Optimal Control with Rational Expectations and Time Inconsistency Problem

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I. INTRODUCTION

The time inconsistency problem related to the optimal control technique under the assumption of rational expectations is that a rational agent will anticipate the future expected changes resulting from government policy. The agent will reflect that information in making decisions based upon future expectations, so that the parameters of the structural equation will be changed and the optimal solution may not be the best. However, this time inconsisteny problem is not the problem of the optimal control technique itself but a matter of specification of the econometric model. The problem can be solved by specifying the optimal control model with an open loop so that the new optimal value feeds into the initial value, the agent's parmeters will continously reflect new expectations and the new optimal path based upon the renewed structural change will give us a consistent and best optimal path. This result has shown the renewed solution.

II. POLICY INVARIANCE PROPOSITION WITH RATIONAL EXPECTATIONS

This paper explores how monetary policy is affected by the assumption of rational expectations. Under the strong version of rational expectations, the effects of monetary policy will be nil, whereas under a weak version of rational expectations with imperfect information, monetary policy will be effective. In section A, the policy Invariance Proposition with a strong version of rational expectations will be discussed, and the effects of a weak

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version of rational expectations will be discussed in section B. In section C, the Time Inconsistency problem, which an OPTIMAL computed from the time variant structural equations, may be the first best because economic agents change their behavior based upon future expectations. In order to see how the results are different when one gets consistent parameters, the program is split into two : one for the government and the other for the private agent

Adaptive expectations have the disturbing implcation that they allow individuals to make systematic forecasting errors period after period, without requiring any amendment to the forecasting rule itself. The point of departure of Rational Expectations is that individuals should not make systematic errors. This does not imply that individuals invariably forecast accurately in a world in which some random movements are inevitable ; rather, the assertion is that guesses about the future must be correct on average if individuals are to remain satisfied with their mechanism of expection formation. of expectations formation. [Steven Sheffrin, 1983].

When such information is widely available, individuals will be assumed to know the entire structure of the model and previous value of all the relevant variables within that model. For example, individuals knowing the are assumed to guess the price level next period, thereby inferring the expected inflation rate. The price level next period is merely one endogenous variable at that date.

The hypothesis of Rational Expectations asserts that the unobservable subjective expectations of indivduals are exactly the mathematical conditional expectations implied by the model itself.

$$X^e_{t-1}=E[X_t/\Omega_{t-1}] \tag{1}$$

Individuals act as if they have a model and form expectations accordingly. By forming their best current guesses about the values of exogenous variables next period, individuals may use the model to solve for the expected values of endogenous variables including the price level in the example. When uncertainty is absent, and information complete, govemment monetary policy does not affect output except unanticipated changes of money supply. This is called the Policy Invariance Proposition of Rational Expectations. To explain this strong assumption of rational expectations, the rational expectationists [Lucas(1973), Barro(1976). Sargent(1973)] explain the logic with the Lucas supply equation in logarithmic form.[1]

$$Y_t = YF + \beta[p_t - p^e_{t-1}(\Omega)] \quad (2)$$

where Y = current output level

YF = full employment level of output

p_t = current price level,

The expected price $p^e(\Omega)$ is log of the price level that the public expectations will occur in time t viewed from period(t—1).

Information set Ω_{t-1} is global information of (t—1), so that economic agents can make decisions by global optimization, whereas the information set 1_{t-1} is partial information of (t—1)period so that economic agents can make decision by local optimization, that is, 1_{t-1} is a subset of Ω_{t-1}

$$1_{t-1} \in \Omega_{t-1} \quad (3)$$

$$\text{and } p^e_{t-1}(1_{t-1}) = p^e(\Omega) - s \quad (4)$$

The aggregate demand function in log form is defined

$$M_t + V_t = P_t + Y_t \quad (5)$$

and a monetary rule will be expressed

$$M_t = \alpha_1 Y_{t-1} + \varepsilon \quad (6)$$

For simplicity, from now on $P^e{}_{t-1}$ will be denoted as P^e, Substituting both(1—2) and (1—6) into(1—5).

$$\alpha_1 Y_{t-1} + \varepsilon + V_t = P_t + YF + \beta[P_t - P^e_{t-1}] \quad (7)$$

When we take the mathematical expectation

$$\alpha_1 Y_{t-1} + V_t = P^e + YF + \beta [P^e - P^e] \quad (8)$$

$$P^e(\Omega) = \alpha_1 Y_{t-1} + V_t - YF \quad (9)$$

Substitute P^e above equation (1—9) into (1—8).

$$\alpha_1 Y_{t-1} + \varepsilon V_t = P_t + YF + \beta [P_t - \alpha_1 Y_{t-1} - V_t + YF]$$

$$P_t(1+\beta) = \alpha_1 Y_{t-1}(1+\beta) + V_t(1+\beta) - YF(1+\beta) + \varepsilon_t$$

$$P_t = \alpha_1 Y_{t-1} + V_t - YF + \varepsilon /(1+\beta)$$

$$(P_t - P^e(\Omega)) = \varepsilon_t/(1+\beta) \quad (10)$$

Substitute (1—10) into (1-2) and obtain the following equation.

$$Y_t = YF + \varepsilon /(1+\beta) \quad (11)$$

Only the unanticipated part of money supply (ε) affects output(Y). There is no money term (M_t) which influences output systematically, Therefore, the monetary policy is ineffective on income.

