Measuring Labor-Force Participation and the Incidence and Duration of Unemployment

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March 16, 2019
Revised: March 25, 2019

Abstract

The underlying data from which the U.S. unemployment rate, labor-force participation rate, and duration of unemployment are calculated contain numerous internal contradictions. This paper catalogs these inconsistencies and proposes a reconciliation. We find that the usual statistics understate the unemployment rate and the labor-force participation rate by about two percentage points on average and that the bias in the latter has increased since the Great Recession. The BLS estimate of the average duration of unemployment overstates by 50% the true duration of uninterrupted spells of unemployment and misrepresents what happened to average durations during the Great Recession and its recovery.

Keywords: unemployment rate, labor-force participation rate, unemployment duration, measurement error

[*] The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. We thank Alessandro Barbarino and Travis Berge for comments on an earlier draft of this paper and Jesse Wedewer for excellent research assistance. Data and software to reproduce results in this paper available at http://http://econweb.ucsd.edu/~jhamilton/AH2_code.zip.

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1 Introduction.

The Current Population Survey (CPS) is the primary source of information about the labor-force participation rate, unemployment rate, and duration of unemployment for the United States. There are multiple internal inconsistencies in the data from which the fundamental statistics are calculated– if one reported number is correct, another must be wrong. In this paper we catalog these inconsistencies and propose a reconciliation.

One source of inconsistency is rotation-group bias. In any given month, some households are being visited for the first time (rotation 1), others are being interviewed for the second time (rotation 2), with 8 different rotations contributing to the statistics reported for that month. One would think that in a random sample, the numbers calculated from different rotations for a given month should all be the same. But as documented by Bailar (1975), Solon (1986), Halpern-Manners and Warren (2012) and Krueger, Mas, and Niu (2017), the reported unemployment rate differs significantly across rotations. In our sample (July 2001 to April 2018), the average unemployment rate among those being interviewed for the first time is 6.8%, whereas the average unemployment rate for the eighth rotation is 5.9%. Even more dramatic is the rotation-group bias in the labor-force participation rate. This averages 66.0% for rotation 1 and 64.3% for rotation 8 in our sample.

We reconcile this by modeling statistically the way in which people's answers change the more times they have been interviewed. We interpret households in different rotations as being surveyed using a different interview technology. We calculate the answer to the following counterfactual question: if a group of households in rotation j in month t were being interviewed for the first time instead of the jth time, how would their answers have been different? We find that the tendency of individuals who would have been counted as U in rotation group 1 to be counted instead as N in later rotation groups has increased over our sample.

A second source of inconsistency is that missing observations are not random. As discussed by Abowd and Zellner (1986) and Meyer, Mok and Sullivan (2015), households in the CPS have become increasingly less likely to answer surveys or to provide all answers, and nonresponses are not random. The standard approach is to calculate statistics for a given month based only on individuals for whom there is an observation that month. But if missing observations are not randomly drawn from the overall population, this may be an increasing source of bias in CPS estimates.

Our solution is to add a fourth category of labor-force status. We regard an individual in any month as either employed (E), unemployed (U), not in the labor force (N), or missing (M). On this basis we construct a data set in which all identities relating stocks and flows are respected; for example, the sum of EE, NE, ME, and UE transitions between $t-1$ and t exactly equals the total number of E at t. Combining this with our description of rotation-group bias allows us to produce the first fully reconciled description of stocks and flows in the CPS data. Moreover, by looking at how ME, MN, and MU transitions differ from the rest of the population, we are able to adjust the treatment of missing observations based on what we know about those individuals when data are collected from them. We find that missing individuals are more likely than the general population to be unemployed. In addition, the biases introduced by missing observations have increased over time and are bigger when the labor market is slack. Our paper is the first to document the cyclical features in the bias coming from nonrandom missing observations.

Our adjustment for missing observations is similar in spirit to that in Abowd and Zellner (1986), though there are a number of important differences. For any month t they make one adjustment looking backward in time based in MX transitions and a second adjustment looking forward in time based on XM transitions, giving them potentially two different estimates for each month t. By contrast, we take a unified approach to the full data set. Abowd and Zellner's adjustments do not deal with the problem of rotation-group bias or the other measurement issues for which we also develop solutions. And they only calculate average unemployment rates over what is now a historically old sample. By contrast, we adjust estimates month-by-month up to the present.

A third problem in the CPS is inconsistency between the unemployment duration that an individual reports at t and the labor-force status recorded for that individual at $t-1$. For example, consider those individuals who were counted as N when in rotation 1 in month $t-1$ and U when surveyed in rotation 2 in month t. In the second survey, the individual would be asked how long he or she has been looking for work. Two-thirds of these individuals say that they have been looking for longer than 4 weeks, and 16% say it has been one or two years. Either these individuals are inaccurately reporting their duration of unemployment at t, or they have a personal concept of a spell of unemployment that is inconsistent with the recorded status of N in $t-1$.

A related anomaly is the inconsistency between unemployment hazard rates and the reported duration of unemployment. In 2011, when the mean duration of unemployment reached a record-high 40 weeks, the average monthly unemployment-exit probability measured by matching individuals observed for two consecutive months– also known as labor-force flows data– was about 0.3 and that of those unemployed longer than 6 months was about 0.25. Such outflow probabilities would be consistent with a mean duration of unemployment of about 4 months (1/0.25) on average in 2011, which is less than half of what is claimed in the official estimates. Our paper is the first to identify and resolve the inconsistency between unemployment hazards and the conventional measure of average unemployment duration.

Our resolution of these inconsistencies is to adopt a broader concept of U than that used by BLS. We propose to reclassify those who transition from N at $t-1$ to U at t with reported job search at t of longer than 4 weeks as having been U at $t-1$. In addition to helping reconcile the inconsistencies noted above, this is also supported by the observation that re-employment probabilities for those who make NU transitions with reported search durations exceeding 4 weeks are similar to those with UU continuations. We also find that someone with a UUU history is similar to that for someone with UNU whose reported search durations when U are consistent with those for the UUU individual.

This leads us to interpret some UN transitions as UU continuations. This adjustment goes a long way to reconciling the inconsistencies between reported unemployment durations and UU continuation probabilities. Our adjusted UU continuation probabilities also lead to an alternative estimate of average unemployment durations.

A final source of inconsistency arises from people's preference for reporting certain numbers over others. On average there are more people who say they have been looking for work for 6 months than say they have been looking for 23 weeks, though the fraction of those unemployed for 23 weeks should be greater than that of those unemployed for 6 months. In addition, people are more likely to report an even number of weeks than an odd number for shorter spells. Our resolution of this problem is to postulate a flexible latent distribution of perceived durations that is then reported by individuals with a certain structure of number-reporting preference; for related approaches see Baker (1992), Torelli and Trivellato (1993), and Ryu and Slottje (2000). Our approach is completely new compared to these studies in that our parameterization allows direct linkage of data on stocks, flows, and durations and in that both digit and interval preference are jointly considered. Our framework describes the reported values extremely accurately, and the adjustment on net revises down the mean duration of unemployment.

The importance of these concerns is illustrated in Panel A of Figure 1. This asks a very fundamental question: if someone is unemployed at $t-1$, what is the probability that person will still be unemployed at t? Researchers have used the CPS data to answer this question in two different ways. A measure based on reported unemployment durations calculates the ratio of individuals who are unemployed at t with a reported duration greater than 4 weeks to the total number of individuals unemployed at $t-1$. Variants of this calculation have been used by van den Berg and van der Klaauw (2001), Elsby, Michaels and Solon (2009) and Shimer (2012). This measure is plotted as the solid black line in Panel A. An alternative measure based on labor-force flows looks at the subset of individuals who are U at $t-1$ and either E, N, or U at t and calculates the number of UU continuations as a fraction of the sum. Variants of this approach were used by Fujita and Ramey (2009) and Elsby, Hobijn and Şahin (2010). The flow-based measure is plotted as the dashed green line. If all magnitudes were measured accurately the two estimates should give a similar answer. But in practice they are wildly different. The duration-based measure averages 70.7% over our sample, while the flow-based measure averages 53.7%.

These differences are caused by the multiple inconsistencies mentioned above. The flow-based measure underestimates the true continuation probability because (1) some UN transitions are a result of rotation-group bias and (2) some UN and UM transitions should be interpreted as UU continuations. On the other hand, the duration-based measure overestimates the probability. One reason for this, as shown by Kudlyak and Lange (2018), is that a substantial number of people interpret the duration of job search to include on-the-job search rather than a continuous spell of unemployment. Our reconciled estimate is shown in the dotted blue line in Panel A, and falls in between the other two estimates. The flow-based estimate was a little more accurate at the beginning of the sample, whereas the duration-based measure is closer to our adjusted series today.

Our adjusted estimate of the unemployment rate is compared with the BLS measure in Panel B. Our measure is 1.9% higher on average, and the gap increased during the Great Recession. The gap recovered gradually after the recession and has only recently returned to its pre-recession level. The gap between our measure and the BLS measure of the labor-force participation rate (Panel C) is 2.2% on average. It also increased in the Great Recession and remained elevated as of 2018.

We conclude that labor-force participation has declined slower over this period than suggested by the BLS estimate.

Our adjusted estimate of the unemployment-continuation probability from Panel A implies a much lower value for the average duration of unemployment than is reported by BLS, as seen in Panel D. Our corrected duration did not rise as much during the Great Recession as suggested by the BLS series, and subsequently recovered to pre-recession levels, whereas the BLS estimate has not.

Estimates of the employment-population ratio are also influenced by rotation-group bias, but are unaffected by missing observations and misclassification of the long-term unemployed. For some purposes this ratio might therefore serve as a more robust measure of economic slack than the unemployment rate.

A number of important studies have approached the problem of measurement error in the CPS data in a very different way from ours. A common assumption is that the reported data differ from latent true values, with identification coming from assumptions about the joint dynamics of the true values and measurement error. Prominent examples include Biemer and Bushery (2000), Feng and Hu (2013), and Shibata (2016). These studies did not deal with rotation-group bias, nonrandom missing observations, inconsistency between reported duration and the previous labor force status, or reporting errors of unemployment duration in their correction. Biemer and Bushery (2000) and Shibata (2016) assumed that true labor-force transitions were first-order Markov. Feng and Hu (2013) relaxed this assumption, though Shibata (2016) noted that their approach generates implausible transition probabilities. By contrast, our approach does not impose any Markov assumptions and produces plausible transition probabilities and unemployment durations that are consistent with these probabilities. Our approach also explains well the non-Markov predictability of labor-force status documented by Kudlyak and Lange (2018). Although our methods and assumptions are very different from these studies, we nevertheless reach the same broad conclusions that the BLS measures significantly underestimate the average unemployment rate and overestimate the average duration of unemployment.

The plan of the paper is as follows. Section 2 describes the structure of the CPS survey and our data set. Section 3 uses averages over the complete sample to document the various inconsistencies in the CPS data and develops the statistical framework that will form the basis for our reconciliation. Section 4 describes the steps we use to reconcile these inconsistencies in the full-sample averages. Section 5 describes how we use this approach to come up with better estimates for each individual month in the sample, explaining the calculations behind the adjusted series plotted in Figure 1. Section 6 briefly concludes.

2 Data construction.

Since 2001, each month around 60,000 housing units are included in the Current Population Survey. An effort is made to contact each address and determine the number of individuals aged 16 or over who are not in the armed forces or in an institution such as prison or a nursing home. An individual is counted as employed (E) if during the reference week of the survey month the individual did any work at all for pay, for their own business, or were temporarily absent from work due to factors like vacations, illness, or weather. People are counted as unemployed (U) if they were not E but were available for work and made specific efforts to find employment some time during the previous 4 weeks. Individuals who are neither E nor U are counted as not in the labor force (N). One person in the household can provide separate answers for each of the individuals living at that address.

The next month and each of the following two months, the interviewer attempts to contact the same address to ask the same questions. In any given month there are around 7,500 (60,000/8) qualifying households being interviewed for the first time (denoted rotation 1), and another 7,500 each being interviewed for the second, third or fourth time (rotations 2, 3, or 4). After the fourth month the household is not interviewed for the next 8 months, but is reinterviewed again 1 year after the first interview (rotation 5) and again for each of the following 3 months (rotations 6, 7, and 8). For data since 1994, if an individual was unemployed in two consecutive months, the interviewer does not ask again the duration of unemployment the second month, but simply adds time elapsed since the previous interview to the previous answer. Thus new unemployment duration data are only collected in rotations 1 and 5, or in the other rotations for someone who was E, N or missing from the sample the month before.

The survey is imperfect for purposes of tracking the experience of an individual across months due to various measurement problems; for discussion of these see Madrian and Lefgren (2000) and Nekarda (2009). Each address has a unique identifier, and an effort is made to associate an individual person within that household with a particular 2-digit number. Our study is unique in treating missing (M) as a separate observed category for someone whose information is not available in a particular rotation or is inconsistent from the information reported for that individual in other rotations. As in Abowd and Zellner (1986), we will use information about that individual in months where it is available to correct for the fact that individuals who are sometimes missing (which could come in part from households that are more prone to reporting errors or to having people moving in or out) may differ in systematic ways from individuals for whom 8 separate months of data are available. We check if an individual with the same household and personal identifier is reported to have the same gender and an age that does not differ by more than 2 years across rotations. If so, we consider that individual successfully matched. If not, we designate that individual as M in the months for which no status is available or for which the age and gender records are inconsistent with those reported across the majority of the 8 rotations.[1]

The raw data for our study thus consist of $y_{X,t}^{[j]}$, the sum of the number of individuals (multiplied by a weight associated with that individual) who are in rotation $j \in \{1, ..., 8\}$ in month t with reported status $X \in \{E, N, M, U\}$, and $y_{X_1,X_2,t}^{[j]}$, the weighted sum of individuals reporting X_1 in rotation $j-1$ in month $t-1$ and X_2 in rotation j in month t for $j \in J = \{2,3,4\} \cup \{6,7,8\}$. See Table A-1 in the online appendix for a summary of notation used in this study. A key advantage of our approach is that, unlike the values used by most researchers, our data on stocks and flows are internally consistent by construction, always satisfying the accounting identities

$$y_{X_2,t}^{[j]} = y_{E,X_2,t}^{[j]} + y_{N,X_2,t}^{[j]} + y_{M,X_2,t}^{[j]} + y_{U,X_2,t}^{[j]} \tag{1}$$

$$y_{X_1,t-1}^{[j-1]} = y_{X_1,E,t}^{[j]} + y_{X_1,N,t}^{[j]} + y_{X_1,M,t}^{[j]} + y_{X_1,U,t}^{[j]} \tag{2}$$

for all t, X_1, X_2 and $j \in J$.

One drawback of this procedure is that we need 16 months of observations to determine whether to categorize someone as M in a given month. For example, our sample starts in 2001:7. Someone

[1] Nekarda (2009) used race in addition to age and gender and Madrian and Lefgren (2000) also used education. Both these variables are susceptible to ambiguity and could be reported differently for a fixed individual, particularly when a different individual answers the questions for the household. We topcode age at 65 years or older, so an individual in this age group with the same address, same gender, and same identifying number is considered matched.

whose history beginning in 2001:7 was $EEMM - MMMM$ will be counted as M in rotation 3 in 2001:9 by our method, whereas someone who would have had the same history if initially surveyed in 2001:5 would never appear in the sample[2]. This causes the number of individuals who are classified as M to be artificially depressed in the first year of the sample. A similar effect arises at the end of the sample, with individuals whose record would have been $MMEE-EEEE$ not being apparent if their rotations 1 or 2 come would have come at the end of the sample. We therefore adjusted the counts of M and MM at the beginning and end of the sample upward based on the average counts of M for each rotation over the nearest year of complete observations; for details see Appendix A. Since changes in M occur relatively slowly in our sample, this adjustment has little effect on any of the key measures we develop. We made additional adjustments when new households were added and other households dropped in the 2004 and 2014 sample redesigns.[3]

BLS also assigns a weight to each individual. People with characteristics that are underrepresented in a particular month are given a larger weight. These weights are a partial response of BLS to the issue that missing individuals are not a random sample of the population. We want to include this correction to demonstrate the need for additional corrections for missing individuals. We can not use the exact BLS weights to do this because the BLS may assign a given individual different weights in two different months, which is another reason in addition to missing observations why (1) and (2) do not hold in the BLS data. Our approach was to assign a fixed weight for an individual across all 8 possible observations based on the BLS weight for that individual in the first month for which data are recorded for that person, as described in Appendix A.

3 Statistical description of labor-force status data.

In this section we develop statistical descriptions of a number of features of the CPS data.

[2] See Appendix A for detailed examples.

[3] With the expansion of the survey from 50,000 to 60,000 households, beginning in July 2001, some individuals were added and others dropped across a number of rotations, with waves of new individuals added to subsequent rotations 5. Tracking individuals before and after this break is considerably harder than handling the sample redesign in 2004 and 2014. For this reason we simply begin our analysis with the modern design adopted in July 2001.

