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AN ANALYTICAL APPROACH TO THE LIQUIDITY EFFECTS OF MONETARY POLICY*

YOUNG SIK KIM** · MANJONG LEE***

This paper characterizes analytically the persistent liquidity effects of a money injection in a model economy with goods market segmentation as well as financial market segmentation. Households in financial sectors receive money transfers from the central bank and have access to the bond market, while others outside financial sectors do not receive money transfers and have no access to the bond market. Also, a shock that causes households to relocate between financial sectors leads to more consumption expenditure for both types of households. This goods market effect then causes a money injection to diffuse through the economy over time, generating persistent liquidity effects even when the money growth shock is transitory. This is empirically important because persistent liquidity effects have been observed in many countries with a relatively low persistence of money growth.

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I. INTRODUCTION

At least since Friedman (1968), open market operations have been thought to have persistent liquidity effects by which money injections

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** The first author: Department of Economics and SIREF, Seoul National University, E-mail: Kimy@snu.ac.kr
*** Corresponding author: Department of Economics, Korea University, E-mail: manjong@korea.ac.kr

lead initially to a decline in short-term nominal interest rates, eventually rising to normal levels or higher. That is, money injections are thought to steepen the yield curve, lowering long-term rates less than short-term rates. The vector autoregression (VAR) studies have also confirmed these patterns in the data (e.g., Cochrane 1994, Christiano et al. 1998). It is also worth noting that these persistent liquidity effects have been observed even when the persistence of money injections is relatively low. For example, the autoregressive coefficient of money growth process is estimated 0.45 using the quarterly series of $M1$ in the U.S. from 1959 to 2006. The corresponding estimates for Germany, Japan, and Australia are respectively 0.38 (1957-1998), 0.37 (1957-2006), and 0.15 (1957-2006).

The goal of this paper is to derive analytically these persistent liquidity effects in the context of an endowment economy version of Williamson (2008) which captures heterogeneity and distributional effects of monetary policy in a tractable manner. The key features of the model economy are segmented markets for both asset and goods. As analogous to Grossman and Weiss (1983) and Rotemberg (1984), some households are “financially connected” in the sense that they have access to bond market and receive money transfers from the central bank, whereas some are “financially unconnected” so that they do not receive money transfers and have no access to bond market.

After money transfers being made, each consumer in the household (financially connected or unconnected) faces a relocation shock. There is a positive probability with which she will either stay financially connected/unconnected as before or “move” to another location which is financially connected or unconnected. This has the effect of segmenting goods market such that consumers in the financially connected (unconnected) households are more likely to engage in consumption trades with other financially connected (unconnected) households. A relocation shock also causes a money injection by the central bank to diffuse through the economy over time so that it yields persistent distributional effects on real and nominal interest rates as well as on consumption.

The key result of the paper is that a money growth shock decreases equilibrium nominal interest rates immediately and raises them back to normal levels over time, generating persistent liquidity effects. It is shown analytically that, besides the contemporaneous money injections, the lagged money growth shock decreases current nominal interest rates by affecting the current distribution of real balances of money across the economy. This serves a key mechanism by which even a transitory money growth shock, including an i.i.d. process, to generate persistent liquidity effects as observed in many countries.

Intuitively, a positive money growth shock increases consumption of financially connected households and decreases their current marginal utility. As a result, an increase in money growth decreases real interest rates measured as the inverse of intertemporal marginal rate of substitution. Further, a money injection increases initially the price level in financially connected sectors. As the money injection is diffused through the overall economy over time, the price level in connected sectors will decline over time, so deflation is anticipated. As a result, both the real interest rate effect and the Fisher effect of a money injection work in the same direction to generate liquidity effects.

In addition to the liquidity effects of contemporaneous money injections, the diffusion of money injections over time has persistent impact on the distribution of money balances across connected and unconnected sectors. Hence, the lagged money growth shock also decreases current nominal interest rates by affecting the current distribution of money balances across the economy. This generates persistent liquidity effects even when the persistence of money growth shock is relatively low.

Moreover, both the real interest rate effect and the Fisher effect of money injections fall as the money growth process becomes more persistent, say, a random walk. For instance, when the shock to money growth persists, a money injection leads to a permanent increase in both current and future consumptions and hence to a smaller decrease in the real interest rate. As long as the money diffusion takes place for sufficiently long periods of time, the persistent distributional effect of money injections can still generate liquidity effects even when the money growth follows a random walk.

Using an endowment economy version of the model, Williamson

(2008) shows analytically how a permanent level increase in the money supply has the liquidity effect and the Fisher effect on the nominal interest rate with the emphasis on the role of the degree of financial market segmentation and goods market segmentation. This paper differs from Williamson (2008) in that we show analytically the liquidity effect (or real interest rate effect) and the Fisher effect of a money growth shock instead of a one-off level increase in the money supply. A change in the money growth rate is shown to have additional effects on the nominal interest rate by altering the distribution of money balances across the population over time in the presence of goods market segmentation. Hence, we show analytically that, in addition to the real interest rate effect and the Fisher effect of the current money growth, goods market segmentation generates persistent distributional effects of money injections on real interest rate and expected inflation. This analytical approach is complementary to Williamson (2008) in which persistent liquidity effects of a money growth shock are illustrated numerically.

These results also generalize those of cash-in-advance models with or without market segmentation. In Lucas (1982) and Svensson (1985), there is a complete financial connectedness and hence no distributional effects of monetary policy. Money injections increase nominal interest rates by increasing expected inflation with no effect on the real interest rate. Models with distributional implications of monetary policy began with the limited participation models by Grossman and Weiss (1983) and Rotemberg (1984).[1] In the Grossman-Weiss- Rotemberg models, there are always some economic agents who are not participating in financial markets and hence do not receive the first-round effects of an open market operation. A money injection causes a redistribution of wealth, leading to short-run changes in nominal interest rates and the distribution of consumption across the population.