Unlike the pefect foresight case, the weak version of rational expection is "Partly Rational" in the Sargent's(1973) paper

One criticism that has been made of the kind of model presented here is that it seems to require extraordinary amount of wisdom and information of the part of those whose expectations are describey by equation ....

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While assuming such a well-informed public may or may not strain credulity, the key aspect of the theory carry through even if the public is much less wise and knowledgeable.

Causes of "Partly Rational" or "Bounded Rationality"[2] are

(1) Imperfect information due to information costs.

(2) The limit of man's abilities to comprehend and compute in the face of complexity.

(3) Uncertainty about relevant exogenous events events, and inability to

calculate conseqences.

III. POLICY EFFECTIVENESS UNDER THE WEAK ASSUMPTION OF REH

The weak version of rational expectations due to "Partly Rational", the economic agents will optimize locally with the incomplete information (1_{t-1}). Therefore, the Lucas supply equation will be changed.[3]

$$Y_t=YF+\beta[P-P^e(1)] \tag{12}$$

Where $P^e=P^e_{t-1}$

Substituting both eqations (1—12) and (1—6) into (1—5)

$$\alpha_1 Y_{t-1}+\varepsilon+V_t=P_t+YF+\beta[P_t-P^e(1)] \tag{13}$$

Taking the mathematical expectation.

$$\alpha_1 Y_{t-1}+V_t=P^e+YF+\beta\,[P^e(\Omega)-P^e(1)]$$

$$\alpha_1 Y_{t-1}+V_t=P^e(\Omega)\beta\,YF+\beta[P^e(\Omega)-P^e(\Omega)+S]$$

$$P^e(\Omega)=\alpha_1 Y_{t-1}+V-YF+\beta S \tag{14}$$

Substitiute $P^e(\Omega)$ above (1—14) into (1—13)

$$\alpha_1 Y_{t-1}+\varepsilon+V_t=P_t+YF+\beta[P_t-\alpha_1 Y_{t-1}-V_t+YF-\beta S-S]$$

$$P_t(1+\beta)=\alpha_1 Y_{t-1}(1+\beta)+V_t(1+\beta)-YF(1+\beta)+\varepsilon_t+\beta S(1+\beta)$$

$$P_t=\alpha_1 Y_{t-1}+V_t-YF+\varepsilon_t/(1+\beta)+\beta S$$

Subtract $P^e(1)$ from both sides.

$$(P_t-P^e)=\alpha_1 Y_{t-1}+V-YF+\beta S+\varepsilon_t/(1+\beta)-P^e(1) \tag{15}$$

$$(P_t-P^e(1))=\alpha_1 Y_{t-1}+\varepsilon_t-\varepsilon_t-YF$$

$$V_t+\beta S+\varepsilon_t/(1+\beta)-P^e(1) \tag{16}$$

Substitute (1—16) into (1—12)

$$Y_t=YF+\beta[M_t-\varepsilon_t+V_t-YF+\beta S+\varepsilon_t/(1+\beta)-P^e] \tag{17}$$

$$Y_t=YF(1-\beta)+\beta[M_t+V_t-\beta S-P^e(1)-\varepsilon_t/(1+\beta)] \tag{18}$$

Money supply (M_t) appears on the right-hand-side of the last equation so that money affects output systematically. When one assumes imperfect information and "Partly Rational", economic agents cannot globaly optimize. As a conequence, government monetary policy is effective on output.

IV. TIME INCONSISTENCY PROBLEM AND THE GAME THEORETIC MODEL

Kydland and Prescott (1977) argue that optimal control theory is an appropriate planning device for situations in which current outcomes and the movements of the system depend only upon current and past policy decisions and upon the current state. However, for dynamic systems current decisions of economic agents depend in part upon their expectations of future policy actions. Only if these expectations were invariant to the future policy plan would optimal control theory be appropriate.

They argue that control theory is not the appropriate tool for dynamic economic planning because current decisions of economic agents depend upon expected future policy, and these expectations are not invariant to the plans selected when expectations are rational.

