Productivity Distribution and Economic Growth*

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This paper develops a tractable model of economic growth in which heterogeneous households produce capital à la Romer (1986). The paper demonstrates that depending on its varying degrees of persistence, productivity heterogeneity dictates economic growth. A regression analysis based upon a reduced-form version of the model shows that the persistence of human capital is the driving force behind the positive effects of productivity dispersion on economic growth.

I. Introduction

Does growth widen income inequality? Conversely, reduced inequality slow down economic growth? The chain of causality between income equality and economic growth has been a cause célèbre in the growth literature. With this

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motivation, we study a model of growth in the presence of productivity differences across households.

Our focus is on a positive relationship between income inequality and growth. Our explanation for the positive relationship between inequality and growth will proceed along the lines of Kaldor (1957).[1] The economy consists of many infinitely-lived heterogeneous households. Households differ in their ability to produce capital. Here capital is broadly defined as in Romer (1986) and it includes human capital and knowledge as well as physical capital. There are two production technologies: a stochastic technology that transforms consumption goods into capital goods using labor and a traditional good-producing technology to use capital as a factor of production. The production of both capital and final goods takes place according to an *AK* technology (see, for instance, Rebelo, 1991, for further details of the *AK* class of technologies).

In this environment, productivity heterogeneity may affect growth nontrivially and the key to understanding the relationship is whether productivity differences are permanent or not. If the productivity differences are permanent, growth will be accelerated over time and the coefficient of time factor is the variance of productivity distribution. However, if the productivity diffecences are purely temporary, they do not matter for economic growth. Finally if the productivity differences are persistent as in an autoregressive process, growth will be affected by both the persistence and the variance of productivity innovation.

There are at least four well-developed theories on this issue in the literature (see, Aghion, Caroli and García-Peñalosa (1999), Barro (2000), Benabou (1996) and Ferreira (1999) for excellent surveys). However, it seems that their debates on the causal relationship between inequality and growth still remain to be inconclusive.

Alesina and Rodrik (1994), Bertola (1993), and Persson and Tabellini (1994) developed politico-economic models of inequality and growth. If the mean income in an economy is larger than the median income, majority voting tends to favor redistribution of income from the rich to the poor, which involves distortionary taxes and regulations on the one hand and lobbying and buying votes of legislators by the rich on the other hand. Hence inequality tends to reduce economic growth.

Along the lines of those politico-economic theories, inequality may motivate the poor to engage in crime, riots, and other disruptive activities and cause socio-political instability (Alesina and Perotti, 1996; Venieris and Gupta, 1986; Benhabib and Rustichini, 1996; Rodrik, 1997; Bourguignon, 1998). Those criminal activities

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[1] Nicholas Kaldor (1957) develops a model of growth with both capitalists (profit-earners) and workers (wage-earners) who have their different saving rates. He assumes that the saving rate of the capitalists is higher than that of the workers. The steady-state equilibrium of his model is guaranteed by changes in income distribution. For example, if capital/labor ratio rises above its steady state level, the wage/profit ratio will also rise. However, since the saving rate out of wages is lower than the rate out of profits, capital/labor ratio is driven down toward the steady-state equilibrium.

and conflicts among economic classes are a direct waste of resources, a cause of uncertainty, and a threat to the key capitalist institutions like property rights. Hence economic inequality may deter investment and growth.

The other channel of economic inequality affecting growth was proposed in the important contributions by Loury (1981), Galor and Zeira (1993), Aghion and Bolton (1997), Ferreira (1995), Banerjee and Newman (1993), and Picketty (1997). Their models are based on assumptions of imperfect capital market. If minimum fixed costs are required for engaging in more productive activities, any imperfection in credit market will reduce the growth rate. For example, if either missing or imperfect credit market leads to a wedge between lending and borrowing interest rates as shown by Galor and Zeira (1993) or to collateral requirements as postulated by Banerjee and Newman (1993), the resulting equilibrium will be characterized by a group of people who fail to make use of the most productive opportunities and growth will be deterred.

All these models predict the negative relationship between inequality and growth. However, traditional theories on the relationship between inequality and economic growth maintain that inequality should have a positive effect on capital accumulation and growth. In an excellent survey, Aghion, Caroli and García-Peñalosa (1999) provide three notable arguments for such positive effects. First, as Kaldor (1957) postulated, the marginal propensity to save of the rich is greater than that of the poor (See, for example, Stiglitz, 1969 and Bourguignon, 1981). Second, investment projects often involve large sunk costs. Third, more equal distribution may result in incentive problems. Combined together, these factors imply the growth-enhancing effect of inequality. In an empirical study, Park (2006) finds that income inequality measured by human capital dispersion affects economic growth positively.