More recently, Lucas (1990), Fuerst (1992), Alvarez and Atkeson (1997), and Alvarez et al. (2002) also examined asset pricing implications of monetary policy via its distributional effects. However, for tractability, they only considered within-the-period redistribution of wealth and

[1] See Edmond and Weill (2008) for a survey of limited participation models of the liquidity effect.

essentially abstract from the persistence in distributional effects of monetary policy. A money injection by the central bank diffuses through the economy contemporaneously so that within-the-period distributional effects lead to a fall in the real interest rate and an increase in expected inflation. Hence, liquidity effects of monetary policy are generated when the real interest rate effect dominates the expected inflation effect. In particular, as shown in Alvarez et al. (2002), the persistent liquidity effects hinge critically on sufficiently persistent money growth shocks.[2]

This is in a contrast to the implications of the current model where it takes time for a money injection to diffuse through the economy. The persistent impact of a money growth shock on the distribution of money balances plays a key role in generating the persistent liquidity effects even when the money growth follows an i.i.d. process. Therefore, the implied persistent liquidity effects of monetary policy are not only robust to the money growth processes, but also plausible from the point of real world observations as mentioned earlier.

II. MODEL

The background environment is an endowment-economy version of Williamson (2008). There is a continuum of islands with unit mass indexed by $i \in [0,1]$. At each island, there is an infinitely lived household, consisting of a seller and a continuum of consumers with unit mass indexed by $j \in [0,1]$. The preferences of a household on island i are given by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 u\left(c^{ij}\right) d\lambda(j) \tag{1}$$

where $\beta \in (0,1)$ is discount factor, c^{ij} is the consumption of consumer j in household i, and $\lambda(\cdot)$ denotes the measure of consumers in the household. Assume that $u(\cdot)$ is twice continuously differentiable and

[2] Besides Alvarez et al. (2002), models with endogenously segmented markets include Alvarez et al. (2003), Chiu (2005), and Khan and Thomas (2007). For example, Khan and Thomas (2007) allow for endogenous changes in the timing of households' participation in asset markets and show numerically persistent liquidity effects following an open market operation.

strictly concave with $u'(0) = \infty$. Each household is assumed to be endowed with y units of consumption goods.

There is a fraction α of financially *connected* islands and the other fraction $(1-\alpha)$ of financially *unconnected* islands where $0 < \alpha < 1$. At the beginning of period t, the household on island i has m_t^i units of fiat money. All the households on connected islands then receive an identical money transfer Γ_t from the central bank and have access to bond market. Households on financially unconnected islands do not receive transfers and have no access to bond market. Note that $\alpha \in (0,1)$ captures the degree of financial connectedness (or financial market segmentation) and hence distributional effects of monetary policy.

After money transfers being made to the financially connected islands, each consumer in the household on both connected and unconnected islands receives a relocation shock. There is a probability π that a consumer is randomly relocated to another island, and a probability $1-\pi$ that the consumer stays on the same island. The relocation shock of an individual consumer is assumed to be unobservable to the other members of the household, as is her consumption quantity. Further, communication among islands and record-keeping are limited, so that consumption goods must be purchased with money. That is, consumers face cash-in-advance constraints.

After receiving their relocation shocks, consumers are allocated money by the household, and they then go shopping to other islands. While consumers shop for goods, sellers remain at home and sell goods to consumers arriving from other islands. When consumers purchase consumption goods, these goods must be consumed on the spot before the consumers return to their home. Hence, the household's consumers cannot share risk by returning to their home and pooling their consumption goods.

It will be shown below that, in the presence of the relocation shock $\pi \in (0,1)$ as described above, the goods markets are essentially segmented in the sense that consumption trades are more likely to occur between the agents respectively from the financially connected or unconnected islands. That is, π captures the degree of goods market segmentation, which also has the effect of governing the speed with which money injections on connected islands diffuse through the economy over time and hence the persistence in distributional effects of monetary policy.

III. EQUILIBRIUM

In the model economy with segmented markets, consumers at different islands face different prices for consumption goods. Here, an equilibrium will be considered where prices are identical respectively on all connected islands and on all unconnected islands. Let p_{1t} and p_{2t} denote respectively the price of goods in terms of money on connected and unconnected islands.

III-1. Households' Optimization

If the household could observe where each consumer will be located, it would give each consumer a different allocation of money. However, since the relocation shock of an individual consumer is unobservable and each consumer wishes to maximize her own consumption, each consumer will report the relocation shock to the household that implies the largest money allocation. Hence, it is optimal for the household to allocate the same quantity of money to each consumer, since any randomness in money allocation would reduce the expected utility of the risk-averse consumers of the household.

III-1-1. Financially connected households

A representative household on a connected island has m_{1t} units of money balances at the beginning of period t, and receives money transfers Γ_t. Households on connected islands can also trade in the bond market at the beginning of the period. Each bond acquired in period t is a claim to one unit of money in period $t+1$, and let q_t denote its price in units of money.

Let $\hat{m}_{1t}$ denote the quantity of money allocated by a representative household on a connected island to each consumer, and let b_t denote the nominal bonds acquired at $t-1$ by the household that mature in period t. Then, this household faces the following cash-in-advance constraint at t:

$$q_t b_{t+1} + \hat{m}_{1t} \leq m_{1t} + b_t + \Gamma_t. \tag{2}$$

When relocation shocks are realized for the household on a connected island, $[1-(1-\alpha)\pi]$ fraction of consumers in the household stay on connected islands, with each consuming c_t^{11} goods purchased at the price p_{1t}. Also, $(1-\alpha)\pi$ fraction of consumers goes to unconnected islands and consumes c_t^{12} goods purchased at the price p_{2t}. Each consumer wishes to consume as much as possible, so that she spends entire money allocation given by the household:

$$p_{1t} c_t^{11} = p_{2t} c_t^{12} = \hat{m}_{1t}. \tag{3}$$

The seller remains at the home location, selling y units of consumption goods at the unit price p_{1t}. The household also chooses money balances $m_{1,t+1}$ to carry over next period. The household's intertemporal budget constraint is then given by

$$q_t b_{t+1} + \hat{m}_{1t} + m_{1,t+1} = p_{1t} y + m_{1t} + b_t + \Gamma_t. \tag{4}$$

That is, bond purchases and money balances allocated to each consumer of the household plus saved money balances equals total revenue from sales of goods, beginning-of-period money balances and bond holdings, plus money transfers from the central bank. Now, from the preferences given by (1), the representative household on a connected island maximizes the following lifetime expected utility:

$$\mathsf{E}_0 \sum_{t=0}^{\infty} \beta^t \{[1-(1-\alpha)\pi] u(c_t^{11}) + (1-\alpha)\pi u(c_t^{12})\}$$

subject to the cash-in-advance constraints (2)-(3) and the budget constraint (4).