Changes in the social objective function reflected in a change of administration have an immediate effect upon agent's expectations of future policies and affect their current decisions. This is inconsistent with the assumption of optimal control theory.

Suppose that the economy at time t can be described by a vector of state variables y_t, a vector of policy variables Z_t, a vector of decision variable X_t for the economic agents, and a vector of random shocks e_t. The movement over time of these variables is given by the system of equations

$$Y_{t+1}=F(Y_t,Z_t,X_t,e_t) \tag{19}$$

Objective function : $W(x_1,\ldots,x_T,\ z_1,\ldots,z_T)$

Let the feedback government policy rule for future periods be

$$Z_s = Z^f(y_s),\ s > t \tag{20}$$

For certain situations, rational economic agents will, in the future, follow a rule of the form

$$X_s = d^f(y_s\ ;\ Z^f) \tag{21}$$

Changes in policy rule Z^f Change the functional form of d^f a point convincingly made by Lucas(1976), in his critique of current econometric policy—evaluation procedure. The decisions of agents in the current period will depend on Z^f and current Z_t where $Z_t = Z_t(z_1,\ldots,z_{t-1},x_1,\ldots,x_{t-1})$.

Economic agents current decision :

$$X_t = d^c(Y_t, Z\ ;\ Z^f) \tag{22}$$

The best policy rule for the current period $Z^c(Y)$ is functionally related to the policy rule used in the future $Z^f(Y)$.

Government policy rule :

$$Z^c = g(z^f) \tag{23}$$

A stationary policy rule Z is consistent if it is a fixed point of mapping g, for then it is best to use the same policy rule as the one expected to be used in the future, according to their definition of consistency.

> A policy Z is consistent if, for each time period t, Z_t maximizes objective function, taking as given previous decisions, $X_1,\ldots,X_{t-1}$, and that future policy decisions (Z_s for $s > t$) are similarly selected.

If the future decisions are differently selected due to the expectations of different government policy rule to be used, the policy rule Z is inconsistent, since it affects their current decisions.

Lucas(1976) argues that changes in the behavioral relations in the equations of an econometric model arise so that parmeters estimated from sample-period values may not be invariant to arbitrary shifts in policy, because agents change their forecasting schemes to adapt to a new economic enviroment. The structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models. Therefore, existing econometric models are almost useless for traditional analysis since traditional policy analysis assumes the economic structure will not change when different policy is applied.

In this case, the expected future economic changes will influence the current decision making and the structure of the equation will be changed. In order to solve this time inconsistency problem, Chow(1983) modifies the structural equation (1—1) as follows;

$$Y_t = AY_{t-1} + C_1X_{1t} + C_2X_{2t} + b_t + u_t \tag{24}$$

where X_{1t} is the control variable subject to the control the economic agents, and X_{2t} are the control of the government.

A Government policy rule is represented by

$$X_{zt} = {}_{zt} = G_2Y_{t-1} + g_{2t} \tag{25}$$

Given the policy rule, the environment facing the economic agents becomes

$$\begin{aligned} Y_t &= (A + C_2G_2)Y_{t-1} + C_1X_{1t} \\ &\quad + b_t + c_2g_{2t} + v_t \\ &= A_1Y_{t-1} + c_1x_{1t} + b_{1t} + v_t \end{aligned} \tag{26}$$

If the economic agents objective function is specified as

$$W_1 = \Sigma_{t=1}(Y_t - z_{1t})^{\prime}k_{1t}(Y_t - z_{1t}) \tag{27}$$

To get a control of economic agents by minimizing the expectation of (V—15) subject to (V—14).

$$X_{1t} = G_{1t}Y_{t-1} + g_{1t} \tag{28}$$

Where G_{1t} may become time invariant.[4]

The esimation procedure for a dynamic game model consists of two stages applying iterative techniques. First, assuming tentatively that the government adheres to a policy rule ($G2,g_{2t}$), the likelihood estimates of the parameters of equation (V—14) are maximized under the assumption that the private sector behaves optimally. Second, assuming that the private sector adheres to the policy (G_2,g_{2t}) as determined above, the parameters are estimated by the maximunm liklihood method under the assumption of the government behaves optimally. This process will go back to step one iterate back and forth until convergence.

V. OPTIMAL CONTROL WITH TIME PARAMETERS

Franco Modigliani(1977) comments on Prescott's paper(1977) for the time inconsistency problem is,

> "···the results might be quite different if one relies on the much relevant sequential open loop procedure. In each period, the optimal policy of model A is applied to model B ; the, results are used to establish new inital conditions for model A in order to secure the optimal policy to apply to B in the next period."[p. 85]

Figure (V—1) shows an open loop of two programs :

Program A = government

Program B = private agents

Program A takes inital values, targets, and weights from the overall directives from the EMS in order to compute optimal path X_{2t}. Program B takes the optimal path of the government as its initial value and targets in order to compute optimal path X_{1t}.

