Keeping the Listener Engaged: a Dynamic Model of Bayesian Persuasion

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This Paper

- **Classical question:** How (much) can a sender persuade a rational receiver to take a particular action? (e.g., *seller-buyer, media-voters, prosecutor-judge, entrepreneur-investor.....*)
- **An important assumption:** Commitment, achieved by *instantaneous* and *unrestricted* experimentation. We relax the commitment power with a model that has:
- **Main features:**
  - Persuasion takes time and cost: To be informative, it takes real time. Also information is costly for the sender to generate and for the receiver to process.
  - No commitment to future actions: For instance, sender may not be able to credibly carry out sustained persuasion.
- **Key issue:** How to persuade the receiver to listen rather than walk away?
- **Questions:**
  - Is dynamic persuasion possible? What payoffs can be achieved?
  - Behavioral implications: Persuasion dynamics

Model

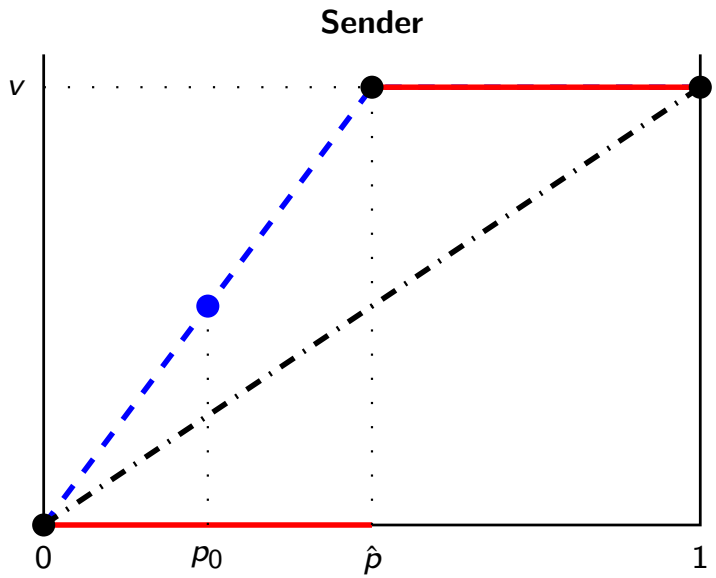
- **Two States:** $\omega \in \{L, R\}$
- **Receiver:**
  - chooses a binary action $a \in \{\ell, r\}$
  - prefers to "match" the state: $u_\ell^L > u_r^L, \quad u_r^R > u_\ell^R$
    - Example: $u_\ell^L = u_r^R = 1$, $u_\ell^R = u_r^L = 0$.
  - Notation:

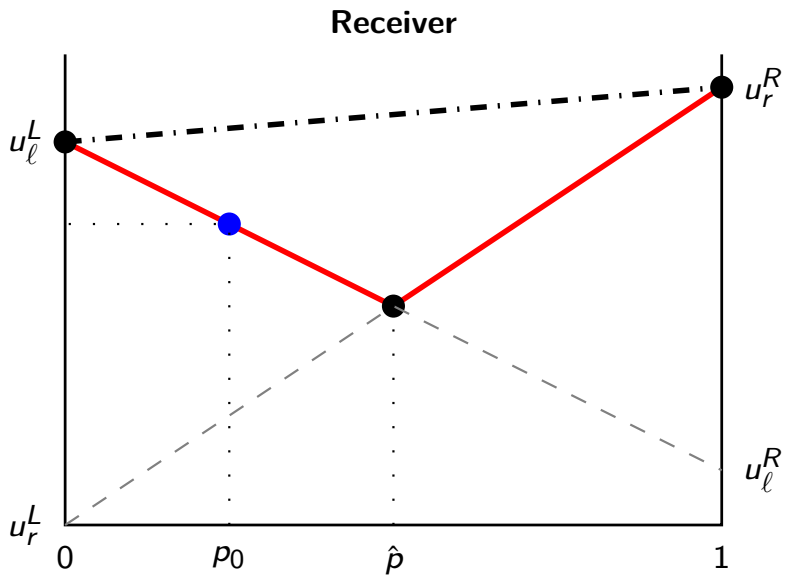
$$U_a(p) = p u_a^R + (1-p) u_a^L, \quad a \in \{\ell, r\}$$
$$U(p) = \max\{U_r(p), U_\ell(p)\} > 0$$

- **Sender:**
  - performs experiments over time to "persuade" receiver.
  - receives state-independent payoff $v \cdot \mathbf{1}_{\{a=r\}}$
- For this talk, focus on the case where $v > u_r^R - u_\ell^R$.

Static Benchmark: Kamenica-Gentzkow Model

- Sender can pick arbitrary Blackwell experiment.
- Let $\hat{p}$ be such that $U_\ell(\hat{p}) = U_r(\hat{p})$. Suppose prior is $p_0 < \hat{p}$.
- **Solution:** Sender maximizes the chance of inducing posterior $\hat{p}$; does so by inducing two posterior beliefs 0 and $\hat{p}$.





Observations

- R-signal sent excessively compared to full information.
- "Fully-revealing of L" in case of L-signal
- **The receiver enjoys no rents.**

Our Model: Dynamic Extension

Timing

At each point $t \geq 0$ in time,

1. Sender picks an experiment (to be described later) or "passes" (= null information)
2. Receiver observes the experiment and its outcome, and either **takes an irreversible action** $a \in \{\ell, r\}$, or **waits**.

- If the receiver waits and the sender performs an experiment, both parties incur $c > 0$ per unit time.
- No costs are incurred by either player if either the receiver does not wait ("game ends") or if the sender "passes."

Markov Perfect Equilibrium

Equilibrium

- **Markov Perfect equilibria (MPE)**: Subgame perfect equilibrium where strategies depend only on the payoff-relevant state p, regardless of the history.
- **Refinement** (motivated by discrete time)
  - Even if sender can persuade receiver only with zero probability, she must do so optimally (to be clarified).

Feasible Experiments: Discrete Time

Feasible "flow" experiment

- For a time period $dt > 0$, sender can choose an experiment of the form:

state/signal	L-signal	R-signal
L	x	$1-x$
R	$1-y$	y

$x, y \in [0,1]$, $1 \leq x+y \leq 1+\lambda dt$.

- We consider: **limit of experiments in this class** as $dt \to 0$: rich set of Poisson signals.

- Sender can split attention $(\alpha, 1-\alpha)$ across two experiments (x,y), (x',y'):

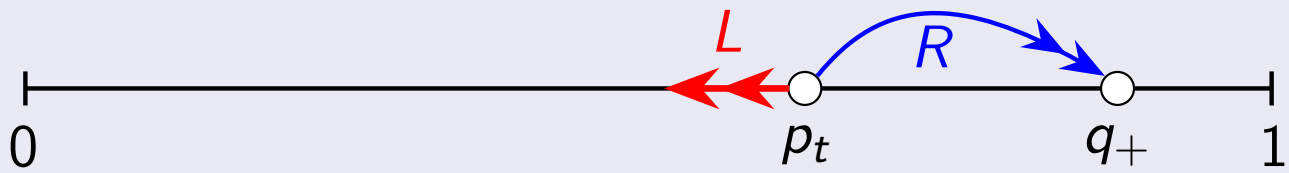
$$x+y \leq 1+\lambda\alpha dt \quad \text{and} \quad x'+y' \leq 1+\lambda(1-\alpha)dt$$

- **Interpretation:** Alternating between experiments/splitting time.