III-1-2. Financially unconnected households

A representative household on an unconnected island begins period t with m_{2t} units of money, and allocates $\hat{m}_{2t}$ to each consumer in the household. Given that households on an unconnected island do not have access to the bond market and receive no transfer from the central bank, the cash-in-advance constraint is given by

$$\hat{m}_{2t} \le m_{2t}. \tag{5}$$

After receiving relocation shocks, $\alpha\pi$ fraction of consumers in the representative household on an unconnected island go to connected islands and each consumes c_t^{21}, while $(1-\alpha\pi)$ fraction of consumers stay on unconnected islands with each consuming c_t^{22}. Like consumers on connected islands, each consumer spends her entire money allocation from the household, so that

$$p_{1t}c_t^{21} = p_{2t}c_t^{22} = \hat{m}_{2t}. \tag{6}$$

The seller at the home location sells y units of consumption goods at the unit price p_{2t}. The household on an unconnected island also chooses money balances $m_{2,t+1}$ to carry over next period. Then the intertemporal budget constraint is given by

$$\hat{m}_{2t} + m_{2,t+1} = p_{2t}y + m_{2t}. \tag{7}$$

That is, money balances allocated for consumption to each consumer of the household and saved money balances equals total revenue from sales of goods plus beginning-of-period money balances. Now, the representative household on an unconnected island maximizes the lifetime expected utility given by

$$\mathsf{E}_0 \sum_{t=0}^{\infty} \beta^t \{\alpha\pi u(c_t^{21}) + (1-\alpha\pi)u(c_t^{22})\}$$

subject to the cash-in-advance constraints (5)-(6) and the budget constraint (7).

Notice that, as long as $\pi < 1$, the fraction of consumers in the connected household who stay on connected islands, $[1-(1-\alpha)\pi]$, is larger than the fraction of consumers in the unconnected household who move to connected islands, $\alpha\pi$. Similarly, the fraction of consumers in the unconnected household who stay on unconnected islands, $(1-\alpha\pi)$, is larger than the fraction of consumers in the connected household who move to unconnected islands, $(1-\alpha)\pi$. Therefore, with the *partial* relocation of consumers across connected and unconnected islands, goods markets are essentially segmented in the sense that the trades of consumption goods are more likely to take place between the agents residing respectively in the financially connected or unconnected islands.

III-2. Market Clearing Conditions

In equilibrium, the cash-in-advance constraints (2) and (5) hold with equality. Also, in the symmetric equilibrium, $b_t = 0$ for all t. Then, since each household spends all of its beginning-of-period money balances on consumption goods each period (and hence no carry-over of beginning-of-period money balances to another location next period), the path for the money stock at each location is exogenous to each other.

Let M_{1t} denote the period-t supply of money per household on each connected island after the transfer is made from the central bank, and let M_{2t} denote the supply of money per household on each unconnected island. At each location on a connected island, during period t, there will be a total of $[1-(1-\alpha)\pi]$ consumers who will arrive from connected islands and each will spend M_{1t} on consumption goods, while $(1-\alpha)\pi$ consumers will arrive from unconnected islands and each will spend M_{2t} on consumption goods. Similarly, at a location on an unconnected island, $(1-\alpha\pi)$ consumers will arrive from unconnected islands with each spending M_{2t}, and $\alpha\pi$ consumers will arrive from connected islands with each spending M_{1t}.

The market-clearing conditions for the money markets in the connected and unconnected islands are respectively:

$$p_{1t} y = [1-(1-\alpha)\pi]M_{1t} + (1-\alpha)\pi M_{2t} \tag{8}$$

$$p_{2t} y = \alpha\pi M_{1t} + (1-\alpha\pi) M_{2t} \tag{9}$$

where $y = [1-(1-\alpha)\pi]c_t^{11} + (1-\alpha)\pi c_t^{21}$ on a connected island and $y = \alpha\pi c_t^{12} + (1-\alpha\pi)c_t^{22}$ on an unconnected island. Further, the money stocks per household at connected and unconnected islands evolve respectively according to

$$M_{1,t+1} = [1-(1-\alpha)\pi]M_{1t} + (1-\alpha)\pi M_{2t} + \Gamma_{t+1} \tag{10}$$

$$M_{2,t+1} = \alpha\pi M_{1t} + (1-\alpha\pi)M_{2t}. \tag{11}$$

Note that, for a given $\alpha \in (0,1)$ which captures distributional effects of monetary policy, π governs the speed of diffusion of a money injection by the central bank. That is, it captures how quickly a money injection becomes diffused through the economy. If $\pi = 1$ in which case all consumers are relocated to another island, then from (8) and (9) the same quantity of money is spent in all locations in each period, so that diffusion occurs within the period of a money injection. As will be shown in the following section, this is the case of Alvarez et al. (2002). If $\pi = 0$, then $M_{2t} = M_{20}$ for all t, whereas M_{1t} is governed entirely by the history of central bank transfers.