In the second period it is still optmal because the new optimization takes care of the time change. In the third period, program A uses the optimal value X_{1t} computed by B as initial values to get a third period optimal which are counted changes in the previous two periods. This iteration process will continue back and forth until the policy maker wants to make a decision. This method with open loops will solve the time inconsistency problem in such a way that each period of time change is reflected in the parameters, and the new optimal value will be best and consistent.

In order to how the results of optimal control are different if rational expectations are assumed, the program is split into two programs : program A and program B. In program A, the government assumes that a higher income growth policy is planned. Program B for the civilian agent assumes an inactive situation which the private economy does not have a particular weights on certain state variable.

All observations quarterly data over the 1970I-1982IV period from the Business Statistics published by the U.S. Department of Commerce.

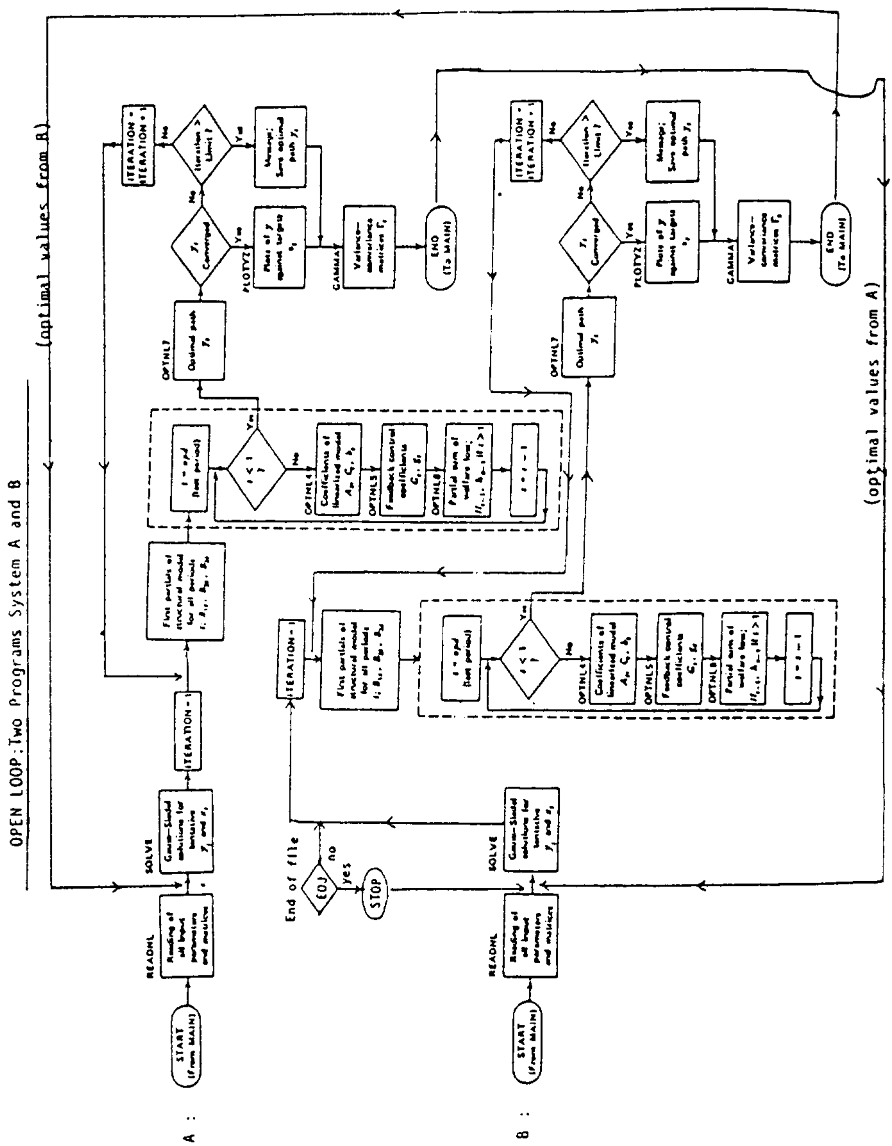
Program A : (Govenment)

(1) Quarterly nominal income growth=2.67%
(2) Quarterly inflation rate=1.92%
(3) Weighting scheme : K[2.1.1].
(4) Government takes initial value(t)=Civilian optimal (t—1), that is, $Y_{ot}(A)=Y_{ot-1}(B)$, $Z_{ot}(A)=Z_{ot-1}(B)$

Program B : (Civilian)

(1) Quarterly nominal income growth=2.48%
(2) Quarterly inflation rate=1.92%
(3) Weighting scheme : K[1.1.1]
(4) Civilian agent takes initial value(t)=Government optimal(t—1), that is, $Y_{ot}(B)=Y_{ot-1}(A)$ and $Z_{ot}(B)=Z_{ot-1}(A)$

Both programs A and B assume perfect foresight so that the government takes the initial value from the Civilian's optimal value of the last period, and the Civilian takes his initial value from the government's optimal of the last period.



