THE KOREAN ECONOMIC REVIEW
Volume 6, Number 1, Summer 1990.

A REEXAMINATION OF THE RELATIONSHIP BETWEEN BERGSON AND ARROW SOCIAL WELFARE FUNCTIONS

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I. INTRODUCTION

It has been argued that the basic difference between Bergson and Arrow social welfare functions(SWF) is that the Bergson SWF deals with only one fixed set of individual preference orderings, whereas and Arrow SWF is a rule which associates a social preference ordering with every possible configuration of individual preference orderings. There seems to be a wide concensus on this.[1] Since some axioms in Arrow's formulation are concerned with how *changes* in individual orderings affect social orderings. The Bergson SWF can be considered immune to the devastating consequence of Arrow's impossibility theorem. Disputes between the two approaches seem to have been settled, even though there still remains a question about the relevance of Arrow's approach to traditional welfare economics.[2]

But recently, new results which have let to a fresh look at the relationship between the two approaches dubbed single-profile and multi-profile approaches, respectively, have been obtained by Parks(1976) and Kemp and Ng(1976) independently and further elaborated by Hammond(1976), Pollak(1979), and Roberts(1980a) among others. Their finding can be summarized as a proposition that there is no Bergson SWF which satisfies a neutrality condition[3] and other essentially technical conditions. But these conditions are closely related with their multi-profile counterparts. For example, the neutrality condition can be derived from the condition of independence of irrelevant alternatives combined with Pareto condition in the multi-profile approach, and plays an essential role in establishing an impossibility theorem.[4] An implication of this result is that the single-profile and multi-profile approaches share essentially the same conditions. It is interesting to note that Samuelson(1977) dismisses this result as totally unacceptable because the neutrality condition is "anything but reasonable", whereas Arrow considers the result illuminating.

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[1] See Little(1952), Samuelson(1967), and Arrow(1981) for typical views on this.

[2] See Arrow(1981).

[3] Formal definition will be given in a later section.

[4] Precise statement and proof of this can be found in Sen(1977) pp. 240-3.

Another recent development in social choice theory which has important implications for the relationship between the two approaches is various axiomatizations of utilitarianism and Rawls' maximin principle, introducing the possibility of cardinal measurement and interpersonal comparisons of utility [Hammond(1976), Strasnick(1976), d'Aspremont and Gevers(1977), Maskin(1978), and Roberts(1980a, b)]. Sen(1977) presented a survey of this new approach which he called informational analysis. In this approach, utility is considered as given independently of value judgements. For Sen, utility is just one of the various factors which must be taken into account in social decision making. Therefore it is understandable for Sen(1979) to criticize the so-called welfarism which results from quite reasonable assumptions of this analysis, because it rules out from consideration other values such as rights, fairness, and etc...

In this paper we want to generalise the framework of informational analysis so that it can be a foundation of social welfare analysis beyond the narrow perspective of welfarism. Having this in mind, we propose a new interpretation of the Bergson SWF quite different from the traditional view. Essentially we interpret the Bergson SWF as a real-valued representation of the Arrow SWF. Under this interpretation, the distinction between the single and multi-profile approaches becomes unimportant. We argue that this perspective provides a coherent conceptual framework for social welfare analysis, maintaining the ordinalist spirit of Arrow. Finally it is noted that our major contribution is in finding a new perspective, not in proving new technical results; most of the technical results are adapted from those of the informational analysis.

This paper is organized as follows. In section II, a new interpretation of the Bergson SWF is presented. Section III sets up a formal model and proves the main results which include an impossibility theorem without the neutrality assumption. Section IV concludes the paper.

II. AN INTERPRETATION OF THE BERGSON SWF

The basic idea is that there are two different ways to represent a functional relationship: one is to specify values at every point in domain and the other is to give a formula summarizing the relationship at a typical point. For example, we have a mapping from (1, 2, 3) to (2, 4, 6) which maps 1 to 2, 2 to 4, 3 to 6. This mapping can also be represented by the formula $y = 2x$ or by the explanation that if one has 2, multiply it by 2 and one gets the value 4 corresponding to 2.