The paper proceeds as follows. In section 2, the economic environment is described. Section 3 solves the model analytically for the equilibrium and obtains equilibrium growth rates. Section 4 puts the implications on test. Section 5 concludes.

II. Economic Environment

Households, indexed by i, are distributed on the interval [0, 1] according to the Lebesgue measure λ. Each agent is interpreted as an "entrepreneur" who operates his own production scheme using his own labor and capital. Preferences are ordered by the utility function:

$$U^i = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\ln((c_t^i) + b \ln(1 - n_t^i)] \right\}, \tag{1}$$

where E_0 is the expectations operator conditional on the initial period information Ω_0. Here c_t^i and n_t^i are the period t consumption and working hours; $0 < \beta < 1$ and b are the utility discounting factor and the preference for leisure, respectively, which are assumed to be identical across the households. This utility specification implies that preferences are additively separable over time and across states of the world; it also implies that preferences are consistent with the balanced growth path (King et al, 1988).

The household i uses goods and labor to produce capital, k_{t+1}^i, which is defined broadly as in Romer (1986). Capital is produced according to an *AK* technology:

$$k_{t+1}^i = (z_t^i n_t^i) s_t^i. \tag{2}$$

z_t^i is the efficiency of capital production, which is considered to be (uninsurable) household-specific risk, while s_t^i is the amount of good used in period t to produce capital. In other words, each individual i has access to a stochastic technology that contemporaneously transforms s_t^i consumption goods into $(z_t^i n_t^i) s_t^i$ units of capital using the random variable z_t^i and his own labor n_t^i.[2] Thus, z_t^i can be interpreted as "entrepreneurial risk."

As in Aiyagari (1994), note that households are ex ante identical in the sense that they have the same preferences and their entrepreneurial risks z_t^i are drawn from the same distribution with is support $\mathbf{Z}$. However, households are ex post heterogeneous in the sense that they experience uninsurable idiosyncratic shock histories $z^t \equiv (z_0, z_1, \ldots, z_t) \in \mathbf{Z}^t \equiv \mathbf{Z} \times \ldots \times \mathbf{Z}$. In this sense, we postulate an incomplete-markets economy.

The initial distribution over capital holdings and "efficiency" shocks is given by

$$\Lambda_0(\mathbf{A} \times \mathbf{B}) = \lambda\{i \in [0,1] : (k_0, z_0) \in \mathbf{A} \times \mathbf{B}\}, \tag{3}$$

where $\mathbf{A} \times \mathbf{B} \subset \mathbf{K} \times \mathbf{Z}$.[3] Since an individual state is now a pair of individual capital holdings and the history of individual shocks, the aggregate distribution over a set of all individual states at time t evolves as follows:

$$\Lambda_t(\mathbf{C} \times \mathbf{D}) = \lambda\{i \in [0,1] : (k_t^i, (z^i)^t) \in \mathbf{C} \times \mathbf{D}\}, \mathbf{C} \times \mathbf{D} \subset \mathbf{K} \times \mathbf{Z}^t.$$

Thus, $\Lambda_t(\mathbf{C} \times \mathbf{D})$ is the measure of households whose capital holdings and shock

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[2] If z_t^i is assumed to be privately observed by the household, then agency issues can be introduced into the environment as in Calstrom and Fuerst (1997). Indeed, letting $n_t^i \equiv 1$, the capital production technology is exactly reduced to the one advocated by Calstrom and Fuerst (1997).

[3] Without loss of generality, it is assumed that $\mathbf{A}$ and $\mathbf{B}$ satisfy suitable measurability conditions.

histories in period t lie in the set $\mathbf{C} \times \mathbf{D}$.

Aggregate good producton takes place according to the following AK technology:

$$Y_t = A_t K_t . \tag{4}$$

A_t is the aggregate productivity, which may fluctuate, and the aggregate capital stock K_t is given by

$$K_t = \int k_t \Lambda_t (dk_t, dz^t) . \tag{5}$$

Each household i faces the following budget constraint:

$$c_t^i + s_t^i = y_t^i , \text{ for each } t \geq 0 \tag{6}$$

where y_t^i is the household i's income. In the line with Aiyagari (1994), borrowing is prohibited. Its income process $\{y_t^i\}$ is generated by the following production technology:

$$y_y^i = A_t k_t^i \tag{7}$$

We also assume that capital depreciates 100 percent each period.