The steps of the programs are as follows:

(1) Both program A and B start out with own initial values of state and target variables:

(2) Run both programs to get optimal paths of each program for the period 1:

(3) In period 1, program A takes the optimal of from program B of the last period, while program B takes its initial value from the optimal of program A of the last period:

(4) Run both programs to get the optimal paths of dependent variables and controls for period 2:

(5) Continue this process until it reaches the desired time period. The plots of time invariant and time variant optimal control programs are shown in the next figures. In the case of the time invariant optimal control, the fitted values almost hit targets at the 2nd, 5th, and 17th periods. The time variant case, the fitted values hit targets in most of the periods. The standard deviations of optimal paths are compared in the following table:

Table 1-1 Standard Deviation of Optimal Control

	1982 III		1982 IV	
	Time inv.	Time var.	Time inv.	Time var.
Y	8.90	3.87	10.32	4.10
P	.93	1.56	1.26	1.76
B	7.99	1.87	7.03	0.78
M	60.51	61.89	59.93	63.59
G	128.12	135.75	124.63	140.27
Welfare cost				
	1394685	1252701	1266208	1178450

The welfare cost in time invariant cases are larger than those of the time variant cases: the time invariant case of 1982 III is 11.3% larger than that of the time variant case in the same period. Similarly, in 1982 IV, the welfare cost of the time invariant case is 7.4% larger than that of the time variant case.

The standard deviations of the time variant case of income and balance of trade are smaller than those of time invariant case. This is consistent with the rational expectations assumption.

The experiment result supposrts the Kydland-Prescott argument of Time Inconsistencey. Parameters of the time variant case ars significantly differ ent from the parameters of time invariant case.

Time invariant case(1982 III):

$$Y_t = 0.30576y_{t-1} + 5.425P_{t-1} - 0.10849B_{t-1} + 7.4457M - 1.5055G - 1165.318 \quad (29)$$

Time variant case(1982 IV):

$$Y_t = 0.1848y_{t-1} + 4.1822P_{t-1} - 0.6885B_{t-1} + 7.932M - 0.924G - 1152.272 \quad (30)$$

Optimal money supply paths are also significantly different.

Time invariant case(1982 III):

$$M_t = -0.01087y_{t-1} + 0.74473P_{t-1} - 0.02982B_{t-1} + 336.359$$

Time variant case(1982 IV):

$$M_t = -0.00455y_t + 0.3537P_{t-1} + 0.0258B_{t-1} + 405.721$$

The optimal paths of income (Y), price (P), balance of trade (B), money supply (M), and government expenditures (G) in the two cases are compared below. The following table shows that the optimal paths of the time variant case are greater than those of the time invariant case.

The past simulated values of time variant and time invariant are shown in tables (1-3) and (1-4). The future optimal paths of time variant and time invariant cases are shown in tables (1-5,6,7).

Table 1-2 Optimal Paths of Time Invariant Case (1983 I)

	Time Inv.	Time Var.	Time Inv.	Time Var.
Y	318.7	3231.3	3201.8	3313.6
P	211.1	213.7	212.8	217.7
B	2.19	31.5	6.0	30.6
M	451.9	458.3	458.8	467.4
G	734.9	751.7	756.1	769.0

Table 1-3 Optimal Paths of Time Invariant Case

	YINV	PINV	BINV	MINV	GINV
19831	3201.80	212.800	6.00000	458.800	756.100
19832	3281.60	215.900	7.30000	466.000	767.400
19833	3361.00	219.200	9.00000	473.700	780.000
19834	3442.00	222.800	10.7000	481.700	793.900
19841	3525.10	226.600	12.5000	490.000	809.000
19842	3610.40	230.600	14.2000	498.600	825.400
19843	3697.90	234.700	15.9000	507.500	742.900
19844	3787.60	239.100	17.4000	516.700	861.400
19851	3879.50	243.600	18.9000	526.200	881.000
19852	3973.70	248.200	20.3000	536.000	901.500
19853	4070.30	253.100	21.6000	546.000	922.900
19854	4169.20	258.000	22.9000	556.300	945.200
19861	4270.40	263.200	24.0000	566.900	968.300
19862	4374.10	268.400	25.1000	577.800	992.200
19863	4480.30	273.900	26.0000	589.000	1016.90
19864	4589.20	279.400	26.9000	600.500	1042.40
19871	4701.30	285.100	27.8000	612.300	1068.50
19872	4817.90	291.000	28.6000	624.600	1095.10
19873	4942.10	297.000	29.5000	637.700	1121.80
19874	5082.30	303.300	30.4000	652.300	1147.90
	1	2	3	4	5

Note: INV denotes the time invariant case.

