Logic of Identification in Dynamic Economics

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1. Introduction

It is recognized that both deductive and inductive inference are necessary in the solution of the problem in the process of setting up hypothesis and vertifying or refuting them.

The general case and the process of establishing it is deductive.

The specific case and the process of establishing it is inductive and constitute the minor premise.

When the minor is established as fact, the conclusion follows deductively from the premises.

An hypothesis is a tentative and provisional thesis put forward upon the basis of accumulated knowledge of the field already possessed by the scientist and the hypothesis performs its function by providing a proposed explanation which will have certain consequences deduced from it.

Hence, these consequence may be confirmed or refuted by testing.

The confirmation or vertification is the final stage in the scientific procedure.

First, as general case, I will state the concept of model and sturcture in (2.1) and the relation between structure and distribution function in (2.2).

Second, as specific case, a priori restrictions on a model in (3)

Third, as conclusion, some concepts of identification in (4)

2. Theory

2. 1 Model and structure

2.11 Definitions

Definition 1. We call model $\mathfrak{S}$ a priori information on a system of equation, $\varphi g(\, y z,\ \alpha(g)=Ug$

$$(g=1,2,\ldots\ldots, G) \text{———} 2.1$$

and on the joint distribution density

$f\ (U\,;\Sigma)$ where $U\equiv(U_1,\ \ldots\ Ug)$; the vector of non-observable random disturbance.

U, z; the vector of observable variables, y as endogenous, z as exozenous.

α, Σ: Parameter's Vectors.

We shall assume that $\mathfrak{S}$ define 1) the form of function φ and f.

2) the a priori restrictions on parameters $\alpha \cdot \Sigma$.

Def. 2. The model $\mathfrak{S}$ is called self-contained if $G=N$

N: number of endogenous variables in $\mathfrak{S}$

The model $\mathfrak{S}$ is said to be sectional if $G<N$

The model $\mathfrak{S}$ is said to be complete if it is self-contained or has the following property.

That is, in subsidiary equations

$$\varphi_{G+k}(y)=u'^{(k)} \qquad k=1, \ldots, K=N-G$$

1) $$\frac{\partial \varphi_{-k}}{\partial y_g}=0 \qquad g=1, \ldots, G; \quad k=1, \ldots, K,$$

2) $f_x(U, U^1)= f(u)\ f_z(u^1)$ where $U^1=U^1_1, \ldots, U^1_L$

Def. 3. We call structure S all properties of the functions and parameters in (2.1) and (2.2).

$$S=(\varphi, f, \alpha, \Sigma)$$

i.e. each structure is defined by the functional forms of the equations and the values of the parameters occuring in (2.3).

Def. 4. When equations (2.3) are thus fully specified we call them structural equations.

A system of structural equations may be composed entirely on the basis of "theory" in which each equation corresponds to a specified law of behavior, to a specific technical law of production or to a specified identity.

2.12 Relation between model and structure.

Any model $\mathfrak{S}$ is a class of structures.

However, when certain a priori restrictions are made on the model $\mathfrak{S}$ the class $\mathfrak{S}$ may be narrowed down to a proper subset of $\mathfrak{S}_0$ of $\mathfrak{S}$

We will consider general definition of a model to help the concepts of model and structure.

Def. 5. An a priori postulate class $\mathfrak{S}_0$ of structures S that is a proper subset of the class $\mathfrak{S}$ of all structures is called a model(1).

It can be easily extended further.

From the concepts above, it is deduced as follows.

The class $\mathfrak{S}$ of all structures are partitioned into a system of subclass $\mathfrak{S}^{(i)}$ such that $\sum_{i} \mathfrak{S}^{(i)} = \mathfrak{S}$

this partition in general need not be finite or denumerable(2). Such process is capable of being developed deductively by means of an a priori restrictions from economic theory or the other sources outside the observations.

Thus, the a priori restrictions on the functions $(\varphi . f)$ and parameters $(\alpha . \Sigma)$ postulate that the structure S which has generated a given distribution function of the observation belongs to a within class $\mathfrak{S}$ of structures. Consequently, it is found that a structure is deduced by analizing the model and that a model performs its function by this process.

2 13 Assumption on model.

