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TEST FOR PARTIAL RATIONALITY

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We test for partial rationality of ASA-NBER surveys by examining whether participants optimally utilized the uncertainty which is one of important informations available at the time their forecasts were made. The ARCH-M model is employed in order to derive the conditional variances, which can be treated as a proxy of uncertainty in testing for partial rationality. It is shown that though direct data or price expectations reported in ASA-NBER Business Outlook surveys are largely free of bias, they are patially irrational and, of course, fully irrational relative to conventional symmetric loss function which is seen to be a special case of asymmetric loss functions. If the model for rationality test is specified well, a by-product is to obtain the optimal forecasts using the survey forecasts.

I. INTRODUCTION

Brown and Maital (1981) analyzed the logical relationships among the properties that characterize rational expectations as follows. Full rationality implies that all available information has been used in an optimal manner. The expectation f_t is said to be fully rational and is optimal in the sense that no other unbiased predictor has small variance, if

(1) $f_t = E\ (P_t/\phi_{t-1})$

where f_t is the predicted percentage change, p_t the actual percentage change, and ϕ_{t-1} the relevant information available at time t-1.

Suppose, now, that the prediction f_t is incomplete, in the sense that ϕ_{t-1} is not fully utilized. Let f_t be based on S_{t-1}, a proper subset of ϕ_{t-1}. Predictions make efficient use of this subset of information when

(2) $f_t = E\ (P_t/S_{t-1})$

This property, which we shall refer to as partial rationality, means that the information partially used-whether or not it is complete-is used efficiently. Partial rationality is a necessary condition for full rationality.

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Meanwhile, a weak test of partial rationality can be constructed without knowing S_{t-1}. We know that $f_t = f\ (S_{t-1})$, whereupon the condition for partial rationality implies

(3) $f_t = E\ (P_t/f_t)$

f_t is said to be an unbiased prediction of P_t if it possesses this property. Unbiasedness is thus a necessary condition for partial rationality.

Since it is assumed implicitly that the forecasts are fully rational, we estimate the regression models as if the error terms follow the appropriate moving average process, thesting for unbiasedness or partial rationality. On the other hand, for full rationlity tests the error term should be the k^{th}-order moving average for k-period ahead forecast if the model is well specified.

Let the forecast be not fully rational. It means that the forecastors utilize the past information incompletely and/or inefficiently. Then the forecast errors are likely to be correlated with the past information. Accordingly, it is conjectured that the error term and the past information may be correlated in the regression model for unbiasedness or the partial rationality test, while for full rationality, they should be uncorrelated under the good specification of model.

Meanwhile, since REH still implies uncorrelatedness of the error term with the forecast, the least square estimates for the test of unbiasedness or partial rationality would be consistent.[1] As long as we consider the consistency of coefficients only, it is not important whether the error terms follow the appropriate moving average process. However, we need precise analysis of the error terms for more efficient estimate of coefficients and for tests of REH.

The American Statistical Association and the National Bureau of Economic Research (ASA-NBER) survey is relatively rich and lengthly enough to be rigorously analyzed, and is promising and reliable in the sense that surveys are carried out in a consistent and scientific manner with particular attention being paid to the models and assumptions the participants employ. This has been largely ignored in the literature, especially in the empirical application studies, and is still surprisingly little known among economists and others who might benefit from the data.

Studies by Su and Su (1975), McNees (1981), Lahiri and Teigland (1982), and Zarnowitz (1985) have made intensive use of the ASA-NBER GNP and IPD (implicit price deflator) expectations. In this article, we seek to determine whether the IPD expectations of the ASA-NBER survey possess the property of partial rationality.

[1]The correlation between the error term and the forecast depend upon whether a past information is utilized efficiently or not. When forecasters are assumed to use a obtained information efficiently, $E(\varepsilon_t/f_t) = 0$. Hence, the least square estimates are consistent. Otherwise, the least square estimates will be inconsistent.

II. SOLUTIONS TO SERIAL CORRELATION

In a number of recently developed models, it was argued that the error term in question is itself uncorrelated and is uncorrelated with all current and past values of the instruments; this corresponds to the assumption that the instruments are predetermined, which is a standard textbook assumption, so standard textbook estimation methods apply.

It is possible that P_{t-1} might not be observed before f_t is formed. This happens when the sampling interval is finer than the interval over which forecasts are made. Thus the serial correlation arises from the fact that the realized values are not yet known when the forecasting is being constructed.

For the one-period (3 months) ahead forecast error of the ASA-NBER survey data, we can postulate a first-order moving average process if the one-period ahead forecasting is fully rational:

$$(4)\ \varepsilon_{t+1} = e_{t+1} + Ce_t$$

The rationale for this is as follows: the forecast for, say, April through June is conducted in May, at time when the most recent data available would be the preliminary estimates for the preceding quarter; since the prelimanary data are likely to be modified, we may interpret it as a true one-period ahead forecast. Then the last known forecast error is the one pertaining to October-December of the last year. Thus the forecast error of the preceding quarter, ε_t, is not available when the forecast is made. Since ε_t is not part of the available information, we cannot rule out the possibility that E $(\varepsilon_{t+1}/\varepsilon_t) \neq 0$. On the other hand, the forecast error ε_{t-k} for $k \geqslant 1$ is observable. Rationality thus requires E $(\varepsilon_{t+1}/\varepsilon_{t-k}) = 0$ for $k \geqslant 1$. Hence, we can conclude that the forecast error will be a first-order moving average.[2]

