

An Econometric Treatment of the Quality of Labor in Production Function

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«Purpose of the Study»

Most empirical studies of production relationships have been based on aggregate indexes of capital and labor inputs. One of the commonly maintained hypotheses in empirical production function studies has been that the aggregate inputs of labor and capital are homogeneous. The progressive sophistication of the theory of production has inevitably generated dissatisfaction with the treatment of labor and capital as homogeneous inputs and has stimulated attempts to disaggregate them.¹⁾

In the literature on functional relationships among economic variables, discussions of the internal structure of production functions have typically been concerned with either (i) whether a function of many arguments could be disaggregated into sub-function (equivalently, whether a set of different inputs in a sub-function could be aggregated into a single measure), or (ii) whether a set of various inputs in a sub-function could be disaggregated into distinct inputs. This study is, however, concerned with (ii) in which disaggregation takes the form of replacing a single aggregate by a set of distinct inputs in a production function.²⁾

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1) See C. Blackorby, D. Primont and R.R. Russell (hereafter called Blackorby et al.), *Duality, Separability and Functional Structure: Theory and Economic Applications* (N.Y.: North-Holland Co., 1978), pp. 41~102.

2) There are two problems in dealing with aggregation. (i) One is to reduce by aggregation various types of inputs (or commodities) to a set of inputs with distinctive characteristics in a production function (or utility function). (ii) The other problem is the case where a single macro-relation is derived by aggregation from a set of micro-relations (e.g., derivation of an aggregate production function from individual firms' production functions, and derivation of the welfare function from individual consumers' utility functions). It should be noted that we discuss only the first problem; i.e., the aggregation problem of inputs in the context of an aggregate production function. The discussion of the problem of aggregation of micro-relations into a macro-relation is beyond the scope of this study.

For further discussions of the aggregation problem of micro-relations into a macro-relation, see R.G.D. Allen, *Mathematical Economics*, Second ed. (London: MacMillan Co., 1964), pp. 694-724; L.R. Klein, "Remarks on the Theory of Aggregation", *Econometrica*, Vol. 14 (1946), pp. 302-12; and H. Theil, *Linear Aggregation of Economic Relations* (Amsterdam: North-Holland Publishing Co., 1954).

Examination of the literature on functional relationships among economic variables reveals that the "functional separability" condition³⁾ must be satisfied in order to permit a disaggregation of the aggregate input (labor) into a set of distinct inputs in a production function. It has been further shown that the functional separability condition can be expressed equivalently in terms of the equality condition of the partial elasticities of substitution (PES) between the inputs in question (i.e., $\sigma_{ik} = \sigma_{jk}$, where i, j and k are the production inputs).⁴⁾

The purpose of this study is to test the functional separability condition for the aggregate input (labor) in order to provide empirical evidence about whether or not the aggregate input adequately represent the sum of inputs which have different skill intensities.⁵⁾

I. Conceptual and Theoretical Framework

I -1. The Concept of Functional Separability⁶⁾

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- 3) W.W. Leontief, "Introduction to a Theory of the Internal Structure of Functional Relationships", *Econometrica*, Vol. 15, No. 4 (October 1947), pp. 361-73; and W.W. Leontief, "A Note on the Interrelation of Subsets of Independent Variables of a Continuous Function with Continuous First Derivatives", *Bulletin of the American Mathematical Society*, Vol. 53, No. 4 (1947), pp. 343-50.
 - 4) E.R. Berndt and L.R. Christensen, "The Internal Structure of Functional Relationships: Separability, Substitutability and Aggregation", *Review of Economic Studies*, Vol. 40 (July 1973), pp. 403-10; and C. Blackorby and R.R. Russell, "Functional Structure and the Allen Partial Elasticities of Substitution: An Application of Duality Theory", *Review of Economic Studies*, Vol. 43 (June 1976), pp. 285-91.
 - 5) As Denny and Fuss clearly stated: "the use of aggregate inputs for postulating a production relationship between output and inputs requires the assumption that the production function is (functionally) separable in these aggregates. Yet until recently, (functional) separability and the existence of aggregate inputs were assumed *a priori* in virtually in all production function studies." See M. Denny and M. Fuss, "The Use of Approximation Analysis to Test for Separability and the Existence of Consistent Aggregates", *American Economic Review*, Vol. 67, No. 3 (June 1977), p. 404.
 - 6) The concept of "functional separability" (the term "separability" is often used instead in the literature) was developed independently by Leontief and Sono. Sono's paper was published in Japanese in 1943, in *Kokumin Keizai Zasshi*, but it remained little known outside Japan until an English translation appeared in 1961. M. Sono presented his results in the context of utility functions. Detailed discussions of Sono's version of functional separability are found in P.T. Geary and M. Morishima, "Demand and Supply Under Separability", in *Theory of Demand: Real and Monetary*, ed. M. Morishima (Oxford: The Clarendon Press, 1973), pp. 89-147. Leontief's work was undertaken independently and appeared in 1946-47. Its context was primarily the theory of production. Follow up discussions of Leontief's version of functional

Suppose that we have a production function

$$Y=F(X)=F(x_1, x_2, \dots, x_n) \quad (1)$$

The set of all n inputs is denoted by $N=(1, 2, \dots, n)$, and it is partitioned into r mutually exclusive subsets $[N_1, \dots, N_r]$, a partition which we shall call R . We denote the first and second partial derivatives of $F(X)$ with respect to x_i and x_j by F_i and F_{ij} : $\partial F_i = \partial F / \partial x_i$; $F_{ij} = \partial^2 F / \partial x_i \partial x_j$; F_i / F_j represents the marginal rate of substitution (MRS) between x_i and x_j . Furthermore, we assume that x_i and x_j are elements in the same subset N_s , and x_k is not an element in the subset N_s .