Table 1-4 Optimal Paths of Time Invariant Case

	YV	PV	BV	MV	GV
19831	3313.60	217.700	30.6000	467.400	769.000
19832	3396.10	221.800	29.9000	475.900	787.900
19833	3480.10	226.000	29.8000	484.600	807.200
19834	3586.00	230.400	29.9000	493.500	826.900
19841	3654.00	234.900	30.3000	502.700	847.000
19842	3744.10	239.400	30.8000	512.100	867.700
19843	3836.40	244.100	31.4000	521.700	888.900
19844	3930.90	249.000	32.0000	531.600	910.700
19851	4027.70	253.900	32.7000	541.700	933.000
19852	4126.90	259.000	33.5000	552.000	955.900
19853	4228.50	264.300	34.3000	562.700	979.500
19854	4332.70	269.600	35.1000	573.500	1003.70
19861	4439.30	275.100	35.9000	584.700	1028.40
19862	4548.60	280.800	36.8000	596.100	1053.90
19863	4660.60	286.600	37.7000	607.800	1080.00
19864	4775.40	292.500	38.7000	619.900	1106.80
19871	4893.40	298.600	39.6000	632.200	1134.20
19872	5015.20	304.800	40.6000	645.000	1162.30
19873	5143.30	311.300	41.6000	658.400	1190.80
19874	5083.80	317.900	42.5000	673.100	1219.20
	1	2	3	4	5

Note : V denotes the time variant case.

Table 1-5 Comparison of Time Variant Income Paths

	YINA	YINV	YV	YEMS	YCMG
19831	3202.10	3201.80	3313.60	3202.20	3202.30
19832	3282.00	3281.60	3396.10	3282.10	3782.54
19833	3361.30	3361.00	3480.10	3361.20	3361.91
19834	3442.30	3442.00	3586.00	3442.10	3442.81
19841	3525.40	3525.10	3654.00	3525.00	3525.74
19842	3610.60	3610.40	3744.10	3610.00	3610.82
19843	3698.00	3697.90	3836.40	3697.30	3698.09
19844	3787.70	3787.60	3930.90	3786.80	3787.60
19851	3879.60	3879.50	4027.70	3878.60	3879.35
19852	3973.70	3973.70	4126.90	3972.70	3973.36
19853	4070.20	4070.30	4228.50	4069.00	4069.58
19854	4169.00	4169.20	4332.70	4167.70	4168.32
19861	4270.20	4270.40	4439.30	4268.90	4269.33
19862	4373.80	4374.10	4548.60	4372.30	4372.78
19863	4479.90	4480.30	4660.60	4478.50	4478.78
19864	4588.80	4589.20	4775.40	4587.50	4587.61
19871	4700.80	4701.30	4893.40	4700.00	4699.85
19872	4817.50	4817.90	5015.20	4817.20	4816.92
19873	4941.90	4942.10	5143.30	4942.50	4942.24
19874	5082.30	5082.30	5283.80	5083.20	5083.40
	1	2	3	4	5

YINV=Optimal income path of an inactive policy

YINV=Optimal income path of an time invariant case

YV=Optimal income path of time variant case

YEMS=Optimal income path of the EMS

YCMG=Optimal income path of the CMG rule

Table 1-6 Comparison of Time Variant Money Supply Paths

	MINA	MINV	MV	MEMS	MCMG
19831	458.800	458.800	467.400	465.500	352.700
19832	466.000	466.000	475.900	472.110	354.580
19833	473.700	473.700	484.600	479.300	357.820
19834	481.700	481.700	493.500	487.000	361.500
19841	490.100	490.000	502.700	495.000	365.430
19842	498.700	498.600	512.100	503.400	369.570
19843	507.600	507.500	521.700	512.100	373.900
19844	516.800	516.700	531.600	521.100	378.420
19851	526.300	526.200	541.700	530.400	383.100
19852	536.000	536.000	552.000	540.000	387.960
19853	546.100	546.000	562.700	550.000	392.980
19854	556.400	556.300	573.500	560.200	398.160
19861	567.000	566.900	584.700	570.800	403.500
19862	557.900	577.800	596.100	581.600	409.000
19863	589.100	589.000	607.800	592.800	414.560
19864	600.500	600.500	619.900	604.200	420.500
19871	612.300	612.300	632.200	616.100	426.550
19872	624.600	624.600	645.000	628.400	432.920
19873	637.700	637.700	658.400	641.500	439.850
19874	652.300	652.300	673.100	656.200	447.860
	1	2	3	4	5

MINA=simulated money supply of an inactive policy

MS=simulated money supply of 3% income growth policy

MK=simulated money supply of weight, K[2.1.1].