Feasible experiments: Continuous Time

L-drifting experiment (with right-jumps $q_+ > p$)

- R-signals: belief jumps to q_+ at arrival rate of $\frac{p(1-p)}{|q_+-p|}\lambda$
- L-signals: belief drifts to the left at rate $\dot{p}_t = -\lambda p(1-p)$

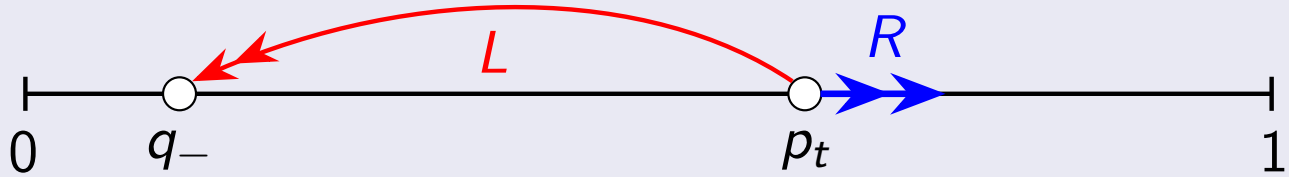


- Sender may choose the "precision" of R-evidence.
  - For example: can target $q_+ = \hat{p}$.
- **Tradeoff**: More precise signals are slower to generate.

Feasible experiments: Continuous Time

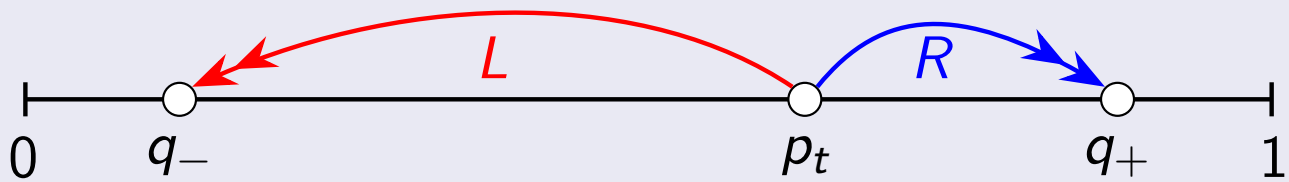
R-drifting experiment (with left-jumps to $q_- < p$):

- L-signals: belief jumps to q_- at rate $\frac{p(1-p)}{|q_- - p|}\lambda$
- R-signals: belief drifts toward right at rate $\dot{p}_t = \lambda p(1-p)$



"Stationary" Experiment

- Splitting attention ($\alpha = 1/2$), we obtain **2 jumps and no drift**
- Jumps to q_- and q_+ at rates $\frac{\lambda p(1-p)}{2|q_\bullet - p|}$,—no drift.



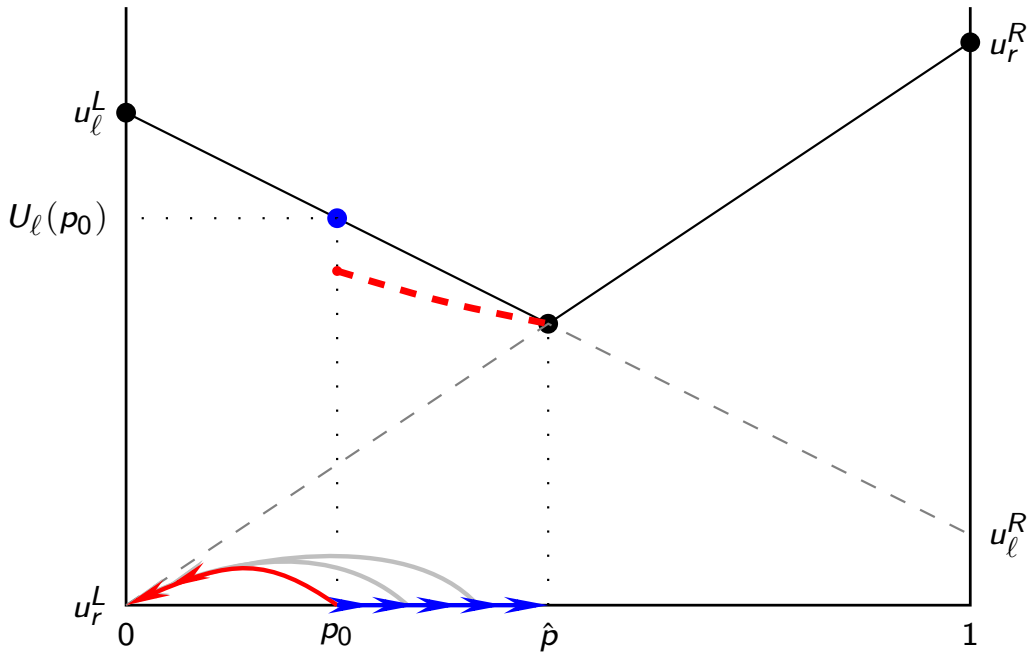
Literature

- Bayesian Persuasion: Kamenica and Gentzkow (2011,...), ..., Aumann/Maschler (1995)
- Wald Decision: Wald (1947), Arrow, Blackwell, and Girshick (1949), Moscarini and Smith (2001), Che and Mierendorff (2017), Nikandrova and Pancs (2015), Mayskaya (2017), Zhong (2018), Henry and Ottaviani (2017), McClellan (2017)
- Dynamic/Repeated Persuasion/Communication: Renault, Solan and Vieille (2013), Margaria and Smolin (2015), Ely (2017), Kremer, Mansour and Perry (2014), Che and Hörner (2018), Bizzoto, Rudiger and Vigier (2017), Best and Quigley (2017), Mathevet, Pearce, and Stachetti (2018), Orlov, Skrzypacz and Zryumov (2018).

  Difference: Permanent state, MPE, slow learning.

Dynamic Implementation of KG Solution

- Fix $p_0 < \hat{p}$.
- Can replicate KG: dynamic experiment that leads to beliefs 0 and $\hat{p}$
- For example: R-drifting experiment until belief reaches $\hat{p}$.

