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TECHNICAL PROGRESS AND UNEMPLOYMENT IN A SMALL OPEN ECONOMY

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I. INTRODUCTION

One of the main source of economic expansion is the occurrence of technological progress in the industries. Recent development of economic phenomena has been characterized by remarkable technical improvement in most developed or developing countries in the world.

Over the last some decades, many economists have tried to explore the implications of these technical advances for the outputs, factor proportions, real factor rewards and other economic variables. Despite the recent rapid economic growth due to such technological progress, however, unemployment has been a problem for most countries and has not revealed any sign of fundamental improvement of this problem.

Since Harberler (1950) has introduced unemployment problem in the standard two–sector trade model where full employment is generally assumed, many economists such as Johnson (1965), Bhagwati (1968), Batra and Pattanaik (1971), Findlay (1973), Brecher (1974), and Batra and Seth (1977), have tried to examine the cause and effect of unemployment in the international trade and, especially, they have concentrated on the impact of rigid factor prices on small countries' gains from trade. They have tried to investigate some aspects of international trade theory in the presence of unemployment. However, no one has considered the technical progress and unemployment problem simultaneously to date.[1]

In the present paper, therefore, we utilize Batra–Seth model of general unemployment and try to explore this problem mainly, and also the impacts of technical progress on the other variables in a small open economy. In particular we follow the Hicks' classification of technical progress for simplicity of analysis. We also consider a trading country which is very small in the sense that its purchases and sales do not affect world prices.

In section II, we introduce the assumptions and basic model, and in section III,

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[1]Kemp arranged the implications of technological progress for the overall economy under the competitive market (1969) and Hazari analyzed the relationship between the factor market distortions an technical progress (1975).

introduce three types of technical progress and analyze their effects under the general unemployment. As a result, we have suggested some propositions and compared with other models in section IV, and, finally, we summarize major findings of this analysis.

II. THE MODEL

Although there are many theories of unemployment, we assume here that rigid real wage rate results in unemployment of labor. Our model is a standard two sector model with two factors but the production functions are no longer linearly homogeneous and are assumed to show the diminishing returns to scale in both sectors.

To introduce technical progress in our model, we assume that it involves simply a new way of combining existing factors in the production of one or both goods. There are two commodities (sectors) x and y, and two factors of production K (capital) and L (Labor). We assume that technical progress takes place only in sector x. Other assumptions include the profit maximization on the part of producers, perfect competition in the product and factor markets except in the labor market, perfect factor mobility, inelastically supplied but fully employed capital (K), and concave production functions in both factors. Thus the production function for sector x is

$$x = x(\beta K_x, \lambda L_x) \tag{1}$$

where K_x and L_x are the capital and labor inputs utilized in sector x, and β and λ are shift parameters each of which is initially equal to unity.[2]

All marginal products are positive but diminishing. That is, $x_L > 0$, $x_K > 0$, $x_{LL} < 0$, $x_{KK} < 0$ and $x_{KL} > 0$. By the assumption of concavity, $(x_{KK}x_{LL} - x_{KL}^2)$ is positive: In sector y, the production function can be defined as

$$y = y(K_y, L_y) \tag{2}$$

where K_y and L_y are the capital and labor inputs utilized in sector y.

It shows that there is no technical improvement in sector y. The properties of production function y are same as those of x except the property of technical progress.

Then, the marginal conditions for competitive producers maximizing profit become

$$\bar{w} = x_L(\beta K_x, \lambda L_x) \tag{3}$$

$$\bar{w} = p y_L(K_y, L_y) \tag{4}$$

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[2]According to Hicks, an increase in $\beta(\lambda)$ indicates the capital-saving or labor-using (labor-saving or capital-using) technical progress.

$$r = x_K(\beta K_x, \lambda L_x) \tag{5}$$

$$r = py_K(K_y, L_y) \tag{6}$$

where p represents the relative price of y in terms of x, and w and r imply the real wage rate and the rental of capital respectively. Here, w is rigid ($\bar{w}$).