According to Leontief the production $F(X)$ is said to be functionally separable with respect to the partition R if the MRS between any two inputs x_i and x_j from any subset $N_s, s=1, \dots, r$ is independent of the quantities of inputs outside of N_s . That is, the necessary and sufficient condition for the MRS between x_i and x_j to be independent of a third variable, say x_k , is:⁷⁾

$$\frac{\partial}{\partial x_k} \left(\frac{F_i}{F_j} \right) = 0 \text{ for all } i, j \in N_s, \text{ and } k \notin N_s \quad (2)$$

Thus, the necessary and sufficient condition for the grouping of variables (i.e., "functional separability"⁸⁾) requires the MRS between any two

separability are found in: R.H. Strotz, "The Utility Tree-A Correction and Further Appraisal", *Econometrica*, Vol. 27, No. 3 (July 1959), p. 482-88; W.M. Gorman, "Separable Utility and Aggregation", *Econometrica*, Vol. 27, No. 3 (July 1959), pp. 469-81; and S. Goldman and H. Uzawa, "A Note on Separability in Demand Analysis", *Econometrica*, Vol. 32, No. 3 (July 1964), pp. 387-98. Throughout this study, we shall follow Leontief's definition of functional separability.

- 7) The relationship (2) can be rewritten as:

$$\frac{\partial}{\partial x_k} \left(\frac{F_i}{F_j} \right) = \frac{F_{ik}F_j - F_{jk}F_i}{(F_j)^2} = 0 \text{ or, } F_{ik}F_j - F_{jk}F_i = 0$$

This relationship implies that if the MRS between x_i and x_k is independent of x_j and the MRS between x_j and x_k is independent of x_i , then the MRS between x_i and x_j is independent of x_k .

- 8) The interpretation of the term "functional separability" corresponds to what has come to be called "weak separability". The terminology of weak and strong separability was established by Strotz. See R.H. Strotz, "The Utility Tree", pp. 482-83. The production function $F(X)$ in equation (1) is said to be *strongly* separable with respect to the partition R if the MRS between any two inputs from the two subsets (N_s and N_t) does not depend on the quantities of inputs outside of N_s and N_t ; i.e., $\partial / \partial x_k (F_i / F_j) = 0$ for all $i \in N_s, j \in N_t, k \notin N_s \cup N_t$, and $N_t \neq N_s$. It is interesting to compare the conditions of strong separability and weak separability. That is, for weak separability, $i, j \in N_s, K \notin N_s$; for strong separability, $i \in N_s, j \in N_t$, and $K \notin N_s \cup N_t$. Strong separability implies weak separability, but weak separability implies strong separability only when the partition R is limited to two subsets. It is thus clear that the condition of weak separability is not applicable to the case where three subsets (e.g., labor, capital, raw materials, etc.) are involved. Throughout this study we follow the concept of functional (weak) separability since we shall be dealing with only two functional groups (labor and capital).

variables in a group to be a function only of the variables in that group and therefore independent of the value of any variable in any other group.

In general it may be stated that a group of variables is said to be "functionally" separable from the remaining variables in a production function if the MRS between variables in that group is independent of the values of variables outside the group.⁹⁾

1-2. Theoretical Relationships Between the Functional Separability Condition and Partial Elasticities of Substitution

Shephard¹⁰⁾ showed that the cost function $C=f(Y; P_1, \dots, P_n)$, which is dual to any homothetic production function $Y=F(X)=F(x_1, \dots, x_n)$, can be written in the following form:

$$C=f(Y; P_n)=H(Y), G(P) \quad (3)$$

where Y is output, P_n is a set of the n inputs prices, X 's are the inputs of production, $H(Y)$ is a function of output only, $G(P)$ is a function of the n inputs prices only.

9) There is yet another way to characterize the functional relationships among economic variables. "The separability of a group variables from its complement is equivalent to the possibility of forming an aggregate function for that group, which can be aggregated consistently into a macro function." See C. Blackorby et al., *Duality, Separability and Functional Structures*, p.100. Green also discusses the concept of consistent aggregation. In his example, it is assumed that each household's consumption is a function of its income alone. Aggregate income is defined as the sum of household incomes, and aggregate consumption as the sum of each household's consumption. Under what conditions is aggregate consumption, as defined, a function of (that is, uniquely determined by) aggregate income? If there are no restrictions on the distribution of income the necessary and sufficient condition is that all marginal propensities to consume are constant and equal. Thus, this property of the marginal propensities ensures that the aggregation procedure described is consistent. H.A.J. Green, *Aggregation in Economic Analysis-An Introductory Survey* (Princeton: Princeton University Press, 1964), p. 12.

10) R.W. Shephard, *The Theory of Cost and Production Functions* (New Jersey: Princeton University Press, 1970), pp.143-46. For historical notes on the concept of duality and its economic applications, see W.E. Diewert, "Applications of Duality Theory", in *Frontiers of Quantitative Economics*, Vol. II, eds. M.D. Intriligator and D.A. Kendrick (Amsterdam: North-Holland Publishing Co., 1974), pp.106-75. Shephard shows that the separable form of the cost function, (3), exist if and only if the parent production function is homothetic. See R.W. Shephard, "Applications of Duality Theory: A Comment", in *Frontiers of Quantitative Economics*, Vol. II, eds. M. Intriligator and D. Kendrick (Amsterdam: North-Holland Publishing Co., 1974), p. 202-204.

Then, the functional separability condition in the context of the cost function (3) requires that¹¹⁾

$$G_{ik}G_j - G_{jk}G_i = 0 \text{ for all } i, j \in N_1, K \notin N_1 \quad (4)$$

where G_i and G_{ik} are the partial derivatives of $G(P)$:

$$G_i = \partial G / \partial P_i, \quad G_{ik} = \partial^2 G / \partial P_i \partial P_k.$$

We now illustrate that the functional separability condition of the cost function (4) requires σ_{ik} to be equal to σ_{jk} , in the following steps: First, In the three-input case the PES is written as follows:

$$\sigma_{ij} = \frac{\sum_{h=1}^n X_h f_h}{X_i X_k} \cdot \frac{F_{ik}}{|F|}, \quad \sigma_{jk} = \frac{\sum_{h=1}^n X_h F_h}{X_j X_k} \cdot \frac{F_{jk}}{|F|} \quad (5)$$

where $\sigma_{ik}(\sigma_{jk})$ is the PES between the inputs i and k (j and k), f_h is the marginal productivity of the h th input, x_h is quantity of the h th input, F_{ik} is the co-factor of the elements f_{ik} in the bordered Hessian determinant $|F|$ in the production function $F(X)$ in equation (1).