The same idea can be applied to social welfare functions which represent various moral principles or group decision rules,[5] mapping from individual preferences

[5] The concept of social welfare function in our formal framework is as formal as Arrow's definition. Therefore it can be interpreted as an ethical judgement or as a group decision rule. But the distinction becomes important when we evaluate normative criteria of desirable SWFs.

to a social preference. By a moral principle we mean a systematic account of ethical value judgments, e. g., utilitarianism.

To fix ideas, let us consider a simple situation where there are two individuals, A and B, and two social states, x and y. An Arrow SWF assigns a social preference to each and every combination of individual preferences between x and y. An example of a SWF is the following: $x >_A y$, $y >_B x$, $x > y$; $x >_A y$, $x >_B y$, $x > y$; $y >_A x$, $x >_B y$, $y > x$; $y >_A x$, $y >_B x$, $y > x$.[6] A problem with this representation is that the moral principle behind the rule is not clear. In the above example, it is not clear whether individual A is a dictator or it happens that he is given a priority for some reason, e. g., because he is poor, and therefore giving him one dollar is morally good.

Another approach is to describe explictly the moral principle applied to a particular profile of individual preferences. For example, when $x >_A y$ and $y >_B x$, the social preference is $x > y$, because A is poor and giving a dollar to A (x) is socially better than giving it to B (y). This principle is universally applicable, and it needs not be repeated for every possible profile of individual preferences. Now the problem is how to represent the verbal descriptions of moral principles in an abstract framework. An elegant solution is the idea of real valued representation of an Arrow SWF. A moral principle is translated into a method of assignment of real numbers to alternatives and a functional form which aggregates these numbers. In the above example, one such representation is that we assign 10 to x and 1 to y for individual A, and 2 to y and 1 to x for B, and add the numbers assigned to each alternative such that society prefers the alternative which yields a greater value. Here x gets 11 and y, 3. Therefore x is socially preferred to y. Another possible representation is that the numbers assigned to x and y for individuals A and B are 5, 1, 2, and 3, respectively. Maximizing the minimum of the two numbers assigned to each alternative, x is socially preferred to y since the minimum for x is 2 and the minimum for y is 1.

A real valued representation can convey all the information required to describe a moral principle in an abstract way and a Bergson SWF is indeed a real valued representation of a social preference ordering at a fixed set of individual preference orderings. This interpretation is different from the traditional view in Bergson(1938)'s original formulation, where the functional from of SWF and utility representation of individual preferences have no intrinsic meaning except that they are arbitrary real valued representations of preference pre-orders unique up to monotone transformations. If we change utility representations of individual preferences by nomotone transformations, then the functional form must be changed accordingly.[7] A serious difficulty with this formulation is that we cannot repre-

[6] Indifferences are ignored for simplicity. As for the notation, A implies "is preferred by A to", and implies "is preferred by the society to".

[7] Mathematical derivation of this change is found in Arrow(1981) pp. 176-7.

sent a moral principle or ethical judgments properly. For example, given that x $>_A$ y and y $>_B$ x, a SWF in Bergson's original formulation only tells us that x y *without any explanation*. There are too many explanations which support this SWF. To take a few examples, individual A is a dictator, or A has the right to make decision about his own private matter, and etc... We cannot distinguish among these in Bergson's framework. But one may argue that an economist as a scientist has no interest in distinguishing various ethical value judgments but only in their economic consequences. Bergson's original formulation surely can be justified in this way.

Nevertheless, we believe that our new interpretation improves upon Bergson's original formulation and is a superior method of representing a moral principle. Economists have indeed been very much interested in the search of proper ethical principles which extend Pareto optimality, and this search culminated in Arrow's pathbreaking work. Moreover, economists are now actively involved in interdisciplinary research including ethics, the theory of justice, decision theory, political theory, and game theory. Welfare economics must be able to provide a formal framework for 'value criticism' as well as the representation of various moral principles. Our new interpretation therefore entails a number of important consequences for the conceptual framework of social welfare analysis, which we now consider in turn.

