

Micro-based Estimates of Demand Functions for Local Public Goods Incorporating Productivity and Benefit Differences

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I. Introduction

An individual's preferences for local public goods are theoretically determined by individual perceptions of the benefits and costs of expenditures on public goods derived from maximizing his utility. The continuous true preferences, however, are not directly observable, because an individual is not free to choose the level of local public goods. Instead, the provision of public goods by local government is regarded as a collective outcome which is made in a political process aggregating citizens' preferences given community characteristics. Furthermore, output units and prices of local public goods are not measured with well defined characteristics. From these attributes, a study of the demand for local public goods are somewhat different from that for common private goods.

Traditionally, the demand analysis of local public goods has been proceeded with aggregate cross-sectional data by the specification of models which integrate collective choice theory via the median voter hypothesis¹ and utility maximization. Assuming that local government expenditures reflect the desires of the median voter, economists have widely adopted the median voter hypothesis in the empirical demand analysis which relates the aggregate outcomes produced through the local political process to the community characteristics and the median ideal points of residents' characteristics.² But aggregate studies using the median voter hypothesis

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fail to explain the effects of individual characteristics distribution.

A qualitative micro-based approach using survey data, which was originally developed by Bergstrom, Rubinfeld and Shapiro(1982), offers an alternative method which avoids the median voter assumption on local political process and the assumption on characteristics distribution. This new approach relates an individual's discrete preferences on local public goods revealed on survey to his individual and community characteristics in estimating the demand functions for local public goods. The individual's discrete preferences on local public goods are observed from a set of questions whether he wants more, about the same, or less local spending and taxes in his jurisdictions. They assume that the responses depend on income, tax price, and other individual and community characteristics. Combining the discrete response data and the individual and community characteristics variables, they derived demand parameters from estimating choice probabilities by a qualitative response model.

Most demand analyses for local public goods using the aggregate or micro-based approach, as discussed above, do not explicitly take into account the different roles of community and individual characteristics in the demand framework. However, it is important to realize that the roles of community and individual characteristics are different. That is, the community characteristics directly affect the productivity in the provision of local public goods and indirectly influence the demand through their prices. On the other hand, the individual characteristics which represent individual tastes for local public goods directly affect the distribution of benefits from the community output given the community characteristics.

Bradford, Malt and Oates(1969) combine the production technology into the argument of the objective utility function thus incorporating the effects of socioeconomic characteristics on the productivity in the demand framework. But they do not distinguish individual characteristics from community characteristics by using a representative individual's characteristics as observed by the community characteristics. Their point is that production functions of local public goods contain as arguments not only the local government's purchased inputs of labor and capital in a narrow budgetary sense, but also the characteristics of local population and en-

vironment. Following Bradford, Malt and Oates, Hamilton(1983) argued the income-as-input hypothesis that community income(except grants), as a proxy for all community socioeconomic characteristics variables, enters directly the production function of local public goods as an input and also serves as a median taste variable for local public goods. Assuming this dual role of income, he showed that the income elasticity of demand for local public goods may be biased downward.

In order to estimate the demand functions for local public goods correctly, the bias problems should be solved in the modeling process. In this context, this paper develops a micro-based demand model for local public goods(the case of local public education) incorporating productivity differences and benefit distributions caused by individual and community characteristics, and disentangles their effects on the demand for local public education. The demand model is estimated by the discrete choice analysis using the Michigan Tax Limitation Survey data.

The remainder of this paper is organized as follows. Section II describes the micro-based demand model incorporating productivity differences and benefit distributions. Section III explains the probit estimation method applied to the demand model. Section IV provides an empirical analysis of the demand for local public educational expenditures using survey data. Section V presents concluding remarks.

II . Model

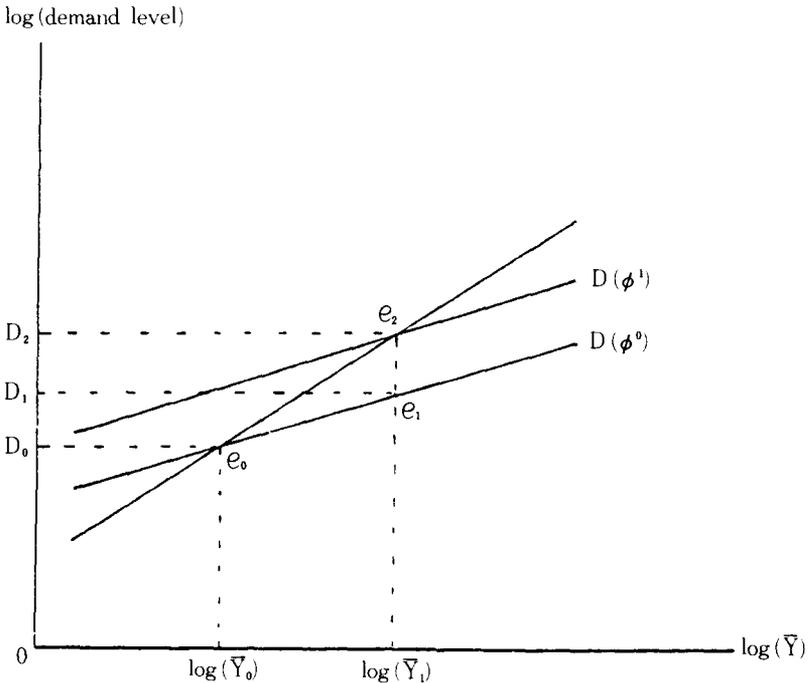
1. Effects of Productivity and Benefit Differences

Community characteristics are considered as important inputs for the production of local public goods along with the usual purchased inputs of labor and capital. That is, the community characteristics directly affect the productivity of purchased inputs and consequently they influence the individual demand for local public goods through their prices. As long as the prices of local public goods derived from their production functions depend on community income and other socioeconomic characteristics, the demand for local public goods rises not only by an increase in income(a rise in purchasing power and a rise in tastes), but also by a price reduction

associated with the increased endowment of the non-purchased input. Therefore, a failure to recognize the community characteristics' effects on productivity lead to a serious bias in estimates of demand functions for local public goods.