In general, for the k-period ahead forecasts, the corresponding forecast errors $\varepsilon_{t+k-s} = P_{t+k-s} - f_{t+k-s}$ for $s = 0,1,\ldots, k$ are not observable. Since $\varepsilon_t, \varepsilon_{t+1},\ldots, \varepsilon_{t+k}$ are not part of the available information, E $(\varepsilon_{t+k}/\varepsilon_{t+k-s}) \neq 0$ or that

$$(5)\ \text{cov}\ [\varepsilon_{t+k}, \varepsilon_{t+k-s}] \neq 10 \text{ for } s = 0,1,2,\ldots, k,$$

We suppose the two processes $\{P_{t+k}\}$ and $\{f_{t+k}\}$ are jointly stationary and ergodic. Then $\{\varepsilon_{t+k} = P_{t+k} - f_{t+k}\}$ will be covariance stationary, and we can write

$$(6)\ \text{cov}\ \{\varepsilon_{t+k}, \varepsilon_{t+k-s}\} = \begin{cases} \sigma^2 \lambda_s & \text{for } s = 0,1,2,\ \ldots,\ k \\ 0 & \text{for } s > k \end{cases}$$

[2]Under the assumption that the past information is known precisely at time t when the forecasts are made, if the prediction is optimal, the one-period ahead froecast error should be white noise, while for the k-period ahead froecast the error term should be MA(k-1).

The k-period ahead forecast error should follow the k^{th}-order moving average process.

In view of the serially correlated errors, it seems logical to base any inferences on generalized least squares procedures. Unfortunately, since the regressors in each case are not strictly exogenous, GLS is likely to yield inconsistent estimates. The reason is that in effect GLS transform the model to estimate the serial correlation in the residuals. But the transformed residuals for some particular period will be linear combinations of the original residuals with their lagged values. These in turn are likely to be correlated with the transformed data for the same period because these include values of the variables in the information set.

1. Previous Estimation Methods

Brown and Maital derive the asymptotic distribution for these estimators and show that they are efficient within a certain class. These estimates are called "finite-order efficient" (FOE). They suggest the following model for the correct inference.

The model may be represented more compactly as

$$(7)\ Y = X\beta + \varepsilon$$

where Y is the $n \times 1$ vector of all observations on y_t, X is the $n \times k$ matrix of all observations on the x's, and ε is the $n \times 1$ vector of distributions. Under the null hypothesis of full rationality,

$$(8)\ E\,(\varepsilon\varepsilon') = \Omega$$

will satisfy (6).

Hansen (1982) has shown the asymptotic properties of $\hat{\beta}$, the OLS estimator of β as follows:

$$(9)\ n^{1/2}\,(\hat{\beta} - \beta) \xrightarrow{a} N\,(0, V)$$

where $V = \text{plim}\ (X'X/n)^{-1}\,(X'\Omega X/n)\,(X'X/n)^{-1}$

Then under the null hypothesis that $\beta = \beta_0$, the chi-square statistic is like this:

$$(10)\ q = n\,(\hat{\beta} - \beta_0)'\,V^{-1}\,(\hat{\beta} - \beta_0) \xrightarrow{a} \chi^2_k$$

If the extremely large values of q are observed, the null hypothesis will be rejected.

Since Ω and hence V are not known, they use the consistent estimates, $\hat{V} = n\,(X'X)^{-1}\,X'\hat{\Omega}X\,(X'X)^{-1}$. Thus an asymptotically appropriate test statistic for the joint null hypothesis is

$$(11)\ \hat{q} = (\hat{\beta} - \beta_0)^{-1}X'X\,(X'\hat{\Omega}X)^{-1}\,X'X\,(\hat{\beta} - \beta_0)$$

Thus Hansen (1982) and Cumby, Huizinga and Obstfeld (1983) derive consistent estimators for models like these without a seperate, explicit "correction for serial correlation" in the estimation process.

2. Forward Filtered Method

Hayashi and Sims (1983) suggest the Forward Filtered (FF) Method in which we can derive the consistent and efficient estmates with a nearly ideal instrument vector. They show that the ideal instrument vector is the linear combination of current and lagged instrumental values and that, as the number of lagged values of the instruments used increases, the asymptotic covariance matrix of coefficient converges to the asymptotic covariance matrix of the optimal estimator. (This is the reason they call their estimator "nearly efficient").

The estimator proposed here uses the condition that the error term is uncorrelated with all past values of the instruments because the k^{th}-order moving average type in the error term should be handled by moving the instrument set forward in time.

We will describe fully the Forward Filtered Method developed by Hayashi and Sims, and the nearly ideal instrument vector below.