III-3. Competitive Equilibrium

A *competitive equilibrium* consists of a sequence of prices $\{p_{1t}, p_{2t}\}_{t=0}^{\infty}$ that satisfy (8) and (9) where $\{M_{1t+1}, M_{2t+1}\}_{t=0}^{\infty}$ are determined by (10) and (11) with given (M_{10}, M_{20}), and consumption quantities $\{c_t^{11}, c_t^{12}, c_t^{21}, c_t^{22}\}_{t=0}^{\infty}$ given by

$$c_t^{11} = \frac{M_{1t}}{[1-(1-\alpha)\pi]M_{1t} + (1-\alpha)\pi M_{2t}} y \tag{12}$$

$$c_t^{12} = \frac{M_{1t}}{\alpha\pi M_{1t} + (1-\alpha\pi)M_{2t}} y \tag{13}$$

$$c_t^{21} = \frac{M_{2t}}{[1-(1-\alpha)\pi]M_{1t} + (1-\alpha)\pi M_{2t}} y \tag{14}$$

$$c_t^{22} = \frac{M_{2t}}{\alpha\pi M_{1t} + (1-\alpha\pi)M_{2t}} y. \tag{15}$$

A sequence of equilibrium bond prices $\{q_t\}_{t=0}^{\infty}$ is obtained after substituting these in the following bond-pricing equation:

$$q_t \left\{ [1-(1-\alpha)\pi]u'(c_t^{11}) + (1-\alpha)\pi u'(c_t^{12}) \frac{p_{1t}}{p_{2t}} \right\}$$
$$= \beta \mathsf{E}_t \left\{ [1-(1-\alpha)\pi]u'(c_{t+1}^{11}) \frac{p_{1t}}{p_{1,t+1}} + (1-\alpha)\pi u'(c_{t+1}^{12}) \frac{p_{1t}}{p_{2,t+1}} \right\}. \tag{16}$$

Let the gross growth rate of M_{it} be μ_t for $i=1,2$. Then the period-t money transfers received by each household on connected islands becomes:

$$\Gamma_t = (\mu_t - 1)\left[M_{1,t-1} + \left(\frac{1-\alpha}{\alpha}\right) M_{2,t-1} \right]. \tag{17}$$

Further, let δ_t denote the period-t ratio of per-household money balances on connected islands to per-household money balances on unconnected islands:

$$\delta_t \equiv M_{1t} / M_{2t}. \tag{18}$$

This captures the distribution of money balances across connected and unconnected islands. Assuming that the cash-in-advance constraints (2) and (5) hold with equality and using (10), (11), (17) and (18), the law of motion for δ_t becomes

$$\delta_{t+1} = \frac{\alpha[\mu_{t+1} - (1-\alpha)\pi]\delta_t + (1-\alpha)(\mu_{t+1} - 1 + \alpha\pi)}{\alpha^2 \pi \delta_t + \alpha(1-\alpha\pi)} \equiv \delta(\mu_{t+1}, \delta_t). \tag{19}$$

It is clear from (19) that δ_t is serially correlated and hence there is persistence in the distribution of money balances across islands. This is due to $\pi < 1$ so that it takes time for a money growth shock to diffuse through the economy. When $\pi = 1$ so that the diffusion occurs within the period, (19) is reduced to

$$\delta_t = (\mu_t - 1 + \alpha) / \alpha$$

where the distribution of money balances across the population (δ_t) in period t is determined by the contemporaneous money growth rate (μ_t), and hence there is no persistence.

Now, in a competitive equilibrium, the period- t consumption allocations (12) through (15) can be rewritten as a function of δ_t which evolves according to the law of motion given by (19):

$$c^{11}(\delta_t) = \frac{\delta_t y}{[1-(1-\alpha)\pi]\delta_t + (1-\alpha)\pi} \tag{20}$$

$$c^{12}(\delta_t) = \frac{\delta_t y}{\alpha\pi\delta_t + 1 - \alpha\pi} \tag{21}$$

$$c^{21}(\delta_t) = \frac{y}{[1-(1-\alpha)\pi]\delta_t + (1-\alpha)\pi} \tag{22}$$

$$c^{22}(\delta_t) = \frac{y}{\alpha\pi\delta_t + 1 - \alpha\pi}. \tag{23}$$

Substituting these in the bond-pricing equation (16) yields the period-t equilibrium bond prices $q_t = q(\mu_{t+1}, \delta_{t+1}, \delta_t)$ which satisfies the following:

$$\begin{aligned} &q_t\{[1-(1-\alpha)\pi]u'[c^{11}(\delta_t)] + (1-\alpha)\pi u'[c^{12}(\delta_t)]\mathbf{P}_{12}(\delta_t)\} \\ &\quad = \beta \mathsf{E}_t\{[1-(1-\alpha)\pi]u'[c^{11}(\delta_{t+1})]\mathbf{P}_{11}(\mu_{t+1}, \delta_t) \\ &\quad + (1-\alpha)\pi u'[c^{12}(\delta_{t+1})]\mathbf{P}_{12}(\mu_{t+1}, \delta_t)\} \end{aligned} \tag{24}$$

where

$$\mathbf{P}_{12}(\delta_t) \equiv \frac{p_{1t}}{p_{2t}} = \frac{[1-(1-\alpha)\pi]\delta_t + (1-\alpha)\pi}{\alpha\pi\delta_t + (1-\alpha\pi)}$$

$$\mathbf{P}_{11}(\mu_{t+1}, \delta_t) \equiv \frac{p_{1t}}{p_{1,t+1}} = \frac{[1-(1-\alpha)\pi]\delta_t + (1-\alpha)\pi}{\kappa_1(\mu_{t+1}, \delta_t)}$$

$$\mathbf{P}_{12}(\mu_{t+1}, \delta_t) \equiv \frac{p_{1t}}{p_{2,t+1}} = \frac{[1-(1-\alpha)\pi]\delta_t + (1-\alpha)\pi}{\kappa_2(\mu_{t+1}, \delta_t)}$$

$$\kappa_1(\mu_{t+1}, \delta_t) \equiv \frac{1-(1-\alpha)\pi}{\alpha}\{\alpha[\mu_{t+1} - (1-\alpha)\pi]\delta_t + (1-\alpha)(\mu_{t+1} - 1$$

$$+\alpha\pi)\} + (1-\alpha)\pi(\alpha\pi\delta_t + 1 - \alpha\pi)$$

$$\kappa_2(\mu_{t+1}, \delta_t) \equiv \pi\{\alpha[\mu_{t+1} - (1-\alpha)\pi]\delta_t + (1-\alpha)(\mu_{t+1} - 1 + \alpha\pi)\}$$
$$+(1-\alpha\pi)(\alpha\pi\delta_t + 1 - \alpha\pi).$$