Similar to Hamilton(1983), Figure 1 illustrates the direction of bias in the income elasticity of demand estimated with aggregate data when the community characteristics' effects on productivity are ignored. Let $D(\phi^0)$ represent the demand level as a function of income (community income as a proxy for all community characteristics) with a given community price ϕ^0 , and $D(\phi^1)$ represent the demand level corresponding to a price $\phi^1 (\phi^1 < \phi^0)$. Holding price constant at ϕ^0 , an increase in income from \bar{Y}_0 to \bar{Y}_1 leads to an increase in demand from D_0 to D_1 which results in an increase in expenditures from $\phi^0 D_0$ to $\phi^0 D_1$. But as long as an increase in the

Figure 1. Direction of Bias Caused by Ignoring the Community Characteristics' Effects on Productivity



endowment of community characteristics(from \bar{Y}_0 to \bar{Y}_1) affects productivity and therefore reduces the price from ϕ^0 to ϕ^1 , the demand level resulting from the change in income will rise from D_0 to D_2 which includes the income effect(D_0D_1) at a given price(ϕ^0) and the price effect(D_1D_2) caused by productivity. The curve, as a kind of Engel curve, which incorporates the productivity effects, will therefore be a line connecting e_0 and e_2 which is more elastic than the $D(\phi^0)$ holding price constant at ϕ^0 .

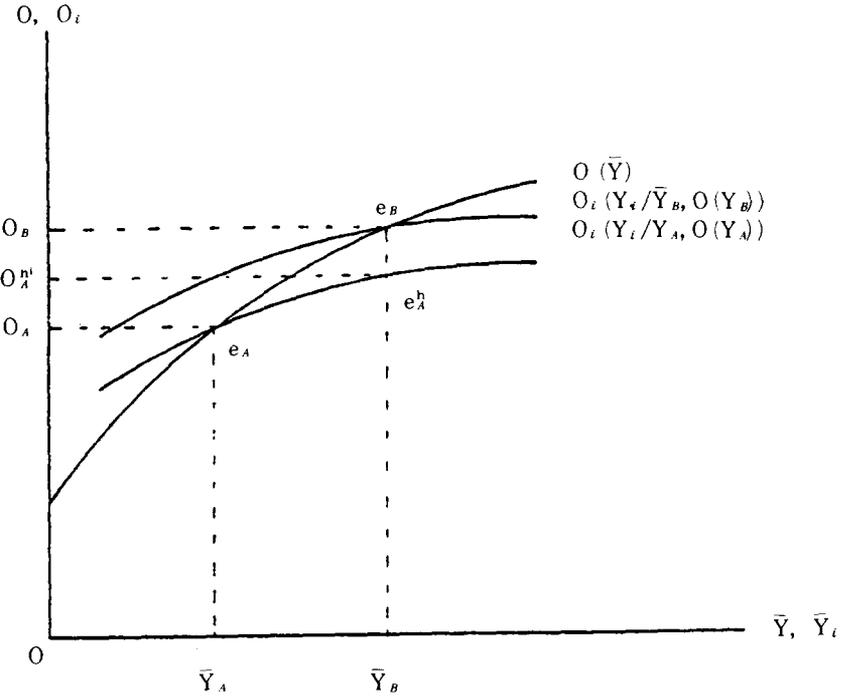
Individual characteristics which represent the tastes and the ability to pay for local public goods may directly influence the distribution of individual benefits and costs from the production of local public goods given the community characteristics. For simplicity, suppose the production of local public goods depends only on the community income, and community and individual characteristics except community income and household income are constant. Then the individual's perceived consumption of local public goods, \tilde{O} , can be represented as the following way :

$$\tilde{O}_i = \tilde{O}_i(Y_i / \bar{Y}, O(\bar{Y})) \tag{1}$$

where Y_i is the household income, \bar{Y} is the community median income, and O is the community output of local public goods. In this specification, the ratio of household income over the community median income(Y_i / \bar{Y}) is assumed to represent individual benefit distribution effects relative to the neighborhood.

The benefit distribution with respect to Y_i / \bar{Y} is illustrated in Figure 2. Let \bar{Y}_A and \bar{Y}_B be the community median household income in community A and B respectively, O_A and O_B be the community average output in community A and B respectively, $O(\bar{Y})$ is the community educational production function, $\tilde{O}_i(Y_i / \bar{Y}_A, O(\bar{Y}_A))$ and $\tilde{O}_i(Y_i / \bar{Y}_B, O(\bar{Y}_B))$ be the individual perceived consumption level associated with household income Y_i at the given community output $O(\bar{Y}_A)$ and $O(\bar{Y}_B)$ respectively. If a person lives in community A with the household income equal to community A 's median household income \bar{Y}_A , his perceived consumption level will be the same as the community average output(O_A). As drawn in Figure 2, if a person with the household income equal to community B 's median household income \bar{Y}_B lives in community A , his perceived consumption level(\tilde{O}_A^*) will be higher

Figure 2. Individual Benefit Distribution from Community Output



than the community A 's average output (O_A) but lower than the consumption level (O_B) which he can receive by moving to community B . Whether or not a move to a higher income community results in larger personal perceived consumption level depends upon the relative size of the benefit effects and the productivity effects. Therefore, a failure to recognize the individual perceived benefit differences as well as productivity differences may also lead to a bias in the estimates of demand functions for local public goods.

From the above context, a demand model for local public goods should explicitly incorporate the productivity differences and the benefit distributions associated with community and individual characteristics in order to estimate the true parameters of demand functions and to disentangle the productivity and benefit effects on demand.

2. Individual Consumption of Educational Output

The demand model assumes that each individual determines his desired level of educational consumption by maximizing his utility function subject to the household budget constraint. In a given locality, each individual's preferences are assumed to be represented by the following utility function :

$$U_i = U_i(C_i, \tilde{O}_i) \tag{2}$$

where C_i is the individual's consumption of the composite private good, \tilde{O}_i is the individual's perceived consumption resulting from the production and distribution of the publicly provided educational output, and the utility function is assumed to be the continuous and strictly quasi-concave utility function.