First, even though only a single profile of individual preference pre-orders is *formally* taken into consideration in a Bergson SWF, in face all possible profiles are considered by the general application of the rule which is represented by a functional form and real number assignments to alternatives. In this interpretation the Bergson SWF can be classified as a multi-profile approach as well and is susceptible to Arrow's impossibility theorem. This fact has a very important implication for the debate between Bergson and Arrow about the relevance of Arrow's work on traditional welfare economics. More discussion on this will appear later.

Second, since the functional form of SWF conveys information about moral principles, it must be invariant over all possible profiles of individual preference ordering and under all permissible transformation of real numbers assigned to alternatives. This fact will be restated formally in the next section as the Invariance Axiom, the consequences of which are fully discussed in that section also. Note Invariance Axiom is a purely formal requirement and does not contain any value judgements. Nevertheless the axiom is shown to be powerful enough to precipitate and impossibility theorem.[8] It turns out that the Invariance Axiom is closely related to the neutrality condition assumed in proving the "Bergson-Samuelson Impossibility Theorem"[9] and in recent axiomatization of utilitarianism mentioned in the Introduction. In fact one of the purposes of this paper is to show that many important

[8] See theorem 2 in Section III.

[9] For this terminology, see Sen(1977) p.251.

results in the recent developments in social choice theory just mentioned can be obtained from the formal requirements of a proper conceptual formal framework for social welfare analysis, without assuming any substantial value judgements such as the neutrality condition. We hope to provide that framework for social welfare analysis by refining Bergson welfare economics.

Third, real numbers assigned to alternatives could be interpreted as utilities. But this is not the only possible interpretation nor legitimate one in our formal framework. A rule assigning numbers to alternatives is part of a moral principle, as is a functional form which aggregates the numbers. Mathematically speaking, a real valued represenation of the underlying moral principle based on the observation of individual preferences and an associated social preference is a problem of conjoint measurement. Therefore analogy with expected utility theory is obvious. Our position is quite similar to that of ordinalists in expected utility theory in that the numbers assigned to alternatives are not interpreted as a measure of intensity of utility, but as a measure of risk aversion in risky situations.[10] This point has been ignored and a lot of confusion arose in the literature of welfare economics. For example Sen's criticism on welfarism of traditional welfare economics can be avoided in our framework, since the numbers attached to alternatives are not utilities in essence.[11] More importantly, the recent axiomatizations of utilitarianism and Rawls' maximin principle based their theory on the possibility of various kinds of measurability and comparability of utilities. But a correct conceptual framework can be developed without assuming any possibility of measurement and interpersonal comparison of utility. Arrow's reluctance to any interpersonal comparison is also, we believe, based on confusion about this matter. Bergson is correct when he says that "the criterion must be ethical in character. This does not by itself rule out empirical comparability, but it means that even with this supposition one must establish why the criterion is ethically compelling."[12] In other words, whether the data about interpersonal comparison of utility should be used as a criterion, even if it is empirically feasible, is an ethical question in nature. But if interpersonal comparison of utility is not made, some other form of interpersonal comparison based on other ethically relevant empirical information must necessarily be made in any ethical value judgement.[13] We have to distinguish between interpersonal comparison as a formal requriement of ethical

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[10] There are two different interpretations of utility numbers in expected utility theory, i. e., cardinalist and ordinalist views. Cardinalists including Harsanyi(1955) argue that the numbers measure the intensity of satisfaction and this measure is meaningful in other applications than decision making under uncertainty. Pure ordinalist position which we believe Arrow(1963) takes on, denies this application outside uncertainty situation, not to speak of application to the ethical problems.

[11] See Sen(1979) for Sen's criticism on welfarism.

[12] Bergson(1954) pp. 250-1.

