A Dynamic Competitive Labor Market under Asymmetric Information

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I. Introduction

In a labor market a firm is often concerned about types of its wokers and efforts taken by them, which are normally unknown to the firm. Most of the studies dealing with these issues have focused on the impacts that each of these informational problems has on the euilibrium in the labor market.

In this paper I considered a dynamic competitive labor market where individual workers have private information both about their types and about the effort levels they choose, which are not directly known to their employers. I followed Shapiro-Stiglitz(1984) in that firms cannot implement intertemporal wage structures, and that the moral hazard of workers is controlled partly by the unemployment pool. Here firms pay wages to their workers at the biginning of each period. This wage policy, which can be justified by the lack of perfect capital market on the part of workers, seems to be the most common one firms adopt in reality. The problem for firms is then how to set up a rule of determining wage offers to their workers in each period. In this model there are two mechanisms through which firms can screen different types of workers or control the possible shirkings of workers. First, workers can choose some exogeneous signals (e.g., education levels as in Spence model) before they start their careers, so that firms initially could have some indirect information about the types of individual workers. Second, a firm can use an on-the-job monitoring technology to observe the output produced by its worker, which is a function of his productivity and the effort level he chooses.

The observed output is assumed to be public to the worker and his firm, so that we do not have to worry about the moral hazard of firms in offering their wages. Therefore the wage offer to a worker by his employer will signal his type to the potential employers in the market, although the observed output of the worker is not known to the other firms.

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Ths basic framework of this model is following. In each period a firm competitively offers its contract T which specifies the prerequisites (in terms of signals and previous contracts they have chosen and the previous wage offers they have received) applicants need to show to apply for the contract, the size of their employment, the initial wage offer and the subsequent wage offers based on the outputs produced in the previous periods of employment. Given a set of contracts offered by firms, workers choose contracts to apply for and choose the levels of efforts to take during the periods of their employment. In doing these workers, who quit their jobs with some probability b in each period for exogeneous reasons, have to care about how the other firms' expectations about their types would be affected by their choices. In particular, workers are assumed to share a set of common beliefs about these future expectations, and then make their optimal choices based on these beliefs. The optimal choices of signals, contracts and effort levels by workers given a set of common beliefs generate a relationship between expected types of workers and their employment histories. And the market is supposed to realize this relationship, which is called a market expectation.

I can show that there always exists an equilibrium despite the competition among firms for contracts to offer, which can be contrasted with R-R-S (Riley(1979), Rothschild-Stiglitz(1976)). Intuitively, this is because the competition among firms that are concerned about workers' moral hazards would impose an additional contraint on the set of wage offers firms can make without incuring losses, which will reduce the scope of profitable deviant contracts. In particular, under the conditions where there does not exist any competitive Nash equilibrim in R-R-S, we have a unique equilibrim, which involves the most efficient pooling in the choices of signals by workers. Finally I showed that in any stationary equilibrim the unemployment rate among the high productivity workers is lower than that among the low productivity workers.

A basic model is presented and its equilibrim is defined in Section 2. The set of possible equilibria is characterized in Section 3, which is followed by some concluding remarks.

Ⅱ. Model

Consider a multiperiod competitive labor market, in which a group of new young workers enters in each period. I assume that workers live infinitely so that the labor force increases at the rate g. There are two types of workers, and each type of a worker differs from the other in productivity $z(z_H > z_L)$. One of the informational problems that characterize this model is that each new worker has private information abuout his type which is unknown to his firm. There are some signalling activities (e.g., education levels) available to new workers, however, so that each new worker can choose a signal which may reveal his type before he starts his career. I assume for simplicity that the signalling technology is constant returns to scale. Then, as in other signalling models, the critical assumption is that

$$c_H < c_L,$$

where c_i is the marginal cost of signalling for a z_i-type worker (i=H,L).

The utility function of a z-type worker is assumed to be the following.

$$U(\{w_t, e_t\}, s; z_i) = \sum_{t=1}^{\infty} (u(w_t) - e_t)(1+r)^{-t} - c_i s,$$

where w_t, e_t are the wage he receives and the level of efforts he exerts in the t-th period of his career, and $u(o)=0$, $u' > 0$ and $u'' < 0$. Here I introduce another informational asymmetry. That is, a firm cannot observe directly the level of efforts taken by its workers. I assume for simplicity that there are two possible levels of efforts for workers to take: e(non-shirking) and 0(shirking).

In each period a firm assigns a project to each worker. The project fails if the worker shirks, and it works out otherwise. So there will not be any productivty difference between the two types of workers when they shirk. When they do not shirk, however, the output z_H produced by a z_H-type worker is greater than the output z_L produced by a z_L-type worker. Although a firm cannot directly identify the types of its workers and the effort levels they take, it can use an on-the-job monitoring technology to observe the outputs produced by them and to get information about their types or actions. First of all, a firm is assumed to be able to see with probability 1 whether a project fails or not. So a firm can detect a shirker with probability 1 at the end of each period. But this alone cannot resolve the moral hazard problem on the part of workers, because each firm is supposed to prepay wages to its workers at the beginning of each period due to the lack of perfect capital market available to the workers. That is, workers will get their wages for the period when they shirk although they may be fired at the end of the period. When a worker does not shirk, however, the on-the-job monitoring technology may not be perfect in identifying the output he produced.

Here each firm is assumed to have a very simple monitoring technology, which figures out the output of a worker with probability p and does not figure out the output at all with probability 1-p. So when a worker does not shirk, the monitoring technology is not informative at all with probability 1-p while it identifies his type with probability p. Thus the monitoring technology is imperfect to the extent that $p<1$.

At the end of each period, the market opens. The market has an applicant pool which consists of new workers who have no previous job experience and many experienced workers who have been fired or quit from their previous jobs. I assume that each worker voluntarily leaves his job with probability b for some exogeneous reasons. Once a worker quits, he will join the unemployment pool (or applicant pool) together with those fired and new workers who have just entered the market.