MSK=simulated money supply of 3%+K[2.1.1].

MEMS=simulated money supply of the EMS

MCMG=simulated money supply of the CMG rule

Table 1-7 Comparison of Time Variant Price Paths

	PINA	PINV	PV	PEMS	PCMG
19831	212.800	212.800	217.700	212.800	212.800
19832	215.900	215.900	221.800	216.000	215.910
19833	219.200	219.200	226.000	219.400	219.270
19834	222.800	222.800	230.400	223.000	222.840
19841	226.600	226.600	234.900	226.800	226.620
19842	230.600	230.600	239.400	230.800	230.600
19843	234.700	234.700	244.100	235.000	234.760
19844	239.100	239.100	249.000	239.400	239.090
19851	243.600	243.600	253.900	243.900	243.590
19852	248.200	248.200	259.000	248.500	248.250
19853	253.100	253.100	264.300	253.400	253.060
19854	258.000	258.000	269.600	258.300	258.020
19861	263.100	263.200	275.100	263.400	263.130
19862	268.400	268.400	280.800	268.700	268.380
19863	273.800	273.900	286.600	274.100	273.770
19864	279.400	279.400	292.500	279.600	279.300
19871	285.100	285.100	298.600	285.300	284.980
19872	290.900	291.000	304.800	291.200	290.820
19873	296.900	297.000	311.300	297.200	296.860
19874	303.200	303.300	317.900	303.500	303.190
	1	2	3	4	5

PINA=optimal price path of an inactive policy
PINV=optimal price path of time invariant
PV=optimal price path of time variant
PEMS=optimal price path of the EMS
PCMG=optimal price path of the CMG rule

Table 1-8 Comparison of Time Variant Bot Paths

	BINA	BINV	BV	BEMS	BCMG
19831	6.00000	6.00000	30.6000	6.70000	6.14000
19832	7.30000	7.30000	29.9000	8.90000	7.67000
19833	9.00000	9.00000	29.8000	11.2000	9.45000
19834	10.7000	10.7000	29.9000	13.4000	11.2800
19841	12.5000	12.5000	30.3000	15.4000	13.1000
19842	14.2000	14.2000	30.8000	17.3000	14.8500
19843	15.8000	15.9000	31.4000	19.0000	16.5200
19844	17.4000	17.4000	32.0000	20.7000	18.1000
19851	18.9000	18.9000	32.7000	22.1000	19.5800
19852	20.3000	20.3000	33.5000	23.5000	20.9600
19853	21.6000	21.6000	34.3000	24.7000	22.2400
19854	22.8000	22.9000	35.1000	25.9000	23.4300
19861	23.9000	24.0000	35.9000	27.0000	24.5300
19862	24.9000	25.1000	36.8000	28.0000	25.5500
19863	25.9000	26.0000	37.7000	28.9000	26.5000
19864	26.8000	26.9000	38.7000	29.3000	27.4000
19871	27.6000	27.8000	39.6000	30.6000	28.2400
19872	28.5000	28.6000	40.6000	31.4000	29.0700
19873	29.3000	29.5000	41.6000	32.3000	29.9300
19874	30.3000	30.4000	42.5000	33.2000	30.9000
	1	2	3	4	5

BINA=optimal BOT of an inactive policy
BINV=optimal BOT of time invariant
BV=optimal BOT of time variant
BEMS=optimal BOT of the EMS
BCMG=optimal BOT of the CMG rule