We consider an equation

$$(12)\ y_t = x_t\beta - \varepsilon_t$$

with the identifying assumption that ε_t is uncorrelated with current and lagged instruments and that the serial correlation in ε does not depend on Z, i. e.

$$(13)\ E(\varepsilon_t\varepsilon_s/z_t,\ z_{t-1},\ \ldots) = E(\varepsilon_t\ \varepsilon_s) \quad \text{for } t>s$$

Let Ω be the $T\times T$ matrix $E(\varepsilon\varepsilon')$. If W is chosen so that $W\Omega W' = \sigma^2 I$, then $W\varepsilon$ has a scalar covariance matrix. If we number the elements so their subscripts increase from top to bottom, a conventional correction for serial correlation by "filtering" corresponds to choosing W to be lower triangular. Each element of $v = W\varepsilon$ is a linear combination of current and past values of ε. In general, with such a choice of W each element of v will be correlated with current and past values of the instrument variables.

The idea of the forward-filtered estimation is simple: Choose W not to be lower triangular but to be upper triangular. It is well known that we can always choose W to be upper triangular while satisfying $W\Omega W' = \sigma^2 I$, and with such a choice of W, each element of v becomes a linear combination of current and future ε's, thus remaining orthogonal to current and past values of instrument variables Z.

It should be now clear how we will define the forward-filtered estimator. Let $Y^* = WY$ and $X^* = WX$. Then the forward-filtered estimator is

(14) $\beta_{FF} = (X^{*\prime}\ Z\ (Z')^{-1} Z'\ X^{*})^{-1}\ X^{*\prime}\ Z\ (Z'Z)^{-1}\ Z'Y^{*}$

Since Z is a vector of instruments which meets the conventional conditions for predeterminedness in the transformed equation

(15) $Y^{*} = X^{*}\beta + v$

there is no novelty in the asymptotic distribution theory for β_{FF} in the case where Ω is known a priori. The conventional theory tells us that the asymptotic covariance matrix of $T^{1/2}$ $(\beta_{FF}$-$\hat{\beta}$ is given by

(16) Plim $\alpha^{2}T\ (X^{*\prime}Z\ (Z'Z)^{-1}\ Z'X^{*})^{-1}$

In practice, we ordinarily do not know Ω, so instead of W we use an estimated matrix, call it Q, which satisfies $Q\hat{\Omega}Q' = \sigma^{2}I$ only approximately. In the conventional model of serial correlation, where one assumes v uncorrelated with current and past Z but allows contemporaneous correlation of Z with ε, estimates of β based on estimates of W have a different asymptotic distribution from that of estimates based on the true W. But as long as the estimates Q are always strictly upper triangular, the asymptotic distribution is the same whether X* and Y* are formed using W or Q, under reasonable regularity conditions.[3]

Estimates of coefficients (GLS estimates) based on the true variance-covariance matrix is definitely superior(on the best linear unbiasedness criterion) to OLS estimates in the general linear regression(GLR) model. But GLS estimates based on the estimated variance-covariance matrix might not be. Monte Carlo results reported in Griliches and Rao(1969), for example, indicate that for sample size 20, when the estimated first-order autocorrelation coefficient of the residuals is less than one-third in absolute value, OLS estimates are more efficient than GLS estimates. Thus the nonsphericalness of the error term must be quite severe to make GLS estimates superior to OLS estimates.

Specifically, in the FF method there are some more reasons that β_{OLS} may be more efficient than β_{FF}. Since the explanatory variable could not be strictly exogeneous and GLS is likely to yield inconsistent estimates, we nedd the instrument variable vector which is uncorrelated in the limit with the transformed error term, but which is highly correlated with the transformed explanatory variables. The real difficulty in practice, of course, is actually finding this ideal instrument variable vector. With only a small correlation between the transformed explanatory variables and the instrumental variable, the sampling variances of the instrumental variable estimator may be unduly large and so we may be paying a very high price for consistency.

[3]See Hayashi and Sims(1983) for the proof.

Second, in our model the error term follows the appropriate moving average process under the assumption of full rationality. If the presence of underutilized information leads us to reject the hypothesis of full rational expectations, the error term may not be MA(K), but ARMA(P,Q). We know that if the error term was not completely filtered, the partially filtered error term could be still serially correlated. Then unfiltered estimator (β_{OLS}) may be better than the partially filtered estimator (β_{FF}) based upon the Forward Filter method with respect to efficiency.

In summary, the necessary conditions for β_{FF} to be superior to β_{OLS} are like this:(1) Estimates of W are always strictly upper triangular;(2) The instrument variable vector is uncorrelated in the limit with the transformed error term but fairly correlated with the transformed explanatory variables;(3) The error term is completely filtered. We, in practice, are not likely to satisfy these conditions. Specifically, it is difficult to find the ideal instrument variable vector.

-An Example

Consider the one-period ahead forecast error under the assumption of the full rationality. If, knowing C, we filtered P_t and f_t through $(1+CL)^{-1}$, the resulting filtered version would have a serially uncorrelated error, $(1+CL)^{-1}\varepsilon_t = e_t$. However, while the original (12) had error ε_t which involved only e_t and e_{t-1}, the filtered error involves ε_{t-s} for all non-negative S, in general. We will find that P_{t-s} for $s \geqslant 2$ which were uncorrelated with ε_t, are correlated with the filtered error e_t. Since f_t is a linear combination of these lagged P's and other variables, it too will be correlated with e_t. The usual procedure of transforming the data to eliminate serial correlation to obtain exact or approximate GLS estimates therefore produces inconsistent estimates in this case.