Each firm competitively offers its contract T to profitably attract workers from the applicant pool. When offering a contract T, a firm should specify the prerequisite R an applicant needs to show to apply for the contract T. The R will include the level of signal he chose, previous contracts and the wage offers he has had. The contract also specifies the size of employment N, the initial wage offer W_1 and the subsequent wage offers $W_t(Y_{t-1})$based on the set Y_{t-1} of observations of outputs produced up the (t-1)st period of employment under the contract T. Then a contract T for the workers whose employment history is R can be written as follows:

$$T(R)=\{N;W_1,\ [W_t(Y_{t-1})]\}\ .$$

Now I will characterize the set of output-dependent wage offers $W_t(Y_{t-1})$.

First of all, it is always optimal for firms to fire a worker who has shirked during the previous period. So

$W_t(Y_{t-1})=0$ whenever Y_{t-1} contains $\{0\}$

In determining output-dependent wage offers for nonshirking workers, firms will consider the information conveyed by the observed outputs Y_{t-1}. Then we can classify the sets of observed outputs Y_{t-1} into the three groups: Y_H, Y_L and Y_o, where Y_H or Y_L contains the observed output z_H or z_L, and Y_o does not contain any revealing observed output z_H or z_L. Without loss of generality we can restrict the set of output-dependent wage offers to the following:

$W_t(Y_{t-1})=[W_t(Y_o)=W_1,\ W_t(Y_i)=W_i$ for $i=H,L]$ for all t.

Then as far as the observed output does not fully reveal the type of a worker, the initial wage W_1 will continue to be offered. Thus any output-dependent wage offer $[W_t(Y_{t-1})]$ can be characterized by $[W_H, W_L]$, so that

$T(R)=\{N;\ W_1,\ [W_H,W_L]\}$.

A set of output-dependent wage offers can be subdivided into the two categories: separating output-dependent wage offers (SODW) where $W_H > W_L$, and pooling output-dependent wage offers (PODW) where $W_H=W_L$. In particular, a competitive SODW w^* is defined as a SODW $[W_H,W_L]$ where $W_H=z_H$ and $W_L=z_L$.

In fact, it is not a specific employment history R but the expected productivity Z(R) implied by R that firms care about when they offer contracts. As I will explain later, the market (or firms) forms a common expectation Z(R) about the productivity for each employment history R, which is called the market expectation. Thus any contract offered by a firm can be specified as follows.

$T(Z(R))=\{N;\ W_1,\ [W_H,\ W_L]\}$.

Since firms treat applicants of different market expectations differently, the market will be segmented according to different expected types. A segmented submarket in which workers of market expectation z are applying for jobs is going to be called a z-submarket. And the submarket for new workers is going to be called a s-submarket. When the size of unemployment pool in a z-submarket is greater than the total demand for those workers by firms, the job acquisition rate a(z), the probability that a worker can get a job in the z-submarket, will be less than 1. In Appendix, I characterized a stationary state in each submarket and showed that the job acquisition rate a(z) is negatively related to the unemployment rate in a z-submarket.

Now let us turn to the response of workers to a set of contracts offered by firms. The crucial aspect of this model is that even when a worker is not fired, there is some chance that he leaves his current job for exogeneous reasons and joins the unemployment pool in a certain submarket. And his future utility from the time he leaves his current job with employment history R will depend on which submarket he will belong to, which will be determined by the market expectation Z(R) about his type (that is formed on the basis of his employment history R). So the expected utility $V_i(T)$ of a z.-type worker from choosing a contract T(Z) ($= \{N; W_1, \{W_H, W_L\}\}$) offered for the workers of the market expectation z will depend not only on the wage offers $\{W_1, \{W_H, W_L\}\}$ made by T but also on the set of $\tilde{Z}(R(T); T)$ about his his future market expectation Z(R(T); T) associated with each employment history R(T) under T and the set of beliefs $\{\tilde{V}_{iu}(z)\}_z$, about his expected utility $\{V_{iu}(z)\}_z$ from joining an applicant pool in the submarket associated with each market expectation z. The employment history R(T) under T is just a set of output-dependent wage offers made by T. Then there are four kinds of sets of output-dependent wage offers: R_H, R_L, R_1 and R_0, where R_H or R_L is a set of output-dependent wage offers that contains W_H or W_L and R_1 or R_0 is a set of output dependent wage offeis that contains W_1 only or 0 only. Note that it is impossible that both W_H and W_L are in R(T) because the observed output is public to the worker and his firm.

I will denote a set $\{Z(R_1; T), Z(R_0; T), Z(R_H; T), Z(R_L; T)\}$ of the future market expectations of all employment histories R(T) under T by Z(T), and denote a set of beliefs $\{\tilde{Z}(R_1; T), \tilde{Z}(R_0; T), \tilde{Z}(R_H; T), \tilde{Z}(R_L; T)\}$ about them by $\tilde{Z}(T)$. So when a z_i-type worker evaluates the expected utility $V_i(T)$ of being hired under T, he has to care about a set $Z(T)$ of future market expectations for the contract T and a set $\{V_{iu}(z)\}_z$ of his expected utility of joining the unemployment pool in each submarket as follows :

$$V_i(T)=\text{Max}\,\{V_i^N(T),\ V_c^s(T)\}\quad,$$

where

$$V_i^N(T)=u(W_1)-e+\frac{1-p}{1+r}\{(1-b)V_i^N(T)+b\tilde{V}_{iu}(\tilde{Z}(R_1\ ;\ T))\}+\frac{p}{1+r}$$
$$\{(1-b)\tilde{V}_i(\tilde{Z}(R_i\ ;\ T)\ ;\ T_2)+b\tilde{V}_{iu}(\tilde{Z}(R_i\ ;\ T))\}\qquad \text{(A)}$$

$$V_i^S(T)=u(W_1)+\frac{1}{1+r}\tilde{V}_{iu}(\tilde{Z}(R_0\ ;\ T)).\qquad \text{(B)}$$

$V_i^N(T)$ or $V^S{}_i(T)$ is the expected utility of a z_i-type worker from choosing a contract T when he does not shirk or when he shirks during the first period of his employment under T, respectively. And $\tilde{V}_i(\tilde{Z}(R_i\ ;\ T)\ ;\ T)$ is the conjectured expected utility for a revealed z_i-type worker who continues to work under T. When workers choose their effort levels during the periods of their employment, they also have to care about the possible information $Z(T)$ that their employment history under the contract T can generate. In particular, I assume that all workers share a set of common beliefs $\tilde{Z}(T)$ about the future market expectations for each contract T and a set of common beliefs $\{\tilde{V}_{iu}(z)\}_{z,i}$.