3.1 Unemployment durations reported in rotations 1 and 5.

First we consider the durations of unemployment that are reported on average over our sample by people who are being interviewed for the first time (rotation 1). The blue bars in the top panel of Figure 2 plot the fraction of unemployed reporting the indicated duration of job search in weeks. Clearly there are some significant reporting errors arising from number preference. Respondents are more likely to report spells as an integer number of months, and for longer spells as either 6 months, 1 year, 18 months, or longer than 99 weeks. For shorter spells, people are more likely to report an even number of weeks instead of an odd number; for example, on average there are more people reporting 2 weeks than 1 and 6 weeks than 5. Respondents are extremely unlikely to report a duration of zero weeks, and for this reason we group the 0-week and 1-week observations together into a category of reported duration less than or equal to one week.

To interpret these numbers in an internally consistent way, we impose the restriction that the only way an individual could have been unemployed for τ weeks would be if the individual had been unemployed for $\tau - 1$ weeks the week before. Thus if $\pi_U^\dagger(\tau)$ denotes an internally consistent summary of the fraction of the population who have been searching for τ weeks, the function $\pi_U^\dagger(\tau)$ must be monotonically decreasing in τ. For our baseline specification we propose to represent this function as a mixture of two exponentials with decay rates p_1 and p_2, respectively. We form a (99×1) vector $\pi_U^\dagger$ whose τth element for $\tau = 1, 2, ..., 98$ is an internally consistent representation of the fraction of the working-age population who perceive having been unemployed for a duration of τ weeks at a fixed point in time, while the 99th element is the fraction with perceived duration greater than 98 weeks:

$$\pi_U^\dagger = \pi_{1U}^\dagger + \pi_{2U}^\dagger \tag{3}$$

$$\underset{(99\times1)}{\pi_{iU}^\dagger} = \pi_U w_i (1-p_i) \begin{bmatrix} 1 & p_i & p_i^2 & \cdots & p_i^{97} & p_i^{98}/(1-p_i) \end{bmatrix}' \quad \text{for } i = 1, 2. \tag{4}$$

Here π_U denotes the fraction of the population who are unemployed and w_i the fraction of those individuals who are type i. Such a distribution would be the outcome of a steady state in which there was a fraction $\pi_U w_1 (1 - p_1)$ of the population who lose their jobs each week and for each of whom the probability of continuing unemployed in any subsequent week is p_1, and an additional inflow of $\pi_U w_2(1 - p_2)$ individuals with continuation probability p_2.[4]

We allow for the various forms of number preference noted above by introducing a (99×99) matrix $A(\theta_A)$ whose elements are determined by a (13×1) vector θ_A. The first element $\theta_{A,1}$ allows a preference for reporting short durations as an even rather than an odd number of weeks, assuming that someone whose true duration is $\tau = 1, 3, 5,$ or 7 in fact reports duration 2, 4, 6, or 8 with probability $\theta_{A,1}$. The value of $\theta_{A,2}$ represents the probability that someone will round their duration up or down by a week to reach an integer number of months for durations within one week of 1, 2, 3 or 4 months, while someone two weeks away from either of two months is presumed to round down with probability $\theta_{A,3}/2$ and up with probability $\theta_{A,3}/2$. As we move to longer durations we allow for the possibility that the rounding tendencies become stronger, introducing new pairs of parameters for durations between 5-7 months, 8-11 months, or 12 or more months. The last elements of θ_A allow for preferences for integer multiples of 6 months for longer durations. For each τ the τth column of A sums to unity and characterizes the probability that someone whose true duration category is τ will report each of the possible categories i between 1 and 99, where i or $\tau = 99$ is interpreted as true or reported durations longer than 98 weeks. Appendix B provides more details on the structure we use to represent the matrix A. Note that our framework does not impose the assumption of the existence or magnitude of any particular reporting error, as it includes as a special case no reporting error of any kind when $\theta_A = 0$.

Let $y_{X,t}^{[1]}$ be the number of individuals in rotation group 1 sampled at date t who report status X for X one of E (employed), N (not in labor force), M (labor-force status for that individual is missing), or U (unemployed). We summarize further detail in the last category in terms of $y_{U,t}^{[1]}(\tau)$ which is the number of unemployed who report having been looking for work for τ weeks for $\tau = 1, ..., 99$.[5] We compare the observed values $y_{U,t}^{[1]}(\tau)$ with the predicted values represented by the (99×1) vector

$$\dot{\pi}_U = A\pi_U^{\dagger}. \tag{5}$$

We also let π_X denote the overall fraction of the population reporting status $X \in \{E, N, M, U\}$.

[4] We will later examine some testable implications of such an interpretation by looking at the actual unemployment-continuation probabilities for different individuals and also look at alternative functional forms. But for now we propose (3) and (4) as a simple but flexible parametric functional form with which to impose monotonicity on $\pi_U^{\dagger}(\tau)$.

[5] The duration is top-coded at 99 weeks in our data.

If we treated observations as independent across months t the log likelihood of the rotation 1 observations alone would then be

$$\begin{aligned} \ell_X^{[1]}(\lambda_X) &= \textstyle\sum_{t=1}^T [y_{E,t}^{[1]} \ln \pi_E + y_{N,t}^{[1]} \ln \pi_N + y_{M,t}^{[1]} \ln \pi_M] \\ &\quad + \textstyle\sum_{t=1}^T \sum_{\tau=1}^{99} y_{U,t}^{[1]}(\tau) \ln \dot{\pi}_U(\tau). \end{aligned} \tag{6}$$

We can maximize this with respect to $\theta_A, p_1, p_2, w_1, w_2, \pi_E, \pi_N, \pi_M, \pi_U$ subject to the constraint that all probabilities are between 0 and 1 and sum to unity.[6]

Estimates are reported in in column 1 of Table 1, along with quasi-maximum-likelihood standard errors in column 2 which allow for the possibility that $y_{X,t}^{[1]}$ is correlated across time (calculated as described in Appendix C). The predicted reported values $\dot{\pi}_U(\tau)$ are compared with the average reported values in the top panel of Figure 2.[7] This framework is able to describe the reported values extremely accurately. The estimated latent function $\pi_U^{\dagger}(\tau)$ along with its two contributing components are plotted as a function of τ in the bottom panel of Figure 2. We also considered an alternative functional form based on a Weibull distribution, as discussed in Appendix D. The mixture of exponentials has a much better fit to the data than that for the Weibull specification, and we will use it in our baseline analysis.

For rotations 2-4 and 6-8, BLS imputes a duration to those reporting UU continuations, making durations for these individuals a hybrid of perceived and imputed quantities. This can create a downward bias in the number of individuals unemployed for less than 5 weeks as discussed by Abraham and Shimer (2001) and Shimer (2012) and blurs the inconsistency between perceived and imputed durations. Since our goal is to characterize perceived durations separately from objective durations, we do not use the imputed duration in the second month in unemployment. However, there are no imputations for unemployment durations for those people in rotation 5. We therefore repeated the analysis with $y_{X,t}^{[1]}$ in (6) replaced by $y_{X,t}^{[5]}$. Parameter estimates and standard errors are reported in columns 3 and 4 of Table 1. These are very similar to those inferred from the

[6] Maximum likelihood estimates of some parameters are known analytically. Let $y_X = \sum_{t=1}^T y_{X,t}$ denote the total number of observations in category X and $n = (y_E + y_N + y_M + y_U)$ the total number of observations. Then $\hat{\pi}_X = y_X/n$ for $X \in \{E, N, M, U\}$. These values can be substituted into expression (6) and the resulting concentrated likelihood then maximized with respect to θ_A, p_1, p_2, w_1 with $w_2 = 1 - w_1$.

[7] As noted in the previous footnote, by the nature of the maximization problem, the estimated values $\hat{\pi}_X$ for $X = E, N, M$ exactly match the historical fractions $y_X/(y_E + y_N + y_M + y_U)$.

rotation 1 observations alone.

3.2 Characteristics of NU, EU, and MU transitions.

Next consider the status of individuals in rotation 2 who had been counted as not in the labor force when surveyed in rotation 1. Figure 3 focuses on the subset who in the second month (when they were in rotation 2) reported being unemployed, giving the percentage reporting each duration of job search. Two-thirds of these people say they have been actively looking for a job for longer than 4 weeks, despite the fact that the previous month they did not report actively looking for a job and so were counted as out of the labor force. Eight percent of NU individuals say that they have been looking for a job for a full year and another 8% report having been looking for work for two years or longer. Although the question is intended to measure the spell of continuous active job search, just how actively an individual was looking for work the previous month is potentially subjective. But it is clear that many of those counted as N the previous month perceived themselves to have been looking for a job at the time despite the N classification.

Of those people who report right after an NU transition that they have been looking for work for more than 4 weeks, what distribution characterizes their perceived duration of job search? We represent the probability of transitions from N to E, N, M, or U with parameters $\pi_{NE}, \pi_{NN}, \pi_{NM}, \pi_{NU}$, respectively, where these four numbers sum to unity. Of those who make an NU transition and report an unemployment duration greater than 4 weeks, suppose that their perceived duration can again be represented by a mixture of two exponentials with decay parameters $p_{1,NU}$ or $p_{2,NU}$. We assume that some fractions $q_{1,NU}, q_{2,NU}, q_{3,NU}$, and $q_{4,NU}$ of those making the NU transition will perceive their unemployment duration to be 1,2,3, or 4 weeks respectively, treating these values of $q_{j,NU}$ completely unrestrained. A fraction $q_{5,NU}$ perceive a duration greater than 4 weeks drawn from an exponential distribution with parameter $p_{1,NU}$ and a fraction $q_{6,NU}$ are characterized by $p_{2,NU}$, with $\sum_{j=1}^{6} q_{j,NU} = 1$. We thus calculate

$$\pi^{\dagger}_{NU}(\tau) = \begin{cases} q_{\tau,NU} & \text{for } \tau = 1,2,3,4 \\ q_{5,NU}(1-p_{1,NU})p_{1,NU}^{\tau-5} + q_{6,NU}(1-p_{2,NU})p_{2,NU}^{\tau-5} & \text{for } \tau = 5,6,...,98 \\ q_{5,NU}p_{1,NU}^{94} + q_{6,NU}p_{2,NU}^{94} & \text{for } \tau = 99 \end{cases} . \qquad (7)$$

The predicted probability of each reported duration is then given by $\dot{\pi}_{NU} = \pi_{NU} A \pi^{\dagger}_{NU}$.

Let $y^{[2]}_{NX,t}$ denote the number of individuals who counted as not in the labor force in rotation 1 in month $t-1$ and reported status X at date t where $X \in \{E, N, M, U\}$. Let $y^{[2]}_{NU,t}(\tau)$ denote the number of NU who report unemployment duration $\tau \in \{1, ..., 98, \geq 99\}$ in rotation 2. Then the contribution to the likelihood for months $t = 1, ..., T$ from rotation 2 NX transitions is

$$\begin{aligned} \ell^{[2]}_{NX}(\lambda_{NX}) &= \textstyle\sum_{t=1}^{T}[y^{[2]}_{NE,t} \ln \pi_{NE} + y^{[2]}_{NN,t} \ln \pi_{NN} + y^{[2]}_{NM,t} \ln \pi_{NM}] \\ &\quad + \textstyle\sum_{t=1}^{T} \sum_{\tau=1}^{99} y^{[2]}_{NU,t}(\tau) \ln \dot{\pi}_{NU}(\tau). \end{aligned} \tag{8}$$

This expression can then be maximized with respect to $\lambda_{NX} = (\theta'_{A,NU}, p_{1,NU}, p_{2,NU}, \pi_{NE}, \pi_{NN}, \pi_{NM}, \pi_{NU}, q_{1,NU}, q_{2,NU}, ..., q_{6,NU})'$ subject to the constraints that all parameters fall between 0 and 1, $\pi_{NE} + \pi_{NN} + \pi_{NM} + \pi_{NU} = 1$ and $\sum_{j=1}^{6} q_{j,NU} = 1$.

Quasi-maximum-likelihood estimates $\hat{\lambda}_{NX}$ are reported in column 5 of Table 1 and predicted values $\dot{\pi}_{NU}$ compared with historical average values for y_{NU} in Figure 3. Note that θ_A was estimated in column 1 solely from individuals who were recorded as being unemployed in rotation 1, in column 3 solely from individuals who were unemployed in rotation 5, and in column 5 solely from individuals who were recorded as being out of the labor force in rotation 1 and unemployed in rotation 2. Although the vector θ_A was estimated from very different data, the estimated values are quite similar. Likewise $\hat{p}_{1,NU}$ and $\hat{p}_{2,NU}$ turn out to be very close to the values $\hat{p}_1$ and $\hat{p}_2$ estimated from rotations 1 and 5.

Next consider the status in month t of individuals who were recorded as employed when sampled in rotation 1 in month $t-1$. Twenty-nine percent of those who make EU transitions report durations longer than 4 weeks. Unlike the NU transitions, we do not interpret these as necessarily implying an inaccuracy in either the E or U designation. Kudlyak and Lange (2018) noted these could represent records of individuals who were employed in $t-1$ but were engaged in on–the-job search for a new job.[8] It is nevertheless interesting to characterize transitions from employment using the same framework as above, replacing NX in (8) with EX. Parameter estimates and standard errors are reported in columns 7 and 8 of Table 1. Much fewer EU transitions perceive themselves

[8] Ahn and Shao (2017) further documented that on-the-job search constitutes a non-negligible fraction of aggregate job search. Hall and Kudlyak (2019) found that many job losers make frequent transitions between short-term employment, unemployment, and out of the labor force before finding a long-term job.

as long-time job seekers ($q_{6,EU} = 0.17$ versus $q_{6,NU} = 0.51$). Interestingly, the estimates of p_1, p_2 and θ_A are similar to those inferred in columns 1, 3, and 5; unemployed individuals in each of these categories can be broadly characterized in terms of the same two types.

Finally, we consider the status in rotation 2 of individuals who were missing in rotation 1, replacing EX with MX. Quasi-maximum-likelihood estimates are reported in column 9 of Table 1. Notably, again we find very similar estimates for p_1, p_2 and θ_A as in the other data sets, with $q_{6,MU}$ much closer to $q_{6,NU}$ than to $q_{6,EU}$.

3.3 Characteristics of UX transitions.

We next examine UX transitions. We have modeled the fraction of the population that reports being unemployed with duration τ as given by the τth element of the vector $\xi_1+\xi_2$ where $\xi_i = A\pi^{\dagger}_{iU}$ for $\pi^{\dagger}_{iU}$ given in (4). If we observe someone reports a duration of τ, the specification allows us to calculate the probability that the individual is type i using the formula

$$\eta_i(\tau) = \xi_i(\tau)/[\xi_1(\tau) + \xi_2(\tau)] \tag{9}$$

for $i = 1$ or 2. The function $\eta_2(\tau)$ is plotted in Figure 4.[9] Someone who reports a duration of $\tau = 1$ week is quite unlikely to have come from the second distribution, whereas someone who reports a duration greater than 40 weeks is almost certain to have come from the second distribution. The function dips down at duration $\tau = 26$ weeks because, given the tendency of answers to clump at this value, this observation includes many individuals whose true duration is less than 26 weeks and accordingly contains a higher mix of type 1 relative to those reporting 25 weeks.

Though the function $\eta_i(\tau)$ can be motivated on the basis of a particular parametric model of the distribution of reported unemployment durations, the measure can alternatively be viewed as a simple device to allow for the possibility that different outcomes are expected for individuals who report longer spells of unemployment relative to those who report shorter spells.[10]

[9] For purposes of this graph, this function was calculated using the values of w_1, p_1, p_2, θ_A from Table 3, which pool all observations from all rotations to estimate these parameters.

[10] One could alternatively try to get at this idea by setting $\eta_2(\tau) = 0$ for $\tau \leq K$ and unity for $\tau > K$. That kind of simple dichotomization into short-term and long-term unemployment would have the drawbacks that it requires picking an arbitrary cut-off K and implies an abrupt discontinuity in outcomes expected for individuals slightly below K relative to those slightly above K. By contrast, the approach we follow here uses a smooth function $\eta_2(\tau)$ relating reported duration τ to expected outcomes.