Each individual maximizes the lifetime utility (1) subject to (6) given the initial distribution Λ_0, a sequence of aggregate distributions $\{\Lambda_t\}$ and the aggregate random process $\{A_t\}$.[4] The first-order conditions for the individual's decision problem are given by the following:

$$\frac{b}{1 - n_t^i} = \beta E_t \left\{ A_{t+1} z_t^i s_t^i \cdot \frac{1}{c_{t+1}^i} \right\} \tag{8}$$

and

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[4] A natural concept of equilibrium for this framework is the analogue of a sequential equilibrium for Bewley-type models with aggregate shocks as in Krebs and Wilson (2004) and Miao (2006). In particular, Miao (2006) proves the existence of sequential (competitive) equilibria for Bewley-type models with aggregate shocks under fairly general conditions. It can be easily shown that our model construct satisfies Miao's (2006) sufficient conditions for the existence of sequential equilibria by construction and thus his existence proof is directly applicable to the construction of our sequential equilibrium. Furthermore, the linearity of production technologies and the assumption of 100-percent capital depreciation in our model construct deliver the aggregation result, requiring no large endogenous state variables as in Krusell and Smith (1998). For greater details, see Kim (2016).

$$\frac{1}{z_t^i n_t^i} \cdot \frac{1}{c_t^i} = \beta E_t \left\{ \frac{A_{t+1}}{c_{t+1}^i} \right\}. \tag{9}$$

(8) equates the marginal cost of labor supply with its marginal benefit and (9) equates in an analogous way the marginal cost of capital production with its marginal benefit. Combining (8) and (9), we have the following:

$$\frac{b}{1-n_t^i} = \frac{s_t^i}{c_t^i n_t^i} \tag{10}$$

From (6), (9), and (10), we have the following equations for optimal consumption, investment and working hours of the household i:

$$c_t^i = (1-\beta)A_t k_t^i \tag{11}$$

$$s_t^i = \beta A_t k_t \tag{12}$$

$$n_t^i = \frac{\beta}{\beta + (1-\beta)b} = n \tag{13}$$

Hence consumption and saving (investment) are fixed proportion of income and working hours, which are identical across the households, are fixed. We will denote the fixed working hours as n. The saving rate of household i obtains as follows:

$$\frac{s_t^i}{y_t^i} = \beta. \tag{14}$$

Notably, (14) implies an identical saving rate across households. Furthermore, the growth rate of the household i's saving is given by the following:

$$s_t^i = \beta A_t k_t^i = \beta^2 A_t z_{t-1}^i s_{t-1}^i n \tag{15}$$

$$\Leftrightarrow \frac{s_t^i}{s_{t-1}^i} = \beta^2 n A_t z_{t-1}^i$$

Thus, the growth rate of household i's saving depends positively on aggregate productivity, household-specific productivity, and working hours. That is, the more productive is a household in capital production, the higher is its growth rate of saving.

The aggregate capital stock can be obtained by aggregating (2) in the following

way:[5]

$$K_{t+1} = \int k_{t+1}^i \Lambda_{t+1}(dk_{t+1}, dz^{t+1}) \tag{16}$$
$$= \int (z_t^i n_t^i) s_t^i \Lambda_t(dk_t, dz^t)$$
$$= \beta n A_t \int z_t^i k_t^i \Lambda_t(dk_t, dz^t).$$

By iteration, we also have the following:

$$k_{t+1}^i = z_t^i s_t^i n_t^i \tag{17}$$
$$= \beta n A_t z_t^i k_t^i$$
$$= [\Pi_{\tau=0}^t \beta n A_\tau z_\tau^i] k_0^i$$
$$= (\beta n)^{t+1} k_0^i [\Pi_{\tau=0}^t A_\tau][\Pi_{\tau=1}^t z_\tau^i].$$

Then the aggregate capital stock can be rewritten as

$$K_{t+1} = (\beta n)^{t+1} (\Pi_{\tau=0}^t A_\tau) \int k_0^i [\Pi_{\tau=0}^t z_\tau^i] \Lambda_t(dk_t, dz^t) \tag{18}$$
$$= (\beta n)^{t+1} (\Pi_{\tau=0}^t A_\tau) \int k_0^i [\Pi_{\tau=0}^t z_\tau^i] Q_t(dk_t, dz_t \mid k_{t-1}, z^{t-1}) \Lambda_{t-1}(dk_{t-1}, dz^{t-1})$$
$$= (\beta n)^{t+1} (\Pi_{\tau=0}^t A_\tau) \int k_0^i [\Pi_{\tau=0}^t z_\tau^i] Q_t(dk_t, dz_t \mid k_{t-1}, z^{t-1})...Q_1(dk_1, dz_1 \mid k_0, z_0) \Lambda_0(dk_0, dz_0)$$

where $\{Q_t\}$ is a sequence of suitably defined stochastic kernels. The (gross) growth rate of the economy can be obtained as follows:

$$\frac{K_{t+1}}{K_t} = \beta n A_t \frac{\int k_0^i [\Pi_{\tau=0}^t z_\tau^i] \Lambda_t(dk_t, dz^t)}{\int k_0^i [\Pi_{\tau=0}^{t-1} z_\tau^i] \Lambda_{t-1}(dk_{t-1}, dz^t)} \tag{19}$$

If k_0^i and the household-specific productivities are stochastically independent, we further simplify (19) in the following way:

$$\frac{K_{t+1}}{K_t} = \beta n A_t \frac{\int [\Pi_{\tau=0}^t z_\tau^i] \Lambda_t(dk_t, dz^t)}{\int [\Pi_{\tau=0}^{t-1} z_\tau^i] \Lambda_{t-1}(dk_{t-1}, dz^t)} \tag{20}$$

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[5] Miao's (2006) Lemma 2 is directly applicable to our iterated integrals. For greater details, see Kim (2016).