[13] Theorem 2 in Section III conforms this. More discussion about interpersonal comparison of utility will appear in a later section.

value judgement and empirical data, based on which it is made. In our interpretation, numbers assigned to alternatives represent the former and utilities independently measured from empirical evidence are included in the latter category.

III. FORMAL FRAMEWORK

We start with the distinction between a formal framework and substantive theories. A formal framework concerns itself only with formal characteristics and logical consistency of ethical judgements, whereas systematic evaluations of them are in the domain of substantive theory. This distinction seems arbitrary at first sight. For example, Pareto condition is an ethical value judgement and may not be regarded as a formal requirement. But what make this relevant is to construct a conceptual framework or scenario in which the meaning of any particular assumption becomes clear. Bergson(1938) was undoubtedly concerned with a formal framework. Our goal is to provide a more consistent formal framework for social welfare analysis by reexamining Bergson's formulation. This will help clarify the relationship between Bergson's and Arrow's approaches.

The nautre of the problem can be illustrated by the example introduced in the previous section, i. e., a society with two individuals(A, B) and two alternatives(x, y). Suppose both A and B prefer x to y. There is no conflict of interest and they both agree that the society choose x over y. When there is unanimity, the problem of social decision becomes trivial. In this sense, the Pareto condition can be considered as a formal requirement rather than as a substantive value judgement which should be justified by a moral argument.[14] It is only when there exists a conflict of interest that a moral principle or a decision rule becomes necessary. Specifically, when individuals have conflicting preferences, a moral principle determines the priority or relative importance of preferences. Therefore a moral principle can be defined as a consistent rule assigning the priority or relative importance to preferences over any pair of alternatives. A consistent rule is required to guarantee the transitivity of social preferences.

With this in mind, we now can introduce a formal model.[15] Consider a society consisting of a set of individuals $N = \{1, \dots, n\}$ where n is assumed to be finite. Individual i has a preference ordering Ri which is a complete, reflexive, and transitive binary relation defined over the set X of social states. The cardinality of X is assumed to be *at least two and finite.*[16] The set of all preference orderings

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[14] sen(1970, 1976) questions the validity of Pareto condition by examining the situation where Pareto condition and liberty principle are in conflict. But we think that Pareto condition can be shown to be robust by logically consistent formulation of concepts of individual preferences and social alternatives. A full discussion of this problem will take another paper.

[15] It is formal in the sense of abstract mathematical symbolism.

[16] Since X is finite, we don't need any topological assumptions to guarantee the existence of a real valued representation of a pre-order.

over X is denoted by D.·A *profile* is a specification of individual preference orderings $(R_i)=(R_1,\ldots, R_i,\ldots R_n)$ which is an element of D^n. xRy (xPy) implies that x is preferred or at least indifferent(strictly preferred) to y in the ordering R. An *Arrow social welfare function* f (ASWF) is a mapping from D^n to D.

AXIOM P (Pareto). Suppose an ASWF maps (R_i) into R. For any x, y$\in$X, xRy if for all i, xR_iy. If xR_iy for all i and xP_jy for some j, then xPy.

THEOREM 1. Suppose an ASWF f satisfies Axiom P. Then there exist a continuous real valued function W over R^n and for each profile(R_i), real valued functions $u=(u_1,\ldots, u_n)$ over X such that for all i, $u_i(x) \geqslant u_i(y)$ iff xR_iy, and $W(u(x))\geqslant W(u(y))$ iff xRy, where $R=f(R_i)$.

PROOF. Since X is finite, the proof is elementary. Let Rij denote the preference ordering of ith individual in jth profile (j$\in$k, where K is the finite set of all preference profiles). Then there exist real valued functions u_{ij} on X such that $u_{ij}(x)\leqslant u_{ij}(y)$iff $xR_{ij}y$, and for all x,y$\in$X and j$\neq$k$\in$K, $(u_{1j}(x),\ldots, u_{nj}(x)) \neq (u_{1k}(y)\ldots, u_{nk}(y))$. Let $M=\{(u_{1j}(x)\ldots u_{nk}(x))\in R^n|j\in K, x\in K\}$. A social preference ordering associated with each profile induces a partial order on M, which can be represented by a real valued function W. W is extended continuously over R^n. Notice that in this representation, Axiom P is used because if x and y are indifferent for every individual, utility measure of x and y should be equal and therefore x and y should be indifferent for the society also. (Therefore weak version of Pareto axiom is sufficient to prove the theorem: xI_iy for all i implies xIy.)