Let the scalar $\gamma_{i,UX}$ be the probability that an individual of type i makes a transition from unemployment in rotation group 1 to status $X = E, N, M$, or U in rotation 2, so $\gamma_{i,UE} + \gamma_{i,UN} + \gamma_{i,UM} + \gamma_{i,UU} = 1$ for both $i = 1$ and $i = 2$. Let $\dot{\pi}_{UX}$ denote a (99×1) vector whose τth element is the probability that someone who reports duration τ in month t has status X in month $t+1$. Under the above assumptions this would be predicted to be

$$\dot{\pi}_{UX} = \eta_1 \gamma_{1,UX} + \eta_2 \gamma_{2,UX}. \tag{10}$$

For $y^{[2]}_{UX,t}(\tau)$ the observed number of individuals who report U with duration τ in rotation 1 and status X in rotation 2, we then have the likelihood function

$$\begin{aligned} \ell^{[2]}_{UX}(\lambda_{UX}) = \textstyle\sum_{t=1}^{T} \sum_{\tau=1}^{99} \quad & [y^{[2]}_{UE,t}(\tau) \ln \dot{\pi}_{UE}(\tau) + y^{[2]}_{UN,t}(\tau) \ln \dot{\pi}_{UN}(\tau) \\ & + y^{[2]}_{UM,t}(\tau) \ln \dot{\pi}_{UM}(\tau) + y^{[2]}_{UU,t}(\tau) \ln \dot{\pi}_{UU}(\tau)]. \end{aligned} \tag{11}$$

We fixed η_2 to be the function plotted in Figure 4 and maximized (11) with respect to $\{\gamma_{i,UE}, \gamma_{i,UN}, \gamma_{i,UM}, \gamma_{i,UU}\}_{i=1,2}$ subject to the constraint that $\gamma_{i,UE} + \gamma_{i,UN} + \gamma_{i,UM} + \gamma_{i,UU} = 1$ for $i = 1, 2$.

Quasi-maximum-likelihood estimates and standard errors are reported in rows 1 and 2 of Table 2. Type 1 individuals have a 32% probability of being employed next month, whereas the probability for type 2 individuals is only 12%. Type 1 individuals have a 37% probability of being unemployed next month, whereas for type 2 the probability is 58%. We also repeated the analysis using only data for individuals who were unemployed in rotation 5, with very similar results.

Consider what we would have expected these coefficients to have been if perceived unemployment durations matched with actual unemployment-continuation probabilities. If type 1 individuals truly had a weekly unemployment-continuation probability of $p_1 = 0.8094$, we would expect to observe a monthly continuation probability of $0.8094^{4.33} = 0.40$. If we condition on missing observations having the same distribution as observed E, N and U, this value would be broadly consistent with the value we'd predict from Table 2 of $\gamma_{1,UU}/(1 - \gamma_{1,UM}) = 0.41$. By contrast, the perceived long-term unemployed are another story. Their perceived weekly unemployment-continuation probability of $p_2 = 0.9734$ would imply a monthly continuation probability of $0.9734^{4.33} = 0.89$, far larger than the estimate $\gamma_{2,UU}/(1-\gamma_{2,UM}) = 0.63$. Even more dramatically, a monthly continuation probability of 0.63 would mean a probability of remaining unemployed for 6 months of $0.63^6 = 0.06$. But in the BLS data, the fraction of those unemployed who report durations over 26 weeks averages 27%. Far fewer people than are reported in the data should be unemployed longer than 6 months if people left the pool of long-term unemployed at anything like the rate implied by $\gamma_{2,UU}$. The observed unemployment continuation probabilities are not consistent with the distribution of reported unemployment durations.

The result is robust whether one uses our parametric model or any other. For example, Appendix D derives the analogous result using a Weibull characterization of durations. Any model that accurately describes the cross-section of durations– and ours does so quite well– is going to predict an unemployment-continuation probability similar to the stock-based measure plotted as the solid line in Figure 1, which we noted is inconsistent with the flow-based measure. The main advantage of our parametric approach is that it highlights that this inconsistency between the stock-based and flow-based measures comes entirely from those whom we have characterized as the perceived long-term unemployed.

For a broad summary of the features of the data, we pool together all observations for all rotations but allowing $\gamma_{i,UU}$ to be completely independent of the value of p_i, while treating the values of θ_A, p_1, and p_2 as the same across all rotation groups. This summary of the full data set was obtained by maximizing the full-sample likelihood

$$\ell = \ell_X^{[1]} + \ell_X^{[5]} + \sum\nolimits_{j\in J}\left(\ell_{EX}^{[j]} + \ell_{NX}^{[j]} + \ell_{MX}^{[j]}\right) + \ell_{UX}^{[2]} + \ell_{UX}^{[6]}. \tag{12}$$

These full-sample estimates are reported in Table 3.

3.4 Rotation-group bias.

Another source of error in the CPS data is the difference across different rotations in the reported labor-force status. Table 4 reports the monthly average number of sampled individuals with measured labor force status E, N, M, or U for each of the 8 rotation groups.[11] Column 6 shows that the average unemployment rate declines sharply as a function of rotation group, starting

[11] For example, the entry in the first row and column is $T^{-1}\sum_{t=1}^{T} y_{E,t}^{[1]}$.

out at 6.8% for rotation 1 but falling all the way to 5.9% for rotation 8. Column 7 reveals another interesting fact that appears not to have been noticed by other researchers: the measured labor-force participation rate falls even more sharply. Column 3 documents a third tendency– individuals are much more likely to be missed in rotation 1 and 5 compared to other groups.

We can summarize these tendencies with some simple regressions. Let $x_t^{[j]} = 100y_{X,t}^{[j]}/\left(y_{E,t}^{[j]} + y_{N,t}^{[j]} + y_{M,t}^{[j]} + y_{U,t}^{[j]}\right)$ denote the percentage of individuals in rotation group j sampled in month t with measured status $X = E, N, M$, or U; thus $e_t^{[j]} + n_t^{[j]} + m_t^{[j]} + u_t^{[j]}$ exactly equals 100 for every j and every t. Consider an 8-variable panel regression with time fixed effects where the dependent variable is $n_t^{[j]}$, $j = 1, ..., 8, t = 1, ..., T$:

$$n_t^{[j]} = \alpha_{nt} + \delta_n j + \alpha_{n1} d_{1t} + \alpha_{n5} d_{5t} + \varepsilon_{nt}^{[j]}. \tag{13}$$

Here α_{nt} is the time fixed effect for month t, δ_n captures a linear trend across rotations (with increased fraction of N in later rotations captured by $\delta_n > 0$), $d_{1t} = 1$ if $j = 1$ and 0 otherwise allows for something special about the first rotation group, while $d_{5t} = 1$ if $j = 5$ serves a similar function for rotation 5. The estimates of these parameters along with standard errors are reported in column 2 of Table 5, and their implications are plotted as the thin red curve in Figure 5. These coefficients summarize the large number of N in rotation groups 1 and 5 and a tendency for the percentage of individuals classified as N to increase sharply across rotation groups.

Coefficients for panel regressions in which $e_t^{[1]}$,...,$e_t^{[8]}$ are the 8 dependent variables are reported in column 1 of Table 5 and plotted as the thick black curve in Figure 5. Coefficients when unemployment is the dependent variable are in column 3 and plotted as the dashed blue line. The rising trend across rotations in N ($\delta_N = 0.0011$) is accounted for by falling trends in E and U ($\delta_E + \delta_U = -0.0011$). The bulges in M in rotation 1 ($\alpha_{M1} = 0.0159$) and rotation 5 ($\alpha_{M5} = 0.0149$) are accounted for by drops in E and N in those rotations.[12]

4 Reconciling the inconsistencies.

In this section, we propose methods to reconcile the inconsistencies identified in Section 3.

[12] These findings are consistent with Krueger, Mas, and Niu's (2017) finding that rotation-group bias is associated with nonresponses and with Bailar's (1975) conclusion that the rotation-group bias of the unemployment rate can be explained by the participation margin.

4.1 Rotation-group bias.

We have seen that a given household can give different answers depending on the number of times the household has previously been interviewed. We interpret this as differences in interview technology: the process by which data are obtained differs across rotations, and the numbers should be interpreted as meaning different things. As a first step we summarize these differences in the form of a counterfactual question: if an individual in rotation j had instead been interviewed using the technology i, how would their answers have differed? We initially show how to answer this question for $i = 1$ and then find the answer for any i. We then ask, which interview technology i should be used to summarize the data? We identify several reasons why we prefer to use the answers that people give the first time they are interviewed ($i = 1$).

Modeling the differences in interview technology. For each rotation $j = 1, 2, ..., 8$, let $\pi^{[j]} = (\pi_E^{[j]}, \pi_N^{[j]}, \pi_M^{[j]}, \pi_U^{[j]})'$ denote the fraction over the full sample of individuals who reported status $X^{[j]}$ when interviewed in rotation j. For each $j \in J = \{2, 3, 4\} \cup \{6, 7, 8\}$, of the individuals who reported status X_1 in rotation $j-1$, some fraction $\pi^j_{X_1,X_2}$ are observed to report status X_2 in rotation j for $X_i \in \{E, N, U, M\}$; thus $\pi_{XE}^{[j]} + \pi_{XN}^{[j]} + \pi_{XU}^{[j]} + \pi_{XM}^{[j]} = 1$ for all X and $j \in J$. Collect these observed probabilities in a matrix

$$\Pi^{[j]} = \begin{bmatrix} \pi_{EE}^{[j]} & \pi_{NE}^{[j]} & \pi_{ME}^{[j]} & \pi_{UE}^{[j]} \\ \pi_{EN}^{[j]} & \pi_{NN}^{[j]} & \pi_{MN}^{[j]} & \pi_{UN}^{[j]} \\ \pi_{EM}^{[j]} & \pi_{NM}^{[j]} & \pi_{MM}^{[j]} & \pi_{UM}^{[j]} \\ \pi_{EU}^{[j]} & \pi_{NU}^{[j]} & \pi_{MU}^{[j]} & \pi_{UU}^{[j]} \end{bmatrix} \qquad j \in J.$$

Notice that each column of $\Pi^{[j]}$ sums to unity. For example, for the first column, if someone reported status E when interviewed in rotation $j-1$, they must have had one of the statuses E, N, M, or U in rotation j.

For an individual who reported status $X^{[j]}$ in rotation j, consider the counterfactual answer that individual would have given if interviewed using the interview technology that was used for rotation 1:

$$r^{[j]}_{X^{[j]},X^{[1]}} = \text{ Prob(would have answered } X^{[1]} \text{ using technology 1 given answered } X^{[j]} \text{ using technology } j).$$

Collect these counterfactual probabilities in a matrix

$$R^{[j]} = \begin{bmatrix} r_{EE}^{[j]} & r_{NE}^{[j]} & r_{ME}^{[j]} & r_{UE}^{[j]} \\ r_{EN}^{[j]} & r_{NN}^{[j]} & r_{MN}^{[j]} & r_{UN}^{[j]} \\ r_{EM}^{[j]} & r_{NM}^{[j]} & r_{MM}^{[j]} & r_{UM}^{[j]} \\ r_{EU}^{[j]} & r_{NU}^{[j]} & r_{MU}^{[j]} & r_{UU}^{[j]} \end{bmatrix} \qquad j \in J.$$

Notice that each column of $R^{[j]}$ sums to unity. For example, for the first column, given that an individual reported status E when interviewed in rotation j, they would have to have given one of the answers E, N, M, U if interviewed using the technology of rotation 1. From the analysis above, we expect $r_{NU}^{[j]} > 0$; some of the individuals who report labor status N in rotation j would have reported status U if they had been interviewed for the first time. We also expect $r_{EM}^{[j]} > 0$ and $r_{NM}^{[j]} > 0$; some of the individuals who were reported as status E or N in rotation j would have been missing using the interview technology of rotation 1.

Notice that

$$R^{[j]}\pi^{[j]} = \pi^{[1]} \qquad \text{for } j \in J. \tag{14}$$

For example, the first row states

$$r_{EE}^{[j]}\pi_E^{[j]} + r_{NE}^{[j]}\pi_N^{[j]} + r_{ME}^{[j]}\pi_M^{[j]} + r_{UE}^{[j]}\pi_U^{[j]} = \pi_E^{[1]}.$$

This equation states that the fraction who reported E in rotation 1 can be viewed as the fraction who reported $X^{[j]}$ in rotation j times the probability someone reporting $X^{[j]}$ would have reported E using technology 1, added across the four possible $X^{[j]}$.

In Section 3.4 we found that the decline in U across rotations is accounted for by a corresponding trend up in N and that differences in M in rotations 1 and 5 correspond to matching drops in E and N. We propose to capture the key differences in interview technology using three parameters

$\theta^{[j]} = (\theta^{[j]}_{EM}, \theta^{[j]}_{NM}, \theta^{[j]}_{NU})'$:

$$R^{[j]} = \begin{bmatrix} 1-\theta^{[j]}_{EM} & 0 & 0 & 0 \\ 0 & 1-\theta^{[j]}_{NM}-\theta^{[j]}_{NU} & 0 & 0 \\ \theta^{[j]}_{EM} & \theta^{[j]}_{NM} & 1 & 0 \\ 0 & \theta^{[j]}_{NU} & 0 & 1 \end{bmatrix}. \tag{15}$$

Note we can estimate $\theta^{[j]}$ for $j = 2, 3, ..., 8$ immediately from rows 1, 4, and 2 of (14):

$$1 - \theta^{[j]}_{EM} = \pi^{[1]}_E / \pi^{[j]}_E \tag{16}$$

$$\theta^{[j]}_{NU} = (\pi^{[1]}_U - \pi^{[j]}_U)/\pi^{[j]}_N \tag{17}$$

$$1 - \theta^{[j]}_{NM} - \theta^{[j]}_{NU} = \pi^{[1]}_N / \pi^{[j]}_N. \tag{18}$$

These values for $\theta^{[j]}$ are plotted in Figure 6. The left panel shows that 1-2% of the individuals who get counted as employed in rotations 2-4 or 6-8 would have been missing from the survey if the rotation 1 interview technology had been used. On the other hand, rotation 5 (which follows an 8-month break) reports similar numbers of E as rotation 1 ($\theta^{[5]}_{EM}$ near 0).[13] The middle panel captures a rising tendency for those who would have been counted as N in later rotations to have been counted as U in the first interview. The right panel indicates that a large and rising fraction of those not in the labor force in later rotations would have been missing under the rotation 1 interview technology.

For purposes of what we are doing so far (summarizing average values over the full sample), expression (14) along with (16)-(18) is just an accounting identity that holds exactly in the observed data. By choosing the three parameters $\theta^{[j]}_{EM}, \theta^{[j]}_{NU}, \theta^{[j]}_{NM}$ we can match the three free values in $\pi^{[j]}$ exactly (the fourth being pinned down by the fact that elements of $\pi^{[j]}$ sum to unity).[14] Our

[13] The estimate of $\theta^{[5]}_{EM}$ from equation (16) is actually very slightly negative (-0.0049). The values plotted in Figure 6 and used in the calculations below set $\theta^{[5]}_{EM} = 0$. This makes essentially no difference for any results.

[14] One can show that equations (16)-(18) imply that row 3 of (14) also holds. Add rows 1, 2, and 4 of (14) together to deduce

$$\pi^{[j]}_E + \pi^{[j]}_U + \pi^{[j]}_N - \theta^{[j]}_{EM}\pi^{[j]}_E - \theta^{[j]}_{NM}\pi^{[j]}_N = \pi^{[1]}_E + \pi^{[1]}_U + \pi^{[1]}_N.$$

Subtracting both sides from 1 gives

$$\pi^{[j]}_M + \theta^{[j]}_{EM}\pi^{[j]}_E + \theta^{[j]}_{NM}\pi^{[j]}_N = \pi^{[1]}_M$$

as required by the third row of (14). In general, since each column of $R^{[j]}$ sums to unity, if elements of π sum to representation will have substantive implications when we now consider the matrix of transition probabilities and in Section 5 when we develop a time-varying generalization of this approach.

Notice that the averages over the full sample exactly satisfy the following accounting identity:

$$\Pi^{[j]}\pi^{[j-1]} = \pi^{[j]} \qquad j \in J. \tag{19}$$

Premultiply (19) by $R^{[2]}$ for $j = 2$ and use result (14):

$$R^{[2]}\Pi^{[2]}\pi^{[1]} = R^{[2]}\pi^{[2]} = \pi^{[1]}.$$

In other words, $R^{[2]}\Pi^{[2]}$ could be used as the counterfactual transition matrix Π^* if people were interviewed with the same technology in rotation 2 as in rotation 1, satisfying the requirement for consistency between transition probabilities and marginal probabilities that $\Pi^*\pi^{[1]} = \pi^{[1]}$ for $\Pi^* = R^{[2]}\Pi^{[2]}$. More generally, premultiplying (19) by $R^{[j]}$ we see

$$R^{[j]}\Pi^{[j]}(R^{[j-1]})^{-1}R^{[j-1]}\pi^{[j-1]} = R^{[j]}\pi^{[j]}$$

$$R^{[j]}\Pi^{[j]}(R^{[j-1]})^{-1}\pi^{[1]} = \pi^{[1]} \qquad \text{for } j \in J \tag{20}$$

where $R^{[1]}$ is defined to be the identity matrix. Thus $\Pi^{*[j]} = R^{[j]}\Pi^{[j]}(R^{[j-1]})^{-1}$ satisfies the internal consistency requirement $\Pi^{*[j]}\pi^{[1]} = \pi^{[1]}$ for each j.