Each individual is assumed to perceive the educational output produced by the local government with a certain systematic rule determined by his socioeconomic characteristics given community characteristics. The relationship between the individual perceived consumption and the educational output produced by the local government can be expressed as the following way :

$$\begin{aligned} \tilde{O}_i &= \delta_i O \\ &= d_o \left(\frac{Y_i}{\bar{Y}} \right)^{d_1} \exp \left[\sum_{k=2}^n d_k S_{ik} \right] N_s^{-d_n} O \end{aligned} \tag{3}$$

Where O is the educational output per pupil produced by the community and δ_i is the individual benefit distribution function from the community educational output. δ_i will depend on the individual benefit effects relative to the neighborhood (Y_i / \bar{Y}), individual characteristics which affect the individual technology to produce consumption units that directly enter the household budget constraint, and the crowding parameter of the publicly provided educational output. And Y_i is the household income, \bar{Y} is the community median household income, S_{ik} represent the individual or the household characteristics such as level of education, number of school age children, public or private school attendance,³ race, sex, age, and recipient of transfer payment which are generally expressed as binary variables, N_s is the number of pupils in the community, exp is the exponential operator,

and d_1 , d_n and d_k are the parameters.

In this specification, the parameter d_1 of Y_i/\bar{Y} can be interpreted as the income effect on benefit distribution. If $d_1 > 0$ ($d_1 < 0$), the benefit distribution is positively (negatively) related to household income in a given community; and if $d_1 = 0$, the benefit is independently distributed with household income in a given community. The coefficient of the number of pupils in the community, d_n , indicates the degree of publicness of local publicly provided education. Borcharding and Deacon (1972) and Bergstrom and Goodman (1973) derived the degree of publicness from consideration of the effects of crowding on an individual's opportunity to benefit from the publicly provided output. If the crowding parameter d_n is zero, educational output is a purely public good in the Samuelsonian sense (Samuelson 1954). But as d_n increases, the educational output becomes more and more crowded. If d_n is 1, the educational output displays purely private characteristics in that each pupil secures only $1/N_s$ share of the community educational output.

3. Production, Cost, and Price of Education

Assuming that a community educational production process is theoretically thought of as producing homogeneous intermediate educational output for use in individual consumption activities and that the school board's production activities are efficiently operate on the production frontier, the standard production theory can be applied to the educational production process in order to find the technical relationship between community educational output and inputs. However, as long as community educational output is a complex term with heterogeneous characteristics, the estimation of educational production function⁴ using a single output index may yield erroneous results.

In order to avoid the estimation problem in the production function and its dual cost function, the implicit price of education is indirectly derived from the production and cost functions. This approach is described below.

Let us assume that there exists a twice differentiable production function relating the community educational output to the inputs which include both the purchased inputs (labor and capital) and the community characteristics. The production function is assumed to be multiplicatively separable

with respect to the purchased inputs and the community characteristics, and to be of Cobb-Douglas technology and constant return to scale with respect to the purchased inputs. The community characteristics variables are assumed to affect the productivity of purchased inputs systematically. From the above assumptions, the community educational production function can be specified as the following form :

$$\begin{aligned}
 O &= G(Z) F(L, K) = G(Z) L^{a_1} K^{a_2} \\
 &= Y^{b_1} \exp \left[\sum_{i=1}^n b_i Z_i \right] L^{a_1} K^{a_2}, \quad a_1 + a_2 = 1 \quad (4)
 \end{aligned}$$

where O is the community output per pupil, Z is a vector of community characteristics, L is the labor input per pupil, K is the capital input per pupil, and a_1 and a_2 are the coefficients of L and K , respectively. \bar{Y} is the community median household income, Z_i represent the other community characteristics such as education level, racial composition, poverty level, and owner occupied housing ratio, b_1 is the effect of community income on productivity, and b_i is the other production parameter.

The community costs for the production of educational output depend on the expenditures on the educational purchased inputs(L and K). If each purchased input is available in perfectly elastic supply, the cost function and average cost function for educational output can be expressed as

$$\begin{aligned}
 E &= wL^* + rK^* \\
 &= \left[(a_2 / a_1)^{a_1} + (a_2 / a_1)^{-a_2} \right] G(Z)^{-1} w^{a_1} r^{a_2} O \quad (5 a)
 \end{aligned}$$

$$\phi = (E/O) = \left[(a_2 / a_1)^{a_1} + (a_2 / a_1)^{-a_2} \right] G(Z)^{-1} w^{a_1} r^{a_2} \quad (5 b)$$

where E is the educational expenditures per pupil in a community, w is the wage rate in educational industry, r is the rental price of capital, and L^* and K^* are the derived factor demand functions for L and K from the given production function(equation 4). The resulting average cost function is the supply curve of education by the community since the community is a

special monopolist that does not seek to maximize its profit but instead to provide educational output to the residents at cost. Assuming that capital is perfectly mobile across political jurisdictions, the rental price of capital can be treated as constant. Then the implicit price of education in a community, ϕ , can be expressed as

$$\begin{aligned} \phi &= (E/O) \\ &= a_0 G(Z)^{-1} w^{a_1} = a^0 \bar{Y}^{-b_1} \exp \left[-\sum_{i=2} b_i Z_i \right] w^{a_1} \end{aligned} \quad (6)$$

where a_0 is the constant value. From the dual role of production and cost functions, the implicit community price of education is derived indirectly without measuring output.

4. Demand Model for Public School Expenditures Using Survey Responses

The demand side and the supply side should be simultaneously considered in order to determine the individual desired expenditure level since the two sides are interdependent. This model couples the demand side with the supply side through combining the production technology into the argument of the objective utility function and the individual implicit price of education.