Second, following Samuelson,¹²⁾ we substitute the relation

$$\lambda \frac{X_i}{P_k} = \frac{F_{ik}}{|F|}, \quad \lambda \frac{\partial X_j}{\partial P_k} = \frac{F_{jk}}{|F|}, \quad F_k = \frac{P_k}{\lambda} \quad (6)$$

where λ is interpreted as the marginal cost of output. Substituting (6) into (5), we obtain

$$\sigma_{ik} = \frac{\sum_{h=1}^n P_h X_h}{X_i X_k} \cdot \frac{\partial X_i}{\partial P_k}, \quad \sigma_{jk} = \frac{\sum_{h=1}^n P_h X_h}{X_j X_k} \cdot \frac{\partial X_j}{\partial P_k} \quad (7)$$

Third, using Shephard's Lemma,¹³⁾ we obtain

11) Shephard and Lau proved the theorem that if the production function is functionally separable, then the unit cost function, which is dual to the production function, is also functionally separable in the same partition. See R.W. Shephard, *The Theory of Cost and Production Functions*, pp.145-46; and L.J. Lau, "Duality and the Structure of Utility Functions", *Journal of Economic Theory*, December 1969, pp.384-85. Further, it is interesting to compare the functional separability condition in the context of the production function $F(X)$ with that in the context of the cost function $G(P)$. That is, for the production function, $F_{ik}F_j - F_{jk}F_i = 0$; and for the cost function, $G_{ik}G_j - G_{jk}G_i = 0$.

12) P. Samuelson, *Foundations of Economic Analysis* (N.Y.: Atheneum, 1965), pp. 60-69. Samuelson states the theorem that "in order for total costs to be a minimum for given output, the price of each factor must be equal to marginal physical productivity multiplied by the marginal cost."

13) Shephard's Lemma can be stated as follows: If we differentiate the cost function with respect to the price of a factor, we obtain the derived factor demand functions. For a proof of Shephard's Lemma, see W.E. Diewert, "An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function", *Journal of Political Economy*, Vol. 79 (May/June 1971), pp.495-97.

$$\begin{aligned}
X_i &= \partial C(Y, P) / \partial P_i = H(Y) \partial G(P) / \partial P_i = H(Y) G_i \\
X_j &= H(Y) G_j \\
X_k &= H(Y) G_k \\
\partial X_i / \partial P_k &= H(Y) \partial G_i / \partial P_k = H(Y) G_{ik} \\
\partial X_j / \partial P_k &= H(Y) G_{jk} \quad \dots \quad (8)
\end{aligned}$$

where $C(Y, P)$ is a total cost function which is dual to the homothetic production function, $Y = F(X) = F(X_1, \dots, x_n)$, $H(Y)$ is a function of output, G is a unit cost function, $G = G(P)$, $G(P)$ means that G is a function of n inputs prices, G_i is the first derivative of G with respect to the i th input price ($\partial G / \partial P_i$), and G_{ik} is the second derivative of G with respect to the input prices of i and k ($\partial^2 G / \partial P_i \partial P_k$).

Thus, from these three steps and the linear homogeneity of the cost function $G(P)$ in the input prices,¹⁴⁾ we arrive at

$$\sigma_{ik} = \frac{G G_{ik}}{G_i G_k} \quad \sigma_{jk} = \frac{G G_{jk}}{G_j G_k} \quad (9)$$

where σ_{ik} (σ_{jk}) is the PES between the inputs i and k (j and k), and G is a unit cost function, $G = G(P)$.

As a result, it is clear that $\sigma_{ik} = \sigma_{jk}$ if and only if¹⁵⁾ $G_{ik} G_j - G_{jk} G_i = 0$. Alternatively, it may be stated that the equality condition of the PES which was defined in (9) can be rewritten as

$$\frac{G_{ik}}{G_i} = \frac{G_{jk}}{G_j} \quad (10)$$

Also, the functional separability condition which was defined in (4) can be rewritten as

$$G_{ik} G_j / G_i G_k = 1 \quad \text{or} \quad \frac{G_{ik}}{G_i} = \frac{G_{jk}}{G_j} \quad (11)$$

We have thus shown that the functional separability condition is expressed equivalently in terms of the equality condition of the PES between the inputs in question, i.e. (10) and (1) are clearly equivalent

14) This implies that the cost function is homogenous of degree one in the prices of the inputs, and the input demand function is homogeneous of degree zero.

15) This discussion has been based on the assumption that the behavior of the prices and quantities is unrestricted. In other words, the theorem discussed rules out the following possibilities: when P_i and P_j move proportionally, the PES is infinite; when x_i and x_j move proportionally, the PES is zero. See H. Wold, *Demand Analysis: A Study in Econometrics* (N.Y.: John Wiley and Sons, Inc., 1953), pp. 108-10.

to each other.

II. Methodology

II-1. Hypotheses to be Tested

For testing the functional separability condition of an aggregate input of labor (L) with two types of labor inputs, production (L_1) and nonproduction (L_2) workers, the hypotheses to be tested are:

$$H_0: \sigma_{L_1K} = \sigma_{L_2K} \quad H_A: \sigma_{L_1K} \neq \sigma_{L_2K}$$

II-2. Model

The CRESH (constant ratios of elasticity of substitution and homothetic) production function is defined implicitly by the following equation:

$$F(Y, X_i) = \sum_{i=1}^n D_i [X_i/h(Y)]^{d_i} - 1 \equiv 0 \quad (12)$$

where Y is the output (value added), X_i is a vector of n inputs, D_i 's are the distribution parameters, and d_i 's are the substitution parameters.