Hence the growth rate depends on the rate of time preference, working hours, aggregate productivity and the distribution of the household-specific productivity. Especially, the growth rate depends crucially on how persistent the productivity differences are. We will turn to this issue in the next section.

III. Productivity Distribution and Growth

The relationship between productivity distribution and economic growth depends crucially on whether the productivity differences are permanent or not. If they are permanent, we have growth rate increasing with time and the coefficient of time factor is the variance of the distribution. On the contrary, if they are purely temporary, productivity dispersion does not matter for economic growth. If the productivity differences are persistent as in an autoregressive process, economic growth depends on the mean and variance of the productivity distribution.

3.1. Permanent Productivity Differences

Assume that the productivity differences are permanent. That is, the household-specific productivity is not time-dependent.

$$z_t^i = z^i \tag{21}$$

Using the results in (18) and (20), we have the following:

$$\frac{K_{t+1}}{K_t} = \beta n A_t \frac{\int (z^i)^{t+1} \Lambda_0 (dk_0, dz_0)}{\int (z^i)^t \Lambda_0 (dk_0, dz_0)} \ . \tag{22}$$

In addition, assume that z^i is log-normally distributed: i.e.

$$\ln[z^i] \sim N\left(\mu_z - \frac{1}{2}\sigma_z^2, \sigma_z^2 \right). \tag{23}$$

Then the mean of z^i is $\exp(\mu_z)$ and the variance is $\exp(2\mu_z)[\exp(\sigma_z^2)-1]$. Hence any increase in σ_z^2 is a mean preserving spread in the sense of Rothchild and Stiglitz (1970, 1971). Now the following equation can be obtained from the distributional assumption on z^i described before:[6]

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[6] Its derivation is available upon request.

$$\int (z^i)_0^{t+1} \Lambda_0(dk_0, dz_0) = \exp\left[\mu_z(t+1) + \frac{1}{2}\sigma_z^2[t(t+1)] \right].$$ (24)

The growth rate of the economy also obtains using (24) as follows:

$$\frac{K_{t+1}}{K_t} = \beta n A_t \exp(\mu_z + \sigma_z^2 t)$$ (25)

The growth rate of the economy depends on the productivity distribution. If productivity differences are permanent, the growth rate of the economy increases exponentially over time. The reason that the growth rate of the economy grows over time is the following. As we can see in (14) and (15), the saving rate is identical across the households. However, the growth rate of the saving of a more efficient household is higher. In other words, although all households save the same proportion of income, more productive household utilize the saving more efficiently and hence the saving grows faster. This differences in saving rate makes the economy grow faster over time.[7]

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[7] It is obvious that growth rate depends crucially on the distributional assumption. Suppose z^i has the following gamma distribution.

$$\int [z^i] = \frac{\lambda[\lambda z^i]^{p-1}}{\Gamma(p)} \exp[-\lambda z^i],$$

where λ and p are parameters determining the shape of the distribution. With this distribution, we have the following:

$$\int (z^i)^{t+1} \Lambda_0(dk_0, dz_0) = \left(\frac{1}{\lambda^{t+1}}\right)\frac{\Gamma(p+t+1)}{\Gamma(p)}.$$

Hence the growth rate of the economy:

$$\frac{K_{t+1}}{K_t} = \beta A_t \left(\frac{p+t}{\lambda}\right)$$

$$= \beta A_t \left[\frac{p}{\lambda} + \left(\frac{p/\lambda^2}{p/\lambda}\right)t\right]$$

$$= \beta A_t \left[\frac{p}{\lambda} + \left(\frac{Var(z^i)}{E(z^i)}\right)t\right]$$

In other words, the growth rate is increasing at a constant rate which is determined by λ. Note that λ is the ratio of the variance of z^i relative to its mean. The coefficient of t is larger with the variance of the distribution and thus the result is qualitatively the same as in (25). However, the