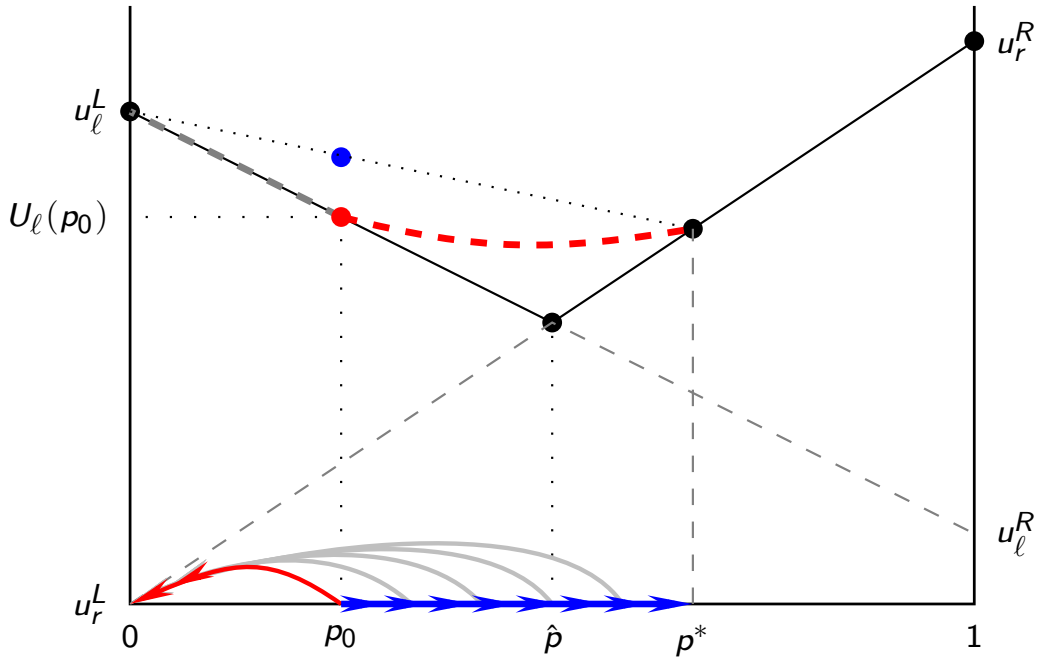


- **Problem:** Receiver does not wait if she does not get rent that compensates for flow cost.

  $\Rightarrow$ **KG experiment can't persuade receiver to listen.**

Fix: Dynamic Commitment

- *Solves the problem if Sender can commit to future experiments*
  - Example: Commit to R-drifting until the belief reaches $p^* > \hat{p}$.



- Similar to KG except to offer "rents" to compensate for Receiver's flow cost. Can approximate KG if $c \to 0$.
- *But will this work without commitment?*

Is persuasion possible without commitment?

- **No**
  - There is an MPE with total persuasion failure regardless of $c > 0$.
  - For any $c > 0$ and any MPE, there always exists a range of prior beliefs for which "no persuasion" occurs.

- **Yes**
  - For each $p_0 < \hat{p}$, some dynamic commitment can be supported as MPE if c is low enough.
  - As $c \to 0$, a KG experiment as well as full revelation (and anything in between) is dynamically credible. $\Rightarrow$ **Folk Theorem**

MPE: Persuasion Failure

Theorem (Persuasion Failure MPE)

For any $c > 0$, there exists a MPE in which no persuasion occurs.

Proof.

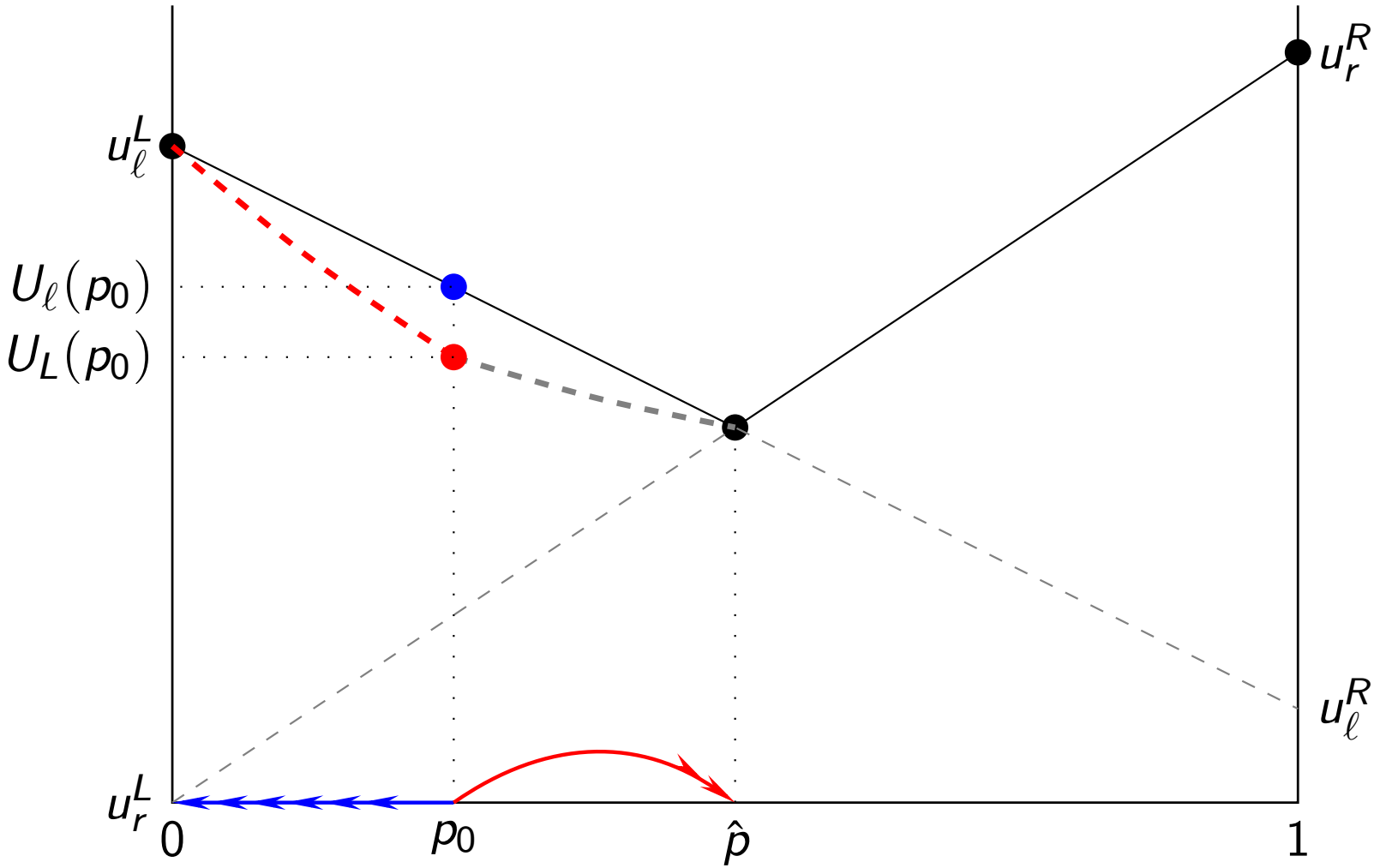
MPE strategy profile:

- Receiver never waits—he picks r if $p \geq \hat{p}$ and ℓ for $p < \hat{p}$.
- Sender
  - "passes" if $p \geq \hat{p}$ or $p < \pi_{\ell L}$, where

$$c = \frac{\lambda \pi_{\ell L}(1 - \pi_{\ell L})}{\hat{p} - \pi_{\ell L}}$$

  - performs an L-drifting experiment with jumps to $\hat{p}$ if $p \in [\pi_{\ell L}, \hat{p})$.