Now I will specify how the market expectation $Z(T)$ for a contract T is formed. After workers optimally choose the contracts to apply for or the siganls and effort levels during the periods of their employment under contracts given a set of common beliefs $\tilde{Z}(T)$ and $\{\tilde{V}_{iu}(z)\}_{z,i}$,market will realize a relationship $Z(R(T)\ ;\ T)$ between each employment history $R(T)$ under the contract T and the associated expected type.

Before defining an equilibrium in this model I will make an important assumption, which is that a set of common beliefs $\{\tilde{Z}(T)\}_T$ and $\{\tilde{V}_{iu}(z)\}_{z,i}$ of workers is self-fulfilling. Basically this model is the same as R-R-S in that firms compete for contracts to offer before workers move. In this type of models, each worker is supposed to make his optimal response to any contract offered. Then we can justify the above assumption because the responses of workers to any contract cannot be optimal unless the underlying beliefs are self-confirmed. Given this type of optimal responses by workers to a set of contracts offered, firms compete with each other with repect to contracts to profitably attract workers in each period. If T(s) and T(z) are a contract for the s-submarket and a contract for the z-submarket, we can define an equilibrium as follows:

An eqilibrium is a set of contracts $D^*(=[\{T^*(s)\}_s\{T^*(z)\}_z])$ such that when workers respond to D^*given a certain set $\{\tilde{Z}(T)\}_{T^*}$, $\{\tilde{V}_{iu}(z)\}_{z,i}$ (= B)of self-fulfilling beliefs,

1) any $T^*(s)$ or $T^*(z)$ yields nonnegative profits for every s and z,

2) when all firms offer T^*, there does not exist any deviant contract T'(s) or T'(z) that can make positive profits by attracting some workers in the s-submarket or in the z-submarket given B and a certain set $\tilde{Z}$(T') of self-fulfilling beliefs about the future market expectations for the contract T'.

Since this model is a dyamic one, the equilibrium defined above should be a stationary one. The distinctive feature of this equilibrium is that an equilibrium requires that if any deviant contract T' is to break the original equilibrium, it should be profitable given B and a set $\tilde{Z}$(T') of self-fulfilling beliefs for the contract T'. Since the profitability of an out-of-equilibrium contract is based on the optimal response of workers to it and the optimal response entails a set of self-fulfilling beliefs for the out-of-equilibrium contract, this equilibrium concept is consistent with a competitive Nash eqiuilibrium in R-R-S. Note that the introduction of T' would not affect the existing set B of beliefs, because any individual firm is so small that it cannot affect the whole market.

Ⅲ. Characterization of an Equilibrium

To figure out a stationary equilibrium in this model, we have to see which submarket exists in stationary state and then characterize an equilibrium contract for each submarket. Since the s-submarket always exists, an equilibrium contract for the s-submarket is important in determining the submarkets that exist in stationary state and the corresponding equilibrium set of contracts. If an equilibrium contract for the s-submarket is a separating one(i.e., each type of a worker is fully revealed to the market by his choice of preemployment signal), an equilibrium will entail z_H-submarket and z_L-submarket as well as the s-submarket, because then a certain population of fully revealed z_i-type workers will be created in each period. (see Appendix for its description) Then we need to characterize the equilibrium contracts for these submarkets. If an equilibrium contract for the s-submarket is a pooling one, the $\bar{z}$-submarket for the workers whose types are not fully revealed yet (where $\bar{z}=(1-q)z_H+qz_L$) also exists in stationary state. If the pooling contract for the s-submarket or an equilibriom contract for the z-submarket involves SODW, both z_H-submarket and z_L-submarket will exist in stationary state because some workers get their types revealed through SODW in each period. Then we have to find out the equilibrium contracts for those submarkets too. If the pooling contract for the s-submarket and an equilibrium contract for the $\bar{z}$-submarket does not involve any SODW but PODW, there will not exist any z_H-submarket or z_L-submarket in stationary state. But I can show that that will not be the case.

Proposition 1

In equilibrium, there always exists in each period some population of workers who gets their types revealed.

⟨proof⟩

Suppose not. Then an equilibrium pooling contract for the submarket or any equilibrium contract for the $\bar{z}$-submarket will entail PODW with the wage equal to $\bar{z}$. But each firm is able to know the types of some of its workers by its monitoring technology. Then each firm can increase the average productivity of its workers by hiring a worker from the applicant pool when one of its z_H-type workers quits and by not hiring any worker when one of its z_L-type workers quits. Since an entrant firm can make positive profits by adopting this employment policy, the PODW cannot constitute an equilibrium.