Let π^* denote the counterfactual fraction if everyone was interviewed with rotation technology 1 and Π^* the counterfactual transition probabilities. We have seen that the matrices $\hat{\Pi}^{*[j]} = R^{[j]}\Pi^{[j]}(R^{[j-1]})^{-1}$ give us different estimates of Π^* for different rotations. We propose to estimate Π^* as the value that minimizes the errors in predicting $\Pi^{[j]}$ on the basis of Π^*. Specifically, taking $R^{[j]}$ as given we choose Π^* so as to minimize the sum of squared elements of

$$\Pi^{[j]} - (R^{[j]})^{-1}\Pi^* R^{[j-1]} \quad \text{for } j \in J. \tag{21}$$

Note we do not have a matrix of transition probabilities into rotation $j = 1$, and we do not use the transition probabilities from rotation 4 to rotation 5 because this 8-month transition is

unity, then the elements of $R^{[j]}\pi$ also sum to unity: $1'R^{[j]}\pi = 1'\pi = 1$ for 1 a vector of four ones.

a fundamentally different object from the other 1-month transition probabilities. Instead for rotations 1 and 5 we compare the observed fractions $\pi^{[1]}$ and $\pi^{[5]}$ with the unconditional probabilities implied by Π^*

$$\pi^{[1]} - \pi^* \tag{22}$$

$$\pi^{[5]} - (R^{[5]})^{-1}\pi^* \tag{23}$$

where π^* is calculated as in Hamilton (1994, eq. [22.2.26]):

$$B = \begin{bmatrix} I_4 - \Pi^* \\ 1' \end{bmatrix} \tag{24}$$

$$\pi^* = (B'B)^{-1}B'e_5 \tag{25}$$

where $1'$ denotes a (1×4) vector of ones and e_5 denotes column 5 of I_5.

We have an estimate of $R^{[j]}$ from (15)-(18), while $\Pi^{[j]}$ and $\pi^{[j]}$ are observed data. Our approach is then to estimate the elements of Π^* (subject to the constraints that every element is between 0 and 1 and that columns sum to unity) so as to minimize the sum of squares of the $96 = 16 \times 6$ elements in (21) plus the sum of squares of the 8 elements in (22) and (23). The resulting estimates of π^* and Π^* are reported in Tables 6 and 7.

The above framework predicts that the fraction of individuals reporting status E, N, M, or U when interviewed using technology j would be given by

$$\hat{\pi}^{[j]} = (\hat{R}^{[j]})^{-1}\pi^*. \tag{26}$$

These predicted shares are compared with the actual shares reported for each rotation in Figure 7. Our representation fits the values in each $\pi^{[j]}$ essentially perfectly.

Our approach also implies a predicted value for the observed fraction of individuals with measured transitions from $X^{[j-1]}$ to $X^{[j]}$:

$$\hat{\Pi}^{[j]} = (\hat{R}^{[j]})^{-1}\Pi^*\hat{R}^{[j-1]}. \tag{27}$$

Figure 8 plots these predicted values along with the actual reported fractions for $j \in J$.[15] These show a reasonable fit, though not perfect. One could try to model in more detail features such as the tendency for those missing in rotation 1 to be reported as employed in rotation 2 and for those not in the labor force in rotation 1 to be missing in rotation 2. Notwithstanding, our simple parsimonious framework does a reasonable job of capturing transitions.

We defined the value of π^* in terms of the rotation 1 technology. But now that we've found π^*, we can also calculate the answer using any other technology. For example, $(\hat{R}^{[5]})^{-1}\pi^*$ gives the answer in terms of the rotation 5 technology. The BLS approach, which simply averages the rotations together, is implicitly reporting results in terms of an "average" technology, which in our formulation would be described as $(1/8)\sum_{j=1}^{8}(\hat{R}^{[j]})^{-1}\pi^*$.

Where does rotation-group bias come from? The framework above allows us to reconcile stocks and flows in the CPS data and summarize that reconciliation using any interview technology or combination of interview technologies. In practice we need to choose a particular technology for purposes of that summary. Which one we choose depends on what we think is the source of the bias.

Halpern-Manners and Warren (2012) suggested that admitting to have failed in finding a job for consecutive months may produce feelings of stigma or shame, which could lead people to say "no" in follow-up interviews when asked if they are still actively looking for a job. Those who report U are asked "what are the things you have done to find work during the last 4 weeks?" and "how long had you been looking for a job?" Halpern-Manners and Warren also raised the possibility that, having learned the questions that follow up if they report U, respondents may believe they would be asked fewer or less onerous follow-up questions if they instead report N or E. Hapern-Manners and Warren concluded that these two factors lead to a downward bias in estimates of the unemployment rate based on later rotations.

We saw in Figure 5 that most of the decrease in U across rotations is accounted for by an increase in N. And the long durations of unemployment search that people report suggest that significant numbers of those classified as N in fact view themselves as looking for work. For these reasons, we think that the answer people give the first time they are asked whether they are

[15] Note we do not offer a predicted value for transitions from $X^{[4]}$ to $X^{[5]}$ since there are 8 intervening months between rotations 4 and 5.

unemployed is the best one to use for purposes of reconciling the full set of observed data, and we will use π^* as our preferred reconciliation of rotation-group bias. However, we emphasize that our framework could be used to calculate an alternative reconciled estimate $\pi^{*[j]} = (\hat{R}^{[j]})^{-1}\pi^*$ from the perspective of any specified technology j.[16]

4.2 Nonrandom missing observations.

The conventional approach simply throws out missing observations, which amounts to assuming that those missing from the survey are just like those included. However, our reconciled probabilities in Table 7 show that someone who is employed has a 6.2% probability of being missing in the next month, whereas someone who is unemployed has 8.7% probability. Of those making ME, MN, or MU transitions, 6.2% are unemployed, although the unemployed only comprise 4.5% of the observed E, N, or U on average. In addition, of those making MU transitions, 65% claim that they have been searching for work longer than 4 weeks. In sum, missing individuals are more likely to be unemployed than a typical person in the observed data.

To correct for the bias coming from nonrandom missing observations, we impute a labor-force status in month $t-1$ to individuals observed to make ME, MN, or MU transitions into period t. Suppose that some fraction m_E of those missing in month $t-1$ are just like those who were counted as employed that month in terms of their transition probabilities, while fractions m_N or m_U share the same transition probabilities as those counted as N or U. We regard the remaining $m_M = 1 - m_E - m_N - m_U$ as "dormant observations" in the sense of having zero probability of being recorded as E, N, or U in month t.[17] The probabilities of observing ME, MN, and MU transitions would then be given by

$$\begin{bmatrix} \pi^*_{ME} \\ \pi^*_{MN} \\ \pi^*_{MU} \end{bmatrix} = \begin{bmatrix} \pi^*_{EE} & \pi^*_{NE} & \pi^*_{UE} \\ \pi^*_{EN} & \pi^*_{NN} & \pi^*_{UN} \\ \pi^*_{EU} & \pi^*_{NU} & \pi^*_{UU} \end{bmatrix} \begin{bmatrix} m_E \\ m_N \\ m_U \end{bmatrix}.$$

[16] Krueger, Mas and Niu (2017) found that survey nonresponse is an important source of rotation-group bias. In our approach, we separately model the role of rotation-group specific nonresponses (as represented by the third column and third row of $R^{[j]}$), the effect of the nonrandom nature of nonresponse rates (as represented by the parameters m_X in Section 4.2), and the contribution of changes over time in each of these factors (Section 5).

[17] This would include people who are in the military, incarcerated, moved away from the address, or yet to move in, for example.

This system of equations can be solved to find $(m_E, m_N, m_U) = (0.0951, 0.0465, 0.0121)$. Our suggested correction for nonrandom missing observations is then

$$\begin{bmatrix} \pi_E^* + \pi_M^* m_E \\ \pi_N^* + \pi_M^* m_N \\ \pi_U^* + \pi_M^* m_U \end{bmatrix}.$$

4.3 Reinterpreting flows from N into longer-term unemployment.

As shown in Section 3.2, a majority of the people making NU transitions interpret their status at $t-1$ as part of an extended period of job search. Our proposal is to characterize individuals who do so as having been U rather than N at $t-1$. The average fraction of the population that is in this category is given by

$$m_N^\sharp = T^{-1} \sum_{t=1}^T \left\{ \frac{\sum_{\tau=5}^{99} \sum_{j \in J} y_{N,U,t}^{[j]}(\tau)}{\sum_{j \in J} \left(y_{E,t-1}^{[j-1]} + y_{N,t-1}^{[j-1]} + y_{M,t-1}^{[j-1]} + y_{U,t-1}^{[j-1]} \right)} \right\} = 0.0038. \qquad (28)$$

Our adjustment subtracts $m_N^\sharp$ from π_N^* and adds it to π_U^*.

In addition to the individual's own perception, this adjustment is supported by a number of other facts observed in the data. Someone who was counted as N in month $t-2$ and unemployed with duration 5 weeks or greater in month $t-1$ has a 13% probability of being employed in month t. This turns out to be quite close to the 15% probability that someone with a UU history in $t-2$ and $t-1$ will be employed in month t, consistent with our conclusion that both groups of individuals should have both been counted as U in $t-2$.[18]

Similar conclusions emerge from looking at longer and more detailed histories. The first column of Table 8 examines UUU continuations in months $t-3, t-2$, and $t-1$ for which the reported durations would be consistent with a true UUU continuation.[19] As we go down the rows, the history is consistent with a longer initial duration in month $t-3$. Our framework would predict that the employment probability in month t would decrease as we move down the rows. This is because type 2 individuals, who have a lower probability than type 1 of becoming employed at t,

[18] Our analysis of the importance of labor force status history in the reemployment prospect of jobless workers was inspired by Kudlyak and Lange (2018).

[19] For example, $U_{t-3}^{1\text{-}4}, U_{t-2}^{5\text{-}14}, U_{t-1}^{5\text{-}14}$ refers to someone who reported being newly unemployed in $t-3$ and being unemployed between 5 and 14 weeks in $t-2$ and $t-1$.

make up a larger fraction of the pool at $t-1$ as we move down the rows.[20] This is exactly what we observe in the data. The third column looks at individuals with an intervening N status in month $t-2$ but with the same U in $t-3$ and $t-1$ as in column 1. These probabilities also tend to decrease as we move down rows. This provides further support for our conclusion that the status of individuals in $t-2$ is similar across the two columns.[21]

Additional corroboration comes from the subset of N that are designated by BLS as "marginally attached workers." These are people who say they want and are available for work or have looked for a job sometime within the last 12 months, even though they did not claim to have actively searched within the survey month. We find from the American Time Use Survey that among individuals who report positive job search time in the survey, unemployed individuals spend 143 minutes per day for job search and marginally attached workers spend 154 minutes per day on average. Marginally attached workers account for only 2.2% of those typically designated as N but represent 40% of the observed $NU^{5.+}$ transitions. Thus for a little less than half of those we retrospectively reclassify as U at date $t-1$ we have independent corroboration before date t that those individuals could end up behaving like unemployed job seekers. Moreover, over 2003-2015, the marginally attached category only includes 10% of those classified as N who actively search for work, suggesting that the N category includes many job seekers who behave like the unemployed. Ahn and Shao (2017) and Mukoyama, Patterson, and Sahin (2018) documented that among those who spend more than zero minutes searching on a survey day, the time spent searching for a job by those categorized as N is similar to that for the unemployed. Our conclusion that significant numbers of N should instead be viewed as U thus receives support from a variety of other sources of evidence.

[20] See Ahn and Hamilton (2019).

[21] Our adjustments are related to those of Elsby, Hobijn, and Şahin (2015) who reclassified all UNU as UUU and all NUN as NNN. By contrast, we only classify UNU as UUU if the final U reports a duration of job search greater than 4 weeks, and would classify NUN as UUX if the intervening U has duration greater than 4 weeks and where X is allocated statistically based on $m_N^\flat$ in (29). Kudlyak and Lange (2018) noted that UNU are similar to UUU in terms of their probability of finding a job but dissimilar in terms of the wage they subsequently earn. We acknowledge Kudlyak and Lange's conclusion that UNU have some different characteristics from UUU, and indeed our specification predicts differences even within the group of individuals who are UNU. Our predictions are confirmed by the differences observed in period t employment prospects for $U_{t-3}N_{t-2}U_{t-1}$ as we move down the rows of Table 8. The question is, what is the appropriate designation of the labor-force status of these individuals during the intervening N_{t-2}? We base our designation on how people describe their status in $t-2$ when asked in $t-1$. We find that the individuals' own descriptions are confirmed by their objective probability of obtaining a job at t. We regard that objective probability as the most important confirmation of how actively they were really looking for a job. Overall, we interpret the evidence uncovered by both Elsby, Hobijn, and Şahin (2015) and Kudlyak and Lange (2018) as broadly supportive of our approach.

4.4 Recovering unemployment-continuation probabilities.

Here we describe our adjustments to estimates of unemployment-continuation probabilities.

Correcting for rotation-group bias. Recall that $\gamma_{i,UX}^{[1]}$ represents the probability that a type i individual who was reported to be unemployed in rotation 1 would have reported status $X \in \{E, N, M, U\}$ in rotation 2. Our first task is to translate the answers from the rotation 2 interview technology into terms of the rotation 1 technology. This can be done by premultiplying the vector $\gamma_{i,U}^{[1]}$ by $R^{[2]}$. We would likewise premultiply the rotation 5 estimate $\gamma_{i,U}^{[5]}$ by $R^{[6]}$. For greater accuracy, in Table 3 we pooled rotation 1 with rotation 5 transitions for purposes of jointly estimating parameters. Adjusting these pooled estimates for rotation-group bias is achieved by

$$\gamma_{1,U}^{*} = (1/2)(R^{[2]} + R^{[6]})\gamma_{1,U} = \begin{bmatrix} 0.3239 \\ 0.2058 \\ 0.1038 \\ 0.3665 \end{bmatrix} \qquad \gamma_{2,U}^{*} = (1/2)(R^{[2]} + R^{[6]})\gamma_{2,U} = \begin{bmatrix} 0.1168 \\ 0.2164 \\ 0.0831 \\ 0.5837 \end{bmatrix}.$$

The resulting adjustments for rotation-group bias are relatively minor.

Adjusting for misclassified N. We concluded that many people who are currently counted as N are better classified as U. This observation means that some UN observations could really be UU continuations. In Section 3.3 we found that the discrepancy between reported unemployment durations and observed unemployment-continuation probabilities mainly comes from $\gamma_{2,UU}$, the unemployment-continuation probability for type 2 individuals. Here we explore whether a fraction ξ_{UN} of the $\gamma_{2,UN}$ transitions should be regarded as UU continuations. Since type 2 individuals account for 95% of those unemployed for 15 weeks and over (hereafter, $U^{15.+}$), we look for evidence in the observed outcomes in month t of individuals who were $U^{15.+}$ in $t-2$ and N in $t-1$.

Someone with a history $U_{t-2}^{15.+}N_{t-1}$ has a 22.5% probability of being $U^{15.+}$ in t. Since we interpret the last two months ($N_{t-1}U_t^{15.+}$) of the $U_{t-2}^{15.+}N_{t-1}U_t^{15.+}$ sequence as a UU continuation, we are forced to interpret the first two months $U_{t-2}^{15.+}N_{t-1}$ also as a UU continuation, requiring $\xi_{UN} > 0.225$.

We also observe that someone with a $U_{t-2}^{15.+}N_{t-1}$ history has a 7.55% probability of being employed at t, far higher than usually observed for individuals classified as N_{t-1} ($P(E_t|N_{t-1}) = 4.63\%$).

Suppose we view $U^{15.+}_{t-2}N_{t-1}$ individuals as a mixture of two populations, with a fraction ξ_{UN} having the same employment probability in month t as someone who is observed to be $U^{15.+}_{t-2}U^{15.+}_{t-1}$, and the remainder with the same employment probability as someone who is truly out of the labor force in $t-1$ as represented by a history of $N_{t-2}N_{t-1}$:

$$P(E_t|U^{15.+}_{t-2}, N_{t-1}) = \xi_{UN}P(E_t|U^{15.+}_{t-2}, U^{15.+}_{t-1}) + (1-\xi_{UN})P(E_{t+1}|N_{t-2}, N_{t-1})$$

$$0.0755 = 0.1071\xi_{UN} + 0.0209(1-\xi_{UN}).$$

This equation gives an estimate of $\xi_{UN} = 0.633$.