Persuasion failure: illustration



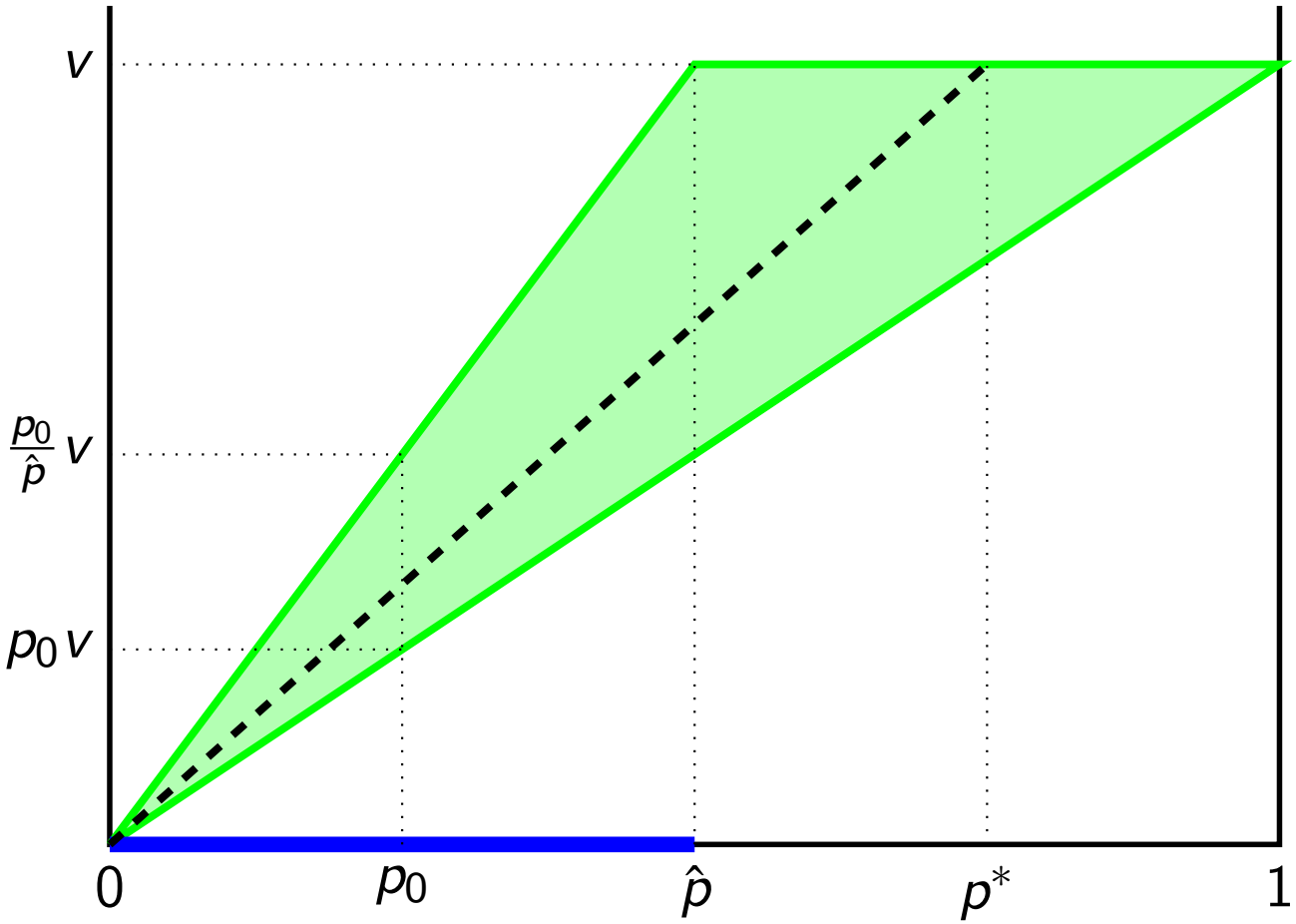
Persuasion MPE: Folk Theorem

More surprisingly, persuasion is possible in MPE. In fact, we can establish a folk theorem.

Theorem (Folk theorem)

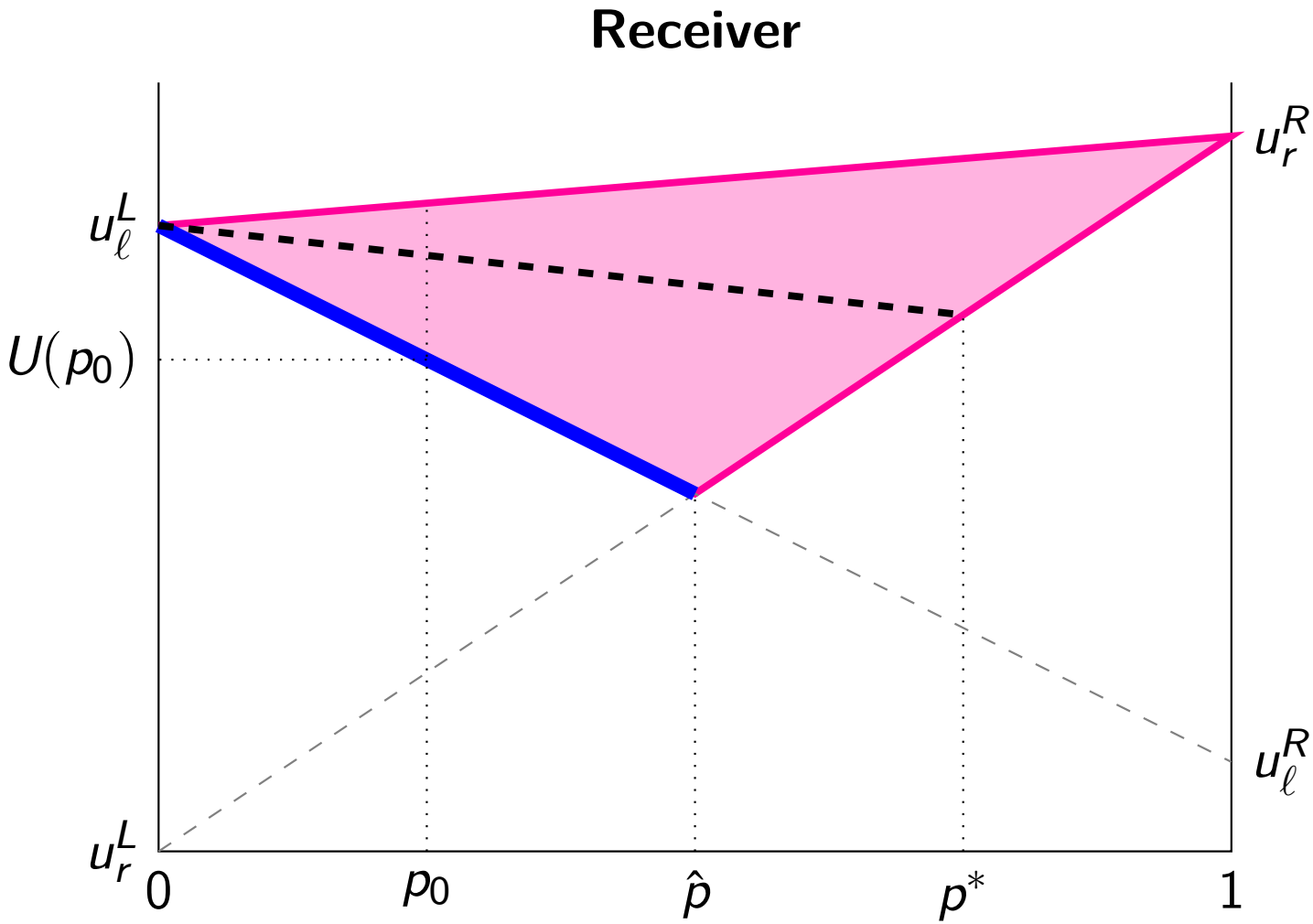
Any payoff vector between KG benchmark and "full revelation" is supported in an MPE for any c sufficiently small.