Since the backward filter $(1+CL)^{-1}$ eliminates serial correlation in ε, the forward filter $(1+CL^{-1})^{-1}$ will also eliminate serial correlation. We cannot apply OLS to the filtered equation because the filtered f_t may be correlated with ε_t and hence with

$$(17)\ m_t = (1+CL^{-1})^{-1}\varepsilon_t$$

But note that $(1+CL^{-1})^{-1}$ will be an exponentially weighted sum of non-positive powers of L. Thus m_t depends only on ε_{t+s} for non-negative s. Since ε_{t+s} for non-negative s is by assumption uncorrelated with f_t, the unfiltered forecast, f_t, is uncorrelated with m_t and is eligible for use as an instrumental variable.

The equation (12) can be reformulated as follows

$$(18)\ (1+CL^{-1})^{-1}P_t = \beta_0(1+CL^{-1})^{-1} + \beta_1 f_t + m_t$$

The resulting estimator is in the class we call FF estimates. The procedure, applying OLS to (12) without a correction for serial correlation, but then correcting the estimated covariance matrix of the parameter estimates for the effects of serial correlation, it is in the class we call FOE. It has been suggested by Hansen (1982), and Cumby, Huizinga, and Obtsfeld(1983).

Hayashi and Sims compared the relative asymptotic efficiencies of β_{FF} and β_{FOE} by considering whether Z or $W^{-1\prime}Z$ is likely to form a better set of instrument variables in the forward-filtered equation. They argued that β_{FF} is asymptotically more efficient than β_{FOE} under certain conditions. If a precise model of the determination of instrument variables is available, it should be clear that we can ordinarily improve on both β_{FF} and β_{FOE}.

Since any linear combination of current and lagged values of f_t is uncorrelated with all current and future values of ε, any such variable is an eligible instrument variable. The ideal instrument vector, then, is that linear combination of current and lagged values of f_t which has maximum correlation with $(1+CL^{-1})^{-1}f_t$. The estimator β_{FF} with this ideal choice of f_t is exactly what Hansen and Sargent (1982) have shown to be the optimal estimator based on the uncorrelatedness of ε_t and current and past f_t.

Consider the following equation:

$$(19)\quad \hat{f}_{t+s} = \alpha_0 + \alpha_1\hat{f}_{t+s-1} + \alpha_2\hat{f}_{t+s-2} + \ \dots\ + \ \alpha_p\ \hat{f}_{t+s-p} \text{ for } s \geqslant 1$$

$\hat{f}_{t+s}$ may be used as the "Nearly Efficient" instrument variable of f_{t+s} for $s \geqslant 1$. Equation (18) can be reformulated using the nearly efficient instrumental variables as follows

$$(20)\quad (1+CL^{-1})^{-1}P_t = \beta_0(1+CL^{-1})^{-1} + \beta_1\ (f_t - C\hat{f}_{t+1} + C^2\hat{f}_{t+2} - \ \dots\ + \ C^n\hat{f}_{t+n}) + m^*_t$$

Since we do not know C, first of all we have to estimate it. In the case of moving average models, we cannot write down the error sum of squares Σm_t^2 as only a function of the observed P_t. What we can do is to write down the covariance matrix of the error term in equation (12) and assuming normality, use the maximum likelihood method of estimation.

An alternative procedure suggested by Box and Jenkins (1976) is the grid search procedure. In this procedure we compute e_t by successive substitution for each value of C given some initial values and $\varepsilon_0 = 0$. We then have

$$(21)\qquad m_1 = P_1 - \mu$$

$$\vdots$$

$$m_t = P_t - \mu - Cm_{t-1}$$

Thus successive values of m_t can be generated and Σm_t^2 can be computed for each value of C. This grid search is computed over the admissible range of values for C $(-1 \leq C \leq 1)$ and the value that minimizes Σm_t^2 is chosen.

But since μ is not given in this case, we cannot apply the Box and Jenkins method directly. Then we regress the equation (20) by OLS, compute the residuals (m_t) by successive substitution for each value C and select the value that minimizes Σm_t^2.

III. TEST FOR PARTIAL RATIONALITY

In some theoretical works, notably Keynes' General Theory, uncertainty is of central importance to economic behavior. In quantitative models, much less importance has traditionally been placed on such effects. Cast in terms of the moments of the probability distributions generating the data, most attention has been paid to the factors influencing the first moment, with only minimal regard for second and higher-order moments.

Even in those instances where higher-order moments enter the analysis, they are generally taken as constant, e.g., as in the CAPM (capital asset pricing models) and portfolio models, and so become absorbed into the slope and intercept terms of a regression.

There now exist a fair number of studies attempting to allow for a changing uncertainty in economic models; a very small sample will include Venderhoff(1983), Mascaro and Meltzer (1983), Lawrence(1983), Engle (1982) and Gylfasson(1981).