In order to derive the input demand equations from the CRESH function we form the augmented objective function for solving an output-constrained cost minimization problem.¹⁶⁾

For minimizing the total cost, $C = \sum_{i=1}^n P_i X_i$, subject to the production function (12), we form the Lagrangian function as follows:

$$M = C - \lambda F = \sum_{i=1}^n P_i X_i - \lambda \left(\sum_{i=1}^n D_i (X_i/h(Y))^{d_i} - 1 \right) \quad (13)$$

where λ is a Lagrange multiplier. For algebraic ease the following

16) There are two ways of obtaining solutions to the problem of minimizing the cost of producing a given output (Y), given fixed input prices (P_i) and a given production function $Y = f(X_i)$. One method is to postulate a functional form for the production function f and then use the Lagrangian multiplier method in order to obtain the derived demand functions, $X_i = f(Y; P_1, \dots, P_n)$. The other is to postulate a functional form for the cost function and obtain the derived demand functions simply by making partial differentiation of the cost function with respect to the input prices, provided that the cost function satisfies the regularity conditions of Shephard's duality theorem. Our approach is to apply the first method.

logarithmic transformation of variables and parameters are made:

$$\begin{aligned}\log h(Y) &= V, \quad \log X_i = Z_i, \quad \log P_i = R_i, \quad \log |D_i d_i| = C_i, \\ \log |\lambda| &= A, \quad \frac{1}{1-d_i} = a_i\end{aligned}\quad (14)$$

The Lagrangian function (13) is now transformed by using the relation (14):

$$M(Z, V, R) = \sum_{i=1}^n \exp(R_i + Z_i) - \lambda \left[\sum_{i=1}^n D_i \exp[d_i(Z_i - V)] - 1 \right] \quad (15)$$

The first order conditions for minimum cost are

$$\frac{\partial M}{\partial Z_i} = \exp(R_i + Z_i) - \lambda D_i d_i \exp[d_i(Z_i - V)] = 0 \quad (16)$$

$$\text{where } (i=1, \dots, n)$$

Equations (16) yield using the definitions of (14):

$$Z_i = a_i(A - R_i - d_i V + C_i) \quad (17)$$

Solving equation (17) for A gives

$$A = (Z_i/a_i) + R_i + d_i V - C_i \quad (18)$$

Substitution of equation (18) into equation (17) yields the following system of equations:

$$Z_i = a_i(C_i - C_1) + \frac{a_i}{a_1} Z_1 + \left(1 - \frac{a_i}{a_1}\right) V + a_i(R_i - R_1) \quad (19)$$

where $i=2, \dots, n$

1 = a particular input of production.

In terms of original variables, equation (19) becomes the input demand equation:

$$\log X_i = a_i(C_i - C_1) + \frac{a_i}{a_1} \log X_1 + a_i \log \frac{P_1}{P_i} + \left(1 - \frac{a_i}{a_1}\right) \log h(Y) \quad (20)$$

Without loss of the properties of equation (20), the following input demand equation for the multi-factor case can be derived:

$$\log \left(\frac{X_i}{X_j} \right) = b_0 + b_1 \log \left(\frac{P_j}{P_i} \right) + b_2 \log \left(\frac{V}{X_j} \right) \quad (21)$$

where $b_0 = a_i(C_i - C_j) = \text{constant}$

$$b_1 = a_i$$

$$b_2 = 1 - \frac{a_i}{a_j}$$

The input demand equation (21) derived from the CRESH function is specified for the purpose of the present study:

For testing the hypothesis of the functional separability condition for the labor input with two types of labor inputs (L_1 and L_2), the following two equations will be used:

$$\log(K/L_1) = b_0 + b_1 \log(w_1/r) + b_2 \log(V/L_1) + u_1 \quad (22)$$

$$\log(K/L_2) = b_0 + b_3 \log(w_2/r) + b_4 \log(V/L_2) + u_2 \quad (23)$$

where V = value added

K = capital stock

L_1 = total man-hours of "production" worker

L_2 = total man-hours of "nonproduction" worker

w_1 = average wage rate per man-hour of L_1

w_2 = average wage rate per man-hour of L_2

r = rental price of capital (rate of return on capital)

b = parameters

u = random disturbance term

II -3. Estimation Method

If we estimate each equation separately and independently by applying the OLS method, we might disregard the information about the interdependence of the disturbances between equations. Subsequently, the efficiency of the OLS estimator becomes questionable. To take into account the interdependence of the disturbances across equations Zellner,¹⁷⁾ in his path-breaking paper, proposed an estimation method which is called a two-stage Aitken method. He demonstrated that this method is more

17) A. Zellner, "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias", *American Statistical Association Journal*, vol. 57 (June 1962), pp. 167-87. A. Zellner, "Estimators of Seemingly Unrelated Regressions: Some Exact Finite Sample Results", *Journal of the American Statistical Association*, vol. 58 (December 1963), pp. 977-92; N.C. Kakwani, "The Unbiasedness of Zellner's Seemingly Unrelated Regression Equations Estimators", *Journal of the American Statistical Association*, vol. 62. (March 1967), pp. 141-42; and J.S. Mehra and P.A.V.B. Swamy, "Further Evidence on the Relative Efficiencies of Zellner's Seemingly Unrelated Regressions Estimator", *Journal of the American Statistical Association*, vol. 71 (September 1976), pp. 634-39. For a list of further empirical evidences, see V.K. Srivastava and T.D. Dwivedi, "Estimation of Seemingly Unrelated Regression Equations, A Brief Survey", *Journal of Econometrics*, vol. 10 (April 1979), pp. 15-32.

efficient than an equation-by-equation application of the OLS. Specifically, Zellner proposed an efficient estimation procedure (hereafter called ZEF) such that the regression coefficients in all equations are estimated simultaneously (joint estimation procedure, to use Zellner's term) by applying Aitken's generalized least squares (hereafter called GLS) method to the system of seemingly unrelated regression equations.

Empirical test results have shown that the two-stage Aitken estimator is asymptotically equivalent to Aitken's GLS estimator and therefore to the maximum likelihood estimator in the linear model. Thus, this estimator is asymptotically efficient.