Suppose that a community provides only one local public good (public education) and the expenditures on public education are entirely financed from local property taxes with a proportional uniform tax rate. Then the budget balance condition for a local government can be written as

$$N_s E = t \sum V_i \quad (7)$$

where N_s is the number of pupils, E is the educational expenditures per pupil, t is the uniform property tax rate, V_i is the assessed residential property value of household, and $\sum V_i$ is the total community assessed property value.⁵ The uniform property tax rate and each household's property tax payment are thus determined from the community budget balance condition as the following way:

$$\text{Household } i\text{'s tax payment} = tV_i$$

$$-\left(\frac{N_s E}{\sum V_i}\right) V_i = \left(\frac{V_i}{\sum V_i / N_s}\right) E = \tau_i E \tag{8}$$

In this equation, the $\tau_i (= V_i / (\sum V_i / N_s))$, which is defined as a household i 's tax share relative to the community average taxes per pupil, measures the marginal tax cost of E to household i .⁶ Therefore, τ_i can be interpreted as the explicit tax price of each household.

From equation(8), a household's budget constraint can be expressed with the tax price of education as

$$Y_i = h_i C_i + \tau_i E \tag{9}$$

where h_i is the size of household,⁷ C_i is the private composite good(as a numeraire) consumed by a household member.

Maximizing the individual utility function, $U_i(C_i, \tilde{O}_i)$, subject to the household budget constraint yields Marshallian demand functions for C_i and \tilde{O}_i :

$$\begin{aligned} &\text{Max } U_i(C_i, \tilde{O}_i) \\ &\text{subject to } Y_i = h_i C_i + \tau_i E \end{aligned}$$

For the maximizing solution with respect to E , the induced utility function and its maximizing first order condition can be written as

$$V_i = U_i\left(\frac{Y_i - \tau_i E}{h_i}, \frac{\delta_i}{\phi} E\right) \tag{10a}$$

$$\frac{\partial V_i}{\partial E} = \frac{\partial U_i}{\partial C_i} \left(-\frac{\tau_i}{h_i}\right) + \frac{\partial U_i}{\partial \tilde{O}_i} \frac{\delta_i}{\phi} = 0 \tag{10b}$$

Given δ_i , τ_i , h_i , and Y_i , the individual desired demand for educational consumption(\tilde{O}_i) is determined by the familiar condition that the marginal rate of substitution be equal to the individual perceived price of public education measured as a numeraire good. The individual perceived price, ϕ , is derived from the equation(10b):

$$\phi = \frac{\partial U_i}{\partial \tilde{O}_i} / \frac{\partial U_i}{\partial C_i} = \frac{\tau_i \phi}{h_i \delta_i} \tag{11}$$

This individualized implicit perceived price ϕ_i measures the marginal price to a consumer of a one unit increase in the individual perceived consumption of educational output, and can be interpreted as the individualized perceived tax price of educational output. The differentiable Marshallian demand function for \bar{O}_i derived from the maximizing solution can be expressed as the following general form :

$$D_i = D_i \left(\phi_i, \frac{Y_i}{h_i} \right) = D_i \left(\frac{\tau_i \phi}{h_i \delta_i}, \frac{Y_i}{h_i} \right) \tag{12}$$

where D_i is the individual demand for \bar{O}_i , and Y_i/h_i is the per household member's income in a household.

Let the demand function be a constant elasticity form, then the individual demand function for educational consumption can be specified as

$$D_i = \beta_0 \left(\frac{\tau_i \phi}{h_i \delta_i} \right)^{\beta_1} \left(\frac{Y_i}{h_i} \right)^{\beta_2} \tag{13}$$

where β_1 is the perceived price elasticity of demand, β_2 is the income elasticity of demand, and β_0 is the constant value. From this demand function and equation (3), the individual i 's desired community provision of education, O_i^* , can be expressed as $O_i^* = \delta^{-1} D_i$.

From equations (3), (6), and (13), individual i 's desired demand function for the community educational expenditures per pupil, E_i^* , can be expressed as the reduced form. For the estimation purpose, the individual demand function for local public spending can be written as the following log-linear form with a random error term ε_i :

$$E_i^* = \phi O_i^* \tag{14a}$$

$$\begin{aligned} \log E_i^* = & \kappa_0 + \beta_1 \log \tau_i + [\beta_2 - d_1(1 + \beta_1)] \log Y_i - \sum_{k=2} (1 + \beta_1) d_k S_{ik} \\ & - (\beta_1 + \beta_2) \log h_i - [(1 + \beta_1)(1 - d_n)] \log N_s + a_1(1 + \beta_1) \log w \\ & + [(1 + \beta_1)(d_1 - b_1)] \log Y - \sum_{l=2} (1 + \beta_1) b_l Z_l + \varepsilon_i \end{aligned} \tag{14b}$$

All independent variables in equation(14b) are expressed as observable

self-explanatory variables, and all structural parameters are exactly identifiable. The estimates of structural parameters can be derived from the estimates of the reduced form parameters. The important identifying restrictions are already imposed in the previous discussions. That is, the individual demand function is a constant elasticity form with respect to the individual's perceived price and the per household member's income, the community educational production function is a constant elasticity form with respect to \bar{Y} and of constant return to scale with respect to purchased inputs, the benefit distribution function is a constant elasticity form with respect to Y_i / \bar{Y} , and the household consumption of composite private good and the household tax burden are equally distributed among the household members (adult equivalent scales). However, the dependent variable, E_i^* , is not directly observable.

Even though each individual has the continuous E_i^* derived from maximizing his utility function, the continuous true preferences are not observable directly because each individual is not free to choose the level of community expenditures. The expenditure or the provision of public services by the local government is regarded as a collective outcome decided in a political process. That is, various residents' needs for local public goods are translated into a collective demand through the collective political decision process. It is assumed that individual vote and respond to surveys in a way that provides a true revelation of their preferences.

In this context, the discrete preferences for local public goods can be observed in the well designed survey even though the continuous preferences are not observable.⁸ In other words, individual discrete preferences (attitudes) toward public goods can be observed from a set of questions asking residents whether they want more, the same, or less local spending and taxes in their jurisdictions. The responses will depend on the relationship between the desired expenditure level (E_i^*) based on individual and community characteristics and the actual community expenditure level (A_i) based on community characteristics. Under the assumption that the utility function is strictly quasi-concave and the constraint set is convex, each individual will answer "more," "the same," or "less" depending on whether $E_i^* > A_i$, $E_i^* = A_i$, or $E_i^* < A_i$, respectively. If there exists a threshold value, Δ , of expenditure that determines the point at which the difference between the desired and actual expenditure levels is sufficient to generate a particular response, then the respondent is assumed to answer "more," "about the same," or "less" respectively as

- More if $E_i^* > A_i + \Delta$
 - Same if $A_i - \Delta \leq E_i^* \leq A_i + \Delta$
 - Less if $E_i^* < A_i - \Delta$
- (15)

On the basis of some assumptions concerning the stochastic nature of the individual desired expenditure level, the choice probabilities of “more,” “about the same,” or “less” can be specified and the demand parameters can be estimated by using the discrete choice model⁹(see Section III).