Persuasion MPE: Folk Theorem — Sender's Payoff Set



as $c \to 0$.

Persuasion MPE: Folk Theorem — Receiver's Payoff Set



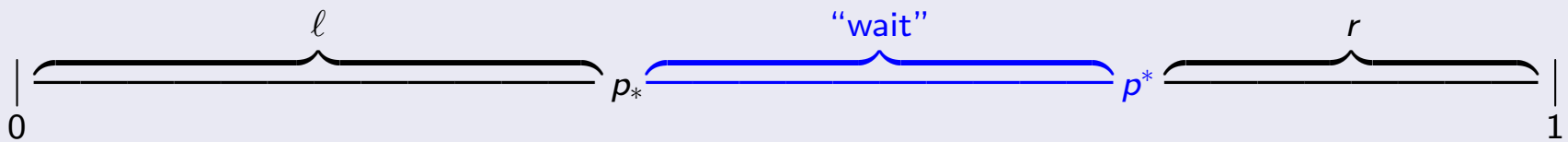
as $c \to 0$.

Illustration of Folk Theorem

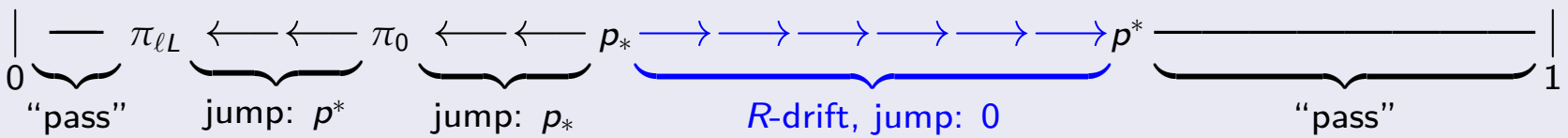
- Fix any $p^* \in (\hat{p}, \eta)$, where $\eta \approx .94325$.
- Then, there exists a persuasion MPE with persuasion target p^*:

Persuasion MPE

Receiver's strategy:



Sender's strategy:



- Approximating KG: Can choose p^* such that $p^* \to \hat{p}$ and $p_* \to 0$ as $c \to 0$.

Intuition: *Power of Beliefs*

- Why is Sender continuing to experiment even after reaching $\hat{p}$? Why not stop at $\hat{p}$?

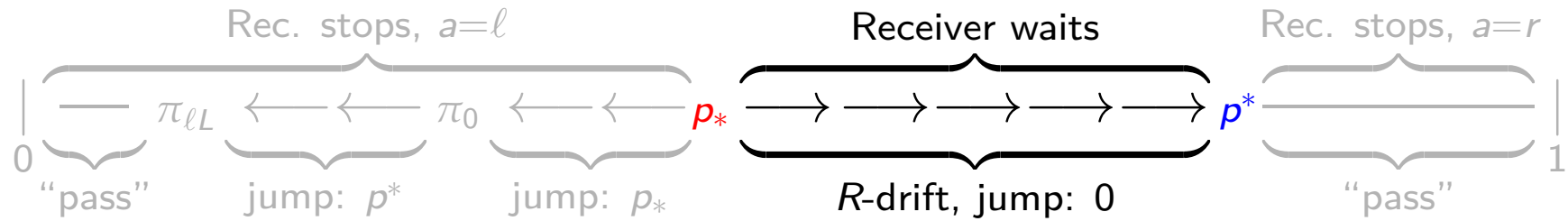
$\Rightarrow$ Because Receiver would never choose r at $\hat{p}$ and would rather wait.