We are assume in (2.1)

(1) First order and second order moment exist

$$EU_{it} = 0$$
$$E(U_{it}\, U_{jt}) = 0$$

(2) Uit have a joint propability distribution function of the form. $(i = 1, \ldots\ldots, G)$

$$\prod_{t=1}^{T} f(U_{1t}, \cdots\cdots, U_{gt})\, dU_{1t} \ldots\ldots dU_{gt}$$

i.e. (i) $(U_{1t}, \ldots\ldots U_{gt})$

is independently distributed in probability sense of

$(U_{1t\pm k}, \ldots\ldots, U_{gt\pm k})$

(ii) All U_{it} are identically dstrbuted for all t

$$f_t(U_{it}) = f(U_{it})$$

(3) f (Uit, . . . , Ugt) is Gaussian multivariate normal distribution,

(4) Exogenous variables are assumed not to be linearly dependent in functional sense.

2. 2 Structure and Joint distribution function of the observations.

2.21 Given structure S (2.3), equation (2.1) can be solved for y in terms of z, involving new parameters λ called parameters of the reduced form. Let reduced form

$$y = \bar{q}_{\varphi}(z, u: \lambda) = \bar{q}_{\varphi}(z, u: \bar{\bar{q}}_{\varphi}(\alpha))$$

The transformation $\bar{q}$ depends on φ, also the transformation $\bar{\bar{q}}$ from α to λ depends on φ .

If the structural function φ are linear, then the functions $\bar{q}$ and $\bar{\bar{q}}$ will be also linear, and the set of parameters $\lambda = \bar{\bar{q}}_{\varphi}(\alpha)$ of the reduced form corresponding to a given structure S will be unique. If the functions φ are non-linear, several sets of parameters α may be compatible with the structure S also in a linear model the reduced form compatible with a given structure S is unique.

2.22 The joint distribution of y and z in (2.1) be written as a product of a conditional and a marginal distributions by the assumption (2.13) such that

$$g(y,z) = g(y/z: \lambda) g_z(z)$$

the distribution of y given z and parameters, depend or the structure

$$S = (\varphi, f, \alpha, \Sigma)$$

$$g(y/z: \lambda) = g_{f.\varphi}(y/z : \lambda)$$

$$\lambda = \bar{\lambda}(\alpha, \Sigma) = {}_{f.\varphi}(\alpha, \Sigma) = (S)$$

Thus, we recognize that structure S determines the form and the parameters of the conditional distribution $g(y/z: \lambda)$ uniquely.

But, S does not determine the distribution $g_z(z)$,

Therefore, for the purpose of estimating S from the observation, we have to consider not the joint distribution $g(y.z)$ but merely the conditional distribution of $g(y/z)$.

This is defined symbolically(3) as follows define G, is a class of all g's generated by the elements of

$$S_1 \in \mathfrak{S}_1 \; ; \; \supset : \exists \, 1 \;\; g_1 \in \mathfrak{G}_1 \ni S_1 .\therefore g_1$$

(For every element s_1 of $\mathfrak{S}_1$ there exist one and only one g_1 in $\mathfrak{G}_1$ such that S_1 is generated by S_1).

However, any set of G independent linear combinations of the structural equations (2.1) can produce the same distribution provided that the joint distribution of the disturbances in replaced by the corresponding distribution of the same linear combinations of the disturbances. This is defined also symbolically such that

$$g_1 \in \mathfrak{G}_1 \; ; \; \supset : \;\; \exists \, S_1 \in \mathfrak{S}_1 \;\; \ni \;\; S_1 .\therefore g_1$$

(For every element g_1 of $\mathfrak{G}_1$, there exist a structure S_1, in $\mathfrak{S}_1$, such that S_1, generates g_1)

It is also possible case that there exist one to one correspondance between structure and distribution function of y given z i.e. for any element g_1 in $\mathfrak{G}$, there exists one and only one S_1 in $\mathfrak{S}_1$ such that s_1 generates g_1

$$g_1 \in \mathfrak{G} \; : \; \supset : \; \exists \, 1 \; S_1 \in \mathfrak{S}_1 \;\; \ni \;\; S_1 .\therefore g_1$$

We extend, further, this concept over submodel as follows.