Theorem 1 states that for any observed relationship between individual preference orderings and social ordering an underlying moral principle or decision rule can always be represented by a real valued function. This fact justifies the view expounded in section II. Now that we have a functional form W and a rule u assigning numbers to social states, we can concentrate on a fixed profile. The functional form W obtained in Theorem 1 is called *Bergson social welfare function*(BSWF). Hereafter, we fix a typical profile $\{R_1,\ldots R_n\}$ in D^n such that for at least one pair of individuals i and j, $R_i\neq R_j$. This is a minimal requirement of diversity of preferences. And we consider a BSWF W on it.

It is worth nothing that real numbers assigned to alternatives in a representation of a moral principle are not utilities which measure the intensity of preferences, as explained in section II. however, as long as there is no confusion, we will call the numbers utility measures, as it is conventional.

From the fact that a BSWF W and utility measure u are simultaneously determined in Theorem 1 and covariant in the sense that if one is to be changed the other must be adjusted accordingly, we can infer that a rule of changes between the two is needed. Suppose we come up with two utility measures which are considered equivalent, i. e., contain the same amount of ethically relevant information. Then to be consistent and to make a functional form W convey a moral principle, the functional form should be the same for the two equivalent utility measures. Or suppose we have an ethically compelling utility measure in, say, an interval scale. In other words, origin and unit of measurement are arbitrary. Then from our framework, the functional form of W should be the same for any choice of origin and unit of measurement. Therefore the next axiom naturally follows from the consistency requirement of our formal framework.

AXIOM I (Invariance). A BSWF W is invariant under a permissible transformation of utility measure, which defines equivalent classes of utility measures. In other words, if u and u′ are in the same equivalent class, then $W(u(x)) \geqslant W(u(y))$ implies $W(u'(x)) \geqslant W(u'(y))$ for all $x, y \in X$.

We now introduce another axiom which formalizes the impartiality requirement of a moral principle.

AXIOM S (Symmetry). A BSWF W is symmetric, i. e., for any permutation σ, $W(u_1,\dots u_n) = W(u_{\sigma(1)}\dots u_{\sigma(n)})$.

In our interpretation of utility measures, all the ethically relevant differences between individuals are already reflected in utility numbers. Therefore W must be symmetric. The axiom can also capture the formal sapect of impartiality of fairness which is common to all moral principles. Hare(1952) showed very persuasively that impartiality is a formal rather than a substantial requirement for a moral principle in the sense that it can be derived from a logical analysis of moral concepts. Hare introduced the concept of universalizability which essentially implies symmetry. What makes various moral principles different is the extent and scope of concrete applications of the formal concept of impartiality.

Now that we have described our formal model, we explore the implications of Axiom P, I, and S taken together. First, we ask ourselves the following question: Suppose we have an ethically relevant utility measure. Then what kinds of functional form W are logically compatible with it[17]

THEOREM 2. Suppose a permissible transformation of a utility measure is any monotone transformation ϕ_i, different for each individual utility measure u_i. Then there is no BSWF W which satisfies Axiom P, I, and S.

PROOF. Easy proof can be obtained by using the result of Theorem 3. There we have $W(u) = \Sigma a_i u_i$. But if we take $\phi_i(u_i = \alpha_i u_i + \beta_i$, then $a_i > 0$ for at most one i, but this is a contradiction to the Axiom S.

This impossibility result is essentially the same as that of Parks(1976) and Kemp and Ng(1976) in the "Bergson-Samuelson Impossibility" literature and that of

[17] Formally, this is the same problem as that of empirical meaningfulness of a statement in the theory of measurement.

