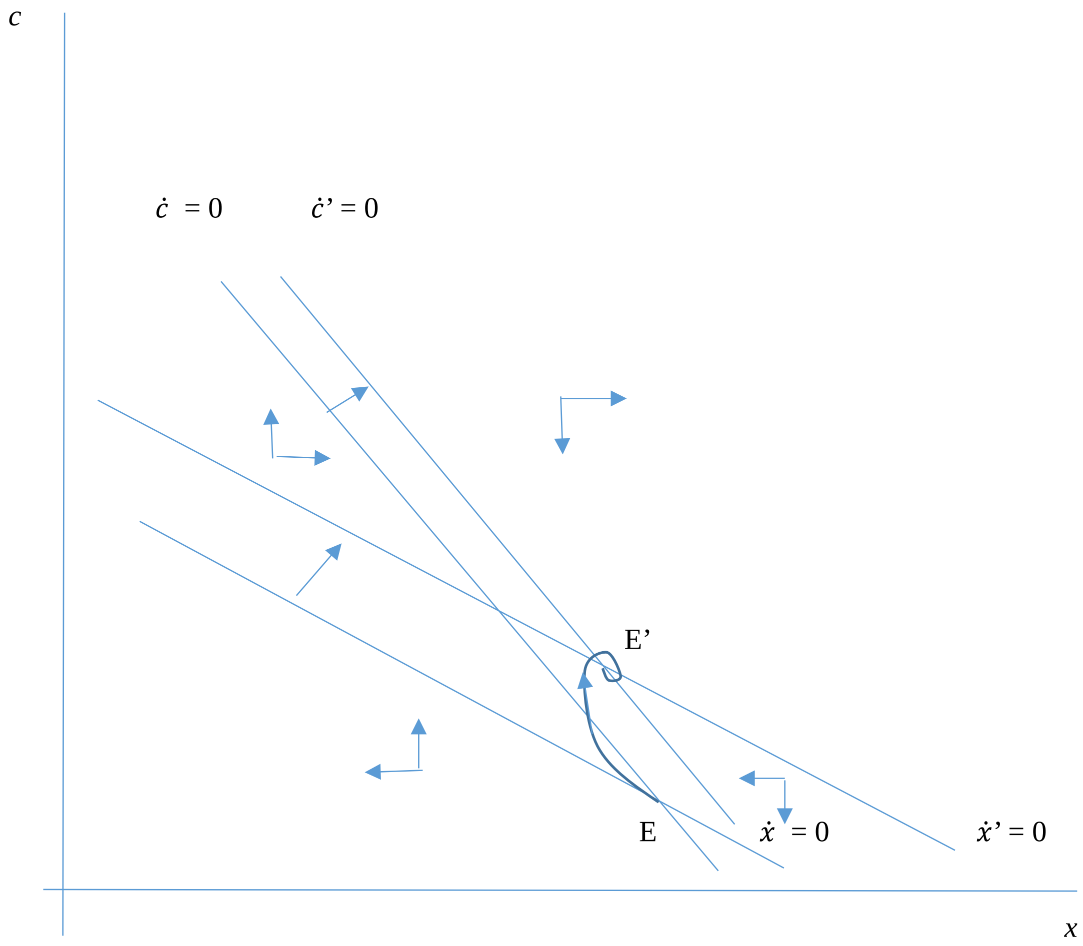

Figure 4 Dynamics of increase in saving rate and capital-augmenting technical progress under stability case

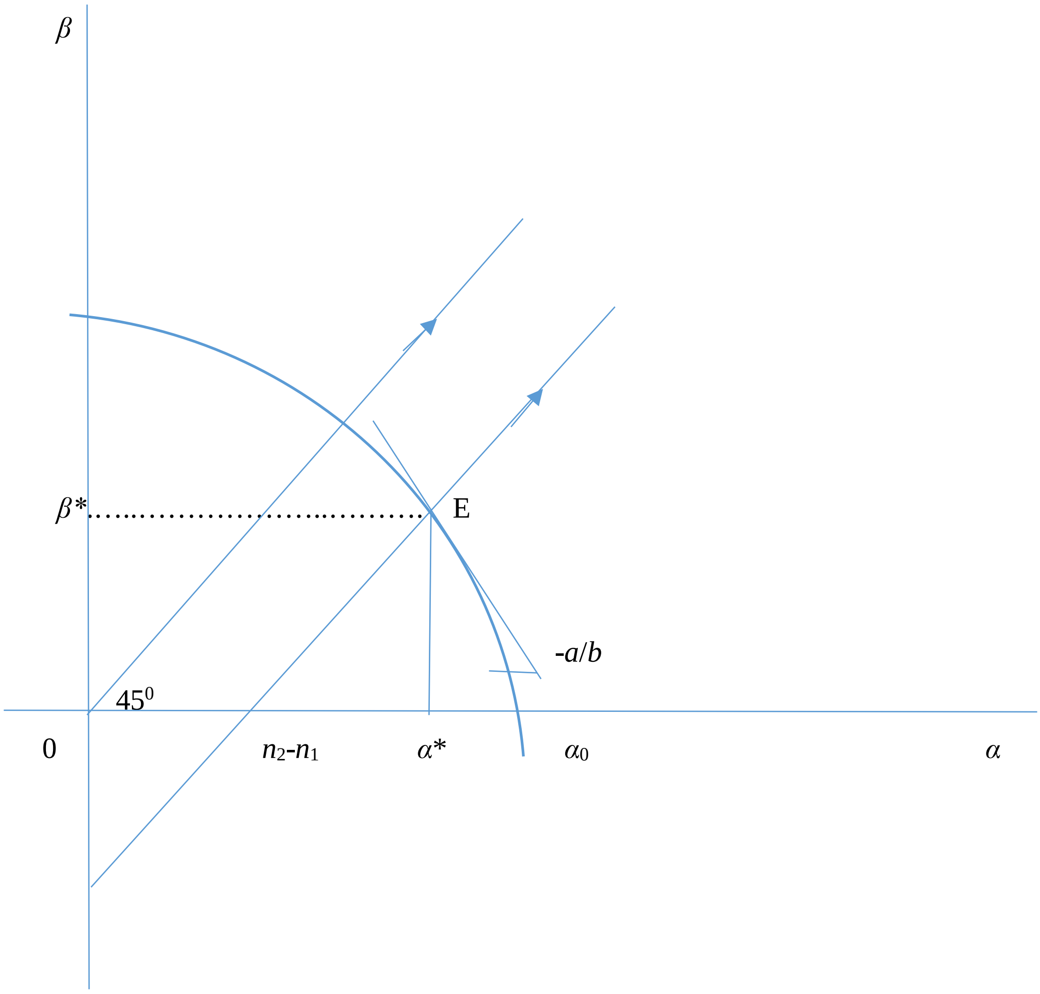

Figure 5 Endogenous skill-biased innovation $\alpha^* > \beta^*$ ($n_2 > n_1$)