$\mathfrak{S}_1$, is partitioned such that $\mathfrak{S}_1 = \mathfrak{S}_{11} + \mathfrak{S}_{12}$ through the identifing restrictions

let S_{11} be an element of $\mathfrak{S}_{11}$ and if S_{11} generates the distribution g_{11} in $\mathfrak{G}_1$, we assume there exists no other element s_1 of $\mathfrak{S}_1$ that also generate g_{11} . then

$$g_{11} \in \mathfrak{G}_1 : \supset : \exists 1 \; S_{11} \in \mathfrak{S}_{11} \ni S_{11} \therefore g_{11}$$

let S_{12} be an element of $\mathfrak{S}_{12}$ and if s_{12} generates g_{12} we assume there exists at least one other s_{12} of $\mathfrak{S}_{12}$ that also generate g_{12} , However g_{12} can not be generated by any element of $\mathfrak{S}_{12}$ then

$$g_{12} \in \mathfrak{G}_1 : \supset : \exists \, S_{12} \in \mathfrak{S}_{12} \ni S_{12} \therefore g_{12}$$

3. A priori restrictions

Further specifications made in (2.3) from economic or other field outside observations call a priori restriction on the model. We have following type of priori restrictions.

1. A priori specifications of the form of each structural equations such that a specifications as to which variable may enter into which structural equations with which possible time lags.
These means information as to which variables are excluded from which behavior equations.
It is remarkable to choose the predetermined variables which have large variances than small variances.

2. We have another type of restrictions in which two coefficients of the variables y_{ir} and $y_{i'r}$ are required to have the same ratio in two different structural equations.

3. A priori restrictions on the distribution of disturbances. In models from which the identities have been removed by the elimination of an equal number of suitably chosen variable, we do require the covariance matrix Σ of the disturbances of the remaining behavior equations to be non-singular. This expresses the equations that, while disturbances in different types of economic decision may be statistically dependent we do not allow the disturbance, in one behavior equation to be linearly dependent on those in other behavior equations.

4. Inequalities

Oftenly on economic considerations in which the signs of coefficients of observable variables is known before hand. Sometime it may be possible to prescribe the sign of the correlations of disturbances in the structural equations.

5. Rules of normalization

A structural equation is not essentially altered if all of its coefficients are multiplied by the same number (different from zero) provided that corresponding adjustment are made in the elements of Σ, to avoid this trivial indeterminancy we add to the a priori restrictions a normalization rule for each equations.

i.e. the parameters including σ_{gh} for each value of g in (2.1) are unaffected by a change in scale usually we have the following rules of normalization

$\beta gigo=1 \qquad (g=1,2,\ldots,G) \qquad$ Coefficient of y

$\sigma_{gg}=1$

as seen (2.2) above, certain consequences might be induced by imposing restrictions on model.

that is one only one structure $s_1 \in \mathfrak{S}$ which has generated a given g_1 belongs to be certain class $\mathfrak{S} \subset \mathfrak{S}$ of structures. or $\mathfrak{S}_1$ contains one and only one S_1 in $\mathfrak{S}$ which generate g_1 in $\mathfrak{G}_1$

In general, let $\mathfrak{S}_1 = \mathfrak{S}_{11} + \mathfrak{S}_{12}$

It might be happen, $\mathfrak{S}_{11}$ contains one and only one S_{11} in $\mathfrak{S}_{11}$ which generate g_{11} in $\mathfrak{G}_{11}$

$\mathfrak{S}_{12}$ contains S_{121}, S_{122} in $\mathfrak{S}_{12}$ which both generate g_{12} in $\mathfrak{G}_1$

4. Identification

4. 1 A hypothetical proposition is made up of two parts. the first being the antecedent and the second the consequent. This hypothetical proposition is affirmed in the way if the antecedent is true, then the consequent is true.

A mixed hypothetical syllogism contains for its major premise a hypothetical proposition and for its minor a categorical whenever we assert a hypothetical proposition and then affirm the truth of the antecedent we are logically require to affirm the truth of the consequent.

4.12 Definition.

Def. 1. If for any element g_1 in $\mathfrak{G}_1$ there exists one and only one S_1 in $\mathfrak{S}_1$ such that S_1 generates g_1, we say that the model $\mathfrak{S}_1$ is uniquely identifing(5).

So, it will be say analitically

Major promise: conditional if ——, then (definition 1)

Minor promise: "For any element g_1 in $\mathfrak{G}_1$, there exist one and only one S_1 in $\mathfrak{S}_1$

Conclusion: $\mathfrak{S}_1$ is uniquely identifing.

If minor promise affirm by means of a priori restrictions as seen in (3) and then affirm the truth of the antecedent of major promise we are logically required affirm the truth of the consequent and if it deny the consequent is false.

Following general definitions of identification are quoted from Prof. L. Hurwich(6).

def. 2. If for any g_1 in $\mathfrak{G}_1$, the set $\mathfrak{S}_1 g$, of all S_1 in $\mathfrak{S}_1$, which generate g_1 is finite or denumerably infinite, the model $\mathfrak{S}$ is said to be completely identifing.