5. Disentangling the Community and Individual Characteristics’ Effects on Demand.

The demand model discussed in the previous sections can be simply rewritten as

$$\text{individual demand } D_i = D_i \left(\phi_i, \frac{Y_i}{h_i} \right) \tag{16a}$$

$$\text{community price } \phi = \phi (\bar{Y}, Z^0, w) \tag{16b}$$

$$\text{benefit distribution } \delta_i = \delta_i \left(\frac{Y_i}{\bar{Y}}, S_i^0, N_s \right) \tag{16c}$$

$$\begin{aligned} \text{perceived price } \phi_i &= \phi_i \left(\frac{Y_i}{\bar{Y}}, S_i^0, N_s, \tau_i, \phi (\bar{Y}, Z^0, w) \right) \\ &= \frac{\tau_i \phi}{h_i \delta_i} \end{aligned} \tag{16d}$$

$$\begin{aligned} \text{desired expenditure } E_i^* &= \phi_i O_i = \phi_i^{-1} \\ &= \phi_i^{-1} D_i \left(\phi_i, \frac{Y_i}{h_i} \right) \cdot \frac{h_i}{\tau_i} \end{aligned} \tag{16e}$$

where S_i^0 is a vector of individual characteristics other than Y_i and Z_0 is a vector of community characteristics other than \bar{Y} .

From the above simplified model, the total effects of changes in individual income on individual desired community expenditures can be decomposed into two parts: one is the direct change caused by an increase in individual income, and the other is the change caused by the individual’s perceived price change associated with the benefit and cost distribution. These two effects can be represented as the following elasticity form:

$$\begin{aligned} \frac{\partial \log E_i^*}{\partial \log Y_i} &= \frac{\partial \log D_i}{\partial \log Y_i} + \frac{\partial \log \phi_i}{\partial \log Y_i} \left(1 + \frac{\partial \log D_i}{\partial \log \phi_i} \right) \\ &= \frac{\partial \log D_i}{\partial \log Y_i} - \frac{\partial \log \delta_i}{\partial \log Y_i} \left(1 + \frac{\partial \log D_i}{\partial \log \phi_i} \right) \end{aligned} \tag{17}$$

Equation(17) shows that changes in individual desired community expenditures(E_i^*) are stimulated not only by the individual household income changes (a rise in ability to pay and a rise in tastes) but also by the individual's perceived price changes. If the public good is normal and individual demand is price inelastic, the direction of bias in income elasticity of demand depends on the income elasticity of perceived price ($\partial \log \phi_i / \partial \log Y_i = - \partial \log \delta_i / \partial \log Y_i$) under the assumption that the community price is not affected by the individual income. That is, when the benefit distribution is positively correlated with individual income ($\partial \log \delta_i / \partial \log \bar{Y} > 0$), the income elasticity of demand is downward biased; when the benefit is negatively correlated with individual income, the income elasticity of demand is likely to be upward biased.

The total effects of changes in community income on individual desired expenditures can also be divided into two parts: the indirect and direct effects caused by the community income effect on individual price. Similar to equation (17), the community income change effects on E_i^* can be written as

$$\frac{\partial \log E_i^*}{\partial \log \bar{Y}} = \frac{\partial \log D_i}{\partial \log \phi_i} \frac{\partial \log \phi_i}{\partial \log \bar{Y}} + \frac{\partial \log \phi_i}{\partial \log \bar{Y}} \tag{18}$$

Equation (18) shows that, unless the relationship between community income, community price (productivity) and benefit distribution are explicitly incorporated into the demand model for local public education, the effects of community income on the demand for education and the educational expenditures can not be disentangled.

III. Probit Specification of the Demand Model

Consider the individual desired expenditure function simplified from equation (14b):

$$E_i^* = \beta_0 + X_i' \beta_1 + \epsilon_i \tag{19}$$

where \tilde{E}_i^* is the logarithm of individual i 's desired level of public educational expenditures per pupil, X_i is a $\kappa \times 1$ vector of individual characteristics and community characteristics, β_0 is a constant, β_1 is a $\kappa \times 1$ coefficient vector of X_i , and ϵ_i is a random error term which is assumed to be normally distributed with $E(\epsilon_i) = 0$, $Var(\epsilon_i) = \sigma_\epsilon^2$ and $Cov(X_i, \epsilon_i) = 0$.

The above E_i^* as an unobserved latent variable can be interpreted as an index of the random utility measure of the most ideal alternative for individual i . If the utility function is strictly quasi-concave and the con-

straints set is convex, then the induced utility function $V_i(E)$ from equation (10a) is single peaked in E . That is, $V_i(E)$ is strictly increasing in E for $E < E_i^*$ and strictly decreasing in E for $E > E_i^*$, for all i . In this context, the random utility model for discrete choice can be applied to the choice probabilities of the "more," "about the same," or "less" conditional on the actual expenditure level and the characteristics variables as the following probit specification:

$$\text{more : } P_{im} = P_r(\bar{E}_i^* > \bar{A}_i + \delta)$$

$$\begin{aligned} &= P_r(\varepsilon_i > \delta - \beta_o + \bar{A}_i - X_i' \beta_1) \\ &= 1 - \Phi \left(\frac{\delta - \beta_o + \bar{A}_i - X_i' \beta_1 - E(\varepsilon_i | X_i, \bar{A}_i)}{\sigma(\varepsilon_i | X_i, \bar{A}_i)} \right) \end{aligned} \quad (20a)$$

$$\text{same : } P_{is} = P_r(\bar{A}_i - \delta \leq \bar{E}_i^* \leq \bar{A}_i + \delta)$$