For one application, Pagan and Ullah(1986) investigate whether the assumption of a zero-risk premium is consistent with the data from the foreign exchange markets. The data for the model constitute the log of the spot rate (S_t) and the log of the 30-day forward rate, W_{t-1}. Thus they test the Canadian-US exchange market efficiency over the period 1970:7 to 1982: 2 using the following equation:

$$(22)\ S_t = \alpha + \beta W_{t-1} + \delta\, \sigma_t^2$$

Coulson and Robins(1985) considered the impact of variability in the inflation rate upon aggregate economic activity over the period 1951:1 to 1979:4. Estimates of the inflation surprises and risk measure, σ_t^2, were taken from Engle(1983), and Y_t was regressed against $\overline{X}_t$ and $\hat{\sigma}_t^2$ where Y_t is the civilian unemployment rate and $\overline{X}_t$ the lagged values of Y_t.

Engle, Lillien and Robins(1987) propose a model of the term structure that incorporates a risk-premium. In this case, the dependent variable is the excess holding yield on three-and six-month Treasury Bills, while the explanatory variables are the yield spread between three-and six-month bills and the risk-premium.

Under the conventional assumption that the uncertainty is constant, there is no mechanism for incorporating changes in the uncertainty. It means that the uncertainty is used inefficiently in forming forecasts indicating that the forecasts will not be fully rational.

Suppose that the uncertainty is allowed to change over time and is common to all forecasters. With this assumption, the uncertainty can be treated as an important element of available information at time t when the forecast was made. In order to test for partial rationality (efficient use of the uncertainty) of the ASA-NBER survey data, we can write down a regression model

$$(23)\ p_t = \beta_0 + \beta_1 f_t + \beta_2\, \sigma_t^2 + \varepsilon_t$$

which will satisfy $\beta_0 = 0$, $\beta_1 = 1$, $\beta = 0$, and $E(\varepsilon_t / f_t) = 0$ if f_t is indeed partially rational. The ARCH-M model introduced by Engel, Lillien and Robins will be applied to the estimation of coefficients.

1. The use of uncertainty as a regressor

It is assumed that an investigator has specified a theoretical relationship of the form

$$(24)\ y_t = X'_t\, \beta + \sigma_t^2 \delta + \varepsilon_t$$

where the term σ_t^2 represents a conditional variance. The literature on uncertainty is somewhat vague about what σ_t^2 represents. The estimation problem in (24) is that no series on σ_t^2 exists.

Suppose, however, that it was possible to construct a series $\psi_t^2 \rightarrow \sigma_t^2\, \forall t$ (and fixed sample size) or the weak property that $E(\psi_t^2) = \sigma_t^2$. Under such circumstances, it seems reasonable that σ_t^2 be replaced by ψ_t^2 in (24). The impact of any such substitution is upon the error term, which changes from ε_t in (24) to e_t in the following equation

$$(25)\ y_t = X'_t \beta + \psi_t^2 \delta + e_t = X'_t \beta + \psi_t^2 \delta + \varepsilon_t + \ (\sigma_t^2 - \psi_t^2)$$

It is not unreasonable to assume that $E(x_t\ \varepsilon_t) = 0$, $E(\psi_t^2 \varepsilon_t) = 0$, and this ensures that the error term in (25) has a zero mean.

If ψ_t possesses the strong property that $\psi_t^2 \rightarrow \sigma_t^2 \forall_t$, it is obvious that the least-square method applied to (25) yields consistent estimates of β and δ. However, if it is characterized only as the weaker property that $E(\psi_t^2) = \sigma_t^2$, then $E\ (\psi_t^2\, e_t) = \delta$ $(\sigma_t^4 - E(\psi_t^4))$, and this will rarely be zero. Thus least-square estimates may be inconsistent under the weak property. This suggests that great care needs to be exercised in estimating δ if ψ_t^2 is substituted for σ_t^2. Only if the strong property for ψ_t^2 can be asserted, or if $\delta = 0$, would the regression of y_t against ψ_t^2 be an adequate procedure, as it is only in those circumstances that no inconsistencies in either $\hat{\delta}$ or its estimated standard error would be observed.

When ψ_t posseses only the weak property, we can use the instrumental variable for the consistent estimates of β and δ. An instrumental variable estimator of β and δ is available that is both consistent and asymptotically normal. See Pagan and Ullah(1986) for details.

An emphasis on the instrumental variables method for handling the errors-in-variable arising when ψ_t^2 replaces σ_t^2 fits well with the recent tendency in macroeconomic research to generate estimates from orthogonality conditions. Some attention needs to be paid, however, to the determination of the instrument variables that are orthogonal to ψ_t^2. The instrument variables must be derived from the conditioning set used in forming anticipation and, thus, should follow the theoretical specification.

If all one was interested in was testing if $\delta=0$, any proxy for σ_t^2 would be entered into a regression of y_t against x_t and the proxy and the least-square results would be used to test if $\sigma=0$. Presumably, the power of the test depends upon the correlation of the proxy and σ_t^2, and so demands a careful choice.

2. Estimation of Uncertainty

There are four main approaches for the estimation of ψ_t^2. One is to base the measure on a moving average of squared "errors" in the data. The fault of this method is the failure to fully specify the information set underlying the construction of σ_t^2, and that variance changes tend to get blurred.

A second utilizes the variance of relative price movements. As we consider the stock market, this method would be desirable. Specifically, with the identical market assumption and no systematic factors, the desirable strong property of a good variance measure alluded to earlier holds, and the least-square estimators are consistent. It should be clear, however, that the strong property fails to hold when markets are not identical.