Table 1-9 Simulation of Time Variant Case

	YV	PV	MV	GV
19701	2829.61	98.3981	432.831	-4.47271
19702	2832.37	99.5987	433.266	3.13237
19703	2835.59	100.718	433.660	10.0313
19704	2839.37	101.409	433.794	15.0174
19711	2841.82	102.580	434.182	23.1343
19712	2849.15	103.897	434.533	31.0556
19713	2854.39	105.137	434.803	40.4108
19714	2857.95	105.921	435.023	45.4024
19721	2862.30	106.755	435.186	51.5160
19722	2869.78	108.035	435.439	60.4136
19723	2875.02	108.715	435.555	64.7050
19724	2879.35	109.512	435.782	69.3328
19731	2886.42	110.753	436.081	77.1955
19732	2894.66	112.112	436.482	84.3710
19733	2899.52	113.818	437.125	94.2449
19734	2904.79	115.509	437.799	103.193
19741	2912.99	117.615	438.439	116.574
19742	2916.53	119.483	439.146	128.342
19743	2925.72	122.107	439.735	148.915
19744	2935.65	124.903	440.500	168.589
19751	2943.58	128.092	441.659	188.372
19752	2943.91	130.982	443.122	203.633
19753	2950.77	132.474	443.578	212.483
19754	2963.44	134.630	443.997	227.540
19761	2972.06	136.832	444.699	240.976
19762	2982.88	137.941	444.620	250.971
19763	2988.95	139.073	444.829	259.391
19764	2995.95	140.593	445.180	270.198
19771	3005.53	142.603	445.653	284.181
19772	3019.32	144.362	445.713	298.989
19773	3030.95	146.485	446.231	312.613
19774	3042.03	148.427	446.709	324.762
19781	3052.04	150.541	447.042	342.119
19782	3060.69	152.341	447.515	353.811
19783	3079.94	155.960	448.552	374.910

19784	3092.11	158.946	449.507	392.832
19791	3106.25	162.156	450.568	410.968
19792	3116.77	165.218	451.645	428.893
19793	3128.56	168.313	452.393	451.787
19794	3142.64	171.439	453.381	469.986
19801	3154.43	174.235	454.053	490.064
19802	3169.11	177.944	455.171	513.662
19803	3173.34	182.189	456.996	539.098
19804	3184.04	185.776	458.433	558.227
19811	3207.08	190.459	459.368	593.041
19812	3230.68	194.969	460.657	619.622
19813	3242.92	197.579	461.134	640.067
19814	3261.68	201.766	462.307	666.873
19821	3269.52	205.805	463.865	691.756
19822	3270.25	207.808	464.687	705.103
19823	3280.22	210.465	465.540	721.486
19824	3291.82	212.277	465.277	743.359

YV=Time variant income

PV=Time variant price

MV=Time variant money supply

GV=Time variant government expenditures

Table 1-10 Simulation of Time Invariant Case

	YINV	PINV	MINV
19701	2233.85	93.9533	391.739
19702	2229.31	95.0161	392.541
19703	2247.50	96.3017	393.216
19704	2252.89	96.9790	393.612
19711	2263.37	98.1564	394.507
19712	2275.69	99.5754	395.032
19713	2293.39	100.861	395.903
19714	2300.81	101.688	396.310
19721	2300.49	102.413	396.815
19722	2311.63	103.704	397.467
19723	2334.74	104.637	397.667
19724	2316.70	105.187	397.969
19731	2337.14	106.609	398.508
19732	2359.03	108.255	398.865
19733	2383.48	110.217	399.810
19734	2394.04	112.044	400.599
19741	2413.95	114.262	401.780
19742	2413.93	116.007	403.069
19743	2438.75	118.514	405.151
19744	2461.86	121.291	407.048
19751	2480.52	124.528	409.077
19752	2478.91	127.391	411.016
19753	2485.78	128.915	411.693
19754	2512.34	131.183	412.797
19761	2542.90	133.661	413.951
19762	2570.69	134.866	414.536
19763	2573.12	135.878	415.227
19764	2579.50	137.307	416.164
19771	2610.54	139.499	417.339
19772	2632.09	141.190	418.352
19773	2643.86	143.320	419.331
19774	2662.36	145.388	420.152
19781	2691.32	147.474	421.759
19782	2712.50	149.408	422.691
19783	2753.79	153.444	424.103

19784	2775.27	156.574	425.594
19791	2818.66	160.271	426.968
19792	2855.47	163.693	428.569
19793	2866.30	166.509	430.770
19794	2884.63	169.774	432.160
19801	2891.95	172.330	433.986
19802	2914.67	176.079	436.054
19803	2918.98	180.178	439.063
19804	2934.13	183.954	440.790
19811	2982.47	188.616	443.787
19812	3046.88	193.788	445.614
19813	3043.60	195.939	447.470
19814	3076.03	200.250	449.677
19821	3122.45	204.643	452.374
19822	3132.16	206.583	454.067
19823	3115.25	208.886	455.513
19824	3079.80	209.466	457.653

YINV=time invariant income
PINV=time invariant price
MINV=time invariant money supply

Footnotes

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(2) Herbert A. Simon, Nobel Speech, December 8, 1978. "Rational Decision Making in Business Organizations." American Economic Review, September 1979. vol.169, no.4, p. 499.

3 Chung, Kyungbae, "Optimal Monetary Rules with an Endogenous Money Supply", Ph.D. Dissertation, University of Pittsburgh, 1986.

4 See Chow(1975) pp. 170-172.

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