3.2. Temporary Productivity Differences

Now suppose that z_t^i is purely temporary in the sense that it is drawn from a distribution which is identical and independent over time. Invoking the law of large numbers in each period, we have the following by aggregation:

$$\int [\Pi_{\tau=0}^{t} z_\tau^i] \Lambda_0(dk_0, dz_0) = \Pi_{\tau=0}^{t} \left(\int_\Delta z_\tau^i \Lambda_0(dk_0, dz_0) \right). \tag{26}$$

If z^i is log-normally distributed as in (23), we have the following:

$$\Pi_{\tau=0}^{t} \left(\int z_\tau^i \Lambda_0(dk_0, dz_0) \right) = [\exp(\mu)]^{t+1} = [E(z^i)]^{t+1} \tag{27}$$

Using (22), we obtain the growth rate of the economy:

$$\frac{K_{t+1}}{K_t} = \beta A_t \exp(\mu) = \beta A_t E[z^i] \tag{28}$$

Now the growth rate of the economy does not depend on productivity distribution. It depends only on its mean.[8]

3.3. Persistent Productivity Differences

We consider the case in which the productivity of a household is not permanent but persistent. Assume the following law of motion for the productivity z^i:

$$z_t^i = [z_{t-1}^i]^\rho \varepsilon_t, \;\; 0 < \rho < 1 \;\; \text{and} \;\; \ln(\varepsilon_t) \sim i.i.d.N\left(\mu_\varepsilon - \frac{1}{2}\sigma_\varepsilon^2, \sigma_\varepsilon^2 \right) \tag{29}$$

where ε_t is an i.i.d. random variable with positive support and z_0^i is given for the individual household i. Also assume that z_0^i is also log-normally distributed and independent of ε_t's: i.e.

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growth rate is increasing with time linearly rather than exponentially.

[8] If the productivity has a gamma distribution as in footnote 7, we have the following:

$$\frac{K_{t+1}}{K_t} = \beta A_t \left(\frac{p}{\lambda} \right) = \beta A_t E[z_t^i].$$

Also note that the growth rate is determined by the mean of the productivity distribution. In other words, the distribution does not matter for the growth of the economy.

$$\ln[z_0^i] \sim N\left(\mu_z - \frac{1}{2}\sigma_z^2, \sigma_z^2\right) \tag{30}$$

By iterated substitution, (29) can be rewritten in terms of the initial productivity and the series of temporary shocks:

$$z_t^i = ((z_o^i)^{\rho^t})[\Pi_{\tau=1}^t((\varepsilon_\tau)^{\rho^{t-\tau}})]. \tag{31}$$

As a result, we have the following:

$$\Pi_{\tau=0}^t z_\tau^i = ((z_0^i)^{\Sigma_{s=0}^t \rho^s})[\Pi_{i=1}^t(\varepsilon_i^{\Sigma_{s=0}^t \rho^s})]. \tag{32}$$

The aggregation of (32) results in the following equation:[9]

$$\int [\Pi_{\tau=0}^t z_\tau^i] \Lambda_t(dk_t, dz^t) \tag{33}$$

$$= \exp\left\{\left(\sum_{s=0}^t \rho^s\right)\left(\mu_z - \frac{1}{2}\sigma_z^2\right) + \frac{1}{2}\left(\sum_{s=0}^t \rho^s\right)^2 \sigma_z^2\right\}$$

$$\cdot \exp\left(\sum_{i=1}^t \left[\left(\sum_{s=0}^{t-i} \rho^s\right)\left(\mu_\varepsilon - \frac{1}{2}\sigma_\varepsilon^2\right) + \frac{1}{2}\left(\sum_{s=0}^{t-i} \rho^s\right)^2 \sigma_\varepsilon^2\right]\right).$$

Using (22) we have the following growth rate of the economy:[10]

$$\frac{K_{t+1}}{K_t} = \beta A_t \exp\left\{\rho^t\left(\mu_z - \frac{1}{2}\sigma_z^2\right) + \frac{\rho^t}{2}\left[\frac{2-(1-\rho)\rho^t}{1-\rho}\right]\sigma_z^2\right\} \tag{34}$$

$$\cdot \exp\left\{\left(\frac{1-\rho^t}{1-\rho}\right)\left(\mu_\varepsilon - \frac{1}{2}\sigma_\varepsilon^2\right) + \frac{1}{2}\left[\frac{1-2\rho^t + \rho^{2(t+1)}}{(1-\rho)^2}\right]\sigma_\varepsilon^2\right\}$$

Taking the limit of (34), the growth rate in the limit is given by

$$\lim_{x\to\infty}\left(\frac{K_{t+1}}{K_t}\right) = \beta A_\infty \exp\left\{\left(\frac{1}{1-\rho}\right)\mu_\varepsilon + \left[\frac{\rho}{2(1-\rho)^2}\right]\sigma_\varepsilon^2\right\}. \tag{35}$$

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[9] Its derivation is available upon request.
[10] Its derivation is available upon request.