The third approach constructs an estimator of the unknown variance from the dispersion of respondents' point forecasts in expectations surveys (e.g., Livingston or ASA-NBER surveys). Levi and Makin(1978, 1979), Makin(1982) and Makin and Tanzi(1982) have all used the Livingston survey data to measure the unobservable market perception about expected inflation and inflationary uncertainty, while Zarnowitz and Lambros(1987) and Lahiri, Teigland and Zaporowki(1986) have made use of actual subjective probability distributions from ASA-NBER surveys.

Finally, some authors explicitly parameterized σ_t^2; e.g., Hansen and Hodrick (1980), Domowitz and Hakkio(1985), Engle(1983) and Engle, Lillien and Robins(1987).

Suppose that for an individual uncertainty is best measured by the probability attached to variables of single valued outcomes. This type of information has typically not been available. The standard deviation of forecasts has been used as proxy for individual forecaster uncertainty; the third method seems to be appropriate to the estimation of uncertainty due to inflation or interest rate fluctuation.

However, the difference between a measure of uncertainty derived from a probability distribution to which values have been consciously assigned by the forecasters and one derived from the dispersion across respondents is a distinct one. The standard deviations derived from the probability distribution represent a more explicit measure of uncertainty while the variance of expectations across respondents is more accurately a measure of disagreement among forecasters.

Recently, Lahiri, Teigland and Zaporowski distinguished explicitly between the average dispersion and the disagreement, concentrating on the individual probability distribution about expected inflation and GNP from the ASA-NBER surveys. They report that the standard deviations of forecasts across respondents are generally smaller than mean standard deviations from probability distributions.

Using this average dispersion as a regressor in the time-series regression models, there are at least two empirical faults. Firstly, Lahiri, Teigland and Zaporowski examined the higher order moments, skewness and kurtosis of the probability distribution of IPD of the ASA-NBER survey data. For the most part, the distributions exhibit an even split between negative and positive skewness, and are overwhelmingly leptokurtic. These results contradict the usual assumption that the distribution is normal. Secondly, since the probability distributions are discontinuous over time, we cannot make use of it for the regression of time-series models. The variance estimated from the ARCH-M model is free from at least the latter problem. Therefore, this estimated variance may be treated as a good proxy for the uncertainly if the ARCH-M model is well specified.

3. Maximum Likelihood Estimation: ARCH-M Model

The ARCH model distinguishes between inflation uncertainty and variability by concentrating on unanticipated inflation. It develops the properties of a natural measure of uncertainty. The advantage is that the ARCH model can be used to generate an estimated time series of K-period ahead conditional forecast variances. For the inflation model, this can be interpreted as the uncertainty about the value of a variable K-period in the future, conditioned on current information. The uncertainty is a function of past information, and so will be treated as an available information when the forecast is made.

Suppose that the conditional variance estimated from the ARCH model represents the true uncertainty. It can be used as a regressor in equation (23). When a function of conditional variance is included as a regressor and so directly affects the mean, the ARCH model is extended by the ARCH-M model.

As the forecast error is white noise, the standard ARCH-M model will work well.[4] For the one-period ahead forecast of ASA-NBER survey data the estimate

[4]For example see Park(1988).

of uncertainty may be parameterized as follow

$$(26)\ h_t = \hat{\sigma}_t^2 = E(\varepsilon_t^2/\phi_{t-2})$$
$$= \hat{\alpha}_0 + \hat{\alpha}_1 E\ (\varepsilon_{t-1}^2/\phi_{t-2}) + \hat{\alpha}_2\varepsilon_{t-2}^2 + \ \dots\ + \ \hat{\alpha}_p\varepsilon_{t-p}^2$$

where $E(\varepsilon_{t-1}^2/\phi_{t-2})$ is employed instead of ε_{t-1}^2 because ε_{t-1}^2 was not fully available at time t-2 when the forecast was made. And the estimation method of the ARCH-M model is very similar to the ARCH model except for the uncertainty being included as a regressor.[5]

As noted by Pagan and Ullah, one problem in the ARCH-M model is its presumption of correct specification. Although it does not seem possible to obtain an estimator of δ that is robust to specification errors in σ_t^2 when the preferred format is ARCH, it is at least possible to devise some specification tests for the adequacy of the ARCH representation.

A test for the validity of the ARCH-M process is to compare the actual heteroskedasticity pattern in $\hat{\psi}_t^2$ with the postulated ARCH-M form. $T^{-1}\ \Sigma\ (\hat{\psi}_t^2 - h_t) \xrightarrow{a} 0$ under the null hypothesis that the model is adequate. Hence, using the residuals $\hat{\psi}_t$ and the ARCH specification for h_t, we can determine how good a representation it is by the size of $T^{-1}\Sigma(\hat{\psi}_t^2 - h_t)$. The estimated variance of $T^{1/2}(T^{-1}\Sigma(\hat{\psi}_t^2 - h_t))$, under the maintained hypothesis, follows as $\psi = 2\hat{V}^4 + (\partial h_t/\partial\theta)\ V_{\theta\theta}(\partial h_t/\partial\theta)$, where $\hat{V}^2$ is the estimated residual variance of $\hat{\psi}_t$, θ's are the parameters in the ARCH-M specification for h_t and $V_{\theta\theta}$ is the asymptotic covariance matrix of $T^{1/2}(\hat{\theta}\text{-}\theta)$. Then $T^{1/2}\ \psi^{1/2}\ \Sigma(\hat{\psi}_t^2 - h_t)$ follows the standard normal distribution. See Pagan and Sabau(1986) for the proof.