The capital is fully employed but inelastically supplied. So,

$$K_x + K_y = \bar{K} \tag{7}$$

In labor market, there is an unemployment because of wage rigidity. So,

$$L_x + L_y + U = \bar{L} \tag{8}$$

where U is unemployment.

Above equations (3)–(6) contain four variables K_x, L_x, L_y and r (because $K_y = \bar{K} - K_x$) and five parameters w, p, K, β and λ.[3] So, this system is determinate.

Let us check the effect of a change in technical progress parameter β and λ on the real factor rewards, employment, outputs and so on. If we consider only three cases of technical progress, a capital–saving technical progress can be represented by an increase in β ($d\beta > 0$) with λ constant ($d\lambda = 0$); a labor–saving technical improvement by $d\lambda > 0$ and $d\beta = 0$; and a neutral technical progress by $d\beta = dt = d\lambda > 0$.

Differentiating equations (3)$-$(6) totally, we obtain

$$\begin{bmatrix} x_{LK} & x_{LL} & 0 & 0 \\ -py_{LK} & 0 & py_{LL} & 0 \\ x_{KK} & x_{KL} & 0 & -1 \\ -py_{KK} & 0 & py_{KL} & -1 \end{bmatrix} \begin{bmatrix} dK_x \\ dL_x \\ dL_y \\ dr \end{bmatrix} = \begin{bmatrix} -x_{LK}K_x d\beta - x_{LL}L_x d\lambda \\ 0 \\ -x_{KK}K_x d\beta - x_{KL}L_x d\lambda \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ b \\ 0 \end{bmatrix}$$

$$\text{where } a = -(x_{LK}K_x d\beta + x_{LL}L_x d\lambda) \tag{9}$$

$$b = -(x_{KK}K_x d\beta + x_{KL}L_x d\lambda)$$

The determinant of the system is simplified into

$$D = -x_{LL}p^2(y_{LL}y_{KK} - y_{LK}^2) - py_{LL}(x_{LL}x_{KK} - x_{KL}^2) \tag{10}$$

which is positive because x_{LL} and y_{LL} are negative and $(x_{LL}x_{KK} - x_{KL}^2)$ and $(y_{LL}y_{KK} - y_{KL}^2)$ are positive from assumptions.

III. THREE TYPES OF TECHNICAL PROGRESS

Accordingly, the above system can be solved in the following way.

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[3]Equations (5) and (6) can be simplified into one equation as the following. $x_K(\beta K_x, \lambda L_x) = py_{LK}(K_y, L_y)$······(5)'. Then there are three variable (K_x, L_x and L_y) and three equations (3), (4) and (5)'. Accordingly, this system can be solved.

(1) The case of capital–saving technical progress ($d\beta > 0$, $d\lambda = 0$)

(a) $dK_x/d\beta = py_{LL}K_x(x_{KK}x_{LL} - x_{KL}^2)/D$

(b) $dL_x/d\beta = p^2x_{KL}K_x(y_{KK}y_{LL} - y_{KL}^2)/D$

(c) $dL_y/d\beta = py_{KL}K_x(x_{KK}x_{LL} - x_{KL}^2)/D$

(d) $dr/d\beta = -p^2K_x(x_{KK}x_{LL} - x_{KL}^2)(y_{KK}y_{LL} - y_{KL}^2)/D$

From our assumptions concerning the production functions, $dL_x/d\beta$ and $dL_y/d\beta$ are positive, whereas $dK_x/d\beta$ and $dr/d\beta$ are negative. From these results, we can conclude that $dx/d\beta$ and $dy/d\beta$ are positive.[4] Immediately, the effect of capital–saving technical progress upon the total income (output) can be determined.

$$dQ/d\beta = dx/d\beta + pdy/d\beta \tag{11}$$

Where Q is the total income in the economy.