Hence the growth rate of the economy is constant. However, the growth rate is increasing with the parameters ρ, μ_ε and σ_ε^2. In other words, as the temporary shocks become more persistent, the growth rate gets larger. In addition, the variance of the productivity distribution affects the growth rate positively.

IV. Empirical Findings

In this section, we illustrate the empirical relevance of our theoretical model in terms of educational attainment levels. Our empirical test is designed to investigate the impact of the average and dispersion indices of human capital on economic growth as in Park (2006). We claim that the average and dispersion indices of human capital are satisfactory proxies for $E[z_t^i n_t^i]$ and $Var[z_t^i n_t^i]$, respectively, in the model. As shown in (2), note that the term $z_t^i n_t^i$ represents the combination of individual productivity and individual "effort" (labor) in producing broadly defined capital goods k_{t+1}^i. Thus, this re-interpretation permits use of $z_t^i n_t^i$ as a proxy for "human capital".[11] It also implies the following approximations:

$$E[z_t^i n_t^i] \simeq \mu_z$$
$$Var[z_t^i n_t^i] \simeq \sigma_z^2 .$$

As in Park (2006), our empirical test uses a pooled 5-year interval time-series data set of 144 countries from 1960 to 2010 compiled from two main sources: Penn World Tables (PWT 7.1) for traditional inputs and output data and Barro and Lee (2010) for educational attainment data. The average and dispersion indices of human capital are calculated using Barro and Lee's (2010) educational attainment data as in Park (2006).

We first estimate the following specification in Model 1:

$$d\log(y_{it}) = c + \beta \cdot d\log(k_{it}) + \delta_m \cdot \mu_{it} + \delta_v \cdot \sigma_{it}^2 + \gamma \cdot \log\left(\frac{A_{it}}{\bar{A}_t}\right) + \varepsilon_{it} . \tag{36}$$

In formulation (36), $d\log(y_{it})$ and $d\log(k_{it})$, respectively, denote the growth rate of real income and physical capital per capita for country i and in period t; μ_{it} it and σ_{it}^2, resepectively, denote the mean and variance of human capital. Lastly,

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[11] However, note that in equilibrium, n_t^i is constant across households, which implies that the average and dispersion indices of human capital should be driven by pure "entrepreneurial risks" z_t^i. In that sense, we cannot exclude the possibility that our empirical setting might be "spurious". In Kim (2016), we consider the general case in which n_t^i is not necessarily constant in equilibrium.

$\log(A_{it} / \overline{A}_t)$ denotes a catch-up variable. The catch-up variable $\log(A_{it} / \overline{A}_t)$ is the average ratio of each country's per capita GDP A_{it} relative to that of the US $\overline{A}_t$ during five years.[12] In Model 2, we first run AR(1) regression on the human capital series of each country i to retrieve its AR(1) parameter ρ_i and its innovation variance τ_i^2; then we include ρ_i and τ_i^2 as explanatory variables instead of σ_{it}^2 to probe the role of productivity persistence on growth. Model 3 include time trends, while Model 4 considers regional dummies for Latin America and Subsaharan Africa. Table 1 reports summary statistics of ρ_i and τ_i^2, while all regression results are reported in Table 2 and 3; Table 2 and Table 3 differ in sample size.[13]

[Table 1] Summary Statistics of Persistence and Innovation Variance

Regions	Persistence (ρ)	Innovation Variance (τ^2)
Advanced Economies	1.0127	0.0079
East Asia and the Pacific	1.0827	0.0140
Europe and Central Asia	0.9681	0.0110
Latin America and the Caribbean	1.1046	0.0063
Middle East and North Africa	1.1806	0.0069
South Asia	1.2244	0.0066
Sub-Saharan Africa	1.1969	0.0046

In Table 2, it merits the emphasis that the growth effect of investment (change in physical capital) is positive and strongly significant across all specificiations. It also varies little in magnitude; across the models, one-percent increase in physical capital per capita results in about 0.3 percent increase in income per capita. However, the other explanatory variables, particularly in Model 1, have correct signs but are not statistically siginificant.[14] Even the mean of human capital is not statistically significant. In Model 2, by contrast, we observe the strongly siginificant growth effects of the mean of human capital and also its persistence ρ_i. As a factual matter, the effect of the persistence of human capital on growth is positive and statistically significant regardless of any model specification. However, the innovation variance still does not have any significant growth effect. The growth effect of the mean of human capital also disappears as time trends are included in Model 3 and Model 4. The regional dummies are significant.

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[12] The specification is the same as in Park (2006). It is basically a log-linearized Cobb-Douglas technology.

[13] In Table 3, we run one more regression using a set of countries with all necessary data between 1970 and 2010, since time series from 1960 include some missing data points across countries.