In order to alleviate the effect of heteroscedastic disturbances in the linear model the GLS estimation method has generally been suggested.¹⁸⁾ An obvious practical difficulty in the application of the GLS estimator is that the true disturbances are not known. More specifically, if the variance-covariance matrix is unknown, the GLS estimator could not be employed. If the variance-covariance matrix of residuals is misspecified, the resulting estimates are not efficient. Moreover, the estimator of the variance-covariance matrix of the estimates of the regression coefficients is biased.¹⁹⁾ Consequently, as noted in the literature, the variance-covariance matrix can be calculated by using the almost unbiased estimator (hereafter called AUE) method introduced by Horn, Horn and Duncan.²⁰⁾ These authors showed that the AUE is consistent, has smaller mean square error than other methods, and nonnegative estimates in the calculation of the variance-covariance matrix. The AUE is not unbiased,

18) A.S. Goldberger, "Best Linear Unbiased Prediction in the Generalized Linear Regression Model", *Journal of the American Statistical Association*, Vol. 57, No. 298 (June 1962), pp.369-77; and T. Amemiya, "GLS with an Estimated Auto-covariance Matrix", *Econometrica*, Vol. 41, No. 4 (July 1973), pp.723-32.

19) T. Amemiya, "Specification Analysis in the Estimation of Parameters of a Simultaneous Equation Model with Autoregressive Residuals", *Econometrica*, Vol. 34, No. 2 (April 1966), pp.283-306.

20) S.D. Horn, R.A. Horn and D.B. Duncan, "Estimating Heteroscedastic Variances in Linear Models", *Journal of the American Statistical Association*, Vol. 70, No. 350 (June 1975), pp.380-85. For a discussion of the properties of alternative estimators of heteroscedastic variances, see S.D. Horn and R.A. Horn, "Comparison of Estimators of Heteroscedastic Variances in Linear Models", *Journal of the American Statistical Association*, Vol. 70, No. 352 (Dec. 1975), pp.872-79.

but it bias does vanish when the weights of sample variances are corrected and in this respect the method, according to the literature, is an almost unbiased estimator. For these reasons, we apply the AUE method for calculating the unknown variance-covariance matrix of the GLS estimator.

II -4. Data Requirements and Sources

This study uses two sets of data: cross-section and time-series data for U.S. manufacturing industries.

For the cross-sectional analysis,²¹⁾ we used the data for the two-digit U.S. manufacturing industries (eighteen manufacturing industries) and total manufacturing industry.

The time-series data contain annual observations for 1957—1976, and were obtained from the Annual Survey of Manufactures, Bureau of the Census, U.S. Department of Commerce, 1957—76. The Commerce Department has published Census of Manufactures for the years 1958, 1963, 1967 and 1972. During intercensal periods, the Annual Survey of Manufactures which is compatible with the Census of Manufactures data provides a continuous series of statistics for the manufacturing industries.

III. Empirical Results

III -1. Cross-Section Analysis

Examination of the empirical results presented in Table 1 Shows:

First, the difference between the two PES, σ_{L1K} and σ_{L2K} , is statistically significant at the .01 significance level (i.e., $\sigma_{L1K} \neq \sigma_{L2K}$) in total manufacturing industry and in all two-digit manufacturing industries, except for the clothing (#23), furniture and fixtures (2#25), leather (#31) and

21) For the cross-section data, under the condition of perfect competition, the factor price ratios for each firm (industry) across states are assumed, on theoretical grounds, to be equal to each other. However, this is an empirical question. Therefore, it is of interest to compare the cross-section results with the time-series results. See T. Mayor, "Some Theoretical Difficulties in the Estimation of the Elasticity of Substitution from Cross-Section Data", *Western Economic Journal*, Vol. 7 (July 1969), pp. 153-63.

instruments (#38) industries. The test result rejects the functional separability condition, which states that production and nonproduction workers are functionally separable from the capital aggregate, in the majority of industries for the cross-section data. It is necessary, therefore, to treat production and nonproduction workers as distinct factors of production for the specification of a production function. This suggests that the internal structure of a production function involving two sub-functions, labor and capital, should be expressed as:

$$Y = F(L_1, L_2, K)$$

where Y is output, L_1 and L_2 are production and nonproduction workers, respectively, and K is assumed to be an aggregate input of capital.²²⁾

The test results, on the other hand, support the functional separability condition that $\sigma_{L_1K} = \sigma_{L_2K}$ in the four industries mentioned above (#23, #25, #31 and #38). This may be due to the fact that a relatively low proportion of skilled workers is required in these industries, so that no significant distinction in the labor quality (the degree of the skill intensity)²³⁾ exists between the production and the nonproduction workers.

In order to show the relationship between the relative degrees of substitution of one type of labor for the other ($\sigma_{L_1L_2}$) and the percentage of skilled workers for each two-digit manufacturing industry, we construct Table 2.

We have found, thus, from the results presented in Table 2, that the industries with a low proportion of skilled workers appear to have a high PES between production and nonproduction workers, except for the

22) For testing the functional separability hypothesis of the labor aggregate, the existence of a capital aggregate is generally assumed. One way to extend this study would be the application of the model to the test of the functional separability hypothesis within the context that labor and capital inputs are disaggregated simultaneously. This would require further study.

23) Three skill intensity indicators (proportion of professional and technical workers; proportion of managerial and administrative workers; and average wage by industry) were used by Teitel. He found evidence of a pattern of statistically significant concordance in the rankings by skill intensity of the manufacturing industries in the countries analyzed. See S. Teitel, "Labor Homogeneity, Skill Intensity and Factor Reversals—An International Comparison", *Journal of Development Economics*, 3, 1976, pp. 355-66.

Table 1. Time-Series Estimates of the PES: Two-Digit Manufacturing Industries

Industry*Code	σ_{L_1K}	σ_{L_2K}	t
Mfg. Total	1.029	.752	19.47
# 20	-1.375	1.757	84.88
# 22	1.436	.297	122.47
# 23	1.242	1.833	1.99*
# 24	1.495	.105	7.68
# 25	2.544	-.519	91.43
# 26	1.169	.781	18.48
# 27	1.005	.525	14.12
# 28	1.744	1.097	46.61
# 29	1.101	.671	29.11
# 30	2.023	4.524	91.95
# 31	1.183	-.875	48.82
# 32	.825	.780	1.59*
# 33	.885	.309	169.41
# 34	.998	.669	36.15
# 35	1.971	.164	72.28
# 36	.757	2.942	227.60
# 37	1.222	.800	48.39
# 38	.842	.902	2.16*

Notes: All industries satisfy the concavity condition.