Unique identification power if $N=I$

Multiple identification power if $N>I$

Def. 3. If for any g_1 in $\mathfrak{G}_1$, the set $\mathfrak{S}_1 g$ of all S_1 in $\mathfrak{S}_1$ which generate g_1 is non-numerable, the model $\mathfrak{S}_1$ is said to be incompletely identifying.

Def. 4. Let $\mathfrak{S}_1 = \mathfrak{S}_{11} + \mathfrak{S}_{12}$

If for any element g_{11} in $\mathfrak{G}_1$ there exist one and only one such that S_{11} generate g_{11} we say that the submodel $\mathfrak{S}_{11}$ is uniquely identifing in the model $\mathfrak{S}_1$ or alternatively it will be said that $\mathfrak{S}$ is uniquely identifing over $\mathfrak{S}_{11}$.

In each case above if some of the total properties are uniquely identifiable we speak of partially unique identification power of the model, unless the model is uniquely identified with regard to all its properties so that totally unique identification power is present.

Def. 5. A structural equation is identifiable if and only if knowledge of its coefficients is implied by the knowledge of the parameters of the distributions function.

If at least a single structural equation is not identifiable the structure is not identifiable.

4. 3 The determination of coefficients of structural equations now, we are in position to seek the conditions under which the minor promise is satisfied: under which conditions the given structure is identifiable. Concerning with this argument we are assuming the case where a model is linear and restricted only by exclusion of certain variable from certain equation. The parameters of the reduced form $\overline{\eta}\varphi\ (\prec, \Sigma)$ constitute a complete set of parameters of this distribution function. Hence the conditions under which given equation is identifiable can be substituted by the conditions under which the coefficients of a structural equation may be determined from the parameters of the reduced form.

The structural equations of the model may be written in the form.

$$\beta y'_t + \Gamma z'_t = U'_t \qquad (\beta \text{ is non-singular matrix})\text{—(1)}$$

Where β, Γ are the coefficient matrixes of the jointly dependent and predetermined variables respectively.

Consider first structural equation

$$\beta y'_t + \gamma Z'_t = U'_{it} \qquad (t=1, \ldots\ldots, T)\text{—(2)}$$

Corresponding to the exclusion of $G^{\Delta\Delta} = G - G^{\Delta}$ and $K^{\times\times} = K - K^{\times}$

as a priori restrictions, the $\beta\ \gamma\ y_t\ Z_t$ are partitioned such that $\beta = [\beta_{\Delta}\ \beta_{\Delta\Delta}]$, $\gamma = [\gamma_{\times}\ \gamma_{\times\times}]$

$y_t = [y_{\Delta t}\ y_{\Delta\Delta t}]$, $Z_t = [Z_{\times}\ Z_{\times\times t}]$

Hence, the reduced form of (1) is obtained

$$y'_t - \Pi Z'_t = V'_t \qquad \text{where } \Pi = -\beta^{-1} \quad \text{——(3)}$$

Now, conversely, the first structural equation (2) is obtained from

i.e. $\beta y'_t - \beta \Pi Z'_t = U'_{it}$

consider the conditions which satisfy $-\beta\Pi = \gamma$

let $\Pi = \begin{bmatrix} \Pi_{\Delta x} & \Pi_{\Delta xx} \\ \Pi_{\Delta\Delta x} & \Pi_{\Delta\Delta xx} \end{bmatrix}$

then $-\beta\Pi = [\beta \quad \beta_{\Delta\Delta}] \begin{bmatrix} \Pi_{\Delta x} & \Pi_{\Delta xx} \\ \Pi_{\Delta\Delta x} & \Pi_{\Delta\Delta xx} \end{bmatrix} =$

$$[\beta_\Delta \Pi_{x}, \beta_\Delta \Pi_{\Delta xx}] = [\gamma, 0]$$

i.e. we have two conditions

$$\beta_\Delta \Pi_{\Delta x} = \gamma_x \text{——(4)}$$

$$\beta_\Delta \Pi_{\Delta xx} = 0 \text{——(5)}$$

Given Π are known and one value of β_Δ is normalized, for solving β_Δ and γ_x

(5) must be $\rho(\Pi_{\Delta xx}) \leq G^\Delta - 1$ (6)

If $\rho(\Pi_{\Delta xx}) = G^\Delta - 1$.

then (5) has unique solution β_Δ

If $\rho(\Pi_{\Delta xx}) < G^\Delta - 1$

then (5) has $K^{xx} - \gamma$ arbitrary values of solution.

of β_Δ and solution of r's β_Δ as linear combination of $K^{xx} - \gamma$ arbitrary values of β_Δ.