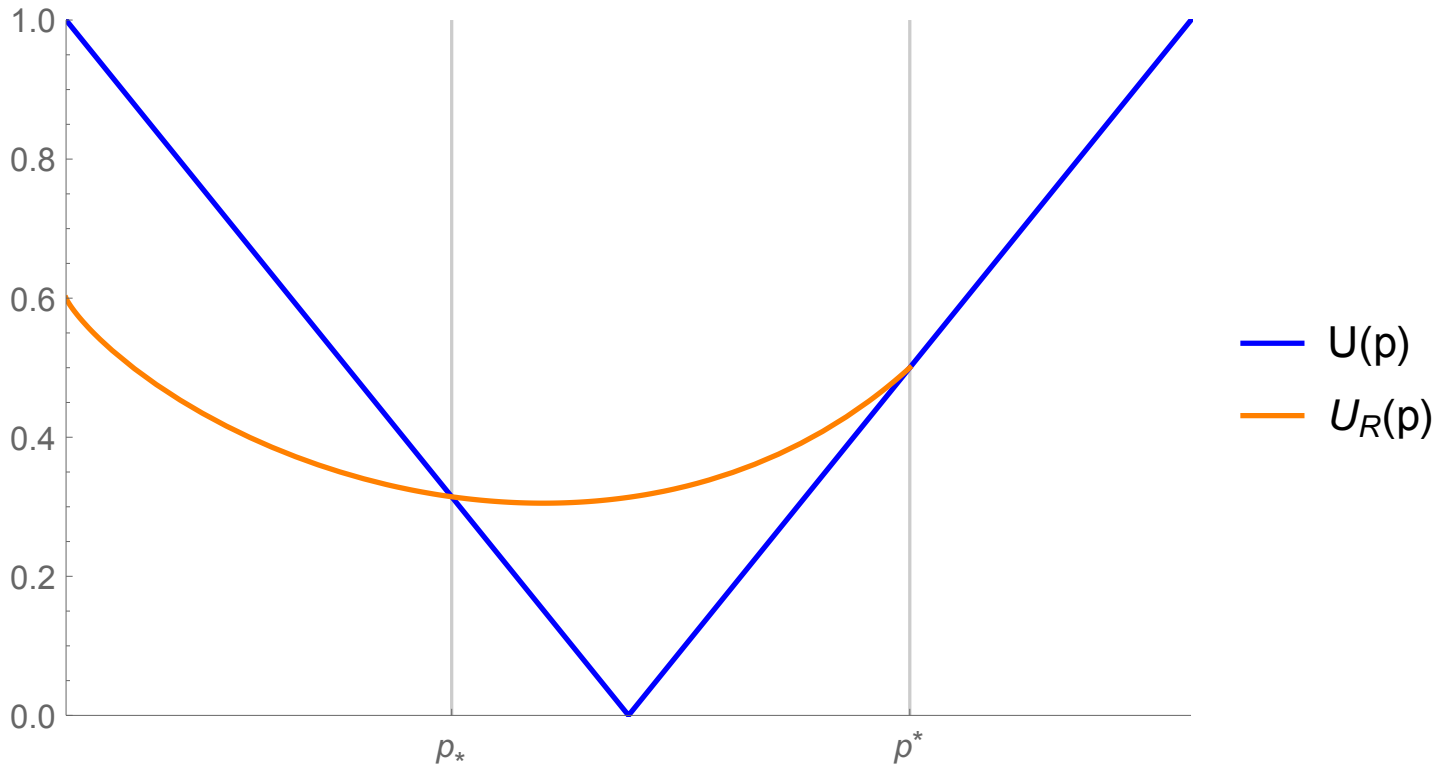
- Why? Why is Receiver waiting even after $\hat{p}$ is reached?

$\Rightarrow$ Because, if Receiver waits, Sender will continue experimenting.

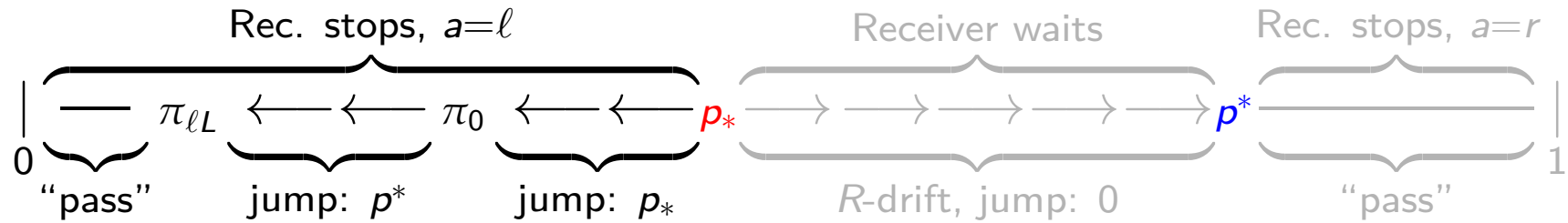
- Power of equilibrium beliefs: reminiscent of Che and Sákovics, ECMA, 2004.

