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THE ECONOMIC EFFECTS OF THE LAND VALUE TAX: IN THE CASE OF A SYSTEM OF CITIES*

DUCK HO LIM**

I. INTRODUCTION

Ever since Mieszkowski(1972) and Grieson(1974) have studied the allocative effects of a property tax on reproducible capital, the nonneutrality of a tax on land value was demonstrated by a number of writers including Skouras (1982), David Mills(1981), Brian Bentick(1982), Brueckner(1986) and Lim (1992). Lim used a two-period model of urban development to confirm their nonneutrality argument. In a two-period model of urban development with perfect foresight, a tax on land rent is neutral but a tax on current land value is nonneutral. Some land is held vacant for more profitable future development, earns zero land rents in the first period, and earns higher land rents in the second period. For taxes on land rents land owners who hold land vacant in the first period pay tax only in the second period. For the land value tax, however, the land owners pay taxes in both periods as current land value reflects both current and future land rents. So, the land value tax is a form of double taxation and distorts the optimal timing of land development.

Lim also analysed the impacts of the nonneutral land value tax on urban structure from the perspective of a single open city located in a large national economy. In this paper, however, I study the effects of the tax from the standpoint of a system of cities, or the nation as a whole. The key difference between the single city analysis and the national perspective is that supplies of labor and capital are more elastic to a single open city, relative to the national economy.[1]

In a single open city the land value tax encourages the development of land in the first period. This premature development decreases industrial output and city size in the second period as less land is available for industrial develop-

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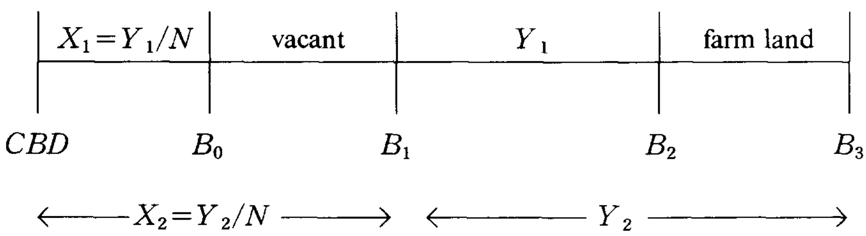
[1] For the analytical simplicity there is no capital in this model.

ment.[2] In contrast, in a system of cities where the supply of labor is inelastic relative to a single open city the imposition of the land value tax by all cities will encourage the development of vacant land in the first period, and while the nation's overall employment will be little affected by this tax, the decrease of the amount of land held vacant in the first period will result in less efficient leapfrog development in the second period. So, even though the land value tax does not affect the wage rate or leisure in the first period it decreases the wage rate and work effort in the second period and decreases output.

II. THE MODEL

Consider a number of identical cities in the nation. In each city population is fixed exogenously in each period. The residential zone is also fixed exogenously, as we assume that every household in the city consumes one unit of residential land. The price of manufactured commodity is also fixed exogenously in both periods; we assume that $P_1=P_2=1$ as a numeraire. In a system of cities a city's growth depends on exogenous population growth rather than price change. As we assume fixed factor-proportions in production, the sizes of the industrial zones are proportional to the sizes of the residential zones : $X_1=Y_1/N$, $X_2=Y_2/N$ in [Figure 1].



where, $i=1,2$

X_i: the sizes of the industrial zones in each period,

Y_i : the sizes of the residential zones(or the number of households) in each period,

N : workers per unit of industrial land.

[Figure 1] Pattern of Urban Land Use in the Case of an Exogenous Population.

Every household supplies the same work hours at all employment sites in the city. Workers who reside close to the central business district(CBD) enjoy

[2] For details see Duck-Ho Lim, "The Nonneutrality of the Land Value Tax: Impacts on Urban Structure," Journal of Urban Economics 32, 186-194(1992).

more leisure as they save on travel time to the *CBD*, relative to workers who live further from the *CBD*. These workers pay more in residential rents and give up some amount of the industrial good in return for additional leisure. Since every worker in the city works the same hours regardless of residential location, workers who live on the outer edge of the residential zone, B_2 in the first period and B_3 in the second period consumes less leisure because of longer travel time, relative to workers who reside on the boundary B_1 in [Figure 1]. The consumption of leisure for workers who live on the outer edge of the residential zone in the first period is shown by

$$L_1(B_2)=L_1(B_1)-T_tY_1$$

where, $L_1(B_i)$: hours of leisure taken by workers in the first period who reside on the boundary B_1 and B_2, respectively,

T_t: travel time taken by workers per unit distance and fixed exogenously in both of two periods.

The leisure for workers who live on the outer edge of the residential zone in the second period is shown by

$$L_2(B_3)=L_2(B_1)-T_tY_2$$

Workers who live on the outer edge of the residential zone in each of the two periods pay zero residential rents, and they consume more of the manufactured good at the cost of leisure, relative to workers who reside close to the *CBD*.

For simplicity we assume that total travel cost, the cost of commuting to work along with a postwork visit to the *CBD*, does not depend on the work location so that the wage rate is constant through the industrial zone. All workers in the city have the same income structure as they supply the same work hours, and earn the same hourly wage rate at all employment sites and receive the same lump-sum rental income.[3] The lump-sum rental income is calculated by dividing total land rents by the number of households in the city, and is distributed equally to workers. Workers at every residential location maximize utility subject to their budget constraint in each period[4] and have the same level of utility. So, we solve the utility maximization problem for a worker who lives on the boundary between the industrial zone and the residential zone. We assume a Cobb-Douglas utility function.