Proposition 1 implies in equilibrium where workers' choices of preemployment signals are not fully revealing, the types of workers are going to be revealed to the market ultimately due to the on-the-job monitoring technology of firms. So an equilibrium will surely consist of a contract $T^*(z_H)$ for the z_H-submarket and a contract $T^*(z_L)$ for the z_L-submarket as well as $T^*(s)$ for the s-submarket. Since any equilibrium entails $\{T^*(z_i)\}_{i=H,L}$, all the possible equilibria can be classified according to the nature for the equilibrium contract $T^*(s)$ for the s-submarket. Then there could be two kinds of equilibria: a separating equilibrium that involves $\{T^*_s(s_i)\}$ $i=$ H,L through which different types of workers choose different signals, and a pooling equilibrium that involves $T^*_p(s)$ through which different types of workers chose the same signal. So a separating equilibrium will be a set of contracts $\{T^*_s(s_i)\}_i$ $\}$ $\{T^*(z_i)\}_i\}$ and a pooling equilibrium will be a set of contracts $\{T^*_p(s), \{T^*(z_i)\}_i, T^*(\bar{z})\}$

The most important factor in characterizing an equilibrium in this model is the contraint imposed by workers' moral hazard on the set of wage offers firms can make. This constraint is related to the set of beliefs $\{\tilde{V}_{iu}(z)\}_{z,i}$ of workers about the expected utility of each type of a worker who leaves his current firm and joins the unemployment pool in the z-submarket, because those beliefs will determine the optimal choices of effort levels by workers during the periods of their employment. So I will characterize the set of beliefs $\{\tilde{V}_{iu}(z)\}_{z,i}$ of workers first. Since those beliefs $\{\tilde{V}_{iu}(z)\}_{z,i}$ are constrained to be self-fulfilling in this model, I will consider a stationary state in each submarket and figure our an equilibrium contract for each submarket to characterize those beliefs. Thus I will derive $\{\tilde{V}_{iu}(z_i)\}_i$ from $T^*(z_H)$ and $T^*(z_L)$ and then derive $\tilde{V}_{iu}(\bar{z})$ from $T^*(\bar{z})$ and $\{T^*(\tilde{V}_{iu}(z_i)\}_i$.

In each z_i-submarket where the type of each worker is fully revealed, a firm is concerned only about possible shirking of its workers. So the equilibrium contract for the revealed z_i-type workers from shirking, and such that an entrant contract cannot make positive profits. The first thing we can say about the equilibrium in z_i-submarket is that each worker should be paid z_i. Then let us suppose all firms offer a following contract $T^*(z_i)$ or T_i in the z_i-submarket:

$$T_i = \{N_i(t);\ z_i,\ w^*\} \quad \text{for } i=H, L, \tag{1}$$

where w^* is a competitive SODW defined before. The size of employment

$N_i(t)$ by each firm will lead to a certain job acquisition rate a_i in each peiod. And this job acquisition rate, as well as the wage offers by T_i, will determine the expected utility V_{iu} of a revealed z_i-type worker joining the applicant pool in the z_i-submarket. Since any belief should be confirmed in stationary state, I will set $\tilde{V}_{iu}(z_i)$ equal to the actual expeted utility $V_{iu}(z_i)$ or V_{iu}. Then the expected utility $V_i(T_i)$ or V_i of an employed z_i-type worker who does not shirk under T_i will be

$$V_i^N = u(z_i) - e + \frac{1}{1+r}\{b\ V_{iu} + (1-b)\ V_i^N\} = \frac{1+r}{b+r}(u(z_i)-e) + \frac{b}{b+r}V_{iu}. \quad (2)$$

And the expected utility $V_i^S(T_i)$ or V_i^S of a z_i -type worker who shirks under T_i will be

$$V_i^S = u(z_i) + \frac{1}{1+r}V_{iu}. \quad (3)$$

Since no type of a worker shirks in equilibrium, the following should hold :

$$V_i^N \geq V_i^S \quad \text{or} \quad \frac{1+r}{b+r}(u(z_i)-e) + \frac{b}{b+r}V_{iu} \geq u(z_i) + \frac{1}{1+r}V_{iu}. \quad (4)$$

(4)is called the nonshirking condition(NSC) for the z_i-submarket. Once (4) is satisfied, workers will choose non-shirking strategies in each z_i-submarket. So the expected utility V_i of a revealed z_i-type worker working under T_i will be equal to V_i. In equilibrium, the unemployment rate in the z_i-submarket (or V_{iu}) should be determined such that the NSC (4) holds with equality. In other words, the NSC (4) should be binding with wage offers(both the initial one W_l and the output-dependent one W_H)being equal to z_i. This is because otherwise there will exist a profitable deviant contract that offers wage less than z_i but does not induce workers to shirk given that all the other firms T_i (or given V_{iu}). The fact that the NSC (4) should be binding with wage offers being equal to z_i is very critical for the existence of a competitive equilibrium, as we will see later (Proposition 5). So we have in equilibrium

$$\left(\frac{1+r}{b+r} - 1\right)u(z_i) - \frac{1}{1+r}e = \left(\frac{1}{1+r} - \frac{b}{b+r}\right)V_{iu} \quad \text{or} \quad u(z_i) - \frac{1+r}{1+b}e = \frac{r}{1+r}V_{iu}. \quad (5)$$

(5)will be called the equilibrium nonshirking condition (NSC) in the z_i-submarket. Here I assume that the disutility e of working is less than the discounted value of its expected benefits $\frac{1-b}{1+r}u(z_i)$ so that $u(z_i) > \frac{1+r}{1+r}e$. When all firms offer T_i, we have

$$V_{iu}=a_i V_i+\frac{1-a_i}{1+r}V_{iu}=\frac{a_i(1+r)}{a_i+r}V_i, \tag{6}$$

where a_i is the job acquistion rate in the z_i-submarket.
From (2) and (6), we have

$$V_i=(\frac{1+r}{r})\ (\frac{a_i+r}{b+r+a_i(1-b)})\ (u(z_i)-e) \tag{7}$$

and

$$V_{iu}=\frac{1+r}{r}\ (\frac{a_i(1+r)}{b+r+a_i(1-b)})\ (u(z_i)-e), \tag{8}$$

or from (5)

$$V_{iu}=\frac{1+r}{r}\ (u(z_i)-\frac{1+r}{1+b}e). \tag{9}$$

Fron (8) and (9) we have another expression for the equilibrium NSC :

$$a_i=1-\frac{e(1+r)}{u(z_i)(1-b)}. \tag{10}$$

We cna see that $a_i > 0$ because $u(z_i) > \frac{1+r}{1-b}e$.