Another way to corroborate this is as follows. On average each month, a fraction $m^{\sharp}_N = 0.0038$ of the population report $NU^{5.+}$ transitions, and we have interpreted $m^{\sharp}_N q_{6,NU}/(q_{5,NU} + q_{6,NU}) =$ 0.0028 of these as long-term UU continuations. In steady state, one would think that the number of recorded NU transitions that are really long-term UU continuations should be roughly balanced by the fraction of the population with recorded UN transitions that are really long-term UU continuations.[22] The fraction of the population that transitions from long-term unemployed to N is $\pi^*_U w_2 \gamma^*_{2,UN}$, and if ξ_{UN} of these are really UU continuations, the fraction of the population with observed UN that is really long-term UU is

$$m^{\flat}_N = \pi^*_U w_2 \gamma^*_{2,UN}\xi_{UN} = (0.0311)(1-0.3920)(0.2164)(0.633) = 0.0026. \tag{29}$$

This is indeed quite close to 0.0028. This supports the inference that the true unemployment-continuation probability for type 2 individuals is $\gamma^*_{2,UU} + \xi_{UN}\gamma^*_{2,UN} = 0.7207$ with $\xi_{UN} = 0.633$.

Adjusting for missing observations. Finally, recall that both the original estimate $\gamma^*_{2,UU}$ and our preferred estimate $\gamma^*_{2,UU} + \xi_{UN}\gamma^*_{2,UN}$ are continuation probabilities with UM transitions regarded as a separate status. In reality, UM transitions must be one of UE, UN, or UU. Allocating UM transitions proportionally to the observed UE, UN, and UU results in our final adjusted estimates

[22] The balance is only approximate because the flows from N to U could alternatively be balanced in steady state by UMN flows, for example.

of true unemployment-continuation probabilities:

$$\tilde{\gamma}_{1,UU} = \frac{\gamma^*_{1,UU}}{1 - \gamma^*_{1,UM}} = 0.4089 \tag{30}$$

$$\tilde{\gamma}_{2,UU} = \frac{\gamma^*_{2,UU} + \xi_{UN}\gamma^*_{2,UN}}{1 - \gamma^*_{2,UM}} = 0.7860. \tag{31}$$

The estimate $\tilde{\gamma}_{2,UU}$ is below $p_2^{4.33} = 0.89$, the value we would have expected based on reported unemployment durations. Nevertheless, the adjustment goes a long way toward reconciling perceived durations with objective continuation probabilities. One source of the remaining discrepancy between our estimate of the objective continuation probability $\tilde{\gamma}_{2,UU}$ and the perceived duration of job search p_2 is on-the-job search. Recall from Section 3.2 that $EU^{5.+}$ transitions account for 29% of EU observations, with many EU individuals reporting duration longer than 6 months. As noted by Kudlyak and Lange (2018), we could interpret these individuals as correctly reporting how long they have been looking for a job or looking for a better job, while still defending the estimate $\tilde{\gamma}_{2,UU}$ as a correct summary of the true probability of remaining unemployed without an intervening spell of employment. A second possible source of discrepancy between $\tilde{\gamma}_{2,UU}$ and p_2 is cognitive bias (Horvath, 1982). Unemployed individuals might have forgotten the exact date when they lost their job, and the discouragement and frustration during the unsuccessful job search period can make them perceive that they have been unemployed longer than is the case. We conclude that our procedure of adjusting unemployment-continuation probabilities up, but not all the way to those implied by reported job-search durations, is the correct way to reconcile the data.

4.5 Unemployment and labor-force participation rates.

We have found that on average in month t a fraction of the population $m_N^\sharp$ in equation (28) is reported as N but should instead be regarded as U based on an their observed $N_t U_{t+1}^{5.+}$ transition. Our calculations also suggest that a fraction of the population $m_N^\flat$ in equation (29) is reported as N_t but should be regarded as U based on their previous observed $U_{t-1}N_t$ transition. There is some double-counting from individuals who would be counted in both groups as a result of $U_{t-1}N_t U_{t+1}^{5.+}$ transitions. We could get an estimate of how large this overlap is by calculating the number of people with labor-force history $U_{t-1}N_t U_{t+1}^{5.+}$ as a fraction of the total population and multiplying by $w_2 \xi_{UN}$, which product we denote $m_N^{\natural} = 0.0006$. Our estimate of the fraction of the population that is truly unemployed is then

$$\tilde{\pi}_U = \pi_U^* + \pi_M^* m_U + m_N^{\sharp} + m_N^{\flat} - m_N^{\natural} = 0.0406. \tag{32}$$

The corresponding adjustments for the fraction who are N or E are

$$\tilde{\pi}_N = \pi_N^* + \pi_M^* m_N - (m_N^{\sharp} + m_N^{\flat} - m_N^{\natural}) = 0.2444. \tag{33}$$

$$\tilde{\pi}_E = \pi_E^* + \pi_M^* m_E = 0.4537. \tag{34}$$

From these we calculate an adjusted unemployment rate and labor-force participation rate:

$$\frac{\tilde{\pi}_U}{\tilde{\pi}_E + \tilde{\pi}_U} = 8.21\% \tag{35}$$

$$\frac{\tilde{\pi}_E + \tilde{\pi}_U}{\tilde{\pi}_E + \tilde{\pi}_U + \tilde{\pi}_N} = 66.91\%.$$

Table 9 summarizes the effects of the various adjustments. The first row reports the average unemployment rate and labor-force participation rate over our sample as reported by the BLS. The second row reports the value if we correct only for rotation-group bias, that is, the calculation from equations (32)-(34) if $m_E = m_N = m_N^{\sharp} = m_N^{\flat} = m_N^{\natural} = 0$. This adjustment alone would add half a percentage point to the unemployment rate and 1.2% to the labor-force participation rate. The third row shows the contribution of also taking account of the nonrandom nature of missing observations (that is, allows for nonzero m_E, m_N, m_U), while the final row shows the effect of all three adjustments. Altogether, our adjustments add 1.9% to the unemployment rate and 2.2% to the labor-force participation rate.

The last column of Table 9 shows that while rotation-group bias matters for the employment-population ratio, the ratio is unchanged after correcting for missing observations or misclassified N. Thus the employment-population ratio could be a robust measure of the labor-market slack in the presence of increasing nonresponses and errors in responses in the CPS.

4.6 Unemployment duration.

Here we calculate average unemployment durations that would be consistent with our estimates of true unemployment-continuation probabilities. To do this we need an estimate of the fraction $\tilde{w}_i$ of total unemployed individuals individuals $\tilde{\pi}_i$ that are of type i. For the first term in equation (32), π_U^*, we know the fraction of type i from the estimate of w_i from Table 3. We assume the same fraction w_i could be used to impute types to the missing unemployed for the second term. The third term in (32), $m_N^\sharp$, is derived from observed $NU^{5.+}$ transitions, for which we estimated directly that the fraction of type 1 is given by $q_{5,NU}/(q_{5,NU} + q_{6,NU})$. The last two terms by construction come solely from type 2 individuals. We thus estimate

$$\tilde{w}_1 = \frac{w_1(\pi_U^* + \pi_M^* m_U) + m_N^\sharp q_{5,NU}/(q_{5,NU} + q_{6,NU})}{\tilde{\pi}_U} = 0.3622$$

and $\tilde{w}_2 = 1 - \tilde{w}_1$.

Next we adapt the weekly formulation (4) to a monthly frequency to calculate a true average duration of unemployment. If the true monthly unemployment-continuation probability for type i individuals is $\tilde{\gamma}_{i,UU}$, then in steady state the average fraction of the unemployed with true duration of n months would be given by

$$\tilde{w}_1(1 - \tilde{\gamma}_{1,UU})\tilde{\gamma}_{1,UU}^n + \tilde{w}_2(1 - \tilde{\gamma}_{2,UU})\tilde{\gamma}_{2,UU}^n.$$

Table 10 uses this expression to calculate the fraction of the truly unemployed $\tilde{\pi}_U$ for whom the true duration is less than 5 weeks (1 month), 5-14 weeks (2-3 months), 15-26 weeks (4-6 months) and longer than 26 months (7 months and over), along with the average duration.[23] Our estimate of the average duration of unemployment is only 16 weeks, about 9 weeks lower than the BLS reports.

Kudlyak and Lange (2018) constructed estimates of the number of newly unemployed as a

[23] These calculations used $\tilde{w}_1 = 0.3653$, $\tilde{\gamma}_{1,UU} = 0.4089$, and $\tilde{\gamma}_{2,UU} = 0.7860$. The fraction between 5 and 14 weeeks was found from

$$\tilde{w}_1(1 - \tilde{\gamma}_{1,UU})(\tilde{\gamma}_{1,UU} + \tilde{\gamma}_{1,UU}^2) + \tilde{w}_2(1 - \tilde{\gamma}_{2,UU})(\tilde{\gamma}_{2,UU} + \tilde{\gamma}_{2,UU}^2).$$

The average duration in weeks is

$$4.33\left(\frac{\tilde{w}_1}{1 - \tilde{\gamma}_{1,UU}} + \frac{\tilde{w}_2}{1 - \tilde{\gamma}_{2,UU}}\right).$$

fraction of total unemployed by (1) counting all $E_{t-1}U_t$ as newly unemployed despite the duration of search reported at t, and (2) also counting all $N_{t-1}U_t$ as newly unemployed. Our estimate of 35.1% is in between their two estimates (29.1% and 46.1%, respectively) because we designate some, but not all, of the $N_{t-1}U_t$ as unemployed at $t-1$. Their two methods produced estimates of 37.5% and 24.1%, respectively, for the fraction of unemployed with duration greater than 14 weeks, with our estimate of 33.4% again in between those two. Although their approach did not allow them to uncover the average duration of unemployment, their calculations confirm our conclusion that the BLS estimates substantially overstate the number of long-term unemployed.

5 Adjusting monthly estimates.

Up to this point in the paper we have used the entire sample to describe our interpretation of full-sample averages. In this section we show how to generalize this approach to allow for changes over time in the various sources of bias.

Our principle in doing so is to start with initial estimates based on month t observations alone, just as the BLS does. We then make adjustments to these estimates using some time-varying parameters θ_t. One approach would be to assume that all the sources of bias that we have identified are constant over time, and simply use the full-sample estimates of parameters θ described earlier to adjust each month's observation. This approach would miss changes over time in these biases which could be important. An alternative approach would be to estimate adjustment parameters θ_t for each month separately, treating the observations for month t as a completely separate sample from the others. This would add so much estimation error that any adjustments would be useless for practical purposes. Our approach uses a hybrid of the two methods. We use exponential smoothing to calculate a weighted average of recent observations through date t to infer how the adjustment parameters θ_t are changing over time. If θ_t denotes an estimate using observations from month t alone, we calculate

$$\overline{\theta}_t = (1-\lambda)\theta_t + \lambda\overline{\theta}_{t-1}.$$

This constructs $\overline{\theta}_t$ as a weighted average of values of θ_t through date t with most recent values given the biggest weight. When $\lambda = 0$ this would correspond to using only date t data to calculate θ_t, whereas when $\lambda = 1$ it yields the full-sample estimate θ when started at $\theta_1 = \theta$. We choose the weighting parameter λ close to unity so that plots of $\overline{\theta}_t$ evolve smoothly smoothly over time, eliminating high-frequency measurement error but still capturing longer-run trends. Note that this smoothing is used only to calculate the adjustment parameters and not to the primary data that forms the initial estimate for any month t.

Rotation-group bias parameters. We first summarize how the rotation-bias parameters $\theta_{EM}^{[j]}$, $\theta_{NU}^{[j]}$, and $\theta_{NM}^{[j]}$ have changed over time. Our first step is to construct weighted moving averages of the counts of individuals in each labor-force status in each rotation,

$$\overline{y}_{X,t}^{[j]} = (1-\lambda)y_{X,t}^{[j]} + \lambda\overline{y}_{X,t-1}^{[j]},$$

where $y_{X,t}^{[j]}$ denotes the observed weighted number of individuals reporting labor status $X \in \{E, N, M, U\}$ in rotation j in month t. We set $\lambda = 0.98$, which means that observations 3 years prior to t receive half the weight of observation t in determining the smoothed count $\overline{y}_{X,t}^{[j]}$.[24] We then calculated the corresponding smoothed fractions as

$$\overline{\pi}_{X,t}^{[j]} = \overline{y}_{X,t}^{[j]} / \left(\overline{y}_{E,t}^{[j]} + \overline{y}_{N,t}^{[j]} + \overline{y}_{M,t}^{[j]} + \overline{y}_{U,t}^{[j]} \right).$$

From these we calculated time-varying rotation-bias parameters as

$$\theta_{EM,t}^{[j]} = \max\left\{1 - \left(\overline{\pi}_{E,t}^{[1]}/\overline{\pi}_{E,t}^{[j]}\right), 0\right\}$$

$$\theta_{NU,t}^{[j]} = \max\left\{\left(\overline{\pi}_{U,t}^{[1]} - \overline{\pi}_{U,t}^{[j]}\right)/\overline{\pi}_{N,t}^{[j]}, 0\right\}$$

$$\theta_{NM,t}^{[j]} = \max\left\{1 - \theta_{NU,t}^{[j]} - \left(\overline{\pi}_{N,t}^{[j]}/\overline{\pi}_{N,t}^{[1]}\right), 0\right\},$$

and exponentially smoothed these as well. For example, $\overline{\theta}_{EM,t}^{[j]} = (1-\lambda)\theta_{EM,t}^{[j]} + \lambda\overline{\theta}_{EM,t-1}^{[j]}$.

The resulting series for $\overline{\theta}_{EM,t}^{[j]}$, $\overline{\theta}_{NU,t}^{[j]}$, and $\overline{\theta}_{NM,t}^{[j]}$ are plotted in Figure 9. The value of $\overline{\theta}_{EM,t}^{[j]}$, which characterizes the tendency to record people as E in rotation j who would have been M in rotation 1, has fallen somewhat over time. By contrast, $\overline{\theta}_{NU,t}^{[j]}$, which governs the tendency of people who would have been counted as U in earlier rotations to be designated as N in later rotations,

[24] That is, $0.98^{36} = 0.48$. We started the recursion by setting $\tilde{y}_{X,1}^{[j]} = (1/36)\sum_{t=1}^{36} y_{X,t}^{[j]}$ the average of the first three years of observations.

has increased over time. The third parameter, $\overline{\theta}_{NM,t}^{[j]}$, which characterizes the tendency of someone who would have been counted as M in rotation 1 to be counted as N in later rotations, has not changed much over time.

Correcting for rotation-group bias. With these values we can calculate that a vector of observed probabilities for those in rotation j in month t, $\pi_t^{[j]}$, would have been reported as $R_t^{[j]}\pi_t^{[j]}$ if the people had been interviewed using the rotation 1 interview technology instead of in rotation j, where

$$R_t^{[j]} = \begin{bmatrix} 1-\overline{\theta}_{EM,t}^{[j]} & 0 & 0 & 0 \\ 0 & 1-\overline{\theta}_{NM,t}^{[j]}-\overline{\theta}_{NU,t}^{[j]} & 0 & 0 \\ \overline{\theta}_{EM,t}^{[j]} & \overline{\theta}_{NM,t}^{[j]} & 1 & 0 \\ 0 & \overline{\theta}_{NU,t}^{[j]} & 0 & 1 \end{bmatrix}.$$

Let Π_t^* denote the counterfactual (4×4) matrix of transition probabilities if all individuals in months $t-1$ and t had been interviewed using the interview technology of rotation 1 in both months. This counterfactual Π_t^* implies a predicted value for the observed $\Pi_t^{[j]}$ given by $(R_t^{[j]})^{-1}\Pi_t^* R_{t-1}^{[j-1]}$. We therefore chose Π_t^* so as to make the elements in the following equations as small as possible:

$$\Pi_t^{[j]} - (R_t^{[j]})^{-1}\Pi_t^* R_{t-1}^{[j-1]} \qquad \text{for } j \in J = \{2,3,4\} \cup \{6,7,8\}. \tag{36}$$

We likewise let π_t^* denote the (4×1) vector of unconditional probabilities if all individuals were interviewed in month t using technology 1. Note these satisfy the accounting identity $\Pi_t^*\pi_{t-1}^* = \pi_t^*$. Thus if we had an estimate of π_{t-1}^* for the previous period, we would predict a value for $\pi_t^{[1]}$ of $\pi_t^* = \Pi_t^*\pi_{t-1}^*$ and predict a value for $\pi_t^{[5]}$ of $(R_t^{[5]})^{-1}\Pi_t^*\pi_{t-1}^*$. This means that Π_t^* should also have the property that it makes all the terms in the following equations small as well:

$$\pi_t^{[1]} - \Pi_t^*\pi_{t-1}^* \tag{37}$$

$$\pi_t^{[5]} - (R_t^{[5]})^{-1}\Pi_t^*\pi_{t-1}^*. \tag{38}$$

Our procedure is to proceed iteratively through the data. We set the initial value of π_t^* for observation $t=1$ as $\pi_1^* = \pi_1^{[1]}$. For each $t = 2,3,...$ we choose the 16 elements of Π_t^* so as to minimize the sum of squares of the 104 terms in (36)-(38) subject to the constraints that each element of Π_t^* is between 0 and 1 and each column of Π_t^* sums to 1. Given Π_t^* we then calculate

$$\pi_t^* = \Pi_t^* \pi_{t-1}^*$$

and proceed to the next observation $t+1$.