IV. LIQUIDITY EFFECTS OF MONETARY POLICY

In order to investigate liquidity effects of monetary policy in the model economy, we need to specify a specific utility function:

$$u(c_t) = c_t^{1-\gamma} / (1-\gamma) \tag{25}$$

where $\gamma > 0$ is the coefficient of constant relative risk aversion (CRRA). Also, let R_t denote the nominal interest rate in period t, the equation for equilibrium bond price (24) becomes

$$\begin{aligned} q_t &= [1/(1+R_t)] \\ &= \beta \mathsf{E}_t \left\{ \frac{[1-(1-\alpha)\pi][c^{11}(\delta_{t+1})]^{-\gamma} + (1-\alpha)\pi[c^{12}(\delta_{t+1})]^{-\gamma} \mathbf{P}_{12}(\delta_{t+1})}{[1-(1-\alpha)\pi][c^{11}(\delta_t)]^{-\gamma} + (1-\alpha)\pi[c^{12}(\delta_t)]^{-\gamma} \mathbf{P}_{12}(\delta_t)} \right. \\ &\qquad \left. \mathbf{P}_{11}(\mu_{t+1}, \delta_t) \right\} \\ &\equiv \beta \mathsf{E}_t \{ \mathrm{m}(\delta_{t+1}, \delta_t) \mathbf{P}_{11}(\mu_{t+1}, \delta_t) \} \end{aligned} \tag{26}$$

where $\mathrm{m}(\delta_{t+1}, \delta_t)$ denotes the intertemporal marginal rate of substitution for consumers on connected islands and δ_{t+1}, the distribution of money balances across the economy, evolves according to (19).

For analytical tractability in the characterization of liquidity effects, it is assumed that the log of money growth in period t is normally distributed with constant conditional variance over time. Further, a log-linear approximation is taken to the equilibrium bond price equation (26) around the constant money growth $\log\mu = \mathsf{E}\log\mu_t$ where E is the unconditional expectation operator. With M_{it} for $i = 1, 2$ growing at a constant rate μ, the ratio of the per-household money stocks on connected and unconnected islands, δ_t as defined by (18), must be constant $\delta_t = \delta$ for all t. The evolution of the ratio of money stocks in

(19) then implies the following constant ratio of money stocks on connected and unconnected islands:

$$\delta = (\mu - 1 + \alpha\pi) / \alpha\pi \tag{27}$$

Now, the first-order linear approximation to the right-hand side of (26) around the constant money growth ($\log\mu$) and the constant ratio of money stocks ($\log\delta$) on connected and unconnected islands, yields the following:

$$[1/(1+R_t)] = \beta \mathsf{E}_t \exp\{\psi_1\hat{\delta}_{t+1} - \psi_1\hat{\delta}_t - \psi_2\hat{\mu}_{t+1} - \psi_3\hat{\delta}_t\} \tag{28}$$

where $\hat{\delta}_t = \log\delta_t - \log\delta$ and $\hat{\mu}_{t+1} = \log\mu_{t+1} - \log\mu$. The first-order linear approximation to the right-hand side of the law of motion of $\delta_{t+1} = \delta(\mu_{t+1}, \delta_t)$ in (19) implies

$$\delta_{t+1} = \exp\{\phi_1\hat{\mu}_{t+1} + \phi_2\hat{\delta}_t\}. \tag{29}$$

Note that in (28) ψ_1 denotes the elasticity of marginal utility on a connected island with respect to an increase in the ratio of money stocks on connected and unconnected islands. Also, ψ_2 and ψ_3 denote respectively the elasticity of inflation on a connected island $1/\mathbf{P}_{11}(\mu_{t+1}, \delta_t)$, with respect to an increase in money growth and with respect to an increase in the ratio of money stocks. In (29), ϕ_1 and ϕ_2 denote respectively the elasticity of money-stock ratio with respect to an increase in the contemporaneous money growth and with respect to an increase in the lagged money-stock ratio.

Let $\hat{\mu}_t$ be conditionally normal. Then, after taking logs, (28) together with (29) becomes

$$\begin{aligned}\hat{R}_t = \bar{R} - \psi_1\phi_1\left[\mathsf{E}_t\hat{\mu}_{t+1} - (1-\phi_2)\hat{\mu}_t\right] + \psi_1\phi_2(1-\phi_2)\hat{\delta}_{t-1} \\ +\psi_2\left[\mathsf{E}_t\hat{\mu}_{t+1} + \left(\frac{\psi_3}{\psi_2}\right)\phi_1\hat{\mu}_t\right] + \psi_3\phi_2\hat{\delta}_{t-1}.\end{aligned} \tag{30}$$

Referring to the standard Fisher equation, the equation (30) implies that a positive money injection $\hat{\mu}_t$ here has two effects on nominal interest rates: (i) the *real interest rate effect* which corresponds to the first line in (30) via its effect on the intertemporal marginal rate of substitution $\mathrm{m}(\delta_{t+1}, \delta_t)$ and (ii) the *Fisher effect* as captured by the second line in (30) via its effect on $1/\mathbf{P}_{11}(\mu_{t+1}, \delta_t)$, the expected inflation (or deflation) on connected islands.

Proposition *(i) For a sufficiently large* $\gamma > 0$, $\psi_1 < 0$, $\psi_2 > 0$, $\psi_3 < 0$, $\phi_1 > 0$, and $\phi_2 > 0$. *(ii) For a sufficiently small* α, $\phi_2 < 1$.