The equilibrium NSC (10) shows that the equilibrium job acquisition rate a_i (or equilibrium size of employment $N_i(t)$ for each t) is uniquely determined. This also implies that the equilibrium contract T_i for each z_i-submarket is unique. Since the equilibrium contract T_i for each z_i-submarket is unique, workers and firms should have the following selffulfilling beliefs : for each i=H,L,

$$\tilde{V}_{iu}=V_{iu}\ \text{in}(9). \tag{11}$$

Finally we can see from (10) that the equilibrium unemployment rate u_H in the z_H-submarket is lower than that u_L in the z_L-submarket. Intuitively this is because the cost of shirking for a z_H-type worker is higher than that for a z_L-type worker given a certain rate of unemployment, since the wage that a shirking z_H-type worker looses during each unemployment period is higher than the wage that a shirking z_L-type worker looses during the same unemployment period. So far we have established the following.

Proposition 2

The equilibrium contract in each z_i-submarket is $T^*(z_i)$ or T_i defined in (1), which leads to the equilibrium unemployment rate as described in(10) And unemployment rate among the hight productivity workers is lower than that among the low productivity workers.

Now let us turn to the belief $\tilde{V}_{iu}(\bar{z})$ or $\tilde{V}_{iu}$ about the expected utility $\tilde{V}_{iu}(\bar{z})$ (or $\tilde{V}_{iu}$) of an unemployed z_i-type worker whose market expectation is $\bar{z}$, where $\bar{z}=(1-q)\ z_H+q\ z_L$, the average productivity of all the workers. Before that, I will make a following assumption about q, the portion of z_L-type workers:

$$q\rangle \bar{q}, \text{ where } \bar{q} \text{ is such that } u(z_H)-u(\bar{z})=\frac{1+r}{1-b}e. \tag{12}$$

I will assume that the required effort level e and the exogeneous quit rate b are very small, such that the lower bound $\bar{q}$ of q is very low. As we will see later, the assumption (12) serves to guarantee that $V_{Hu}(\bar{z})\langle V_{Hu}$, i.e., that the expected utility of a z_H-type worker from being in the unemployment pool of the z_H-type is greater than that from being in the unemployment pool of the $\bar{z}$-susbmarket.

To specify the self-fulfilling belief about the expected utility $\tilde{V}_{iu}(\bar{z})$(or $\tilde{V}_{iu}$), I will first consider a stationary state in the $\bar{z}$-submarket and characterize an equilibrium contract $T^*(\bar{z})$ or $\bar{T}$ for the $\bar{z}$-submarket. Suppose all firms offer a following contract $\bar{T}$ for the workers of market expectation $\bar{z}$:

$$\bar{T}=\{\bar{N}(t)\ ;\bar{z},\ w^*\}\ , \tag{13}$$

where $\bar{N}(t)$ is the employment size of each firm which leads to a certain unemployment rate in the $\bar{z}$-submarket. Note that w^* is the only output--dependent wage offer that an equilibrium contract can entail by Proposition 1. For $\bar{T}$ to be an equilibrium contract for the $\bar{z}$-submarket, the workers should not shirk under $\bar{T}$. Then a set $\bar{z}(\bar{T})$ of self-fulfilling beliefs about the future market expectations for T will be such that

$$Z(R_i\,;T)=z_i,\ i=H,L,\ \text{and}\ Z(R_I\,;T)=z, \tag{14}$$

and

$$Z(R_O\,;T)=z_L \tag{15}$$

Then we can establish the following equilibrium NSC for the $\bar{z}$-submarket.

Proposition 3

Given a set $\tilde{Z}(\bar{T})$ of beliefs (14) and (15) for the contract $\bar{T}$ and $\{\tilde{V}_{iu}\}_i$ in (11), the equilibrium NSC for the $\bar{z}$-submarket entails zero unemployment rate.

⟨proof⟩

Since the type of a worker hired under $\bar{T}$ will be revealed to his firm by the monitoring technology with probability p, the expected utility $V_i^N(\bar{T})$ or $\tilde{V}_i^N$ of a z_i-type worker choosing the nonshirking strategy under $\bar{T}$ will be

$$\tilde{V}_i^N=u(\bar{z})-e+\frac{1-p}{1+r}\{(1-b)\tilde{V}_i^N+bV_{iu}\}\ +\frac{p}{1+r}\{(i-b)V_i+bV_{iu}\}, \tag{16}$$

where V_{iu} is the expected utility of an unemployed z_i-type worker of market expectation $\bar{z}$. Then From (14),(15) and (16) we have

$$\begin{aligned}\tilde{V}_L^N&=u(\bar{z})-e+\frac{1+p}{1+r}\{(1-b)\tilde{V}_L^N+b\tilde{V}_{Lu}\}+\frac{p}{1+r}\{(1-b)V_L+bV_{Lu}\}\\&>u(\bar{z})-e+\frac{1}{1+r}V_{Lu},\ \text{since}\ \tilde{V}_L^N>V_L>V_{Lu}\ \text{and}\ \tilde{V}_{Lu}>V_{Lu}\\&=\tilde{V}_L^S.\end{aligned}$$

Since the nonshirking condition is not binding, the unemployment rate for the z-submarket should be zero in equilibrium.

Since the unemployment rate in the z-submarket is zero, the Proposition 3 says that we do not need positive unemployment rate in the $\bar{z}$-submarket because both of the two types of workers have additional incentives to work. That is, z_L-type workers want to stay in the $\bar{z}$-submarket for longer periods and z_H-type workers want to move to the z_H-submarket soon.