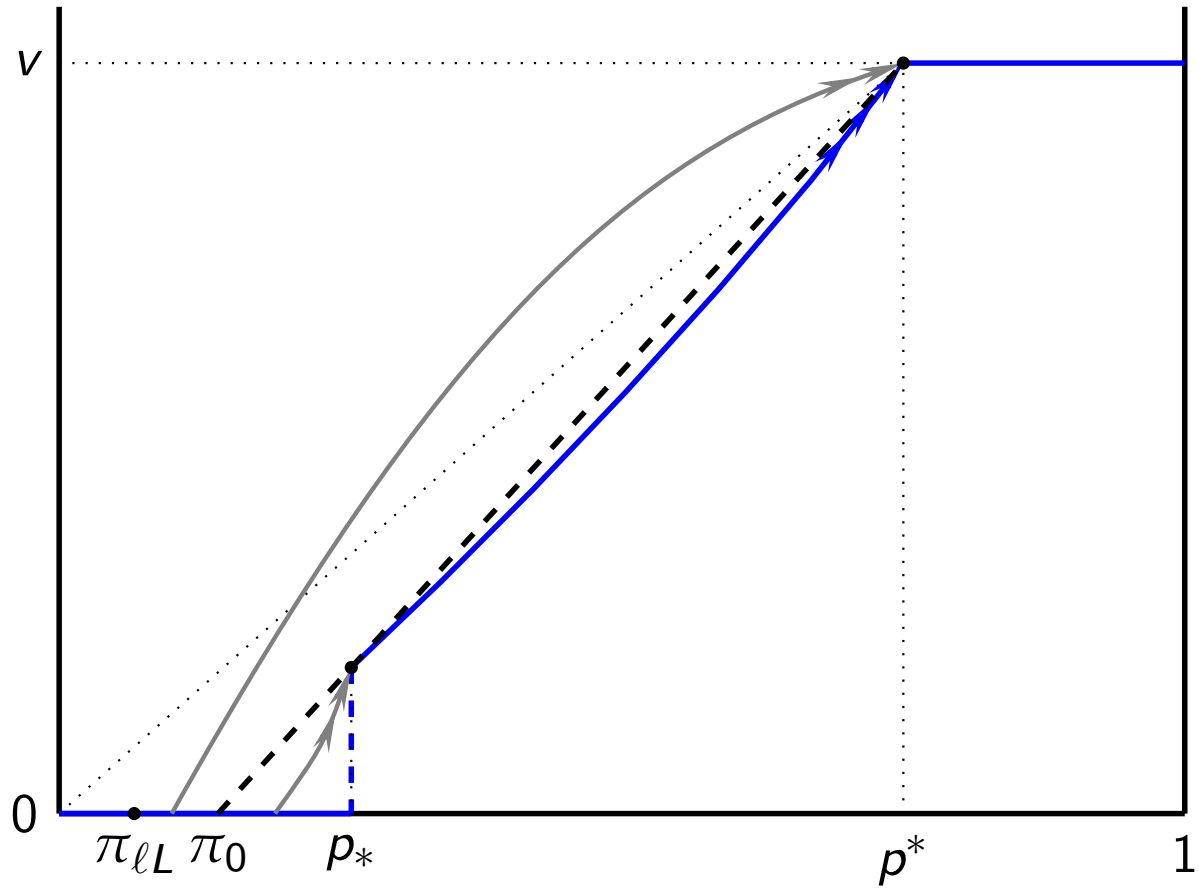
Receiver Incentive



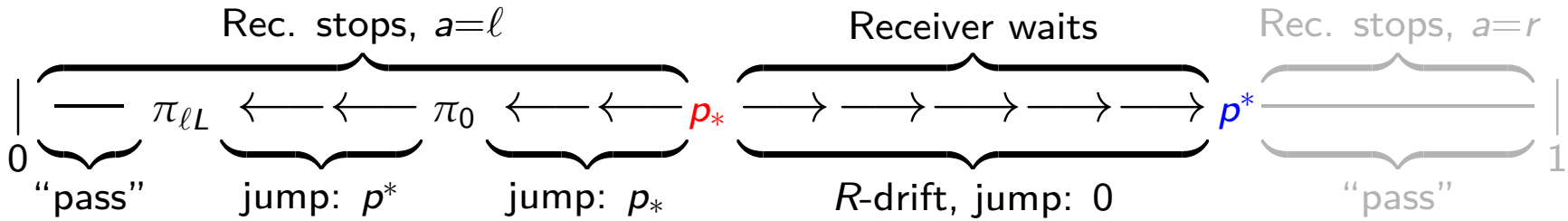


Sender Incentive





Art and Dynamics of Persuasion



- When Receiver is already interested in listening (i.e., $p \in (p_*, p^*)$):

$\Rightarrow$ Slow build-up of reputation, interspersed with rare breakdowns.

$\Rightarrow$ Persuasion is backloaded; no persuasion until some time followed by once-and-for-all persuasion,

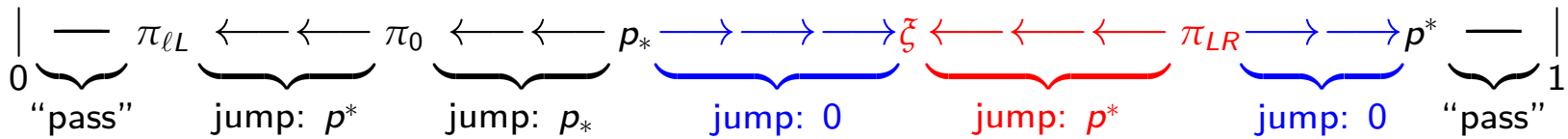
- When Receiver is skeptical (i.e., $p < p_*$):

$\Rightarrow$ Seller throws "Hail Mary" to gain "privilege to be listened to" or "immediate persuasion" (becomes more risk taking as p falls.)

$\Rightarrow$ Persuasion likely fails, except for a rare success.

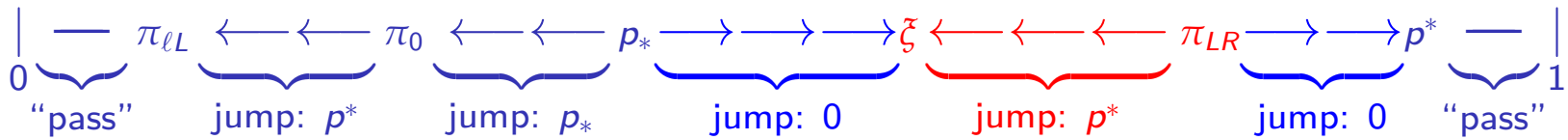
Equilibria that approximate Full Revelation

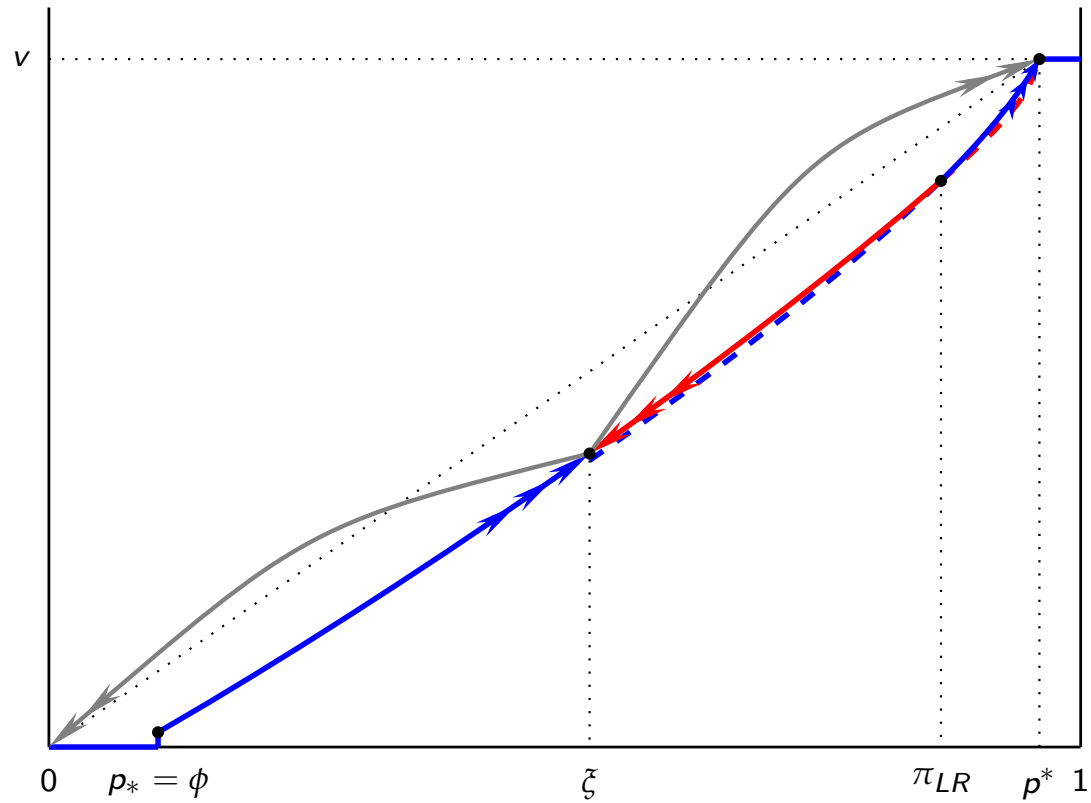
- Fix any $p^* > \eta \approx 0.94325$.
- For $c > 0$ sufficiently small, an MPE with the following sender strategy:



At ξ: stationary strategy with jump targets $q_- = 0$, $q_+ = p^*$.