* Indicates that $\sigma_{L_1K} = \sigma_{L_2K}$ at the .01 significance level.

instruments industry (#38).

Second, an interesting fact found throughout the comparison of the relative magnitudes of two PES is that $\sigma_{L_1K} > \sigma_{L_2K}$. This result seems to reveal a theoretical implication for the hypothesis of capital-skill complementarity which was proposed and tested by Griliches and Fallon-Layard.

Suppose, for example, that to some extent, an industry (firm) substitutes capital for labor in the production process as the price of capital decreases relative to the wage rates of labor inputs. Then those industries in which the σ_{L_2K} is lower than the σ_{L_1K} would tend to substitute capital for nonproduction workers relatively less than for production workers per unit of capital for a given level of output. In other words, it is expected,

Table 2. Cross-Section Estimates of the PES Between Production and Nonproduction Workers and % of Skilled Workers

Industry Code	$\sigma_{L_1L_2}$	% of Skilled Workers
Mfg. Total	6.535	7.55
# 20	8.446	2.63
# 22	-4.543	1.89
# 23	5.058	1.04
# 24	6.369	1.50
# 25	1.569	1.80
# 26	-10.842	5.44
# 27	.592	9.12
# 28	.382	15.51
# 29	1.847	16.25
# 30	1.127	5.94
# 31	17.858	1.04
# 32	35.299	4.96
# 33	7.106	5.06
# 34	4.671	5.34
# 35	.709	9.44
# 36	.582	15.27
# 37	-7.406	12.27
# 38	6.633	13.91

Note: * The proportions of skilled workers are calculated by dividing the number of professional and technical workers by total employed persons (14 years old and over) in manufacturing industry and are obtained from *Census of Population 1960, Detailed Characteristics of Occupations by Industry*, Table 209, Bureau of the Census, U.S. Department of Commerce, 1960.

on theoretical grounds, that as the employment of capital increases because of a decrease in the price of capital, the employment of nonproduction workers is *relatively* greater than production workers, even though the employment of both production and nonproduction workers per unit of capital *absolutely* decreases because of the substitutability between capital and labor.

Consequently, as the price of capital decreases relative to the wage rates of labor inputs, those industries in which the σ_{L_2K} is lower than the σ_{L_1K} would become skill-intensive. Thus, the differences in the PES (i.e., $\sigma_{L_1K} > \sigma_{L_2K}$) might lead to evidence for the capital-skill complementarity

hypothesis that "skilled" workers (nonproduction workers) are more complementary with capital than are "unskilled" workers (production workers).²⁴⁾

Third, the two-digit manufacturing industries can be categorized in two groups, "consumer-oriented" and "producer-oriented" industries.²⁵⁾ In general, consumer-oriented industries tend to be characterized by relatively higher elasticity of substitution between the aggregate inputs of labor and capital, and to operate in more competitive market structures than do producer-oriented industries.

Consequently, in terms of the cross-section estimates of the PES among manufacturing industries, there is no clear cut distinction between "consumer-oriented" and "producer-oriented" industries groups. We find little evidence to support the argument that consumer-oriented industries tend to be characterized by higher elasticity of substitution than do producer-oriented industries.²⁶⁾

24) According to the Bureau of Labor Statistics, U.S. Dept. of Commerce, the category of nonproduction workers (L_2) contains more skillintensive occupations (professional and technical workers) than does the category of production workers (L_1). Therefore, throughout this study, nonproduction workers may be referred to as "skilled" workers and production workers as "unskilled" (low skilled) workers.

25) Following Dhrymes, we classify the eighteen two-digit manufacturing industries into two groups: "consumer-oriented" and "producer-oriented" industries. The consumer-oriented industries group consists of the food (#20), textile (#22), clothing (#23), lumber and wood (#24), furniture and fixtures (#25), paper (#26), printing and publishing (#27), rubber and plastics (#30), leather (#31), and stone, clay and glass (#32) industries. The producer-oriented (or, "investment-oriented" to use Dhrymes' term) industries group consists of the following industries: chemicals (#28), petroleum and coal (#29), primary metal (#33), fabricated metal (#34), machinery (#35), electrical equipment (#36), transportation equipment (#37) and instruments (#38). See P.J. Dhrymes, "Some Extensions and Tests for the CES Class of Production Functions", pp. 364-65.

26) The same result can be found in Griliches' study. Griliches' study. Griliches estimated by using a two-factor CES function, the elasticity of substitution between capital and labor (production workers) for the cross-section U.S. manufacturing industries (two-digit) in 1958. He then concluded that "there is no sharp dichotomy in the estimates of the elasticity of substitution between 'consumer-oriented' and 'produceroriented' industries." However, Dhrymes found that consumer-oriented industries tend to be characterized by higher elasticity of substitution (between aggregate inputs of capital and labor) than do investmentoriented industries. See Z. Griliches, "More on CES Production Functions", *Review of Economics and Statistics*, vol. 49 (November 1967, pp. 608-10.

■ -2. Time-Series Analysis

Based on the empirical results presented in Table 3, three findings are discussed:

First, our time-series results do not support the functional separability condition that the aggregate input of labor does represent the two different categories of labor inputs, production and nonproduction workers in the total manufacturing industry and in all two-digit manufacturing industries, except for the clothing (#23), stone, clay and glass (#32), and instruments (#38) industries. The test results tend to suggest that production and nonproduction workers should be treated as distinctive factors of production, rather than aggregating them into a composite measure of

Table 3. Cross-Section Estimates of the PES: Two-Digit Manufacturing Industries

Industry Code	$\sigma L_1 K$	$\sigma L_2 K$	t
Mfg. Total	3.227	.409	29.92
# 20	.330	2.640	13.71
# 23	.797	.794	.01*
# 24	1.363	.650	33.20
# 25	.798	.950	.84*
# 26	7.507	2.549	2.96
# 27	.736	1.206	25.36
# 28	2.237	1.904	5.84
# 29	3.936	-.839	14.61
# 30	3.479	.483	13.07
# 31	.506	.843	2.18*
# 32	-16.721	2.154	8.84
# 33	.886	.669	3.69
# 34	.362	1.119	33.53
# 35	.161	.297	12.40
# 36	1.410	.713	10.25
# 37	4.525	1.170	4.27
# 38	.744	.697	1.55*
<hr/>			
# 22	1.220	-2.123	20.23

Notes: The industry under the dotted line violates the concavity condition.