Since the unemployment rate in the $\bar{z}$-submarket is zero, the self-fulfilling belief about the expected utility $\tilde{V}_{iu}$ for an unemployed z_i-type worker of market expectation $\bar{z}$ is equal to $\tilde{V}_i^N$.

$$
\begin{aligned}
V_{iu} &= \tilde{V}_i^N \\
&= \frac{1+r}{p+r}(u(\bar{z})-e)+\frac{p(1-b)}{p+r}(V_i-V_{iu})+\frac{p}{p+r}V_{iu} \\
&= \frac{1+r}{p+r}\{u(\bar{z})+\frac{p}{r}u(z_i)-(1-p+\frac{p(1+r)}{r(1-b)})e\}. \qquad (17)
\end{aligned}
$$

Also we have the following.

$$V_{iu}-\tilde{V}_{iu}=\frac{1+r}{p+r}\{u(z_i)-u(\bar{z})-(p+\frac{b+r}{1+b})e\}.$$

So $V_{Lu}-\tilde{V}_{Lu}<0$ and we can see from the assumption (12) that

$$V_{Hu}-\tilde{V}_{Hu}>\frac{1+r}{p+r}\{u(z_H)-u(\bar{z})-(p+\frac{b+r}{1-b})e\}$$

Using the same procedure as before, we can also compute the the conjectured expected utility $\tilde{V}_{Hu}(z_L)$ for a z_H-type:

$$V_{Hu}(z_L)=\frac{1+r}{p+r}\{u(z_L)+u(z_H)-(1-p+\frac{1+r}{1-b})e\}. \qquad (18)$$

Finally we can also figure out the conjetured expected utility $\tilde{V}_{Lu}(z_H)$ for a z_L-type worker who is mistaken for a z_H-type. Since he will continue to be taken as z_H-type as far as he is not identified as z_L-type by the monitoring technology, it will be optimal for him to shirk each time he is hired. So

$$\tilde{V}_{Lu}(z_H)=V^S_{Hu}=V_{Hu}. \qquad (19)$$

So far we have characterized a set of self-fulfilling beliefs $\{\tilde{V}_{iu}(z)\}_{z,i}$ (specified in (11), (17)-(19)) about the expected utility associated with each market expectation z. For notational simplicity, I will denote a set of beliefs $\{\tilde{V}_{iu}(z)\}_{z,i}$ specified in (11) and (17)-(19) by V^E. This set V^E of beliefs plays an important role in determining the choice of a signal, a contract to apply for and the effort level by each type of a worker. In particular, as we will see later, the beliefs $\{\tilde{V}_i\}_i$ in V^E impose an important constraint on the set of wage offers firms can offer without incurring losses because firms have to worry about the possible shirkings of their workers.

Next I will find out the conditions under which we have a separating equilibrium where each type of a worker is fully revealed by his choice of preemployment signal or a pooling equilibrium where both of the two types of workers choose the same preemployment signal. First I will start with describing a separating equilibrium. Since $\tilde{V}_{Lu}(z_H)=\tilde{V}_{Hu}$, the following should hold if different types of workers are to be separated from each other by their choices of signals s_L and s_H.

$$V_Lu-c_L\ s_L\rangle V_{Hu}-c_L\ s_H,$$

or

$$c_L(s_H-s_L)\rangle V_{Hu}-V_{Lu}=\frac{1+r}{r}(u(z_H)-u(z_L)). \tag{20}$$

Suppose firms offer a set of separating contracts $\{T_s(s_i)\}_{\ i=H,L}$ for the submarket such that

$$T_s(s_i)=\{N(s_i)\,;\ z_i,\ w^*\}\ , \tag{21}$$

where $N(s_i)$ is the employment size that leads to the same unemployment rate u_i(as in the z_i-submarket) among the new workers choosing s_i in each period, and s_H and s_L satisfy (20). Then differnet types of workers are going to be separated from each other by their choices of preemployment signals. As in Riley, however, this separating contract $\{T_s(s_i)\}_i$ may be dominated by a profitable pooling contract with a set of selffulfilling beliefs. Let $\bar{s}_L$ and $\bar{s}_H$ be the minimun signals that satisfy (20):

$$\bar{s}_L=O \text{ and } \bar{s}_H=\triangle V/c_L,$$

where $\triangle V=V_{Hu}-V_{Lu}$. Then we can establish the following.

Proposition 4

There does not exist any separating equilibrium $\{\{T_s(s_i)\}_i,\ \{T_i\}_i\}$ if

$$\frac{u(z_H)-u(z_L)}{r}\rangle\frac{1}{p+r}\{u(z_H)-u(\bar{z})-(p+\frac{b+r}{1-b})e\} \tag{22}$$

⟨proof⟩

Suppose all firms offer $\{T_s(s_i)\}_{\ i=H,L}$ in (21) for the new workers, where $s_L=\bar{s}_L$ and $s_H=\bar{s}_H$. Then one firm can offer $T'(0)$ for the new workers choosing zero signal.

$$T'(0)=\{N\,;\bar{z}-\varepsilon\ ,\ w^*\}\ . \tag{23}$$

Suppose workers have the same set $\tilde{Z}(T(0))$ of beliefs about the future market expectation for the contract $T'(0)$ as $\tilde{Z}(\bar{T})$ in (14),(15). Then no worker will shirk under $T'(0)$ for a very small $\varepsilon\rangle O$.The contract $T'(0)$ will attract z_L-type workers because $\bar{z}-\varepsilon\rangle z_L$ given $\tilde{Z}(T'(0))$ and V^E. Since (22) implies that $V_{Hu}-\bar{s}_H\ c_H<V_H$, the contract $T'(0)$ will also attract z_H-type workers for small $\varepsilon>O$ given the set of self-fulfilling beliefs $\tilde{Z}(T'(0))$ and

V^E. Since the initial wage offer $\bar{z}-\varepsilon$ is less than the average productivity $\bar{z}$, the deviant contract will make positive profits given V^E and a set $\tilde{Z}(T'(0))$ of self-fulling beliefs for $T'(0)$.