Approximating Full Revelation

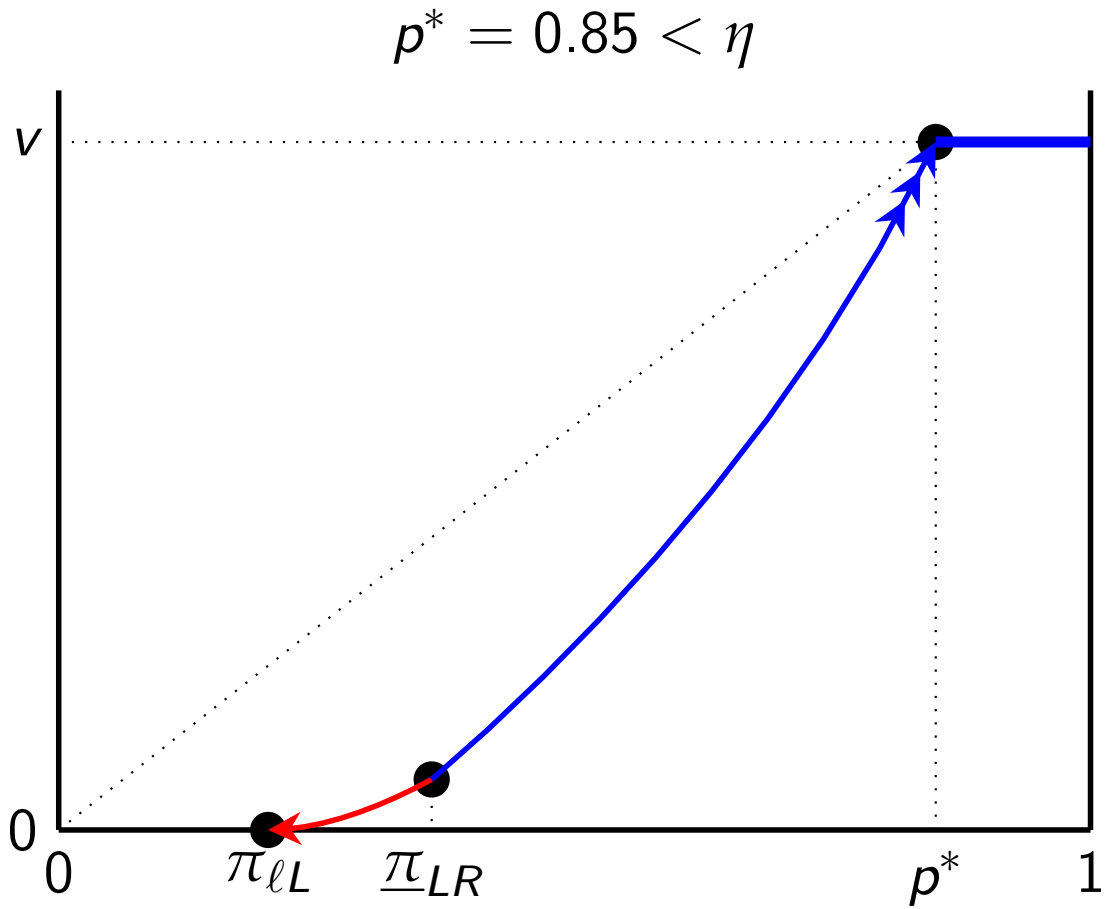




- $p_* \to 0$ as $c \to 0$
- $\pi_{LR} \to 1$ and $\xi \to 1/2$ as $p^* \to 1$.

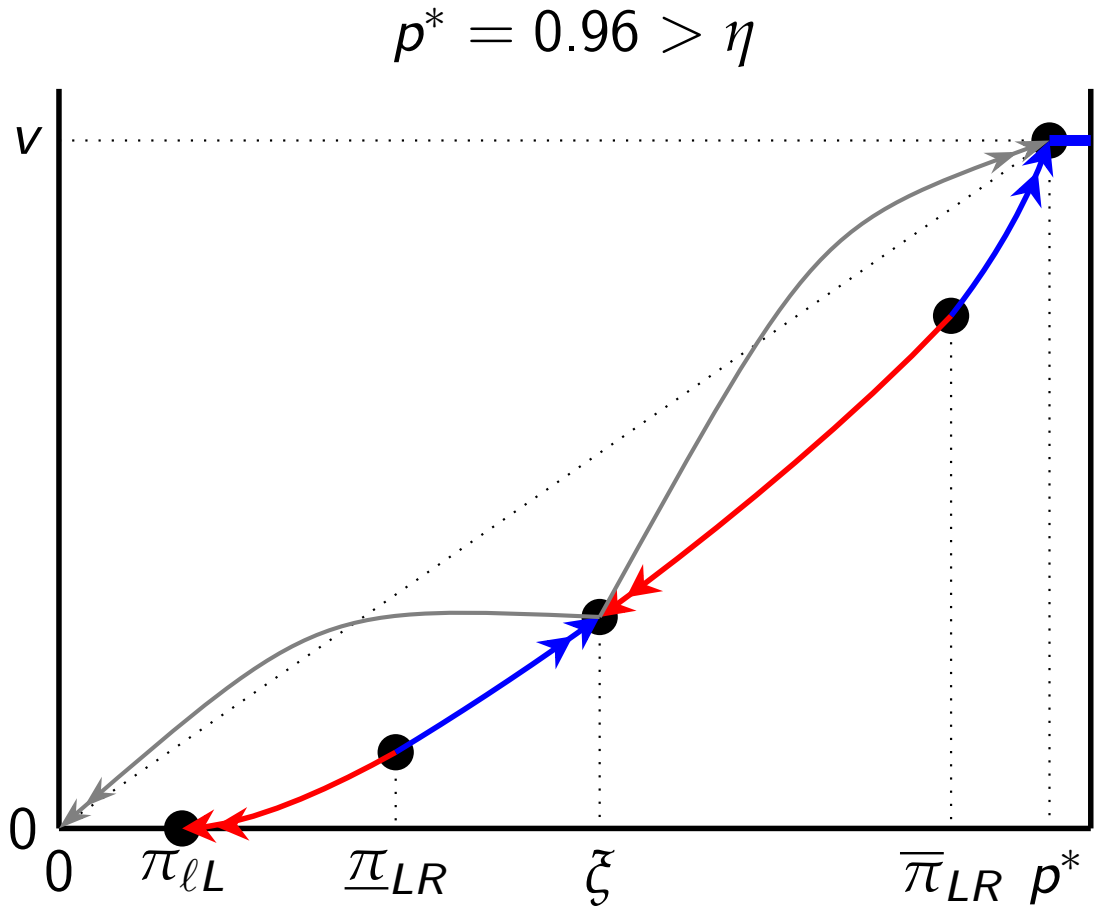
When Sender has Weak Incentive to Persuade

Suppose that $v \leq U_r(p^*) - U_\ell(p^*)$. In this case, the receiver has a stronger incentive to continue than the sender.
$\Rightarrow$ The sender makes a stopping decision, so $p_* = \phi_{\ell L}$.



When Sender has Weak Incentive to Persuade

If $p^* > \eta$, then



Summary and Discussion

Is persuasion possible, and if so, what form does it take, if *persuasion takes time and cost for both sender and receiver*, and *neither party has commitment power*?

- There is a rich set of MPE's including
  - "total failure of persuasion".
  - Persuasive ones that implement (as $c \to 0$, for all prior beliefs):
    - KG outcome,
    - full information
    - and anything in between
    - Power of beliefs makes sustained persuasion possible.
    - **Behavioral Predictions:** Different Persuasion Dynamics
- Tractable model of dynamic strategic information choice.

Thank you!