$$\begin{aligned} &= P_r(-\delta - \beta_o + \bar{A}_i - X_i' \beta_1 \leq \varepsilon_i \leq \delta - \beta_o + \bar{A}_i - X_i' \beta_1) \\ &= \Phi \left(\frac{\delta - \beta_o + \bar{A}_i - X_i' \beta_1 - E(E_i^* | X_i, \bar{A}_i)}{\sigma(\varepsilon_i | X_i, \bar{A}_i)} \right) \\ &\quad - \Phi \left(\frac{-\delta - \beta_o + \bar{A}_i - X_i' \beta_1 - E(\varepsilon_i | X_i, \bar{A}_i)}{\sigma(\varepsilon_i | X_i, \bar{A}_i)} \right) \end{aligned} \quad (20b)$$

$$\text{less : } P_{il} = P_r(\bar{E}_i^* < \bar{A}_i - \delta)$$

$$\begin{aligned} &= P_r(\varepsilon_i < -\delta - \beta_o + \bar{A}_i - X_i' \beta_1) \\ &= \Phi \left(\frac{-\delta - \beta_o + \bar{A}_i - X_i' \beta_1 - E(\varepsilon_i | X_i, \bar{A}_i)}{\sigma(\varepsilon_i | X_i, \bar{A}_i)} \right) \end{aligned} \quad (20c)$$

where P_{im} , P_{is} , or P_{il} is the choice probability that individual i chooses "more," "about the same," or "less" response respectively, \bar{A}_i is the logarithm of the community's actual level of public educational expenditures, δ is the threshold value in terms of logarithm, $E(\varepsilon_i | X_i, \bar{A}_i)$ is the expected value of ε_i conditional on X_i and \bar{A}_i , $\sigma(\varepsilon_i | X_i, \bar{A}_i)$ is the standard deviation of ε_i conditional on X_i and \bar{A}_i , and $\Phi(\cdot)$ is a cumulative standard normal distribution function. In order to evaluate the choice probabilities, assumptions on the covariance matrix of $(X, \bar{A}, \varepsilon)$ are needed for $E(\varepsilon_i | X_i$

, \bar{A}_i) and $\sigma(\epsilon_i | X_i, \bar{A}_i)$. But for simplicity, $E(\epsilon_i | X_i, \bar{A}_i) = 0$ and $\sigma(\epsilon_i | X_i, \bar{A}_i) = \sigma_\epsilon$ are assumed for this paper.¹⁰

The statistical estimation of the parameters of discrete choice models is typically carried out by the maximum likelihood method which has the advantage of producing estimators that are consistent, asymptotically efficient, and asymptotically normal under the usual regularity conditions. With the probit specification, the likelihood function for the randomly observed survey responses of “more,” “about the same,” or “less” is given by

$$L(\theta) = \prod_{i=1}^N P_{i,m}^{y_{i,m}} P_{i,s}^{y_{i,s}} P_{i,l}^{y_{i,l}} = \prod_{i \in \text{more}} P_{i,m} \prod_{i \in \text{same}} P_{i,s} \prod_{i \in \text{less}} P_{i,l} \quad (21)$$

where y_{ij} is 1 if individual i chooses alternative j and zero otherwise, N is the number of individuals, and θ is the parameter of the likelihood function.

The maximum likelihood estimate $\hat{\theta}$ is the value of the parameter vector that maximize $L(\theta)$. Usually it is more convenient to find $\hat{\theta}$ by maximizing the following log-likelihood function $\bar{L}(\hat{\theta})$ because, from the monotonically increasing property of logarithmic function, its maximum coincides with the maximum of $L(\theta)$.¹¹

$$\begin{aligned} L(\theta) &= \sum_{i \in \text{more}} \log P_{i,m} + \sum_{i \in \text{same}} \log P_{i,s} + \sum_{i \in \text{less}} \log P_{i,l} \\ &= \sum_{i \in \text{more}} \log [1 - \Phi(\theta_0^m + \theta_1 \bar{A}_i + \theta_2 X'_i)] \\ &+ \sum_{i \in \text{same}} \log [(\theta_0^s + \theta_1 \bar{A}_i + \theta_2 X'_i) - \Phi(\theta_0^l + \theta_1 \bar{A}_i + \theta_2 X'_i)] \\ &+ \sum_{i \in \text{less}} \log [\Phi(\theta_0^l + \theta_1 \bar{A}_i + \theta_2 X'_i)] \end{aligned} \quad (22)$$

Where θ_0^m is $(\delta - \beta_0) / \sigma$, θ_0^l is $(-\delta - \beta_0) / \sigma$, θ_1 is $1 / \sigma$, and θ_2 is $-\beta_1 / \sigma$. The consistent estimates of the demand parameters $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\delta}$ can be given by some manipulation of $\hat{\theta}$. That is,

$$\hat{\beta}_0 = -\frac{\hat{\theta}_0^l + \hat{\theta}_0^m}{2 \hat{\theta}_1}$$

$$\hat{\beta}_1 = -\frac{\hat{\theta}_2}{\hat{\theta}_1}$$

$$\hat{\delta} = -\frac{\hat{\theta}_0^1 - \hat{\theta}_0^2}{2 \hat{\theta}_1} \quad (23)$$

IV. Empirical Analysis

1. Data

The data used in this study were obtained the 1987 Michigan Tax Limitation Survey¹² designed by Courant, Gramlich, and Rubinfeld. This survey includes 2001 randomly selected households in the State of Michigan, but this study is based on the subsample¹³ of homeowners since the measurement of tax price is essential for the demand analysis. The survey data was supplemented with the aggregate data on community characteristics and school expenditures of 129 school districts in the State of Michigan from which the sample was selected. The community characteristics variables were collected from the 1980 US Census of Population and Housing, and the school input data was obtained from the publications by the Michigan Department of Education (1978, 1980). The list of all variables utilized in the estimation procedure is given in Table 1.