Note there is no smoothing involved in our estimate of Π_t^*, which is based solely on the observed values of $\pi_t^{[1]}$, $\pi_t^{[5]}$, and $\Pi_t^{[j]}$ for $j \in J = \{2,3,4\} \cup \{6,7,8\}$. The smoothing was only used for purposes of calculating $R_t^{[j]}$ under the assumption that changes in the interview technology occur slowly over time.

Correcting for missing observations. From the reconciled transition probabilities Π_t^* we next calculate estimates of the fraction of the M observations for date $t-1$ that, based on their observed status as E, N, or U at date t, should be imputed to E, N, or U at date $t-1$. We do this by finding the values of $m_{E,t-1}$, $m_{N,t-1}$, and $m_{U,t-1}$ that solve

$$\begin{bmatrix} \pi_{ME,t}^* \\ \pi_{MN,t}^* \\ \pi_{MU,t}^* \end{bmatrix} = \begin{bmatrix} \pi_{EE,t}^* & \pi_{NE,t}^* & \pi_{UE,t}^* \\ \pi_{EN,t}^* & \pi_{NN,t}^* & \pi_{UN,t}^* \\ \pi_{EU,t}^* & \pi_{NU,t}^* & \pi_{UU,t}^* \end{bmatrix} \begin{bmatrix} m_{E,t-1} \\ m_{N,t-1} \\ m_{U,t-1} \end{bmatrix}.$$

We also smooth these as

$$\overline{m}_{X,t} = (1-\lambda) m_{X,t} + \lambda \overline{m}_{X,t-1}.$$

The $m_{X,t}$ parameters have more high-frequency movement than terms like $\theta_{EM,t}$. We accordingly use a shorter effective window by setting $\lambda = 0.97$, which gives observations 2 years ago half the weight current observations for purposes of calculating $\overline{m}_{X,t}$. The resulting values of $\overline{m}_{X,t}$ are plotted in the first three panels of Figure 10. Both $\bar{m}_{Nt}$ and $\bar{m}_{Et}$ rise over time, while $\overline{m}_{U,t}$ is highly counter-cyclical without exhibiting a particular trend. The secular rise in $\bar{m}_{Nt}$ and $\bar{m}_{Et}$ suggests that the upward trend in missing individuals likely comes from N and E. The counter-cyclical behavior of $\bar{m}_{Ut}$ tells us that unemployed individuals are more likely to be missed during a weak labor market.[25]

Correcting for transitions from N to long-term unemployment. We proposed that individuals

[25] Krueger, Mas and Niu (2017) also investigated increases in missing observations over time.

who reported status N in month $t-1$ and reported in month t that they were unemployed and had been looking for work for longer than 4 weeks should be counted as U rather than N in month $t-1$. The fraction of the population for whom this is observed to be the case in month $t-1$ is given by

$$m^{\sharp}_{N,t-1} = \frac{\sum_{\tau=5}^{99} \sum_{j \in J} y^{[j]}_{N,U,t}(\tau)}{\sum_{j \in J} \left[y^{[j-1]}_{E,t-1} + y^{[j-1]}_{N,t-1} + y^{[j-1]}_{M,t-1} + y^{[j-1]}_{U,t-1} \right]}.$$

Estimates of monthly transition probabilities. Our main results do not model changes over time in the number-preference parameters θ_A but hold these fixed at the full-sample estimates reported in Table 3 for all months t.[26] However, we estimated all the other parameters θ_t in Table 3 maximizing (12) separately for each month t and smoothed these using $\lambda = 0.9$. Figure 10 plots some of the key magnitudes of interest. The fractions of NU and MU transitions that individuals perceive as continuations of long-term unemployment ($q_{6,NU,t}$ and $q_{6,MU,t}$) rose sharply during the Great Recession and have been slow to return to their historical averages. Both the perceived weekly UU continuation probability for type 1 individuals $\overline{p}_{1t}$ and the objective monthly probability $\bar{\gamma}_{1,UU,t}$ react to seasonal hiring, consistent with the high seasonality in unadjusted short-term unemployment, and both fell during the Great Recession.[27] For type 2 individuals, there is a time trend in perceived $\overline{p}_{2t}$ that is not fully matched by that for the objective $\bar{\gamma}_{2,UU,t}$ probability, though both increased significantly in the Great Recession and were slow to come down afterward. The fraction $\bar{w}_{2t}$ of type 2 workers among the reported unemployed rose through 2011 and has been slowly declining since.

Estimates of unemployment rate and labor-force participation rate. We construct monthly estimates of (29), the fraction of the population with reported UN who are better interpreted as long-term UU, from

$$m^{\flat}_{Nt} = \pi^{*}_{Ut} \bar{w}_{2t} \bar{\gamma}_{2,UN,t} \xi_{UN}$$

[26] We also estimated $\theta_{A,t}$ separately for each month, and found no significant trend or cyclical component in $\theta_{A,t}$. Keeping $\theta_{A,t}$ fixed at θ_A reduces noise and measurement error and does not affect substantive conclusions.

[27] The feature of the data that gives rise to this conclusion is the observation that individuals with unemployment durations of 5-14 weeks were much more likely to remain unemployed during the Great Recession, meaning that more type 1 individuals must have exited unemployment before entering this group. One possible interpretation is that individuals would only voluntarily quit their job in this episode if they knew they could get another job quickly. A drop in p_{1t} during the Great Recession was also found by Ahn and Hamilton (2019, Figure 4 and Table 1). They found that this feature was unique to the Great Recession and was not seen in other recessions.

where we fix $\xi_{UN} = 0.633$ at the full-sample average[28]. We calculate $k^\natural$, the fraction of $m^\sharp_{Nt} + m^\flat_{Nt}$ that comes from double-counting the same individuals, from our full-sample estimate of that fraction:

$$k^\natural = \frac{m^\natural_N}{m^\sharp_N + m^\flat_N} = \frac{0.0006}{0.0038 + 0.0026} = 0.094$$

giving rise to the monthly estimate $m^\natural_{Nt} = k^\natural(m^\sharp_{Nt} + m^\flat_{Nt})$. Our final estimates that correct for rotation-group bias, non-randomly missing observations, and misclassified N are then

$$\begin{bmatrix} \tilde{\pi}_{E,t-1} \\ \tilde{\pi}_{N,t-1} \\ \tilde{\pi}_{M,t-1} \\ \tilde{\pi}_{U,t-1} \end{bmatrix} = \begin{bmatrix} \pi^*_{E,t-1} + \pi^*_{M,t-1}\overline{m}_{E,t-1} \\ \pi^*_{N,t-1} + \pi^*_{M,t-1}\overline{m}_{N,t-1} - m^\sharp_{N,t-1} - m^\flat_{N,t-1} + m^\natural_{N,t-1} \\ \pi^*_{M,t-1}(1 - \overline{m}_{E,t-1} - \overline{m}_{N,t-1} - \overline{m}_{U,t-1}) \\ \pi^*_{U,t-1} + \pi^*_{M,t-1}\overline{m}_{U,t-1} + m^\sharp_{N,t-1} + m^\flat_{N,t-1} - m^\natural_{N,t-1} \end{bmatrix}.$$

Our adjusted estimates of the unemployment rate and labor-force participation rate are

$$\tilde{u}_t = \tilde{\pi}_{U,t} / \left(\tilde{\pi}_{E,t} + \tilde{\pi}_{U,t}\right)$$

$$\tilde{\ell}_t = \left(\tilde{\pi}_{E,t} + \tilde{\pi}_{U,t}\right) / \left(\tilde{\pi}_{E,t} + \tilde{\pi}_{N,t} + \tilde{\pi}_{U,t}\right).$$

Note that these are all seasonally unadjusted magnitudes in order to preserve all the accounting identities associated with observed transitions. To relate these to the usually reported magnitudes, we plotted seasonally-adjusted values[29] for these rates in Panels B and C of Figure 1. In the top panel of Figure 11, we compare our adjusted estimate $\tilde{u}_t$ (in dotted blue) with three different unemployment rates reported by the BLS– the usual U3 unemployment rate (solid black) along with U5 unemployment (dashed red), which includes discouraged workers and all other marginally attached workers, and U6 unemployment (dashed green) which adds people who are employed part-time for economic reasons. Our adjustment includes more individuals than U5, but far less than U6.

Estimates of monthly continuation probabilities. We calculate true monthly unemployment-

[28] We obtained similar results allowing $\xi_{UN,t}$ to change over time.
[29] These were calculated using the X11 instruction in RATS.

continuation probabilities from $\tilde{\gamma}_{1,UU,t} = \overline{\gamma}_{1,UU,t}/(1 - \overline{\gamma}_{1,UM,t})$ and

$$\tilde{\gamma}_{2,UU,t} = \frac{\overline{\gamma}_{2,UU,t} + \xi_{UN}\overline{\gamma}_{2,UN,t}}{1 - \overline{\gamma}_{2,UM,t}}.$$

Our monthly estimate of the the fraction $\tilde{w}_{1,t}$ of type 1 workers among all unemployed is

$$\tilde{w}_{1,t} = \frac{\overline{w}_{1,t}(\pi^*_{U,t} + \pi^*_{M,t}\overline{m}_{U,t}) + m^{\sharp}_{N,t}\overline{q}_{5,NU,t}/(\overline{q}_{5,NU,t} + \overline{q}_{6,NU,t})}{\tilde{\pi}_{U,t}}.$$

The adjusted estimates $\tilde{\gamma}_{2,UU,t}$ and $\tilde{w}_{2t}$ are plotted as dashed red lines in Figure 10. Our estimate of the true monthly continuation probability averaged across all individuals who are truly unemployed is

$$\tilde{w}_{1,t}\tilde{\gamma}_{1,UU,t} + \tilde{w}_{2,t}\tilde{\gamma}_{2,UU,t},$$

which is the series plotted as the dotted line in Panel A of Figure 1.

Estimates of average duration of unemployment. We estimate that a fraction $\tilde{w}_{it}\tilde{\pi}_{Ut}$ of the population are truly unemployed of type $i \in \{1, 2\}$ in month t. Of these, a fraction $\tilde{\gamma}_{i,UU,t+1}$ will still be unemployed the next month, giving rise to $\tilde{V}_{i,t+1} = \tilde{w}_{i,t+1}\tilde{\pi}_{U,t+1} - \tilde{\gamma}_{i,UU,t+1}\tilde{w}_{it}\tilde{\pi}_{Ut}$ as an estimate of the number of individuals of type i who are newly unemployed in month $t + 1$ and

$$\tilde{U}^d_{it} = \tilde{V}_{i,t-d+1}\tilde{\gamma}_{i,UU,t-d+2} \cdots \tilde{\gamma}_{i,UU,t-2}\tilde{\gamma}_{i,UU,t-1}\tilde{\gamma}_{i,UU,t}$$

as the number who have been unemployed for exactly d months as of month t. This implies an average unemployment duration of those who are unemployed in month t of

$$\tilde{d}_t = \frac{\sum_{d=1}^{D} d(\tilde{U}^d_{1t} + \tilde{U}^d_{2t})}{\sum_{d=1}^{D}(\tilde{U}^d_{1t} + \tilde{U}^d_{2t})}.$$

Dividing by 4.33 gives the unemployment duration in weeks plotted as the blue dotted lines in Panel D of Figure 1 and the bottom panel of Figure 11. Our series is much lower on average and less cyclically variable than the BLS measure in black. As additional corroboration we compare our estimate with 12-month moving averages of the average duration of unemployment among individuals collecting unemployment insurance, which is shown as the dashed red line in the bottom panel of Figure 11. This differs conceptually from the object we are trying to measure in two ways.

First, those collecting unemployment insurance are primarily job losers, who have higher durations of unemployment on average than voluntary job leavers and entrants to the labor force. Second, eligibility for unemployment insurance expires for the longest durations. The first factor would bias the insurance-based measure upward and the second would bias it downward relative to our object of interest. Notwithstanding, the insurance-based measure is remarkably similar to our adjusted series.

6 Conclusion.

The data underlying the CPS contain multiple internal inconsistencies. These include the facts that people's answers change the more times they are asked the same question, stock estimates are inconsistent with flow estimates, missing observations are not random, reported unemployment durations are inconsistent with reported labor-force histories, and people prefer to report some numbers over others. Ours is the first paper to attempt a unified reconciliation of these issues. We conclude that the U.S. unemployment rate and labor-force continuation rates are higher than conventionally reported while the average duration of unemployment is considerably lower.

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Table 1. Parameters estimated separately for rotation 1, rotation 5, and NX, EX and MX transitions from rotation 1 to rotation 2.

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
param	rotation 1 only	std error	rotation 5 only	std error	NX only	std error	EX only	std error	MX only	std error
p_1	0.8271	0.0037	0.8272	0.0024	0.7556	0.0096	0.7541	0.0148	0.8338	0.0062
p_2	0.9738	0.0026	0.9735	0.0026	0.9746	0.0022	0.9687	0.0035	0.9744	0.0025
w_1	0.4243	0.0455	0.4009	0.0484						
π_E	0.4256	…	0.4215	…						
π_N	0.2358	0.0054	0.2475	0.0055						
π_M	0.3075	0.0037	0.3030	0.0031						
π_U	0.0311	0.0027	0.0280	0.0024						
π_{XE}					0.0386	0.0016	0.8902	0.0030	0.1266	0.0029
π_{XN}					0.8765	0.0006	0.0317	0.0004	0.0649	0.0030
π_{XM}					0.0594	…	0.0647	…	0.7979	…
π_{XU}					0.0254	0.0016	0.0134	0.0006	0.0105	0.0007
q_1					0.0920	…	0.2145	…	0.0882	…
q_2					0.0779	0.0057	0.1911	0.0139	0.1011	0.0095
q_3					0.0805	0.0052	0.1768	0.0083	0.0784	0.0054
q_4					0.0530	0.0031	0.1236	0.0095	0.0826	0.0058
q_5					0.1883	0.0210	0.1204	0.0032	0.2199	0.0226
q_6					0.5082	0.0433	0.1736	0.0148	0.4298	0.0503
$q_5 + q_6$					0.6965	…	0.2940	…	0.6497	…
$\theta_{A,1}$	0.1227	0.0019	0.1305	0.0074	0.2063	0.0288	0.0930	0.0491	0.0424	0.0336
$\theta_{A,2}$	0.7735	0.0027	0.7385	0.0030	0.7545	0.0060	0.7194	0.0183	0.7400	0.0079
$\theta_{A,3}$	0.4835	0.0097	0.4571	0.0088	0.4894	0.0166	0.3767	0.0352	0.5150	0.0261
$\theta_{A,4}$	0.9268	0.0035	0.8775	0.0071	0.8562	0.0113	0.8260	0.0261	0.8582	0.0107
$\theta_{A,5}$	0.7219	0.0158	0.6790	0.0120	0.7080	0.0166	0.6891	0.0413	0.7718	0.0367
$\theta_{A,6}$	0.9254	0.0084	0.9028	0.0038	0.8740	0.0147	0.8254	0.0185	0.8836	0.0187
$\theta_{A,7}$	0.9605	0.0080	0.9554	0.0022	0.9541	0.0159	0.9729	0.0104	0.9315	0.0220
$\theta_{A,8}$	0.9000	0.0063	0.8521	0.0149	0.7297	0.0344	0.7640	0.0251	0.7941	0.0276
$\theta_{A,9}$	0.9417	0.0083	0.9445	0.0040	0.9488	0.0136	0.9467	0.0398	0.9339	0.0116
$\theta_{A,10}$	0.1637	0.0078	0.1497	0.0059	0.1994	0.0106	0.1359	0.0094	0.1428	0.0075
$\theta_{A,11}$	0.4920	0.0086	0.4985	0.0040	0.5882	0.0102	0.4845	0.0174	0.4939	0.0160
$\theta_{A,12}$	0.8951	0.0155	0.8880	0.0133	0.9214	0.0092	0.9100	0.0195	0.9036	0.0073
$\theta_{A,13}$	0.1595	0.0267	0.0991	0.0317	0.1196	0.0222	0.1519	0.0159	0.0666	0.0330

Table 2. Parameters estimated separately for UX transitions from rotations 1 to 2 and 5 to 6.