Proposition 4 establishes the same point that has been made by R-R-S. The condition (22) just describes the circumstance under which there does not exist any competitive Nash equilibrium in their models where any pooling contract cannot be a competitive Nash equilibrium. However, I will show later that in this situation there exists a unique equilibrium which is a pooling one, and show that is mainly due to the additional constraint (the equilibrium NSC (5) or (10) in the z_i-submarket) imposed by workers' moral hazard on the set of wage offers firms can make. The equilibrium NSC in the z_i-submarket has the following important implication:

Proposition 5

Suppose a set of contracts D is a proposed equilibrium. Then a deviant contract cannot upset the equilibrium by attracting the workers whose types are fully revealing.

⟨proof⟩

Suppose a deviant contract T' attracts z_i-type workers only. Since the set of beliefs $\tilde{Z}(T')$ for the contract should be self—fulfilling, $Z(R(T'))=z_i$ for all $R(T)$. then the z_i-type workers attracted to T' will shirk if any wage offer is less than z_i-type workers attracted to T' will shirk if any wage offer is less than z_i, because of the self—fulfilling beliefs $\{V_{iu}\}$ derived from the equilibrium NSC (5). This implies that the deviant contract T' will not be able to make positive profits.

The proposition 5 is based on the fact that whenever the type of a worker is revealed to the market, the competition among firms that are concerned about their workers' moral hazard will always lead to the wage equal to his productivity, as specified in the equilibrium NSC (5). And this fact reduces the scope of profitable deviant contracts significantly, which is crucial for the existence of a competitive equilibrium as we will see in the next proposition.

Proposition 6

$\{T_p^*(0),\ \bar{T},\ \{T_i\}\ _i\}$ is an equilibrium, where $T_p^*(0)$ is a pooling contract for the s-submarket such that

$$T_p^*(0)=\{N_p\,;\ \bar{z},w^*\}\,, \tag{24}$$

and N_p is the size of employment of new workers by each firm which leads to zero unemployment rate in the s-submarket in each period.

⟨proof⟩

Suppose an entrant firm offers a contract $T'(s)$ for new workers choosing a signals s.

$$T'(s) = \{N^1 ;\ W_l,\ w^*\}\ .$$

For the deviant contract $T'(s)$ to make positive profits, it is necessary that the optimal choice of effort levels by the workers under $T'(s)$ should be nonshirking. Then if the deviant contract $T'(s)$ is to be chosen by both types of workers (so that $\tilde{Z}(R_l : T'(s)) = \bar{z})$), it should be that $W_l > \bar{z}$ because of the alternative $T^*_p(0)$. If $W_l > \bar{z}$, however, the deviant contract will make losses by attracting both types of workers. Now what if $T'(s)$ attracts only z_H-type workers or z_L-type workers? Then by Proposition 5, $T'(s)$ will not be able to make positive profits.

Although firms compete with each other in offering contracts to profitable attract workers as in R-R-S, a pooling contract can be supported as a competitive equilibrium. This is clearly contrasted with R-R-S, in which no pooling contract can constitute an equilibrtium. There are two reasons for this result. The main reason is the fact that the competition among firms in controlling their workers' moral hazard reduces the scope of profitable deviant contracts (given a set of the existing pooling contracts) to nil, as described in Proposition 5. And there is another factor which is favorable to the stability of a pooling equilibrium, although it is not explicit in this analysis. Since a pooling contract reveals the types of workers through its SODW based on the monitoring technology (so that the utility of a z_H-type worker from choosing it is greater than that of a z_L-type worker), the benefit of a z_L-type worker if the choice of the deviant contract T' signals z_H-type. This is because once a z_L-type worker is taken as z_H-type, he can get the same utility as a z_H-type worker can get. In other words, the payoff difference between the two types of workers under a pooling contract is large due to the on-the-job monitoring technology, while the payoff difference under a deviant contract which signals z_H-type will be zero because then the monitoring technology is not effective in revealing the types of workers.

This possibility is not considered in R-R-S, where there is no on-the-job monitoring technology available to firms. Thus the introduction of on-the--job monitoring technology makes empty the set of possible contracts that can attract z_H-type workers only given a set of the existing pooling contracts, although there always exists in R-R-S a possibility that a deviant contract can attract z_H—type workers only given a set of the existing pooling contracts.

Since we have a pooling equilibrium because of the above reasons, the problem of possible nonexistence of equilibrium in R-R-S is resolved in this model. As in R-R-S, however, the same kind of competition among firms with respect to their contracts eliminates many inefficient pooling equilibria.

Proposition 7

Any inefficient pooling contract for the s-submarket does not constitute an equilibrium.

⟨proof⟩

Suppose that all firms offer a pooling contract $T_p(s)$ which is the same as $T_p^*(0)$ except for the required signal s⟩O. Then an entrant firm can offer a contract $T'(0)$ in (23), and workers can have the same set of self-fulfilling beliefs $\tilde{Z}(T'(0))$ for $T'(0)$as that in the proof of Proposition 4. Then the deviant contract will make positive profits as before.

Thus Proposition 4 and Proposition 7 imply that if (22) holds, there exists a unique equilibrium $\{T_p^*(0), \bar{T}, \{T_i\}_i\}$, which is the most efficient pooling one. However, there can be many separating equilibria $\{T_s^*(s_i)\}_i$, $\{T_i\}_i\}$ if (22) does not hold. This is because any set of separating contracts $\{T_s(s_i)\}_i$ for the s-submarket cannot be dominated by another separating contract due to the constraint described in Proposition 5. So any separting contract $\{T^*(s_i))\}_i$ which require workers to choose the signals satisfying (20) can constitute an equilibrium unless (22) is satisfied. This can be contrasted with R-R-S, where, if any, there exists a unique separating equilibrium that is the most efficient one. Thus we can say the following about the set of equilibria of this model.

Proposition 8

If the condition (22) holds, there exists a unuque equilibrium, which is the most efficient pooling one $\{T_p^*(0), \bar{T}, \{T_i\}_i\}$. If the condition (22) does not hold, the equilibrium is not unique. The possible equilibria include not only the most efficient pooling equilibrium but also many separating equilibria $\{\{T_s^*(s_i)\}_i, \{T_i\}_i\}$, of which the separating contracts $\{T_s^*(s_i)\}_i$ are not dominated by the pooling contract $T_p^*(0)$.