Roberts(1980b) in the "informational analysis" literature. But the main difference is that we do not use the neutrality assumption[18] which has very unreasonable implications as Samuelson(1977) has shown. Theorem 2 clearly shows that the "Bergson-Samuelson Impossibility" can be established in a formal framework without any substantial assumptions and strongly indicates that the original formulation of Bergson SWF should be reexamined.

Note that Theorem 2 holds even when $\#X=2$, i. e., when intransitivity of preferences cannot arise. In Arrow's framework, the impossibility theorem does not apply when $\#X=2$. For example, majority rule satisfies all the assumptions in Arrow's formulation with $\#X=2$. But here majority rule is not allowed even when $\#X=2$. This is because majority rule is formally a utilitarian rule with a fixed utility unit common to all individuals and preferences. More discussion about the majority rule will appear later. We think that Theorem 2 reveals more clearly the reason why an impossiblilty result obtains. Indeed, our claim that interpersonal comparison of some form must be made for any ethical value judgement, is confirmed by Theorem 2.

THEOREAM 3. Suppose permissible transformations ϕ_i of a utility measure are linear with a common coefficient $\alpha(>0)$ for every individual utility measure u_i, i.e., $\phi_i(u_i(x))=\alpha u_i(x)+\beta_i$. Then the only BSWF W which satisfies Axiom P, I, and S is additive, i. e., $W(u_i)=\Sigma_i$, more precisely, $W(u_i)=g(\Sigma u_i)$ for some increasing function g.

PROOF. First, there exist u and v ($u\neq v$) in R^n such that $W(u)=W(v)$. Next, by Axiom I, $W(\alpha u+z)=W(\alpha v+z)$ for any $\alpha \in R$ and $z\in R^n$. Let $z=(1-\alpha)v$. Then $W(\alpha u+(1-\alpha)v)=W(v)$. This, combined with Axiom P, implies $W(u)=g(\Sigma a_i u_i)$ for an increasing function g. By Axiom S, $W(u)=g(\Sigma u_i)$.

Theorem 3 implies that if we believe in a common good for the society, the only logically feasible form of social decision is to maximize the sum of the common good. This is a very strong case for utilitarianism. But again we have to point out that the real problem with utilitarianism is not the functional form itself but whether we can assume the existence of a common good which is also empirically measurable. This is not a concern of formal analysis but rather a problem of substantive moral theory. As an example of this theory, Harsanyi(1953, 1955) provides a very persuasive justification for utilitarianism, which is itself a substantive moral theory and concerns itself with the systematic examination of substantive value judgements.

Since Theorem 3 is formal in nature, majority rule when there are only two

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[18] Neutrality axiom in the ordinal framework requires that for any two pairs of social states(x, y) and (w, z), and for a given profile (Ri), if for all i, Ri(x, y) = Implication of this axiom is well explained in Samuelson(1977) pp.81-3.

social states can also be interpreted as a form of utilitarianism where the utility unit is the same for all individuals. Then it is easily seen that Theorem 3 implies May(1952)'s characterization of majority rule when $\#X=2$.

THEOREM 4. Suppose permissible transformations ϕ_i of a utility measure are linear with an absolute zero, i. e., $\phi_i(u_i(x))=\alpha_i u_i(x)$, $\alpha_i>0$. Then the only continuous BSWF W which satifies Axiom P, I, and S is multiplicative, i. e., $W(u_i)=\pi u_i$, more precisely, $W(u_i)=g(\pi u_i)$ for some increasing function g.

PROOF. It is enough to show that a continuous W which is invariant under the tranformations $\phi_i(v_i)=v_i+\alpha_i$ is additive. A slight extension of the Theorem 4.3.1 in p.118 of Blackwell and Girshick(1954) proves this.