		$\gamma_{1,UE}$	$\gamma_{1,UN}$	$\gamma_{1,UM}$	$\gamma_{1,UU}$	$\gamma_{2,UE}$	$\gamma_{2,UN}$	$\gamma_{2,UM}$	$\gamma_{2,UU}$
[1]	Rotation 1 estimate	0.3183	0.2179	0.0909	0.3729	0.1153	0.2353	0.0735	0.5759
[2]	Standard error	0.0053	0.0032	0.0025	…	0.0092	0.0087	0.0028	…
[3]	Rotation 5 estimate	0.3379	0.2178	0.0890	0.3554	0.1210	0.2224	0.0686	0.5880
[4]	Standard error	0.0068	0.0019	0.0014	…	0.0080	0.0065	0.0036	…

Table 3. Parameters estimated jointly across all rotations.

	estimate		estimate		estimate		estimate		estimate		estimate
p_1	0.8094	$\theta_{A,6}$	0.8881	$q_{1,\mathrm{EU}}$	0.2235	$q_{1,\mathrm{NU}}$	0.0870	$q_{1,\mathrm{MU}}$	0.1014	$\gamma_{1,UE}$	0.3274
p_2	0.9734	$\theta_{A,7}$	0.9542	$q_{2,\mathrm{EU}}$	0.1878	$q_{2,\mathrm{NU}}$	0.0811	$q_{2,\mathrm{MU}}$	0.0955	$\gamma_{1,UN}$	0.2179
w_1	0.3920	$\theta_{A,8}$	0.8045	$q_{3,\mathrm{EU}}$	0.1967	$q_{3,\mathrm{NU}}$	0.0756	$q_{3,\mathrm{MU}}$	0.0969	$\gamma_{1,UM}$	0.0901
$\theta_{A,1}$	0.1441	$\theta_{A,9}$	0.9394	$q_{4,\mathrm{EU}}$	0.1148	$q_{4,\mathrm{NU}}$	0.0650	$q_{4,\mathrm{MU}}$	0.0693	$\gamma_{1,UU}$	0.3646
$\theta_{A,2}$	0.7355	$\theta_{A,10}$	0.1700	$q_{5,\mathrm{EU}}$	0.1365	$q_{5,\mathrm{NU}}$	0.1847	$q_{5,\mathrm{MU}}$	0.2113	$\gamma_{2,UE}$	0.1181
$\theta_{A,3}$	0.4688	$\theta_{A,11}$	0.5203	$q_{6,\mathrm{EU}}$	0.1408	$q_{6,\mathrm{NU}}$	0.5067	$q_{6,\mathrm{MU}}$	0.4255	$\gamma_{2,UN}$	0.2291
$\theta_{A,4}$	0.8766	$\theta_{A,12}$	0.9014							$\gamma_{2,UM}$	0.0711
$\theta_{A,5}$	0.7103	$\theta_{A,13}$	0.1146							$\gamma_{2,UU}$	0.5817

Notes to Table 3. Also estimated (but not reported) are separate coefficients $\pi_{XE}, \pi_{XN}, \pi_{XM}, \pi_{XU}$ for $X \in \{E, N, M\}$.

Table 4. Average numbers of individuals with indicated status across different rotation groups.

	[1]	[2]	[3]	[4]	[5]	[6]	[7]
rotation	E	N	M	U	total	U/(U+E)	(U+E)/(U+E+N)
1	7,905	4,378	5,708	580	18,570	6.8	66.0
2	8,047	4,590	5,373	566	18,575	6.6	65.2
3	8,049	4,634	5,349	547	18,579	6.4	65.0
4	8,032	4,650	5,367	533	18,581	6.2	64.8
5	7,831	4,598	5,628	522	18,578	6.2	64.5
6	7,939	4,685	5,444	514	18,581	6.1	64.3
7	7,970	4,702	5,409	504	18,585	5.9	64.3
8	8,016	4,724	5,342	507	18,588	5.9	64.3

Table 5. Effects of rotation on fractions reporting indicated labor status (coefficients and standard errors for regression (13).

	[1]	[2]	[3]	[4]
	$X = E$	$X = N$	$X = U$	$X = M$
a_{X1}	-0.0064	-0.0102	0.0007	0.0159
s.e.	(0.0021)	(0.0011)	(0.0002)	(0.0032)
a_{X5}	-0.0104	-0.0042	-0.0003	0.0149
s.e.	(0.0017)	(0.0009)	(0.0002)	(0.0026)
δ_X	-0.0006	0.0011	-0.0005	-8.95E-07
s.e.	(0.0003)	(0.0002)	(2.65E-05)	(0.0005)

Table 6. Estimated average fraction of individuals who would have reported labor status E, N, M, or U if the individual were being interviewed for the first time.

$$\begin{bmatrix} \pi_E^* \\ \pi_N^* \\ \pi_M^* \\ \pi_U^* \end{bmatrix} = \begin{bmatrix} 0.4244 \\ 0.2359 \\ 0.3086 \\ 0.0311 \end{bmatrix}$$

Table 7. Estimated labor force status transition probabilities measured by the rotation group 1 technology.

$$\begin{bmatrix} \pi_{EE}^* & \pi_{NE}^* & \pi_{ME}^* & \pi_{UE}^* \\ \pi_{EN}^* & \pi_{NN}^* & \pi_{MN}^* & \pi_{UN}^* \\ \pi_{EM}^* & \pi_{NM}^* & \pi_{MM}^* & \pi_{UM}^* \\ \pi_{EU}^* & \pi_{NU}^* & \pi_{MU}^* & \pi_{UU}^* \end{bmatrix} = \begin{bmatrix} 0.8997 & 0.0366 & 0.0897 & 0.2007 \\ 0.0255 & 0.8688 & 0.0452 & 0.1992 \\ 0.0621 & 0.0647 & 0.8564 & 0.0870 \\ 0.0126 & 0.0299 & 0.0088 & 0.5130 \end{bmatrix}$$

Table 8. Month *t* employment probabilities for *UUU* and *UNU* histories.

UUU	Probability	*UNU*	Probability
$U_{t-3}^{1.4}, U_{t-2}^{5.14}, U_{t-1}^{5.14}$	0.19	$U_{t-3}^{1.4}, N_{t-2}, U_{t-1}^{5.14}$	0.14
$U_{t-3}^{5.14}, U_{t-2}^{5.14}, U_{t-1}^{15.26}$	0.16	$U_{t-3}^{5.14}, N_{t-2}, U_{t-1}^{15.26}$	0.14
$U_{t-3}^{15.26}, U_{t-2}^{15.26}, U_{t-1}^{15.26}$	0.14	$U_{t-3}^{15.26}, N_{t-2}, U_{t-1}^{15.26}$	0.15
$U_{t-3}^{15.26}, U_{t-2}^{27.+}, U_{t-1}^{27.+}$	0.11	$U_{t-3}^{15.26}, N_{t-2}, U_{t-1}^{27.+}$	0.10
$U_{t-3}^{27.+}, U_{t-2}^{27.+}, U_{t-1}^{27.+}$	0.08	$U_{t-3}^{27.+}, N_{t-2}, U_{t-1}^{27.+}$	0.07

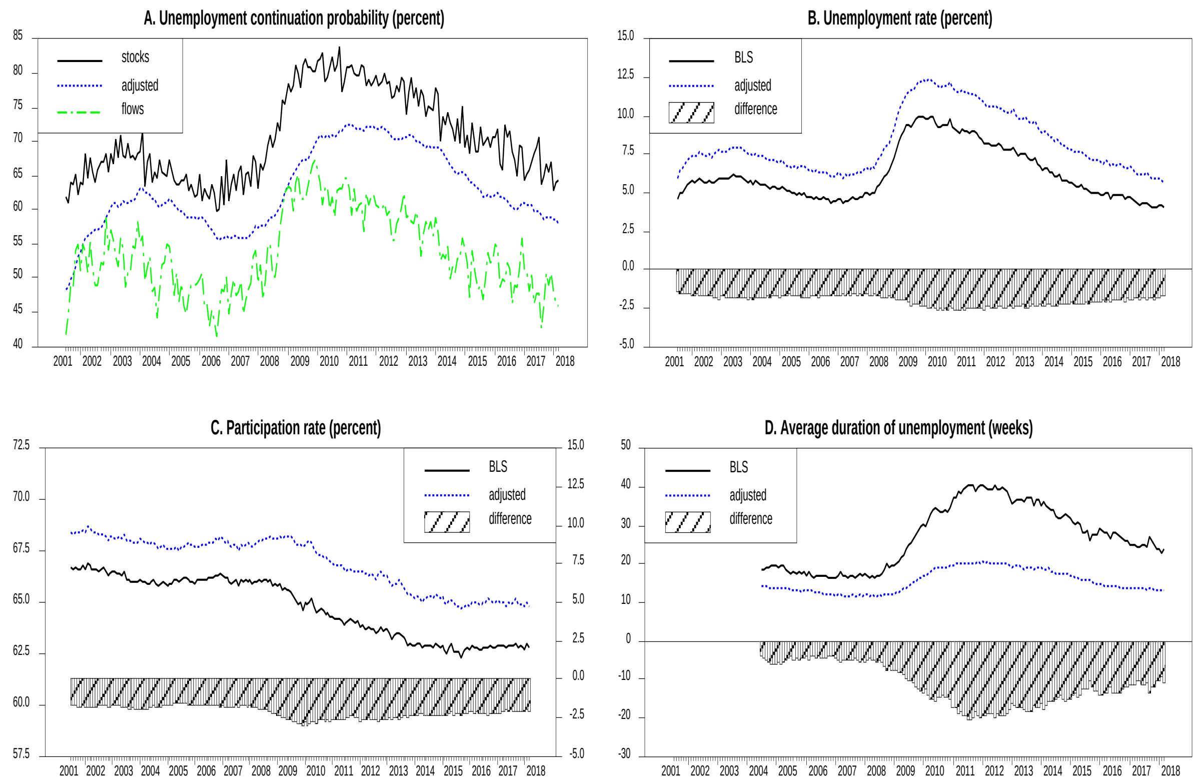
Table 9. Effects of adjustments on unemployment rate and labor-force participation rate.

	Unemployment rate	Labor-force participation rate	Employment-population ratio
Unadjusted BLS	6.3%	64.7%	60.6%
Corrected for rotation-group bias only	6.8%	65.9%	61.4%
Corrected for rotation-group bias and missing observations	7.1%	66.1%	61.4%
Corrected for rotation-group bias, missing observations, and long-term unemployed	8.2%	66.9%	61.4%

Table 10. Adjusted and unadjusted estimates of duration of unemployment

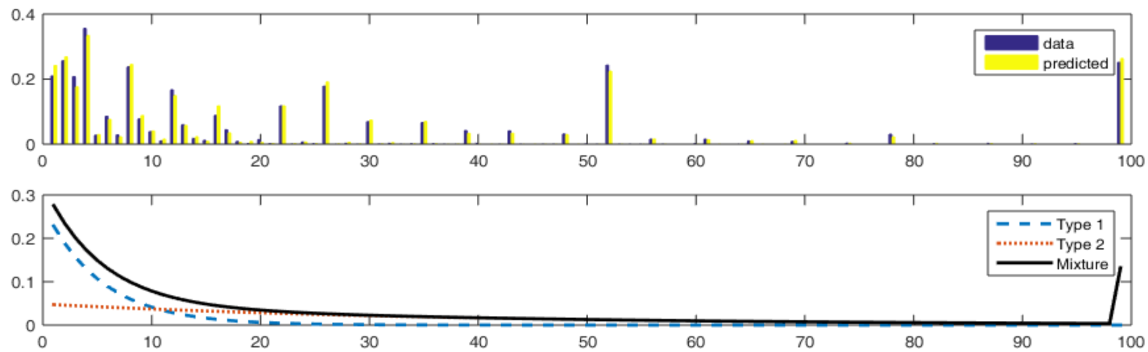
	BLS	Adjusted
< 5 weeks	29.4	35.1
5-14 weeks	27.8	31.5
15-26 weeks	15.6	18.2
> 26 weeks	27.2	15.2
Average duration	25 weeks	16 weeks

Figure 1. Alternative measures of unemployment-continuation probability, unemployment rate, labor force participation rate, and average duration of unemployment, Aug 2001 to April 2018.



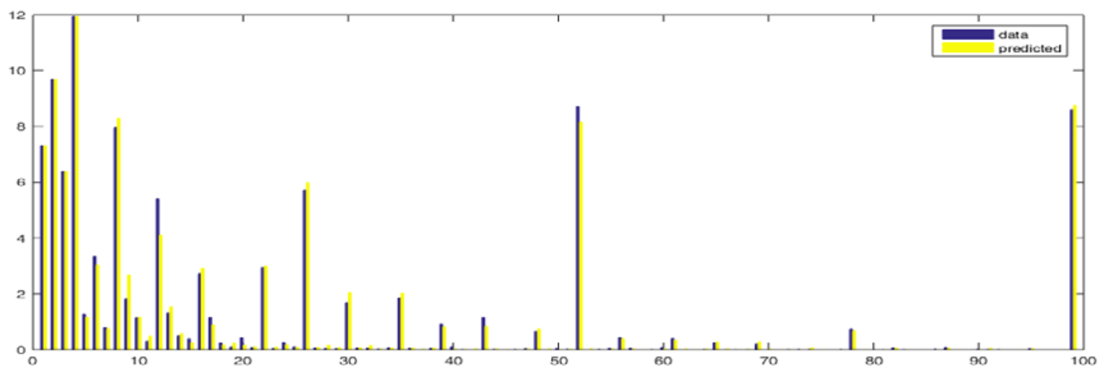
Notes to Figure 1. Panel A: probability that an unemployed individual will still be unemployed next month as calculated by: (1) ratio of unemployed with duration 5 weeks or greater in month *t* to total unemployed in *t* -1 (solid black); (2) fraction of those unemployed in *t* -1 who are still unemployed in *t* (dashed green); (3) reconciled estimate (dotted blue). Panel B: Unemployment rate as calculated by BLS (solid black), adjusted estimate (dotted blue), and difference (bars). Panel C: labor force participation rate as calculated by BLS (solid black), adjusted estimate (dotted blue) and difference (bars). Panel D: Average duration of unemployment as calculated by BLS (solid black), adjusted estimate (dotted blue), and difference (bars). All series seasonally adjusted. Source: the series labled as BLS are from the Bureau of Labor Statistics, and the other series are based on the authors' calculation.

Figure 2. Reported durations of unemployment for individuals in rotation 1.



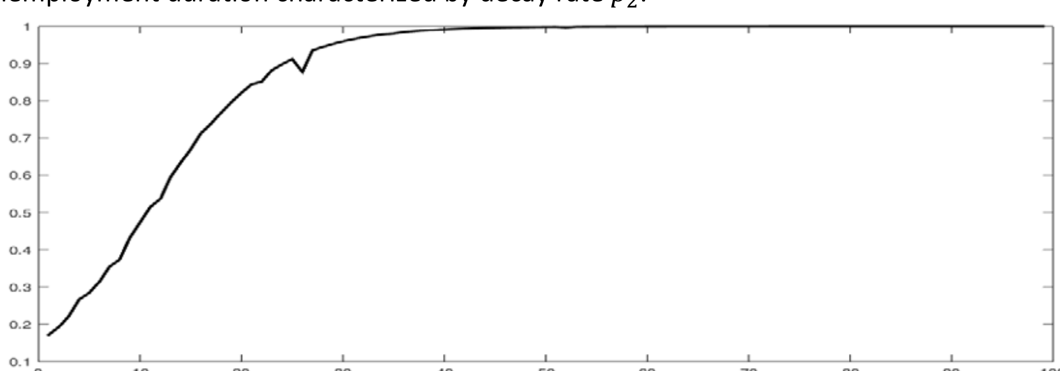
Notes to Figure 2. Top panel: reported fraction (blue) and predicted by equation (5) (in yellow) of unemployed who have been searching for indicated number of weeks. Bottom panel: total fraction of unemployed (in black) who have been looking for work for τ weeks and fraction for each type. Source: authors' calculation.

Figure 3. Reported and predicted unemployment durations in rotation 2 for individuals who were not in the labor force in rotation 1 and unemployed in rotation 2.



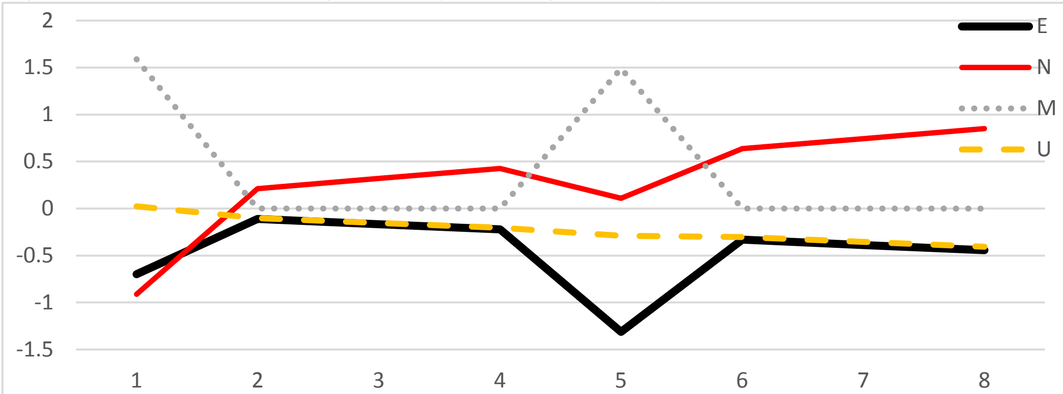
Notes to Figure 3. Horizontal axis: duration of unemployment spell in weeks. Vertical axis: of the individuals who were not in the labor force in rotation 1 and unemployed in rotation 2, the percent who reported having been searching for work at the time of rotation 2 for the indicated duration. Source: authors' calculation.