[14] The result is in stark contrast with Park (2006), who finds that all explanatory variables under consideration are statistically significant across various specifications. However, it merits the emphasis that only 94 countries are considered in his regression and time periods are shorter than in our regression.

[Table 2] The Effect of Productivity Persistence (1)

Variables	Model 1	Model 2	Model 3	Model 4
$d\log k$	0.3140***	0.3139***	0.3083***	0.3044***
	(0.0129)	(0.0129)	(0.0129)	(0.0130)
μ	0.0150	0.0775***	0.0140	0.0375
	(0.0278)	(0.0265)	(0.0321)	(0.0335)
σ^2	0.0452			
	(0.0320)			
$\log(A_0 / \bar{A}_0)$	-0.0027	-0.0019	-0.0081	-0.0013
	(0.0019)	(0.0019)	(0.0020)	(0.0020)
ρ		0.0145***	0.0087*	0.0096*
		(0.0050)	(0.0052)	(0.0053)
τ^2		0.0235	0.0232	0.0094
		(0.0574)	(0.0571)	(0.0573)
Time Trend			-0.0002***	-0.0001***
			(0.00004)	(0.00004)
Latin America				-0.0037**
				(0.0015)
Sub-saharan				-0.0040**
				(0.0016)
R^2	0.4516	0.4778	0.5355	0.5757
Number of Observations	1385	1385	1385	1385
Number of Countries	144	144	144	144

Note: The numbers in parentheses are standard errors.
    * Significant at the 10 percent significance level.
    ** Significant at the 5 percent significance level.
    *** Significant at the 1 percent significance level.

In Table 3, all the explanatory variables except the AR(1) innovation variance, τ^2, are statistically significant. The investment factor is strongly significant as in Table 2 and its growth effects are a little larger across the models in Table 3 than in Table 2. In addition, both mean and variance of human capital affect growth positively.

[Table 3] The Effect of Productivity Persistence (2)

Variables	Model 1	Model 2	Model 3	Model 4
$d\log k$	0.3556***	0.3512***	0.3340***	0.3291***
	(0.0156)	(0.0155)	(0.0155)	(0.0155)
μ	0.0844*	0.0182***	0.0206***	0.0285***
	(0.0451)	(0.0039)	(0.0048)	(0.0049)
σ^2	0.0248*			
	(0.0139)			
$\log(A_0 / \bar{A}_0)$	-0.7669**	-0.8322***	-0.2497***	-0.1886***
	(0.1964)	(0.2835)	(0.306)	(0.0312)

ρ		0.2098***	0.5647***	0.8793***
		(0.0690)	(0.0719)	(0.0723)
τ^2		0.8596	0.6144	0.4348
		(0.6647)	(0.6521)	(0.6525)
Time Trend			-0.0379***	-0.0356***
			(0.0061)	(0.0061)
Latin America				-0.4893***
				(0.1677)
Sub-saharan				-0.4573***
				(0.1731)
R^2	0.4867	0.4965	0.6481	0.6752
Number of Observations	891	891	891	891
Number of Countries	99	99	99	99

Note: The numbers in parentheses are standard errors.

* Significant at the 10 percent significance level.

** Significant at the 5 percent significance level.

*** Significant at the 1 percent significance level.

V. Conclusion

The theme of this paper is in the spirit of Kaldor's (1957) celebrated research. Unlike Kaldor's (1957) model, however, we consider a model of economic growth in which households differ in their ability to produce broadly defined capital including human capital and knowledge. The more efficient a household is, the more it saves in absolute amount and also the more efficiently it uses savings. We have to note that there are no differences in saving rates across the households but the growth rate increases with productivity efficiency, which is a driving force behind the results in the paper.

It turns out that the growth effects of the productivity distribution depends on whether the productivity is permanent or not. If the productivity is not purely temporary, its dispersion affects economic growth. That is, if the productivity is permanent, the growth rate of the economy grows over time and the coefficient of the time factor is the variance of productivity distribution. If the productivity is not permanent but persistent, its distribution affects the growth rate, which is constant over time. However, if the productivity is purely temporary, its dispersion does not matter in terms of growth. Examples of temporary productivity differences in human capital include health, migration (human capital acquired abroad), contracts, schooling, education policy changes and etc.

It is obvious that our conclusion is not a final verdict on the relationship between

income inequality and growth.[15] Depending on its causes, income inequality may affect economic growth either positively or negatively. As Barro (2000) suggests, the relationship between inequality and growth also may vary, depending on stages of economic development.

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[15] As discussed in Section 2, income inequality is implied as an equilibrium outcome, since heterogeneous households generate different income levels based upon the distribution of their capital holdings in equilibrium.