* Indicates that $\sigma L_1 K = \sigma L_2 K$ at the .01 significance level.

Table 4. Time-Series Estimates of the PES Between Production and Nonproduction Workers and % of Skilled Workers

Industry Code	$\sigma_{L_1L_2}$	% of Skilled Workers*	
		1960	1970
# 20	11.428	2.63	4.71
# 22	4.223	1.89	3.31
# 23	9.467	1.04	2.03
# 24	2.079	1.50	2.74
# 25	5.584	1.80	3.06
# 26	5.313	5.44	6.80
# 27	4.166	9.12	12.72
# 28	5.070	15.51	19.01
# 29	2.101	16.25	21.63
# 30	-9.286	5.94	8.74
# 31	4.731	1.04	1.90
# 32	1.858	4.96	6.01
# 33	0.950	5.06	9.10
# 34	3.566	5.34	11.51
# 35	5.800	9.44	11.13
# 36	-7.425	15.27	20.12
# 37	4.595	12.27	14.50
# 38	-20.391	13.91	17.71
Mfg. total	7.214	7.55	9.92

Note: * The proportions of skilled workers are calculated by dividing the number of professional and technical workers by total employed persons (14 years old and over) in manufacturing industry and are obtained from *Census of Population* 1960 and 1970, U.S. Dept. of Commerce, 1960, 1970.

aggregate labor input.²⁷⁾

The functional separability condition, however, is supported in the three industries mentioned above (#23, #32, and #38). This implies that the labor aggregate (L) in these industries for our time-series data does represent well the two components of labor inputs, production (L_1) and nonproduction (L_2) workers; i.e., $L=f(L_1, L_2)$. This may be due to the

27) M. Denny and M. Fuss reached the same conclusion for the U.S. total manufacturing industry. They rejected the hypothesis that "the two types of labor, blue collar and whitecollar workers, are separable from aggregate capital input for the U.S. manufacturing 1929-68." See M. Denny and M. Fuss, "The Use of Approximation Analysis to Test for Separability", pp. 404-17.

fact that production and nonproduction workers are highly substitutable for each other (i.e., $\sigma_{L1L2} > 1$) in these industries. Second, we have found that the substitutability between production workers (unskilled workers) and capital is relatively higher than the substitutability between nonproduction workers (skilled workers) and capital; i.e., $\sigma_{L1K} > \sigma_{L2K}$. This factor substitution behavior seems to reveal an interesting implication for technical progress in the time-series data. As shown in Table 4, over time, the proportions of skilled workers tend to increase over all the manufacturing industries. This result may be due to the fact that most manufacturing industries employ more skilled workers than unskilled workers per unit of capital input which incorporate technical progress over time. Consequently, technical progress tends to be accompanied by an increase in the skill-intensity (an increase in the proportion of the skilled workers relative to the unskilled workers) for a given level of output. A probable reason for this result is advanced by Guisinger:²⁸⁾

"Over time, machines (capital) may displace relatively more easily unskilled workers than skilled workers; sophisticated capital (structures or equipment) often needs sophisticated skilled workers to keep it operating smoothly."

III -3. Comparative Analysis

Based on the comparison of the two results thus far, we conclude that there are no clear cut patterns of factor substitution or complementarity among industries. We find little theoretical basis for explaining why the inconsistency appears only in those industries which were found in the comparison of the cross-section and the time-series results. The most probable reason seems to be the characteristics of the particular data used for the present study rather than of underlying production relations

28) See S. Guisinger, "Wages, Capital Rental Values and Relative Factor Prices in Pakistan", World Bank Staff Working Paper No. 287 (June 1978), Washington, D.C., p. 17.

in those particular industries. Therefore, we shall confine ourselves to a principal factor—differences in the quality of the data between the cross-section and the time-series results.

(i) It is generally expected that technological progress would have a greater impact on the time-series data (over time) than in the cross-section data (among states at a particular point in time). Consequently, we could assume that the time-series estimates of the PES in this study already incorporate substitution effect due to input-price changes along the production function with technical progress (i.e., a shift of the production function) throughout the period considered.²⁹⁾

Thus, the time-series and the cross-section estimates of the PES in this study may be divergent because of the impact of technological progress over time.³⁰⁾ However, the direction and magnitude of such a bias in the time-series estimates compared with the cross-section estimates would be difficult to determine exactly unless a proper specification of non-neutral technical change was compared with a model of the neutral technical change. This falls beyond the objectives of the present study.

(ii) Our estimation of the PES is based on the assumption of full competitive equilibrium state. More realistically, for each year in the time-series data, no guarantee can be given that the industry (firm) adjusts fully its factor proportions with respect to changes in factor

29) For a discussion of the relationship between technical progress and factor proportions, see M.J. Beckman and R. Sato, "Aggregate Production Functions and Types of Technical Progress: A Statistical Analysis", *American Economic Review*, vol.59 (March 1969), p.90; and D.B. Humphrey, "Substitution in an Input-Output Table", *Journal of Economics and Business*, 1978, pp.38-45.