Because $dx/d\beta$ and $dy/d\beta$ are positive, $dQ/d\beta$ is positive. In addition, if we consider the effect of this type of technical progress on the employment (or unemployment), we obtain the following result.

$$dL/d\beta = dL_x/d\beta + dL_y/d\beta \tag{12}$$

Since from (8) $dL/d\beta = dL_x/d\beta + dL_y/d\beta$, $dL/d\beta$ is positive as $dL_x/d\beta$ and $dL_y/d\beta$ are positive. Accordingly, $dU/d\beta = -dL/d\beta$. So, $dU/d\beta$ is negative.

(2) The case of labor–saving technical progress ($d\lambda > 0$, $d\beta = 0$)

(e) $dK_x/d\lambda = py_{LL}(-x_{LL}L_xx_{KL} + x_{LL}L_xx_{KL})/D = 0$

(f) $dL_x/d\lambda = [p^2L_xx_{LL}(y_{KK}y_{LL} - y_{KL}^2) + pL_xy_{LL}(x_{KK}x_{LL} - x_{KL}^2)]/D$

(g) $dL_y/d\lambda = py_{KL}(x_{LL}L_xx_{LL} - x_{LL}L_xx_{KL})/D = 0$

(h) $dr/d\lambda = p^2(y_{KK}y_{LL} - y_{KL}^2)(x_{LL}L_xx_{KL} - x_{LL}L_xx_{KL})/D = 0$

Using our assumptions about the production functions we get $dK_x/d\lambda$, $dL_y/d\lambda$ and $dr/d\lambda$ equal to zero while $dL_x/d\lambda$ is negative. Naturally, $dx/d\lambda$ and

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[4](1) $dx/d\beta = \underset{(-)}{x_K dK_x/d\beta} + \underset{(+)}{x_K K_x} + \underset{(+)}{x_L dL_x/d\beta}$

At first galance, it seems to be indeterminate. However, if we substitute for $dK_x/d\beta$ and $dL_x/d\beta$ and rearrange it, then we obtain the following result.

$$\begin{aligned} dx/d\beta &= x_Kpy_{LL}K_x(x_{KK}x_{LL} - x_{KL}^2)/D + x_KK_x + x_Lp^2x_{KL}K_x(y_{KK}y_{LL} - y_{KL}^2)/D \\ &= p^2K_x(y_{KK}y_{LL} - y_{KL}^2)(x_Lx_{KL} - x_Kx_{LL})/D > 0 \end{aligned}$$

(2) $dy/d\beta = y_KdK_y/d\beta + y_LdL_y/d\beta = -y_KdK_x/d\beta + y_LdL_y/d\beta$ as $dK_y = -dK_x$. Therefore, $dy/d\beta > 0$

$dy/d\lambda$ are also zero.[5] Therefore, $dQ/d\lambda$ is zero. Now, $dL/d\lambda = dL_x/d\lambda + dL_y/d\lambda$. $dL/d\lambda$ is negative, for $dL_x/d\lambda$ is negative while $dL_y/d\lambda$ is zero. So, $dU/d\lambda$ is positive.

(3) The case of neutral technical progress ($d\lambda = dt = d\beta > 0$)

(i) $dK_x/dt = py_{LL}K_x(x_{KK}x_{LL} - x_{KL}^2)/D$

(j) $dLx/dt = [p^2(x_{LK}K_x + x_{LL}L_x)(y_{KK}y_{LL} - y_{KL}^2) + pL_xy_{LL}(x_{KK}x_{LL} - x_{KL}^2)]/D$

(k) $dL_y/dt = py_{LK}K_x(x_{KK}x_{LL} - x_{KK}^2)/D$

(l) $dr/dt = -p^2K_x(x_{KK}xx_{LL} - x_{KL}^2)(y_{KK}y_{LL} - y_{KL}^2)/D$

In a similar way, we can find that dK_x/dt and dr/dt are negative while dL_y/dt is positive and dL_x/dt is indeterminate. Accordingly, dx/dt and dy/dt are positive.[6] However, dL/dt (or dU/dt) is indeterminate because dL_x/dt is indeterminate.[7]

IV. PROPOSITONS AND COMPARISONS WITH OTHER MODELS

Using the above results, we establish the following propositions.