Proof. (i) First, $\psi_1 \equiv \left.\frac{d\log\{[1-(1-\alpha)\pi][c^{11}(\delta_t)]^{-\gamma}+(1-\alpha)\pi[c^{12}(\delta_t)]^{-\gamma}\mathbf{P}_{12}(\delta_t)\}}{d\log\delta_t}\right|_{\mu_t=\mu,\delta_t=\delta} < 0$ if and only if $-\gamma(1-\alpha\pi)(1-\delta)(1-\pi)-\delta[\gamma(1-\alpha\pi)-(1-\pi)] < 0$ which holds for a sufficiently large $\gamma > 0$. Further, $\left.\frac{d\log[\mathbf{P}_{11}(\mu_{t+1},\delta_t)]}{d\log\mu_{t+1}}\right|_{\mu_t=\mu,\delta_t=\delta} < 0$ implies $\psi_2 \equiv \left.\frac{d\log[1/\mathbf{P}_{11}(\mu_{t+1},\delta_t)]}{d\log\mu_{t+1}}\right|_{\mu_t=\mu,\delta_t=\delta} > 0$. Similarly because $\left.\frac{d\log[\mathbf{P}_{11}(\mu_{t+1},\delta_t)]}{d\log\delta_t}\right|_{\mu_t=\mu,\delta_t=\delta}$ $=(1-\alpha)(1-\pi)\{[1-(1-\alpha)\pi]\left(\frac{\mu-1}{\alpha}\right)+\pi\} > 0$, $\psi_3 \equiv \left.\frac{d\log[1/\mathbf{P}_{11}(\mu_{t+1},\delta_t)]}{d\log\delta_t}\right|_{\mu_t=\mu,\delta_t=\delta} < 0.$ Finally, $\phi_1 = \left.\frac{d\log[\delta(\mu_{t+1},\delta_t)]}{d\log\mu_{t+1}}\right|_{\mu_t=\mu,\delta_t=\delta} = \frac{\mu(\alpha\delta+1-\alpha)}{\delta(\mu,\delta)[\alpha^2\pi\delta+\alpha(1-\alpha\pi)]} > 0$ and $\phi_2 = \left.\frac{d\log[\delta(\mu_{t+1},\delta_t)]}{d\log\delta_t}\right|_{\mu_t=\mu,\delta_t=\delta} = \alpha^2\mu(1-\pi) > 0$. (ii) Immediate consequence of the last part of the proof in (i).

IV-1. Real Interest Rate Effect vs Fisher Effect

The proposition above implies that both the real interest rate effect and the Fisher effect of a money injection decrease nominal interest rates. First of all, $\psi_1 < 0$, $\phi_1 > 0$ and $\phi_2 \in (0,1)$ for a sufficiently small α in (30) imply that a positive money growth shock $\hat{\mu}_t$ increases the period-t consumption (c_t^{11}, c_t^{12}) of households on connected islands and hence decreases their marginal utility at *t*. Also, $[\mathrm{E}_t\hat{\mu}_{t+1}-(1-\phi_2)\hat{\mu}_t]$ is decreasing in $\hat{\mu}_t$ as long as the money growth processes are not sufficiently persistent.[3] Hence, an increase in the current money growth

[3] When the money growth follows an autoregressive process given by $\hat{\mu}_{t+1} = \rho\hat{\mu}_t + \varepsilon_{t+1}$ with

$\hat{\mu}_t$ decreases the real interest rate measured as the inverse of intertemporal marginal rate of substitution.

Notice that, in addition to the real interest rate effect of the current money growth, $\psi_1\phi_2(1-\phi_2)\hat{\delta}_{t-1}$ implies the real interest rate effect of $\hat{\delta}_{t-1}$, which is the lagged distribution of money balances across connected and unconnected islands. This represents the persistent distributional effect of money injections on real interest rates. It is also worth noting that the real interest rate effect increases when $|\psi_1|$ is large (that is, the marginal utility is more responsive to a money injection).

Second, $\psi_2 > 0$ and $\psi_3 < 0$ together with $\phi_1 > 0$ in (30) imply that an increase of $\hat{\mu}_t$ decreases the inflation rate on connected islands. That is, an increase of $\hat{\mu}_t$ on connected islands increases their period-t price level p_{1t}. When $\pi < 1$ so that a money injection diffuses through the economy (including unconnected islands) over time, the price level on connected islands will decline over time, so deflation is anticipated. Also, it can be shown that $[\mathsf{E}_t\hat{\mu}_{t+1} + (\frac{\psi_3}{\psi_2})\phi_1\hat{\mu}_t]$ is decreasing in $\hat{\mu}_t$ as long as the money growth processes are not sufficiently persistent.[4] Hence, the Fisher effect of an increase in the current money growth $\hat{\mu}_t$ decreases the expected inflation on connected islands. Notice that, in addition to the current money growth $\hat{\mu}_t$, $\psi_3\phi_2\hat{\delta}_{t-1}$ implies the Fisher effect of lagged money growth $\hat{\mu}_{t-1}$ via $\hat{\delta}_{t-1}$. This captures the persistent distributional effect of money injections on expected inflation.

As noted by Alvarez et al. (2002), the real interest rate effect is smaller as the money growth shock becomes more persistent. When the shock to money growth persists, a money injection leads to a permanent increase in both current and future consumptions and hence to a smaller decrease in the real interest rate, which is the inverse of intertemporal marginal rate of substitution. On the other hand, a transitory money growth shock leads to a temporary increase in current consumption and hence to a relatively large decrease in the real interest rate.

$\rho \in [0,1]$, it can be shown that $[\mathsf{E}_t\hat{\mu}_{t+1} - (1-\phi_2)\hat{\mu}_t]$ is decreasing in $\hat{\mu}_t$ if and only if $\rho < (1-\phi_2)$.