Figure 4. Probability that someone who reports being unemployed with duration τ has perceived unemployment duration characterized by decay rate p_2.



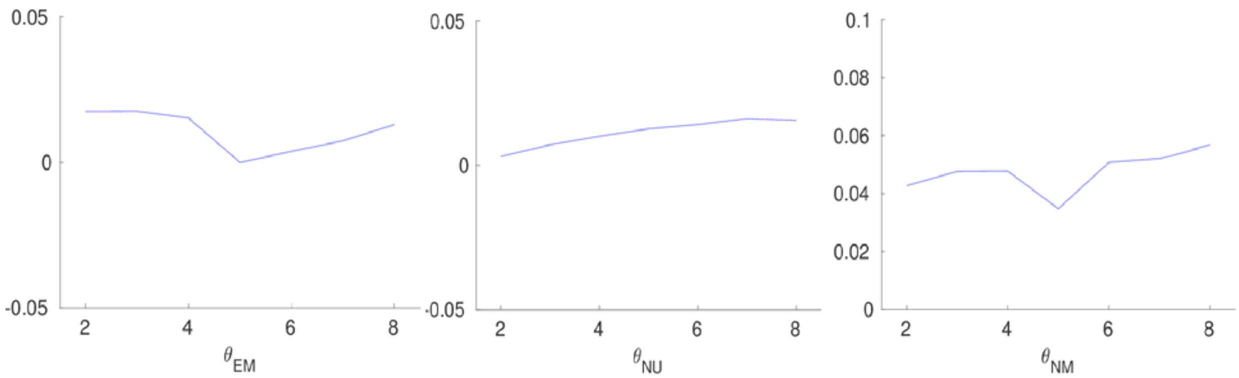
Notes to Figure 4. Horizontal axis denotes the duration τ of reported unemployment spell in weeks and vertical axis is $\eta_2(\tau)$.

Figure 5. Effect of rotation group on percentage of sampled individuals with indicated reported status.



Notes to Figure 5. Graph shows predicted values implied by regression (13).

Figure 6. Parameters capturing rotation bias.



Notes to Figure 6. Probability that someone who reported status E in rotation *j* would have been counted as missing if interviewed using rotation 1 technology (left), that someone who reported status *N* in rotation *j* would have been counted as unemployed if interviewed using rotation 1 technology (middle), and that someone who was counted as not in the labor force in rotation *j* would have been missing if interviewed using rotation 1 technology (right).

Figure 7. Fraction of individuals reporting labor status *E, N, M,* or *U* in each rotation group (solid blue) and fraction predicted to report that status for that rotation according to equation (26) (dashed red).

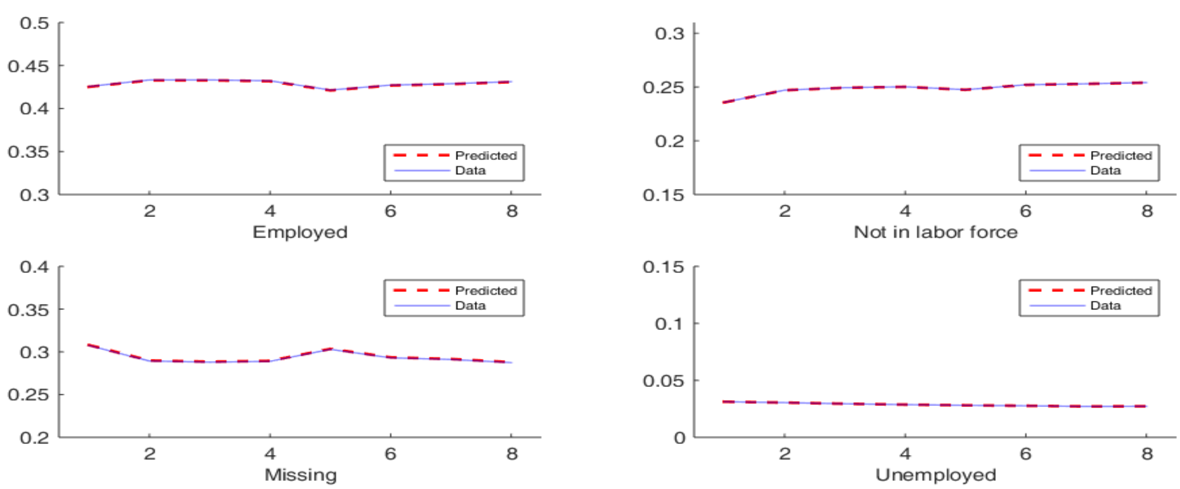


Figure 8. Actual reported transition probabilities for each rotation (solid blue) and fraction predicted by equation (27) (dashed red).

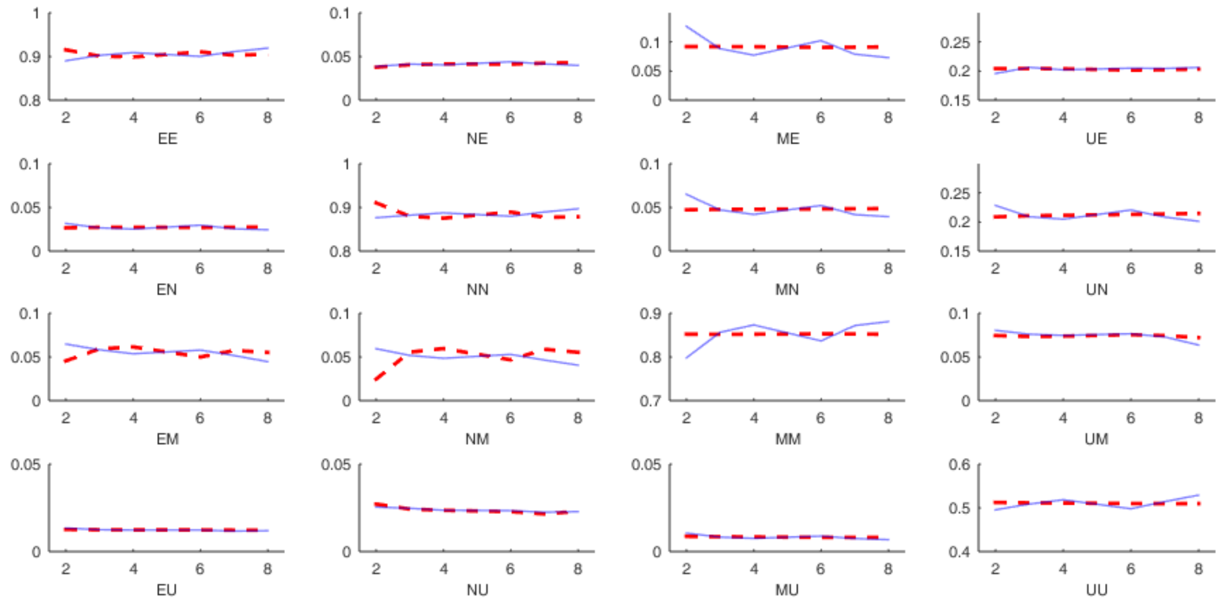


Figure 9. Changes in rotation-group bias parameters over time.

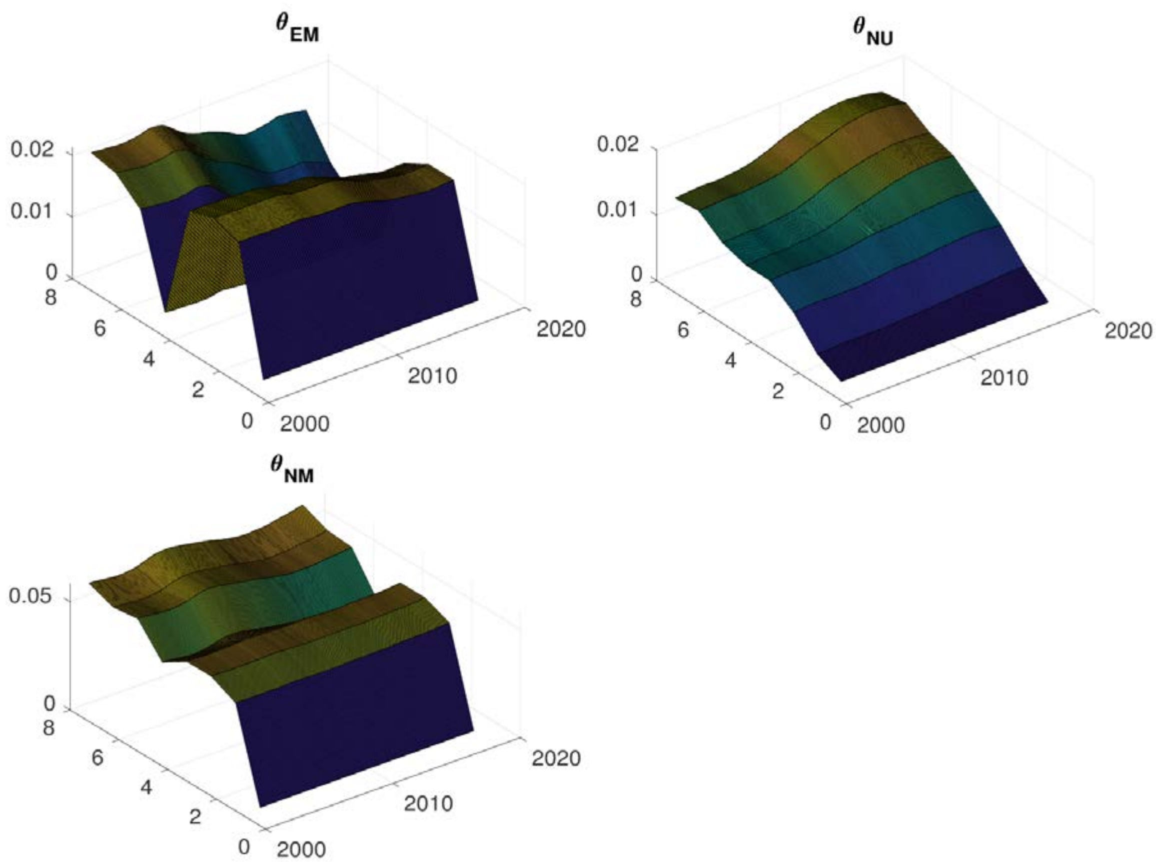
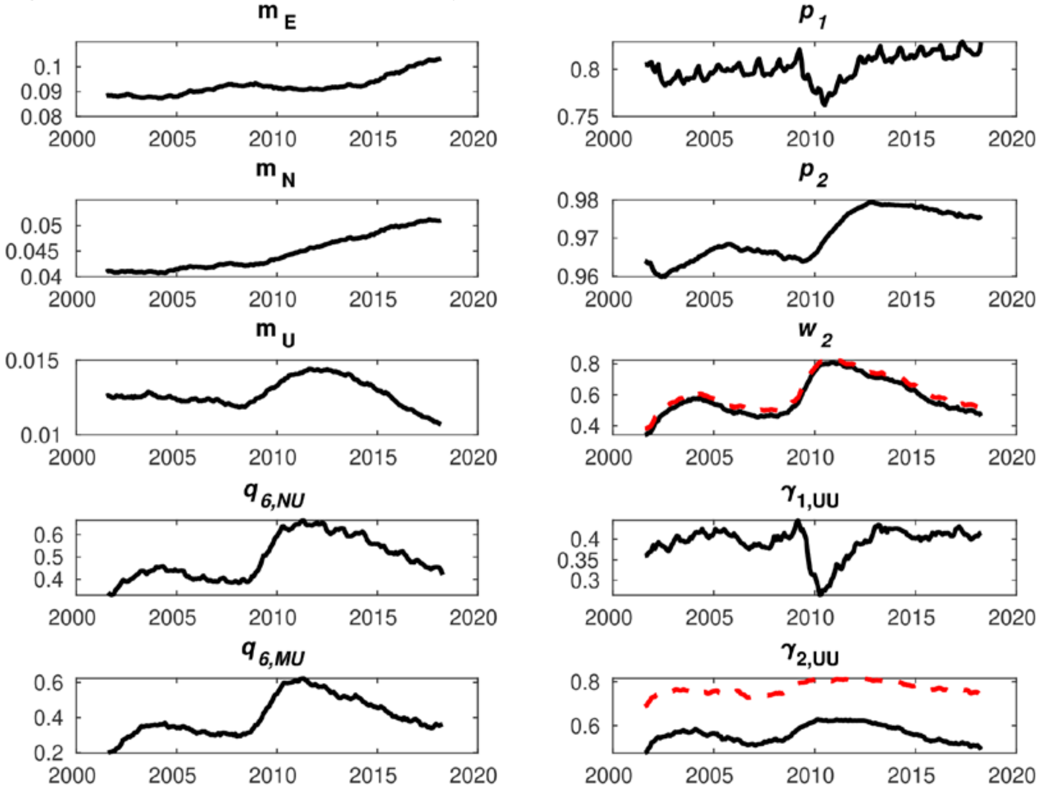
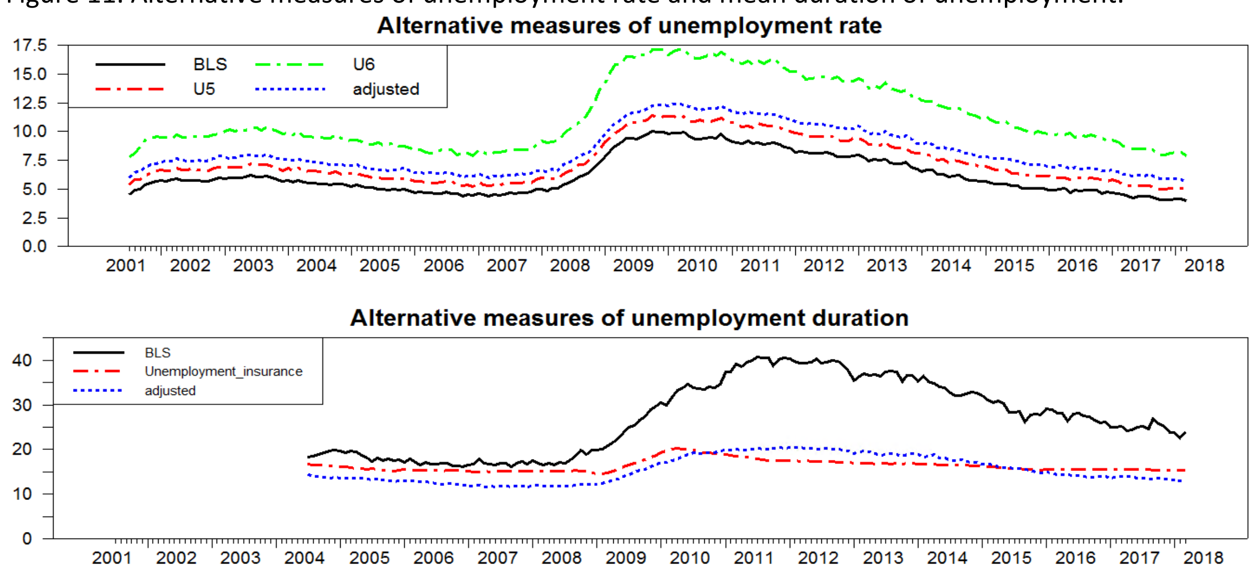


Figure 10. Time variation in selected parameters.



Notes to Figure 10. Black lines denote smoothed data summaries $\bar{\theta}_t$ and red dashed lines denote estimates $\tilde{\theta}_t$ that adjust for rotation-group bias, missing observations, and long-term unemployed.

Figure 11. Alternative measures of unemployment rate and mean duration of unemployment.



Notes to Figure 11. Top panel: adjusted unemployment rate ($\tilde{u}_t$, in dotted blue), BLS unemployment rate (solid black), U5 (dashed red) and U6 (dashed green). Bottom panel: adjusted average unemployment duration ($\tilde{d}_t$, in dotted blue), BLS average duration (solid black), and average duration of those collecting unemployment insurance (dashed red). Source: the series labeled as BLS, and U5 and U6 unemployment rates are from the Bureau of Labor Statistics, average duration of UI receipts is from the Department of Labor, and the rest is based on the authors' calculation.