PROPOSITION 1 In an unemployment economy, a capital-saving technical

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[5] $$\begin{aligned} dx/d\lambda &= x_K dK_x/d\lambda + x_L dL_x/d\lambda + x_L L_x \\ &= x_K\cdot 0 + x_L[p^2x_{LL}L_x(y_{KK}y_{LL} - y_{KL}^2) + py_{LL}L_x(x_{KK}x_{LL} - x_{KL}^2)]/D \\ &\quad -x_L L_x[p^2x_{LL}(y_{KK}y_{LL} - y_{KL}^2) + py_{LL}(x_{KK}x_{LL} - x_{KL}^2)]/D \\ &= x_L L_x[p^2x_{LL}(y_{KK}y_{LL} - y_{KL}^2) + py_{LL}(x_{KK}x_{LL} - x_{KL}^2)]/D \\ &\quad -x_L L_x[p^2x_{LL}(y_{KK}y_{LL} - y_{KL}^2) + py_{LL}(x_{KK}x_{LL} - x_{KL}^2)]/D = 0 \end{aligned}$$

[6] $$\begin{aligned} dx/dt &= x_K dK_x/dt + x_L dL_x/dt + (x_K K_x + x_L L_x) \\ &= px_K K_x y_{LL}(x_{KK}x_{LL} - x_{KL}^2)/D + p^2x_L(x_{KL}K_x + x_{LL}L_x)(y_{KK}y_{LL} - y_{KL}^2)/D \\ &\quad + px_L L_x y_{LL}(x_{KK}x_{LL} - x_{LL}^2)/D + (x_K K_x + x_L L_x)[-x_{LL}p^2(y_{LL}y_{KK} - y_{KL}^2) \\ &\quad -py_{LL}(x_{KK}x_{LL} - x_{KL}^2)]/D \\ &= p^2K_x(y_{KK}y_{LL} - y_{KL}^2)(x_L x_{KL} - x_K x_{LL})/D > 0 \end{aligned}$$

$$dy/dt = y_K dK_y/dt + y_L dL_y/dt = \underset{(+)}{-y_K dK_x/dt} + \underset{(+)}{y_L dL_y/dt} > 0$$

[7] $$\begin{aligned} dL/dt &= dL_x/dt + dL_y/dt \\ &= p^2(x_{KL}K_x + x_{LL}L_x)(y_{KK}y_{LL} - y_{KL}^2)/D + py_{LL}L_x(x_{KK}x_{LL} - x_{KL}^2)/D \\ &\quad + py_{KL}K_x(x_{KK}x_{LL} - x_{KL}^2)/D \\ &= p(x_{KK}x_{LL} - x_{KL}^2)(y_{LL}L_x + y_{KL}L_x)/D + p^2(y_{KK}y_{LL} - y_{KL}^2)(x_{LL}L_x + x_{KL}L_x)/D \end{aligned}$$

Therefore, dL/dt is indeterminate, and dU/dt is indeterminate too.

progress leads the capital intensity of the sector in which technical improvement takes place, to decline, the rental on capital to decrease, and the outputs in both the sectors to increase. In particular, overall unemployment in the economy will decline as a consequence of such a capital-saving technical progress.

PROPOSITION 2 In an unemployment economy, a labor-saving technical progress leads the capital intensity of the sector in which technical progress takes place, to increase and that of the other sector in which it does not take place, not to change. It does not affect rent on capital, outputs in both the sectors and total income in the economy. Consequently, this type of technical progress increases unemployment.

PROPOSITION 3 Neutral technical progress can not explain about the capital intensities of both sectors because those are indeterminate. The effects upon rent, both outputs and the total income are the same as the case of capital-saving technical progress. However, the effect upon unemployment is uncertain under such a neutral technical progress.

From the above three propositions, we can present these results simply in Table 1.