[4] When the money growth follows an autoregressive process given by $\hat{\mu}_{t+1} = \rho\hat{\mu}_t + \varepsilon_{t+1}$ with $\rho \in [0,1]$, it can be shown that $[\mathsf{E}_t\hat{\mu}_{t+1} + (\psi_3/\psi_2)\phi_1\hat{\mu}_t]$ is decreasing in $\hat{\mu}_t$ if and only if $\rho < -(\psi_3/\psi_2)\phi_1$

To see this more specifically, suppose that money growth shocks are i.i.d. so that $\mathsf{E}_t\hat{\mu}_{t+1} = 0$. Then (30) becomes

$$\hat{R}_t = \bar{R} + \left[\psi_1(1-\phi_2) + \psi_3\right]\hat{\delta}_t \tag{31}$$

where $\hat{\delta}_t = \phi_1\hat{\mu}_t + \phi_2\hat{\delta}_{t-1}$. The real interest rate effect corresponds to $\psi_1(1-\phi_2)\hat{\delta}_t$, whereas the Fisher effect is reduced to $\psi_3\hat{\delta}_t$. Now, suppose that money growth shocks follow a random walk such that $\hat{\mu}_{t+1} = \hat{\mu}_t + \varepsilon_{t+1}$ where ε_{t+1} is a normal i.i.d. with mean zero. Then $\mathsf{E}_t\hat{\mu}_{t+1} = \hat{\mu}_t$ and (30) becomes

$$\hat{R}_t = \bar{R} + \left[\psi_1(1-\phi_2) + \psi_3\right]\hat{\delta}_t - \left(\psi_1\phi_1 - \psi_2\right)\hat{\mu}_t \tag{32}$$

where $\psi_1(1-\phi_2)\hat{\delta}_t - \psi_1\phi_1\hat{\mu}_t$ corresponds to the real interest rate effect and $\psi_3\hat{\delta}_t + \psi_2\hat{\mu}_t$ represents the Fisher effect. Notice that both the real interest rate effect and the Fisher effect (or anticipated deflation effect) are smaller than that of the i.i.d. money growth process as in (31). Hence, the liquidity effects of money injections decrease as the money growth process becomes more persistent.

However, (32) implies that a highly persistent money growth shock (e.g., a random walk) can still generate liquidity effects (i.e., a fall in nominal interest rates) in the presence of the persistent distributional effect on real interest rates, $\psi_1(1-\phi_2)\phi_2\hat{\delta}_{t-1}$, and on expected deflation, $\psi_3\phi_2\hat{\delta}_{t-1}$. In particular, noting $\phi_2 = \alpha^2\mu(1-\pi)$ as shown in Proposition, a sufficiently large $\phi_2 \in (0,1)$ implied by a sufficiently small π for a given α would generate liquidity effects due to the persistent effect of lagged distribution of money balances across the economy.

Notice that both the real interest rate effect and the Fisher effect of a money injection on nominal interest rates depend critically on $\pi \in (0,1)$ and $\alpha \in (0,1)$. Here, $\alpha \in (0,1)$ governs the distributional effects of a money injection, whereas $\pi \in (0,1)$ determines its speed of diffusion through the economy and hence the persistence in its distributional effects. When $\alpha = 1$ or $\pi = 0$, $c_t^{11} = y$ in (20) and $\psi_1 = 0$ which implies a zero real interest rate effect in (30) and a positive expected inflation effect. This is the case of standard cash-in-advance models as in Lucas (1982) and Svensson (1985).

When $\pi = 1$ so that a money injection by the central bank becomes diffused within the period, (20) and (21) reduce to $c_t^{11} = c_t^{12} \equiv c_t^1$ where c_t^1 is the period-t consumption on connected islands, and $p_{1t} = p_{2t} = p_t$ from (8) and (9). This then implies the following bond price equation as in Alvarez et al. (2002):

$$q_t = \frac{1}{1+R_t} = \beta \mathsf{E}_t \left\{ \left(\frac{c_{t+1}^1}{c_t^1} \right)^{-\gamma} \frac{p_t}{p_{t+1}} \right\}. \tag{33}$$

Noting $p_{t+1} / p_t = \pi_{t+1} = \mu_{t+1}$, it can be shown that $\psi_1 = -\gamma(1-\alpha)\mu < 0$, $\psi_2 = 1$, $\psi_3 = 0$, $\phi_1 = \mu / (\mu - 1 + \alpha) > 0$, and $\phi_2 = 0$ in (30):

$$\hat{R}_t = \overline{R}_{\pi=1} - \psi_1 \phi_1 \left(\mathsf{E}_t \hat{\mu}_{t+1} - \hat{\mu}_t \right) + \mathsf{E}_t \hat{\mu}_{t+1} \tag{34}$$

where $\psi_1 \phi_1 < 0$ implies that an increase in money growth $\hat{\mu}_t$ decreases real interest rates. On the other hand, if the money growth shock raises expected money growth $\mathsf{E}_t \hat{\mu}_{t+1}$ as well, $\mathsf{E}_t \hat{\mu}_{t+1} = \mathsf{E}_t \hat{\pi}_{t+1}$ implies that an increase in money growth $\hat{\mu}_t$ raises expected inflation and hence increases nominal interest rates. Therefore, liquidity effects are generated when the real interest rate effect dominates the expected inflation effect.

Now, in order to see the inverse relationship between the real interest rate effect and the persistence of money growth, suppose that money growth shocks are i.i.d. Then (34) becomes

$$\hat{R}_t = \overline{R}_{\pi=1} + \psi_1 \phi_1 \hat{\mu}_t \tag{35}$$

and hence the real interest rate effect is $|\psi_1 \phi_1|$ with a zero expected inflation effect. When money growth shocks follow a random walk, $\mathsf{E}_t \hat{\mu}_{t+1} = \hat{\mu}_t$ and (34) becomes

$$\hat{R}_t = \overline{R}_{\pi=1} + \mathsf{E}_t \hat{\mu}_{t+1} \tag{36}$$

which represents a zero real interest rate effect and a unitary expected inflation effect where $\mathsf{E}_t\hat{\mu}_{t+1} = \mathsf{E}_t\hat{\pi}_{t+1}$. Therefore, as shown in Alvarez et al. (2002), the real interest rate effect is dominated by the expected inflation effect if the money growth shock becomes sufficiently persistent (e.g., a random walk). This is in a contrast to the current model with $\pi < 1$ where the liquidity effects are obtained for a sufficiently small π even when the money growth shock follows a random walk.

IV-2. Persistence of Liquidity Effects

Suppose that money injections follow an autoregressive process given by

$$\hat{\mu}_{t+1} = \rho\hat{\mu}_t + \varepsilon_{t+1} \tag{37}$$

where $\rho \in [0,1]$ is the persistence of the money injections and ε_{t+1} is a normally distributed i.i.d. random variable with mean zero and variance σ_ε^2. Then, as noted by Alvarez et al. (2002), the model economy with $\pi = 1$ implies that real interest rates, expected inflation rates, and nominal interest rates have the same persistence as money growth shocks, and hence liquidity effects of money injections depend critically on the persistent money growth shocks, i.e., a sufficiently high ρ in (37).