We have conclusion: The necessary and sufficient condition for the identifiability of the coefficients

β_Δ γ_x in (1) is that $\rho(\Pi_{\Delta xx}) = G^\Delta - 1$ or $\rho(\Pi) = G - 1$

this condition called rank condition i.e. we can form at least one nonvanishing determinant of order G—1 out of those co- efficients, properly arranged, with which the variables excluded from that structural equation appear in the G—1 other structural equations.

The necessary condition is that $\Pi_{\Delta\, xx}$ have at least G—1; columns

or $K^{xx} \geq G^{\Delta} - 1$

or equivalently $K^{xx} + G^{\Delta\Delta} \geq G - 1$

Where $K^{xx} + G^{\Delta\Delta}$ is the total number of dependent and predetermined variables which are excluded from the equation (2)

remark: concerning with these conditions, we should remark as following if structure or a part of it, is not identifiable its estimates is not possible, however numerous the observations on the variable treated as observables in the model. However, observations on other variables may provide additional information such as to make the structure, or its relevant part, identifiable, the failure of a model to identify the structure is not a ground for rejecting the model; rather it calls for additional information to be provided by a new type of observations.

4. 4 The identification problem is a logical problem that precedes estimation. It is therefore neither a problem in statistical inference nor probability problem but a priori problem, arising in the specification and interpretation of the probability distribution of the variables.

We are consider for what purposes the identification is necessary.

4.41 The observational structure S° may be different from some structure S valid for a different periods.

Suppose we know a transformation T carring S° into S i.e.

$$S = T S^{\circ} \;;\quad S = (\varphi, \alpha), \quad S^{\circ} = (\varphi^{\circ}, \alpha^{\circ})$$

If, in addition, we know S° from observation, we can obtain S.

Thus it is possible to evaluate y for a given z by indirect least square method.

4.42 The Policy-maker tries to maximize the "gain" ω.

The gain ω is a certain functions of the observations which must be supposed to be known to him.

denote by $\omega = \omega^{*}(Z;S)$ Z:observables.

$$= \omega^{*}(Z\ ; T S^{\circ}) \text{———} (1)$$

also define T consist in controllable ones T_c and uncontrollable ones T_u.

And z consist in controllable ones z_c and uncontrollable ones z_u then (1) becomes $\omega = \omega^{\times\times}(Z_c Z_u ; T_c T_u ; S^{\circ})$
the best policy depend on the values $\hat{z}_c \hat{T}_c$ of $Z_c T_c$ which maximize ω.

Given Z_u. T_u we can determine the best policies ($\hat{z}_c$, $\hat{T}_c$) provided the observational structure S° is known.

4.43 By the above reasons in (541) (4.42), to estimate the best policy ($\hat{z}_c$ $\hat{T}_c$) for given set (Z_u T_u), we have to have an estimate of the observational structure S° when structural changes are intended or expected.

If we suppose structural changes are neither intended nor expected for the period then T is the identical transformation

Hence, $S = T S^{\circ} = S^{\circ}$

$$\varphi = \varphi^{\circ},\ \alpha = \alpha',\ \pi = \pi^{\circ}$$

(Parameters of reduced form)

4.44 When structural change are neither intended nor expected then we don't have identification problem, because as seen (4.43)

$\varphi=\varphi^{\circ}$, $\alpha=\alpha^{\circ}$ and only Π is obtained from the knowledge of distribution function of observation which based on the same structure not need to prove it be true.

However, when structural changes are either intended or expected to the policy-maker, we have identification problem to prove the true one before its estimation.

So, it is desirable to deal with the model which possess the property of being structure- identifing when knowledge of the structure is need.

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* 1 L. Hurwich: "Generalization of the concept of identification"
  Cowles commission Monograph 10. P 248
* 2 " P 253
* 3 " P 249
* 4 " P 249
* 5 " P 249
* 6 " P 245
* 7 Ed. C. Hood, & C. Koopmans;
  Studies in Econometrics method P 135
* 8 Perlis; Theory of Matrixes P 47

(1) Ed. T.C. Koopmans : Statistical inference in Dynamic Economic models.

(2) Ed. Hood, Koopmans: Studies in Econometric Method.