Table 1. The Implications of Technical Improvement in Industry x

$\text{Type}_{\text{of.T.P}}$	Kx(Ky)	Lx	Ly	L(U)	r	X	Y	Q
Capital-Saving	↘ (↗)	↗	↗	↗(↘)	↘	↗	↗	↗
Labor-Saving	0(0)	↘	0	↘(↗)	0	0	0	0
Neutral	↘ (0)	?	↗	?(?)	↘	↗	↗	↗

Notes: An arrow pointing-up indicates that the variable indicated at the top of the column has increased as a result of that type of technical improvement. A downward pointing arrow indicates that the variable has decreased in value. Zero indicates that the variable has not change at all and a question mark represent that the direction of change can not be determined with our assumptions.

Next, we compare the results of our model with those of other models. Our model is different from Kemp's model in terms of assumptions. Kemp assumes linearly homogeneous and concave production functions with constant returns to scale and competitive system in product and factor markets. He analyzes the effects of three types of technological improvement upon the economy under these assumptions. According to Kemp's model, if the technical improvement is saving of a factor used intensively in the progressive sector, or if the technical advancement is neutral, then the rate of the real reward of the factor used intensively in the progressive sector will rise and the intensity of its use will fall in both sectors, the rate of real reward of the other factor will fall and the intensity of its use will rise in both sectors.

However, if the improvement is saving of the factor used intensively in the other sector (static factor), then the real rate of reward of that factor will fall and the intensity of its use in the other sector will rise, the intensity of its use in the progressive sector may change in either direction or may not change at all, and the real rate of reward of the other factor will rise. The effects of the technical progress upon the outputs are such that if $k_x > k_y$, (sector x is more capital intensive than sector y), capital–saving technical progress in sector x (or neutral technical progress) leads x to increase, while it leads y to decrease. But the effect of labor–saving technical progress is not clear. If $k_y > k_x$ labor–saving or neutral improvement in sector x leads x to increase but y to decrease. This result depends considerably on the capital intensities in both sectors. This point is clearly different from our model.

Besides Kemp, Hazari explored the implications of technical progress for factor intensities, factor prices and output levels for a single country with the same technic, on the assumption that the economy is characterized by an inter–sectoral wage differentials. In his model, technical progress exists for the wage differentials. This factor market imperfections have an important effect on the results. According to his model, the output of the sector in which technical progress occurs may fall as a consequence of a technical progress and such a technical progress in the framework of factor market distortions may lead to "the immiserizing growth". Other results are similar to ours and Kemp's on the whole. The important point is that if we intend to compare with other models, we should examine corresponding assumptions at first, and then compare the results. There are some other models dealing with the technical progress with different assumptions. However, we refrain from considering them to avoid complications.

V. CONCLUSIONS

Using a two–sector, two–factor model of a small open economy with unemployment caused by the rigid wage, if technical progress takes place in one of the sectors, we derived the following conclusions.

(1) In the presence of unemployment in a small open economy, the capital–saving technical progress decreases the capital intensity of the sector in which the technical progress occurs while it is indeterminate for the capital intensity of the other sector. Under the labor–saving technical progress, the capital intensity of the sector in which the technical progress takes place, rises while that of the other sector remains unchanged. The impact of neutral technical progress is uncertain.

(2) When there is unemployment in a small open economy, capital–saving technical improvement in any sector increases employment (drecreases unemploy-

ment) in both the sectors and decreases the real rent on capital. Although the impact of neutral technical progress is uncertain in total employment, it decreases the real rent on capital. However labor–saving technical progress decreases the overall employment because it decreases the labor use of the sector in which technical progress is made but it does not affect the other sector.

(3) Capital–saving and neutral technical progress unambiguously increase the outputs in both the sectors. If the technical progress is labor–saving, both outputs remain unchanged.

(4) Accordingly, capital–saving or neutral technical progress increases the real income (output) in a small open economy. However, the labor–saving technical progress does not affect the real income.

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