However, the estimates of ρ are relatively low, say 0.45 using the quarterly series of $M1$ in the U.S. from 1959 to 2006.[5] The corresponding estimates for Germany, Japan, and Australia are respectively 0.38 (1957-1998), 0.37 (1957-2006), and 0.15 (1957-2006) all using the quarterly series of $M1$ from the International Financial Statistics.

Now, when $\pi < 1$ so that money injections diffuse through the economy over time, the term $[\psi_1(1-\phi_2)+\psi_3]\phi_2\hat{\delta}_{t-1}$ in (31) implies that even an i.i.d. money growth shock does generate persistent liquidity effects via the effects of lagged distribution of money balances $(\hat{\delta}_{t-1})$ on both real interest rates and expected inflation rates. This appears to be empirically important in the sense that the persistent liquidity effects have

[5] Chari et al. (2000) estimated the persistence of money growth to be 0.57, whereas Cooley and Hansen (1989)'s estimate was 0.48 in the U.S.

been observed in the economies where the persistence of money growth is relatively low as described above.

V. CONCLUDING REMARKS

The well-documented liquidity effects of monetary policy (e.g., Friedman 1968) have been characterized analytically in an endowment economy version of Williamson (2008) which features goods market segmentation as well as financial market segmentation. With the cash-in-advance constraints on consumption goods purchases, households in financially connected sectors have access to the bond market and receive money transfers from the central bank, while others outside financial sectors do neither receive money transfers nor have access to the bond market. Household's consumers also face a relocation shock which has the effect of generating more consumption trades among the households respectively within or outside financial sectors, and hence segmenting goods market between financially connected and unconnected sectors. The goods market segmentation then causes a money injection to diffuse through the economy over time, having persistent impact on the distribution of money balances across the economy.

A positive money growth shock decreases real interest rates by increasing consumption of households in financial sectors and decreasing their marginal utility. Further, a money injection initially increases the price level in financial sectors. As the money injection is diffused through the economy, the price level in financial sectors declines over time and hence deflation is anticipated. Therefore, referring to the Fisher equation, both the real interest rate effect and the Fisher effect of a money injection work in the same direction to generate liquidity effects of monetary policy.

In addition to the contemporaneous money injections, the lagged money growth shock decreases current nominal interest rates by affecting the current distribution of money balances across the economy. This serves a key mechanism to generate the persistent liquidity effects even when the money growth shock is transitory, including an i.i.d. process. This is empirically plausible, noting that the persistent liquidity effects have been observed in many countries with a relatively low persistence of money growth such as in the U.S., Germany, Japan, and Australia.

References

Alvarez, F. and A. Atkeson (1997), "Money and Exchange Rates in the Grossman-Weiss-Rotemberg Model," *Journal of Monetary Economics*, Vol. 40, pp. 619-640.

Alvarez, F., A. Atkeson, and C. Edmond (2003), "On the Sluggish Response of Prices to Money in an Inventory-Theoretic Model of Money Demand," NBER Working Paper No. 10016.

Alvarez, F., A. Atkeson, and P. Kehoe (2002), "Money, Interest Rates, and Exchange Rates with Endogenously Segmented Markets," *Journal of Political Economy*, Vol. 110, pp. 73-112.

Baumol, W. J. (1952), "The Transactions Demand for Cash: An Inventory Theoretic Approach," *Quarterly Journal of Economics*, Vol. 66, pp. 545-56.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2000), "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?" *Econometrica*, Vol. 68, No. 5, pp. 1151-1179.

Chiu, J. (2005), "Endogenously Segmented Asset Market in an Inventory Theoretic Model of Money Demand," University of Western Ontario Working Paper.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (1998), "Modeling Money," NBER Working Paper 6371.

Cochrane, J. H. (1994), "Shocks," *Carnegie-Rochester Conference Series on Public Policy*, Vol. 41, pp. 295-364.

Cooley, T. and G. Hansen (1989), "The Inflation Tax in a Real Business Cycle Model," *American Economic Review*, Vol. 79, No. 4, pp. 733-748.

Edmond, C. and P. Weill (2008), "Models of the Liquidity Effect," *The New Palgrave Dictionary of Economics*, 2nd edition, edited by Steven N. Durlauf and Lawrence E. Blume, Palgrave Macmillan, 2008.

Friedman, M. (1968), "The Role of Monetary Policy," *American Economic Review*, Vol. 58, pp. 1-17.

Fuerst, T. S. (1992), "Liquidity, Loanable Funds, and Real Activity," *Journal of Monetary Economics*, Vol. 29, pp. 3-24.

Grossman, S. and L. Weiss (1983), "A Transactions-Based Model of the Monetary Transmission Mechanism," *American Economic Review*, Vol. 73, pp. 871-880.

Khan, A. and J. K. Thomas (2007), "Inflation and Interest Rates with Endogenous Market Segmentation," Working Paper, Federal Reserve Bank of Philadelphia.

Lucas, R. (1982), "Interest Rates and Currency Prices in a Two-Country World," *Journal of Monetary Economics*, Vol. 10, pp. 335-359.

Lucas, R. (1990), "Liquidity and Interest Rates," *Journal of Economic Theory*, Vol. 50, pp. 237-264.

Rotemberg, J. (1984), "A Monetary Equilibrium Model with Transactions Costs," *Journal of Political Economy*, Vol. 92, pp. 40-58.

Svensson, L. (1985), "Money and Asset Prices in a Cash-in-Advance Economy," *Journal of Political Economy*, Vol. 93, pp. 919-944.

Tobin, J. (1956), "The Interest-Elasticity of Transactions Demand for Cash," *Review of Economics and Statistics*, Vol. 38, pp. 241-247.

Williamson, S. D. (2008), "Monetary Policy and Distribution," *Journal of Monetary Economics*, Vol. 55, pp. 1038-1053.