Table 1 Definition of Variables

Variable	Definition
LGEXP	log of general fund school expenditures per pupil in respondent's school district (1977-78 school year)
LGHY	log of respondent's 1977 household income
LGPR	log of respondent's tax price defined as respondent's house value divided by 1977/78 State Equalized Value per pupil
GLTHS	dummy=1 if respondent did not graduate from high school, 0 otherwise
CGRAD	dummy=1 if respondent is a college graduate, 0 otherwise
K05	number of children age 0 to 5 in household
K611	number of children age 6 to 11 in household
K1216	number of children age 12 to 16 in household

PRIV	dummy=1 if respondent has 1 or more children in private school, 0 otherwise
TRANSF	dummy=1 if respondent receives either AFDC or food stamps, 0 otherwise
BLACK	dummy=1 if respondent is black, 0 otherwise
FEMALE	dummy=1 if respondent is female, 0 otherwise
AGE65	dummy=1 if respondent is over age 65, 0 otherwise
LGADEQ	log of adult equivalent household size defined as number of adults plus a half of children under age 18 in household
LGENRL	log of enrollment in school district (1977-78 school year)
LTSAL	log of average public school teacher salary in school district (1979-80 school year)
LMHY	log of community median household income (1979)
PHGRAD	proportion of high school graduates in community (1980)
PROORF	proportion of families below poverty level in community (1979)
POWNH	proportion of owner occupied housing units in community (1980)
PWHITE	proportion of white in community (1980)
DTROIT	dummy=1 if respondent lives in Detroit, 0 otherwise

Note: log represents natural logarithm.

2. Estimated Demand Parameters and Disentangling the Productivity and the Benefit Effects.

The choice probability function of “more,” “about the same.” or “less” response on public school expenditures specified by the probit model has been estimated by the maximum likelihood method. The Table 2 reports the estimation results of probit likelihood function (equation 22).

The structural parameters of the educational demand function including benefit and productivity effects can be derived from the estimated parameters of probit likelihood function in Table 2 with equation (14b), (22), and (23). The estimates of the structural parameters of the model are summarized in Table 3. All parameters and their relationships are consistent and support the theory discussed in Section II.

The price elasticity of demand for public education is estimated to be -0.11 and the income elasticity of demand is estimated to be 0.34. This suggests that the price elasticity and income elasticity of education demand

Table 2 Maximum Likelihood Coefficients of Probit Choice Function

	Estimates of Coeffients	Standard Error
θ_o^L	4.345	4.923
θ_o^M	6.059	4.924
LGEXP	-0.814*	0.336
LGPR	-0.090	0.058
LGHY	0.222*	0.070
GLTHS	-0.236*	0.105
CGRAD	0.077	0.106
K05	0.254*	0.061
K611	0.171*	0.063
K1216	0.027	0.060
PRIV	-0.274*	0.139
TRANSF	-0.239	0.262
BLACK	1.136*	0.202
FEMALE	0.135	0.077
AGE65	-0.120	0.162
LGADEQ	-0.185	0.118
LGENRL	-0.114*	0.056
LTSAL	1.205*	0.547
LMHY	-0.110	0.391
PHGRAD	-0.057	0.707
PPOORF	-0.667	2.561
POWNH	-0.013	0.556
PWHITE	-0.606	0.758
DTROIT	-0.057	0.307
N	963	
-2 log L	1750.8	

Note: Absolute t-ratios greater than 2 are denoted by an asterisk(*). θ_o^L and θ_o^M are constant values in probit choice function which is defined in equation(22). Threshold values(δ) in equation(20) is estimated to be 1.05. N is the number of observations and log L is the value of log likelihood function at its maximum.

Table 3 Demand Parameters and Disentangling Benefit and Productivity Effects

	Estimates
<i>Demand Parameters</i>	
Price elasticity of demand ($\beta_1 = \partial \log D_i / \partial \log \phi$)	-0.111
Income elasticity of demand ($\beta_2 = \partial \log D_i / \partial \log Y_i$)	0.337
Income elasticity of expenditures ($= \partial \log E_i / \partial \log Y_i$)	0.273
<i>Benefit Effects</i>	
Income effect on benefit ($d_1 = \partial \log \delta_i / \partial \log Y_i$)	0.073
Income effect on perceived price ($= \partial \log \phi / \partial \log Y_i$)	-0.073
Crowding parameter(d_n)	0.842
<i>Productivity Effects</i>	
Community income effect on production ($b_1 = \partial \log O / \partial \log \bar{Y}$)	0.225
Community income effect on community price ($\partial \log \phi / \partial \log \bar{Y} = -b_1$)	-0.225
Community income effect on perceived price ($\partial \log \phi / \partial \log \bar{Y} = d_1 - b_1$)	-0.152
Community income elasticity of demand ($\partial \log D_i / \partial \log \bar{Y} = \beta_1(d_1 - b_1)$)	0.017
Community income elasticity of expenditures ($\partial \log E_i / \partial \log \bar{Y} = (1 + \beta_1)(d_1 - b_1)$)	-0.135

Note: Structural parameters are defined in Section II.

are low and the variations in price and income will have little effect on the quantity of education demand. The results support the past evidence that suggests price and income inelastic demands for most local public services. The negative price inelastic demand and positive income inelastic demand implies that public education is a normal good and a necessity and that there are no differences between the demand behaviors for private goods and for public education in this aspects.

The previous studies define the price variables as explicit tax prices

even though each study measures the tax prices differently. But the price variable employed here is defined as the implicit perceived individual price (equation 11) incorporating benefit and productivity differences. Therefore, the income elasticity of demand and the income elasticity of expenditures have different values (see Section II. 5). As long as the income effect on perceived price is negative (or the income effect on benefit is positive) and demand is price inelastic, the income elasticity of expenditures is smaller than the income elasticity of demand. The estimated results show that the benefit is positively correlated with income and the income elasticity of perceived price is -0.07 in the given explicit tax share and community price of education. Consequently, the income elasticity of desired expenditures (0.27) is smaller than the income elasticity of demand (0.34).