Theorem 4 is a formal justification of Nash bargaining solution interpreted as a social welfare function. A difficulty in this interpretation is how to interpret the absolute zero point in a ratio scale measurement of a utility measure. One possibility is the status quo point. The merit of this formal representation is that utility units do not have to be comparable(α_i can be different for each i). The comparability requirement falls onto absolute zero points. There is no substantive moral theory based on this formal representation except the Nash bargaining solution as yet, but it seems quite plausible to construct a systematic account of a substantive moral principle based on it, since the contractarian tradition of moral theory is reluctant to assuming a common good and can be formalized in this framework. In this moral principle, our intuition of fairness can be represented in the following statement: equal percentage change in the strength of preferences are treated equally.

Now we change our viewpoint and ask ourselves if any given observation about a profile of individual preferences and a social preference associated with it can be rationalized as a result of a special moral principle, e. g., utilitarianism.

THEOREM 5. Suppose we observe a profile $\{R_1.....R_n\}$ and a social preference R such that axiom P is satisfied and for all i, R_i is an order i. e., if x and y are indifferent then $x=y$. Then there exist $u_1,\ldots,u_n$ such that u_i is a utility representation of R_i and for all i, xRy iff $\Sigma u_i(x)\geqslant\Sigma u_i(y)$.

PROOF. Elementary proof is possible by a constructive method.

Theorem 5 implies that when the number of social state is finite, the utilitarian representation is always possible without any substantial assumptions if indifferences are not allowed in individual preferences. This is another strong case for utilitarianism. But we note that in this representation, utility numbers are not unique up to a linear transformation.

A refined result is obtained with a little bit technical assumption called the finite cancellation axioms in Narens(1985).[19]

THEOREM 6. Suppose we observe a profile $\{R_1,...R_n\}$ and a social preference R such that R satisfies the finite cancellation axioms. Then there exist $u_1,..., u_n$ such that u_i is a utility representation of R_i and for all i, xRy iff $\Sigma u_i(x) \geqslant \Sigma u_i(y)$.

PROOF. Direct application of the Theorem 2. 3 of Narens(1985) p. 266 proves this.

If the set of social states X is not finite and has some topological structures, then Theorem 6 can be substantially improved by using results from Debreu(1959). The following result was first obtained by Fleming(1952).

AXIOM D (Independence). Consider a profile $\{R_1.....R_n\}$ and R and a partition I, J of N (#N$\geqslant$3). If for all $i \in I$, xI_ix' and yI_iy' and for all and $j \in J$, xI_jy and $x'I_jy'$, then $R(x,y) = R(x', y')$.

Independence axiom extends the idea of Pareto axiom such that any individual who is indifferent between x and y should not be given any priority in making decisions about x and y.

THEOREM 7. Suppose $R_1....R_n$, and R are continuous orderings on a connected space X and satisfy Axiom P and D. Suppose the set of alternatives X is a Cartesian product of commodity spaces. Then there exist $u_1...u_n$ such that u_i is a utility representation of R_i for all i, and xRy iff $\Sigma u_i(x) \geqslant \Sigma u_i(y)$. Moreover, u_i is unique up to a linear transformation with a common coefficient for all i, i.e., $\phi_i(u_i) = \alpha u_i + \beta_i$, $\alpha > 0$.

PROOF. Debreu(1959) gives a rigorous proof of this.

We can also prove that under the same conditions as in the Theorem 7 multiplicative representation is always possible.

IV. CONCLUSION

In this paper, we proposed a new interpretation of Bergson social welfare functions. With this new interpretation, many important results from recent developments in social choice theory were proved in a very formal framework without any substantial assumptions about ethical value judgements. In particular, the 'Bergson-Samuelson Impossibility' result was proved without the neutrality assumption, which Samuelson has rejected as being unreasonable. We consider the impossibility as an indication that original formulation of Bergson-Samuelson should be revised. A formal framework based on our new interpretation was outlin-

[19] See Narens(1985) pp. 262-3. It is too long to be presented here. The implication of this axiom is still unclear.

ed in section III. We have argued that this formal framework which is essentially a refined version of Bergson's original formulation is more general than Arrow's framework, in the sense that we extend an ordinalist position in social welfare analysis in a consistent manner without encountering any impossibility results.

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