References

Aghion, P., E. Caroli and C. García-Peñalosa (1999), "Inequality and Economic Growth: The Perspective of the New Growth Theories," *Journal of Economic Literature*, 37(4), 1615-1660.

Aghion, P. and P. Bolton (1977), "A Theory of Trickle-Down Growth and Development," *Review of Economic Studies*, 64, 151-172.

Aiyagari, S. R. (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109, 659.684.

Alesina, A. and R. Perotti (1996), "Income Distribution, Political Instability, and Investment," *European Economic Review*, 40, 1203-1228.

Alesina, A. and Rodrik (1994), "Distributive Politics and Economic Growth," *The Quarterly Journal of Economics*, 109, 465-489.

Banerjee, A. V. and A. F. Newman (1993), "Occupational Choice and the Process of Development," *Journal of Political Economy*, 101, 274-298.

Barro, R. J. (2000), "Inequality and Growth in a Panel of Countries," *Journal of Economic Growth*, 5, 5-32.

Barro, R. J. and J. W. Lee (2010), "A New Data Set of Educational Attainment in the World, 1950-2010," NBER Working Paper No. 15902.

Bénabou, R. (1996), "Inequality and Growth," *NBER Macroeconomics Annual*, 11 11-73.

Benhabib, J. and A. Rustichini (1996), "Social Conflict and Growth," *Journal of Economic Growth*, 1, 129-146.

Bertola, G. (1993), "Factor Shares and Savings in Endogenous Growth," *American Economic Review*, 83, 1184-1198.

Bourguignon, F. (1981), "Pareto Superiority of Unegalitarian Equilibria in Stiglitz' Model of Wealth Distribution with Convex Saving Function," *Econometrica*, 49(6), 1469-1475.

_____ (1998), "The Measurement of Multidimensional Poverty," Presented at the 1998 Congress of the European Economic Association.

Carlstrom, C. T. and T. S. Fuerst (1997) "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review*, 87, 893-910.

Ferreira, F. H. G. (1997), "Economic Transition and the Distributions of Income and Wealth," manuscript, World Bank.

_____ (1999), "Inequality and Economic Performance: A Brief Overview to Theories of Growth and Distribution," Text for World Bank's Web Site on Inequality, Poverty, and Socioeconomic Performance: http://www.worldbank.org/poverty/inequal/index.htm.

Galor, O. and J. Zeira (1993), "Income Distribution and Macroeconomics," *Review of Economic Studies*, 60, 35-52.

Kaldor, N. (1957), "A Model of Economic Growth," *Economic Journal*, 67, 591-624.

Kim, H. (2016), "Uninsured Production Risk and Economic Welfare," Unpublished.

King, R. G., C. I. Plosser, and S. T. Rebelo (1988), "Production, Growth and Business

Cycles: I. The Basic Neoclassical Model," *Journal of Monetary Economics*, 21, 195-232.

Krebs, T. and B. Wilson (2004), "Asset Returns in an Endogenous Growth Model with Incomplete Markets," *Journal of Economic Dynamics and Control*, 28, 817-839.

Krusell, P., and A. A. Smith (1998), "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867-896.

Loury, G. C. (1981), "Intergenerational Transfers and the Distribution of Earnings," *Econometrica*, 49, 843-867.

Miao, J. (2006), "Competitve Equilibria of Economies with A Continuum of Consumers and Aggregate Shocks," *Journal of Economic Theory*, 128, 274-298.

Park, J. (2006), "Dispersion of Human Capital and Economic Growth," *Journal of Macroeconomics*, 28, 520-539.

Perrson, T. and G. Tabellini (1994), "Is Inequality Harmful for Growth? Theory and Evidence," *American Economic Review*, 84, 600-621.

Picketty, T. (1997), "The Dynamics of Wealth Distribution and the Interest Rate with Credit Rationing," *Review of Economic Studies*, 64, 173-189.

Rebelo, S. (1991), "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy*, 99, 500-521.

Rodrik, D. (1997), "Globalization, Social Conflict, and Economic Growth," *The World Economy*, 21(2), March 1998.

Romer, P. M. (1986), "Increasing Returns and Long-run Growth," *Journal of Political Economy*, 94, 1002-1037.

Rothschild, M. and J. Stiglitz (1970), "Increasing Risk I: A Definition," *Journal of Economic Theory*, 2, 225-43.

Rothschild, M. and J. Stiglitz (1971), "Increasing Risk II: Its Economic Consequences," *Journal of Economic Theory*, 3, 66-84.

Stiglitz, J. (1969), "Distribution of Income and Wealth Among Individuals," *Econometrica*, 37(3), 382-397.

Venieris, Y. P. and D. K. Gupta (1986), "Income Distribution and Sociopolitical Instability as Determinants od Savings: A Cross-Section Model," *Journal of Political Economy*, 94(4), 873-883.