The degree of publicness defined in terms of joint consumption characteristics can be observed with the crowding parameter d_n . The estimated pupils crowding coefficient (0.84) is close to one, indicating that public education is not a public good in the Samuelsonian sense. The previous studies also find that the crowding parameter is close to one. This fact implies that education is not much different from the private goods in consumption characteristics as long as the public education is provided efficiently with residents' property taxes. The estimated degree of publicness suggests that an increase in enrollment decreases the per capita consumption of education proportionately and that total expenditures should be changed by the same percentage change as enrollment in order to maintain the per capita consumption of education.

The effects of community characteristics on demand for local public education can also be derived from the probit results. The community income elasticity of educational production is estimated to be 0.23 , and consequently the community income elasticity of community price of education is -0.23 from the equation (6). This implies that rich communities have a higher productivity than poor communities and therefore rich communities have a relatively lower unit cost for educational production than poor communities. That is, the community characteristics affect the productivity of purchased inputs of labor and capital in the educational production process and further influence the community price of education through the

cost function.

The community income elasticity of individual perceived price, whose value depends on the parameter d_1 in equation (3) and the parameter b_1 in equation (4), is estimated to be a negative value of -0.15 . The negative value implies that the community income effects on productivity are greater than the income effects on benefit distribution. Therefore, people will have a tendency to move from a poor community to a richer community.¹⁴ Furthermore, as the community income influences the individual demand through the individual's perceived price and the price elasticity of demand is small, the community income elasticity of education demand is estimated to be extremely negligible value of 0.02 . This implies that the community income variations have little effect on individual demand.

As long as the community price of education depends on community income through the productivity effect of community characteristics, the community income elasticity of expenditures is not the same as the community income elasticity of educational demand. The community income elasticity of expenditures is estimated to be -0.14 . This negative value implies that if a resident with a given income lives in a higher income community he will desire less educational expenditures than in a lower income community. This fact can be explained by the community income effect on individual perceived price and the price elasticity of demand for education (see equation 18). Since the community income elasticity of individual perceived price is negative (-0.15) and demand for education is price inelastic (-0.11), individual demand for community educational expenditures decreases as community income increases relative to own household income. Even though the community income inversely affects the individual's desired community expenditures, higher income groups will demand higher community educational expenditures as the individual income effects dominate the community income effects on desired community expenditures.

V. Conclusions

This paper has developed a micro-based demand model for local public

education incorporating benefit and productivity differences caused by individual and community characteristics, and has disentangled their effects on the demand for local public education. The demand model has been estimated by the discrete probabilistic choice analysis using survey data. The major findings in this study are as follows.

First, the individual perceived price and income elasticities of individual demand for local public education are estimated to be -0.11 and 0.34 respectively. These results support the past evidence suggesting that the demand for most local public services is inelastic with respect to price and income and that income elasticity is higher than price elasticity.

Second, community characteristics affect the productivity of purchased inputs in the production of local public goods and consequently they influence the community price and further the individual price of local public goods. From this point of view, previous studies that have estimated reduced form educational expenditure functions, while ignoring the community characteristics' effects on productivity, have produced biased demand parameters.

Third, people have a tendency to move from a poor community to a richer community, because the community income effects on productivity are greater than the community income effects on benefit distribution.

Fourth, individual demand for community educational expenditures decreases as community income increases relative to own household income, because the individual's perceived price falls as community income rises and demand for education is price inelastic.

Fifth, the demand behaviors and consumption characteristics of local public education are not much different from those of private goods. These, however, afford no normative indication of the desirability of private versus public provision of education in a view of social welfare because private providers have no instruments to observe the collective demand determined through the political process in a community. Furthermore, the maintenance of equal educational opportunity may justify the public provision of education.

The above findings may raise more questions than they answer. It is, however, expected that this study advances the demand analysis for local

public goods not only by offering partial answers but also by allowing more appropriate questions.

Footnotes

- 1) The median voter model, theoretically developed by Hotelling(1929), Bowen (1943), and Black (1958), suggests that community represents the individual preferences of a hypothetical median voter who has the median ideal points of residents' socioeconomic characteristics.
- 2) Several studies of this type are discussed in a review article by Romer and Rosenthal (1979).
- 3) The attendance of public or private school is treated as an exogenous variable even though the choice of school type might be correlated with individual characteristics. Sonstelie (1979) discusses the choice between public and private schools in relation to the public school quality and the personal reservation school quality.
- 4) Review articles on educational production function can be found in Hanushek(1979) and Bridge, et al. (1979).
- 5) For simplicity, ΣV_i is used as a notation of the total assessed value of all property including non-residential property.
- 6) The budget balance condition for a local government can generally be expressed as
$$N_i E = t \Sigma V_i + N_i R_0$$
where R_0 is the lump-sum intergovernmental grants per pupil as community non-tax revenue, Even though the community budget balance condition is specified as the above, the marginal tax cost of E to a household i is the same as T_i .
- 7) For the discussion of household composition and equivalence scales, see Muellbauer (1974).
- 8) Shapiro(1974), Deacon and Shapiro (1975) Rubinfeld (1977), Bergstrom, Rubinfeld and Shapiro(1982), and Rubinfeld, Shapiro and Roberts(1985) have developed a demand model using voting or survey response behavior. In the same context, the framework for the discrete choice probabilities is formulated.
- 9) For a review of discrete choice models, see Daganzo (1979), Amemiya (1981), Manski and McFadden (1981), Maddala (1983), and Mcfadden (1984).
- 10) The general case of $E(\epsilon_i, X_i, \bar{A}_i) \neq 0$ is discussed in Rubinfeld, Shpiro and Roberts (1985) and Kim (1986) in connection with the Tiebout bias and its control.
- 11) If a maximizing solution to equation(22) exists, it must satisfy the usual first order conditions($\partial \tilde{L} / \partial \theta = 0$) where θ is the parameter vector. These first order conditions are necessary but not sufficient for a maximum of the likelihood function. Therefore, there is no guarantee that such a solution is a global maximum unless the likelihood function $\tilde{L}(\theta)$ is concave. Here the local maximum is assumed to suffice the objective of demand analysis in the following discussion.
- 12) See Courant, Gramlich, and Rubinfeld(1980) for details about the survey.
- 13) A subsample of 963 households were finally selected after missing values were eliminated by listwise deletion.
- 14) See Figure 2 for a graphical illustration.

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