30) Ferguson, for example, argues: "...the use of time-series data to estimate the elasticity of substitution imparts a downward bias that is basically attributable to changes in the quality of labor service, especially during periods of expansion and contraction. ...In recession periods, an increase in unemployment is normally accompanied by an increase in the quality of labor services because the more efficient workers (at each wage rate are the ones retained. ...Thus value added per man-year tends to increase in recession periods. ...The opposite tends to occur in periods of expansion: so on balance the observed slope is less than the true slope." See C.E. Ferguson, "Time-Series Production Functions and Technological Progress in American Manufacturing Industry", *Journal of Political Economy*, Vol. 73 (April 1965), p.142.

price rations.³¹⁾ Therefore, it might be conjectured that our failure to account for lagged adjustment might systematically bias certain estimates of the elasticities of substitution.³²⁾

(iii) We have assumed that the tax rates for capital stock (fixed assets) are constant across the states and over time. More realistically the presence of tax policies alters the price of capital.³³⁾ Tax rates on property and income reduce the after-tax rate of return on capital, while investment tax credits and accelerated depreciation allowances reduce the net acquisition price of the fixed assets. It might be reasonable to assume that at a particular point in time (cross-section data), effective tax rates and capital gains are likely to be equal across states (regions). Over time (time-series data), however, it seems reasonable to expect considerable variation in effective tax rates, capital gains and, in turn, nominal rates of return on capital.

Thus, the assumption of constant rates of tax over time, which might have a substantial impact on the rate of return on capital, might result in an error of measurement in our calculation of the rate of return on capital for each industry.

(iv) One may suggest use of a method of pooling together cross-section and time-series data in order to avoid a possible discrepancy between cross-section and time-series estimations. For the following two reasons, however, this study does not consider the pooling method for the purpose of estimating the elasticities of substitution.

(a) Problems of interpretation may arise from an application of the

31) Stigler has observed diverse sources of disequilibrium over time: a shift in demand, a change in taxes, a change in foreign markets, new regulations by governments, discovery of new technology, and so forth. See G.J. Stigler, *Capital and Rates of Return in Manufacturing*, pp.64-66.

32) Lucas, however, found that "the lagged adjustment hypotheses make essentially no contribution to the reconciling of time-series and cross-sectional evidence of substitution." See R.E. Lucas, Jr., "Labor-Capital Substitution in U.S. Manufacturing", in *The Taxation of Income from Capital*, eds. A.C. Harberger and M.J. Bailey (Washington, D.C.: The Brookings Institution, 1969), pp.223-74.

33) For further discussion of this, see R.M. Coen, "Effects of Tax Policy on Investment in Manufacturing", *American Economic Review*, Vol.58 (May 1968), pp.200-11; and R.E. Hall and R.E. Hall and D.W. Jorgenson, "Tax Policy and Investment Behavior", *American Economic Review*, Vol.57 (June 1967), pp.391-414.

pooling technique. That is, since some of its coefficients represent a relationship to a particular point in time (cross-section) while others represent the relationship to the entire period considered, the problem arises of how to interpret the elasticities estimated from the equation which are based on the pooling data for cross-section and time-series.

(b) There is an econometric estimation problem. The behavior of the disturbances in the cross-section data (states) can not be treated as the same behavior as the disturbances in the time-series data. Kmenta argued that it is not possible to construct the variance matrix which represent different behaviors of the disturbances in both sets of data without imposing restrictions on the variance matrix.³⁴⁾ The problem of economic interpretation of the econometric restrictions which are imposed for the sake of simple estimation still remains. Further, Shupack maintained that the functions estimated from pooling methods are not efficient for prediction.³⁵⁾

IV. Conclusion

In order to investigate the possibility of the disaggregation of the labor aggregate into production and nonproduction workers, we have tested, using the CRESH model, the functional separability hypothesis of whether or not the PES between production workers and capital is equal to the PES between nonproduction workers and capital. The test results show that in most U.S. manufacturing industries, the difference between the two PES is statistically significant at the .01 significance level in both sets of data—cross-section and time-series. This empirical result rejects the functional separability condition that production and nonproduction workers are functionally separable from aggregate capital. Our results tend to support the disaggregation of the labor aggregate into production

34) J. Kmenta, *Elements of Econometrics*, pp.508-17.

35) M.B. Shupack, "The Predictive Accuracy of Empirical Demand Analysis", *Economic Journal*, September 1962, pp.550-75.

workers ("unskilled" workers) and nonproduction workers ("skilled" workers), which have different skill-intensity.

However, our test results support the existence of labor aggregation in some industries within the manufacturing sector. These industries are: clothing (#23), furniture and fixtures (#25), leather (#31), instruments (#38) in the cross-section result; and clothing (#23), stone, clay and glass (#32), instruments (#38) in the time-series result. Except for the instruments industry (#38), these industries tend to be characterized by relatively low proportions of skilled workers and to have relatively high elasticities of substitution between the two categories of labor inputs, production and nonproduction workers.

As a consequence of the test of the functional separability hypotheses for the aggregate input of labor we have also made three interesting observations.

(1) our estimates of the σ_{LK} appear to be relatively lower than those of the σ_{L1K} , indicating that capital works more closely with nonproduction workers ("skilled" workers) than with production workers ("unskilled" workers) in the majority of the manufacturing industries. The complementary relationship between capital and nonproduction workers obtained from the CRESH model is confirmed by the results obtained from Griliches' model. We have confirmed, thus, the findings of Griliches who proposes and tests the hypothesis of capital-skill complementarity.

(2) From the results of the estimated PES, we have recognized that the degrees and the signs of the estimated PES (positive or negative values of the PES) between factors of production vary widely from industry to industry. We have found little evidence to support the argument that consumer-oriented industries tend to be characterized by high elasticities of substitution between factors of production than do producer-oriented industries. We have failed to find any generally valid pattern of the factor substitution behavior among industries. This will require further research to analyze the inter-industry patterns of the estimates of the PES in relation to, for example, the skill-intensity, the

productivity of factors and the capital-labor ratio for the various manufacturing industries.

(3) Due to the impact of technological progress and different rates of capacity utilization over time, it was expected a priori that time-series estimates of the PES would tend to be lower than cross-section estimates. We have found, however, that our results appear to be mixed in some industries within the manufacturing sector. The most probable reason may be due to the fact that the impact of technological progress over time varies widely from industry to industry. We have assumed that the effect of technical change could be separated from that of factor substitution and that it was Hicks-neutral. More realistically, it is difficult to isolate the substitution possibilities without technical progress from economies reaped through technical progress overtime, and alternative specification of technical change may be necessary. In order to reconcile the divergency between the cross-section and time-series estimates, the proper treatment of technological change for the time-series analysis seems to be worthy of further research.

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