Finally, I will identify the conditions that can make a separating equilibrium more likely. First of all, the quality of on-the-job monitoring technology (which is indicated by p) plays an important role in determining the pattern of an equilibrium. If p gets higher (or if the quality of on-the-job monitoring technology increases), the condition (22) will be more likely to be satisfied because then the pooling contract for the s-submarket will pay the initial wage offer $\bar{z}$ to the z_H-type workers for shorter periods. So we can say that a separating equilibrium will be more likely to prevail as the quality of on-the-job monitering technology decreases. The other factors that can determine the pattern of an equilibrium are the relative cost of signalling for each type of a worker and the distribution of types, which are also mentioned in R-R-S. That is, as c_H/c_L gets smaller or as the portion of z_l-type workers (q) is higher, a separating equilibrium will be more likely to emerge. Summarizing these results,

Proposition 9

A separating equilibrium is more likely to emerge if

i the quality of on-the-job monitoring technology decreases

ii c_H/c_L is lower and

iii the portion of the z_L-type workers is higher.

Ⅳ. Concluding Remarks

I considered a dynamic competitive labor market in which the two important informational problems-moral hazard and adverse selection among workers-arise simultaneously. Faced with these informational problems, workers choose some preemployment signals (e.g., educational levels) which may reveal their types before they start their careers. Also firms can use an on-the-job monitoring technology to control moral hazard and adverse selection on the part of workers.

In my model firms competitively offer their contracts which specify some prerequisites (in term of previous wage offers, contracts and signals they have chosen) the applicants need to show, the sizes of their employment, the initial wage offers and the subsequent wage offers based on the observations of outputs produced by workers. Given these contracts, workers choose contracts to apply for and their effort levels during the periods of their employment under their contracts. Although my model shares with R-R-S the common property that firms move first competitively and workers respond to that, it is critically different from their models in that when workers choose contracts to apply for and effort levels during the periods of their employment, they have to care about how the market expectations about their types would be affected by their choices. So workers make their optimal choices to a set of contracts offered, given sets of common beliefs about the future market expectations. However, the set of common beliefs for any contract is constrained to be self-fulfilling in this model, so that the competition among firms in offering contracts is based on the self-fulfilling beliefs on the part of workers. This is consistent with the competitive Nash equilibrium of R-R-S in which workers make optimal (ex post) responses even to an out-of-equilibrium contract, in that the optimal response to a contract entails a set of self-fulfilling beliefs for the contract.

The main result of this model is that although firms compete with each other in offering contracts to profitably attract workers, there always exists a competitive equilibrium. The most important reason for this existence is that the competition among firms that are conerned about possible shirkings of their workers imposes an additional constraint on the set of wage offers firms can make without incurring losses, so that the scope of profitable deviant contracts given a set of the existing pooling contracts shrinks significantly by Proposition 5. In particular, I showed that the most efficient pooling contract for the new workers can be supported as a competitive equilibrium, which can be contrasted with R-R-S. On the other hand, I showed that as in Riley, the competition among firms eliminates many inefficient equilibria although there could be multiple equilibria. Also I showed that there exists a unique equilibrium under certain circumstances where the nonexistence problem arises in R-R-S. Futhermore, I pointed out some factors that can make a separating equilibrium more likely to emerge such as poor on-the-job monitoring technology, lower c_H/c_L and higher portion of z_L-type workers. Finally I showed that in any stationary equilibrium, the unemployment rate among the higher productivity workers is lower than that among the lower productivity workers.

Appendix: Descripton of Stationary State in Equilibrium

Let L_t, N_t, U_t be the size of total labor force, total employment, total unemployment in a z-submarket (or s-submarket) in a certain period t, respectively. Then

$$L_t=(1+g)\ L_{t-1}=N_t+U_t,$$

where g is the rate of increase in the labor force.

(1) Stationary State of z_i-submarket in Separating Equilibium

The size of total applicants A_t in period t will be

$$A_t=bN_{t-1}+U_{t-1}+gL_{t-1}=(b+g)N_{t-1}+(1+g)U_{t-1}.$$

And the size of new employment m_t in period t will be

$$m_t=(b+g)N_{t-1}.$$

So the job acquisition rate a_t in period t will be

$$a_t=m_t/A_t=(b+g)/b+g+(U_{t-1}/N_{t-1})(1+g).$$

Since $U_t/N_t=u_t/(1-u_t)$ (where u_t is the unemployment rate in period t for the z_i-submarket),

$a_t=(b+g)/b+g+(u_t/1-u_t)(1+g)$.

So in stationary state,

$a=(b+g)/b+g+(u/1-u)(1+g)$.

Note that the job acquisition rate a is negatively related to the unemployment rate u.

(2) Stationary state in a Pooling Equilibrium

(A) $\bar{z}$-submarket or s-submarket

The size of applicant pool in this submarket will be

$A_t=(g-p)L_{t-1}+b(1-p)N_{t-1}$,

and the size of employment m_t will be

$m_t=(b+(1-b)p+g)N_{t-1}$.

Since the equilibrium unemployment rate in the this submarket is zero,

$A_t=m_t$ and $L_{t-1}=N_{t-1}$,

so that

$(1+g)N_{t-1}=N_t=L_t=(1+g)L_{t-1}$.

(B) z_i-submarket

The size of applicant pool A_t will be

$A_t=bN_{t-1}+U_{t-1}+g\ L_{t-1}=(b+g)N_{t-1}+(1+g)U_{t-1}$.

And the size of employment m_t will be

$m_t=bN_{t-1}+gL_{t-1}=(b+g)N_{t-1}+gU_{t-1}$.

So the stationary state a will be

$a=m/A=b+g+(U/N)g/b+g+(U/N)(1+g)=b+g+(u/1-u))g/b+g(u/1-u)(1+g)$.

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