This test can be simplified as follows. Observe that a necessary condition for $T^{-1}\ \Sigma(\hat{\psi}_t^2 - h_t) \xrightarrow{a} 0$ is that none of the variation in $\hat{\psi}_t^2 - h_t$ can be explained. In particular, if we regressed $\hat{\psi}_t^2 - h_t$ against a constant and h_t, the two estimated coefficients should not be far from zero under the null hypothesis.

Then, of course, the t-statistics are biased because h_t is a generated regressor. This test method is know to be especially useful if the uncertainty enters in a non-linear way, e.g., as $\log h_t$.

IV. EMPIRICAL RESULTS

In general, most attention has been paid to the factors influencing the first moment, and even in those instances where second-order moments enter the analysis, they are taken as constant. Thus second-order moments(uncertainty) have not been used efficiently in forming the forecasts.

When the uncertainty(conditional variace) is allowed to change over time by ARCH models, the uncertainty can be treated as an available past information.

[5]See Park(1988) for estimation and test of the ARCH-M model.

If uncertainty is common to all forecasters, it seems reasonable to test whether the uncertainty was included efficiently at time t when the forecasts were made: we may use it as a regressor in equation (23) to test for partial rationality.

As noted in the above, the estimation problem in equation (23) is that no series on σ_t^2 exists. Then our first job is to search for the appropriate series of uncertainty. The ARCH-M model is employed in order to derive the proxy of uncertainty(conditional variance). The conditional variance estimated from the ARCH-M model is compared with mean variance or an individual variance computed from the ASA-NBER surveys and is, generally, more fitted to individual variances than mean variance.[6] It is, however, pointed out that the conditional variance is much higher and far more volatile than mean variance or an individual variance.[7]

Meanwhile, in its standard form the ARCH-M model expresses the conditional variance as a linear function of past variances. This argument corresponds precisely to the Mandlebroit(1963) observation: "Large changes then to be followed by large changes- of either sign- and small changes tend to be followed by small changes...." Thus in the context of ARCH-M model, the unusual large variance should yield the unusual large variance of next periods. This point is a limitaion of ARCH-M model.

After conducting some diagnostic tests for the model selection, Park(1988) showed that the model with the log of variance may be superior to that in the variance. So our proferred model does not have h_t as a determinant but $\log h_t$, and this means that we cannot use an instrumental variable approach but are forced into parameterization. Given this restriction it becomes necessary to test the validity of our chosen specification for h_t based upon test statistic $T^{-1}\Sigma(\hat{\omega}_t^2 - h_t)$ suggested by Pagan and Ullah. There is evidence for the presence of ARCH effect: $\ell_1(3)$ is significant at the 90% level with 3 degrees of freedom.[8] And since the estimates were, in fact, -.346 and .139 with t-statistics of -.307 and .309, the parametric on the ARCH process are not rejected.

[6]The list of those who applied to any of the questionnaires of ASA-NBER survey includes approximately 180 names. Respondents are asked to attached subjective probabilities to the potential annual percentage changes in the GNP and GNP-implicit price deflator. The dispersion of forecasts may be as important as the reported median expectation forecasts. It is relevant to consider the implications for model behaviour of the dispersion of expectations about their mean value. For example, the variance of inflationary expectations has been treated as a proxy of uncertainty about inflation. The obtained variance may be used as an independent variable in the regression model to test for partial rationality also.

[7]We computed the rate of inflation at quarterly rate like Engle and Kraft(1984) and regressed the following equation

$$p_t = \beta_0 + \beta_1 p_{t-1} + \beta_2 p_{t-3} + \beta_3 p_{t-4} + \beta_4 p_{t-6}$$

The trend and scale of conditional variance estimated from the above equation seem to be very similar to those of survey variance. But note that survey variance is derived from the probability distribution of annualized survey forecasts.

[8] $l(k)$ is the Lagrange Multiplier test statistic which is asymptotically distributed Chi-square with k degrees of freedom. The statistic $\ell_1(k)$ tests against the alternative of an unresticted Pth-order ARCH model and is computed using the scoring algorithm suggested by Engle(1982). The statistic $\ell_2(k)$ is the test statistic for the validity of k restrictions on the parameters of the ARCH process.

The least squares estimation for the one-period ahead forecast is

$$(27)\ p_t = .04 + .93f_t + 1.28\ \log h_t + \varepsilon_t$$
$$(.06)\quad (10.6)\quad (2.04)$$
(t-statistics in parenthesis)

The maximum likelihood estimation is

$$(28)\ p_t = -.38 + .97f_t + 1.38\ \log h_t + \varepsilon_t$$
$$(-5.9)\quad (12.1)\quad (2.30)$$
(t-statistics in parenthesis)

$\hat{\beta}_0$ and $\hat{\beta}_1$ are equal to zero and to one, respectively, but $\hat{\beta}_2$ is significantly greater than zero. The last fact shows a tendency toward underestimation of inflation which has long been observed in a great variety of forecasts.[9] Thus since there exist systematic errors(uncertainty) in the forecast, the one-period ahead forecast should not be partially rational and, of course, not to be fully rational.

The Q-statistic for the one-period ahead forecast is 26.63, indicating that the null hypothesis is accepted at the 90% significance level and so that the error term follows the first-order moving average process.[10] With this result, the efficient estimates are derived from the model filtered forward with MA(1).

The least squares estimation for the FF model is

$$(29)\ P_t = .39 + .95ft \pm .49\log h_t + \varepsilon_t$$
$$(.60)\quad (9.32)\quad (1.39)$$
(t-statistics in parenthesis)

The maximum likelihood estimation for the FF model is

$$(30)\ p_t = -.04 + .96f_t + .67\log h_t + \varepsilon_t$$
$$(-.08)\quad (12.1)\quad (2.76)$$
(t-statistics in parenthesis)

The least squares estimation for the FOE model is

$$(31)\ p_t = .41 + .96f_t + .50\log h_t + \varepsilon_t$$
$$(.60)\quad (10.6)\quad (2.04)$$
(t-statistics in parenthesis)

[9]Note that the underestimation can arise in unbiased as well as biased predictions. See Park(1988) for details.

[10]The Ljung-Box Q-statistic which provides a measure of the overall serial correlation of residuals is a Lagrange Multiplier test statistic.

The maximum likelihood estimation for the FOE model is

$$(32)\ p_t = -.04 + .97f_t + .73\log h_t + \varepsilon_t$$
$$\quad(-0.7) \qquad (10.8) \qquad (2.14)$$

(t-statistics in parenthesis)

The maximum likelihood estimates differ only slightly from the ordinary least squares estimaties by decreasing the size of β_0 and increasing the sizes of β_1 and β_2. Under REH, the coefficient of the predicted change is equal to one and the constant term(intercept) is zero. One might expect that maximum likelihood estimation for the FF and FOE models provide a better result than the least square estimation and than that for the unfiltered models. These results indicate that the ARCH estimates for filtered models(FF and FOE models) are most desirable.

V. CONCLUSIONS

For the one-period ahead forecast of ASA-NBER survey, the test does not reject H_0: $(\beta_0, \beta_1) = (0, 1)$, indicating that the forecast is unbiased. However, in the FF model, the maximum likelihood estimate for β_2 is .67, generating t-statistic of 2.76: the coefficient of variance is positively significant. Thus since the one-period ahead forecast is systematically underestimated, it seems to be partially irrational and, of course, fully irrational relative to symmetric loss function. Note that in the context of asymmetric loss function, the above result may have different interpretation.

By the Q-statistic test for the one-period ahead forecast, the null hypothesis that the error term follows the first-order moving average process is not rejected. Then the efficient estimates are derived from the model filtered forward with MA(1). Our results show that the maximum likelihood estimation for the filtered models(FF and FOE models) provide a better result than that for the unfiltered models in the sense that β_0 approaches zero and β_1 approaches one.

One by-product is to obtain the optimal forecast by using the survey forecasts. The Q-statistic can give a standard for model selection. As the one-period ahead forecast does not fail to pass the Q-test at the 90% significance level, it is known that equation (23) is well specified. With this result, the corrected survey forecast, $\beta_1 f_t + \beta_2 \log h_t$, might be optimal relative to symmertric loss function.

[Table] Test for Partial Rationality (ASA-NBER): One-period, 1968-86

	$p_t = \beta_0 + \beta_1 f_t + \beta_2 \log h_t^{a)} + \varepsilon_t^{b)}$ $\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	R^2	$\ell_1(3)^{c)}$	$\ell_2(2)^{d)}$
I. FF						
-OLS:	.39	.95	.49	.61		
	(.60)	(9.32)	(1.39)			
-ARCH: 1st iteration:	.02	.96	.69	.60		
	(.04)	(10.8)	(1.99)			
-ARCH: Final iterations:	−.04	.96	.67	.60	7.0	1.30
	(−.08)	(12.05)	(2.76)			
II. FOE						
-OLS:	.41	.96	.50	.61		
	(.60)	(10.6)	(2.04)			
-ARCH:1st iteration:	.00	.96	.72	.61		
	(.00)	(11.2)	(2.16)			
-ARCH: Final iteration:	−.04	.97	.73	.61	7.1	.73
	(−.07)	(10.8)	(2.14)			
Pagan-Ullah Test: $\varepsilon_t^2 - h_t =$	−.346 +	.139h_t				
	(−.307)	(.309)				
Q Test: $\ell(23) = 26.63^{e)}$						

a) $h_t = E(\varepsilon_t^2/\phi_{t-2})$ where $\varepsilon_t = p_t - .53 - .63p_{t-1} - .23p_{t-2} - .12p_{t-4} + .09p_{t-6}$
b) $\varepsilon_t = e_t + .20e_{t+1}$ c) The critical value is: $x^2_{.9}(3) = 6.25$.
d) The citical value is: $x^2_{.9}(2) = 4.61$. e) The critical value is: $x^2_{.9}(23) = 32.0$.
f) There are t-statistics in parenthesis.

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