[3] In the case of a system of cities land owners are profit-maximizing residents and lump-sum rental income appears in household's income equations as the city is characterized as the closed city.

[4] This is not an overlapping-generation model. Workers maximize their utility in each period.

$$\max\ U_i(B_1)=[q_i(B_1)]^b[L_i(B_1)]^{1-b}, \qquad 0<b<1$$
$$\text{s.t. } W_iH_i+TR_i/Y_i=q_i(B_1)+R_i(B_1)+T_mY_2/N \tag{1}$$

where, $i=1,2$,

- $q_i(B_1)$: the amount of the industrial good purchased by workers who live on the boundary B_1,
- W_i : hourly wage rate,
- H_i : work hours in each period,
- TR_i : total rental income,
- T_m: money cost per unit distance in household's transportation costs,
- $R_i(B_1)$: residential rents at distance Y_2/N from the CBD in each period.

From the first order conditions we obtain

$$q_i(B_1)=\frac{b}{(1-b)}W_iL_i(B_1) \tag{2}$$

This condition implies that the return from lump-sum rental income at the given hourly wage rate increases both leisure and consumption.

To calculate lump-sum rental income in equation (1) we derive land-rent functions. Industrial land-rent functions can be derived from firm's zero profit condition. In a competitive market all firms in the industry will have zero profit. So, the product exhaustion condition is

$$P_iNH_iQ_i-cNH_iQ_iX-W_iNH_i-I_i(X)=0$$

where, $i=1,2$

- P_i : the price of the industrial good in each period and is assumed to be equal to 1 as a numeraire,
- Q_i : the amount of the industrial good produced by a worker per unit hour and is assumed to be equal to 1 for the simplicity,
- c : firm's transportation costs per unit distance, per unit of output,
- X : distance from the CBD,
- $I_i(X)$: industrial land rents at distance X from the CBD.

Solving this equation for $I_i(X)$, we obtain the industrial land-rent functions in each period.

$$I_i(X)=NH_i(1-cX-W_i) \tag{3}$$

Wage income is calculated by multiplying total work hours by the hourly wage rate. The price of leisure can be defined to be the opportunity cost of wage income. So, we assume that in each period workers who reside on the outer edge of the residential zone can consume more units of the industrial commodity at the cost of leisure. The equation of the budget constraint for a worker who lives on the outer edge of the residential zone in each period is

$$W_iH_i+TR_i/Y_i=q_i(B_1)+T_tY_iW_i+T_m(Y_2/N+Y_i) \tag{4}$$

In the right hand side of equation (4) the second term represents time cost which a worker on the outer edge of the residential zone has to pay more, relative to a worker on the inner edge, and the third term shows money cost. Since every worker supplies the same work hours and earns the same wage rate and receives the same lump-sum rental income, the right hand sides of equations (1) and (4) must be equal in each period and hence residential land rents on the boundary B_1 is $R_i(B_1)=(T_tW_i+T_m)X$. Since we assume uniform density and zero land rents on the outer edge of the residential zone, the residential land-rent functions can be approximated by

$$R_i(X)=(T_tW_i+T_m)(Y_i+Y_2/N)-(T_tW_i+T_m)X \tag{5}$$

For analytical simplicity, we assume linear land-rent functions as the approximations for the rent gradient. The actual land-rent function will be non-linear as indifference curves are convex.[5]

From the locational equilibrium land-rent functions we can measure lump-sum rental income in each period. In the first period lump-sum rental income is

$$\begin{aligned} TR_1/Y_1 &= \frac{1}{Y_1}\left[\int_0^{Y_1/N} I_1(X)\,dX+\int_{Y_2/N}^{(Y_2/N+Y_1)} R_1(X)\,dX\right] \\ &= \frac{1}{2}[H_1(1-W_1)+(T_tW_1+T_m)Y_1] \end{aligned} \tag{6}$$

In the second period lump-sum rental income is

$$\begin{aligned} TR_2/Y_2 &= \frac{1}{Y_2}\left[\int_0^{Y_2/N} I_2(X)\,dX+\int_{Y_2/N}^{(1+1/N)Y_2} R_2(X)dX\right] \\ &= \frac{1}{2}[H_2(2-2W_2-cY_2/N)+(T_tW_2+T_m)Y_2] \end{aligned} \tag{7}$$

[5] The value of the approximated linear land-rent is very close to the true value of the non-linear land-rent. See Appendix.

For a system of cities we have four unknowns, $L_1(B_1)$, $L_2(B_1)$, W_1, and W_2 and four equations. Substituting equations (2), (5), (6), and (7) into equation (1), we have two equations of the budget constraint in the first and second periods.

$$\frac{1}{2}(1+W_1)H_1=\frac{b}{(1-b)}W_1L_1(B_1)+\frac{1}{2}(T_tW_1+T_m)Y_1+T_mY_2/N \tag{8}$$

$$(1-\frac{1}{2}cY_2/N)H_2=\frac{b}{(1-b)}W_2L_2(B_1)+\frac{1}{2}(T_tW_2+T_m)Y_2+T_mY_2/N \tag{9}$$

The product exhaustion condition states that land rents on the outer edge of the industrial zone in the first period is zero and is, from equation (3), equal to

$$NH_1(1-cY_1/N-W_1)=0 \tag{10}$$

The boundary condition between two land zones, a location that is equally profitable as industrial land in the second period and as residential land in the first and second periods is

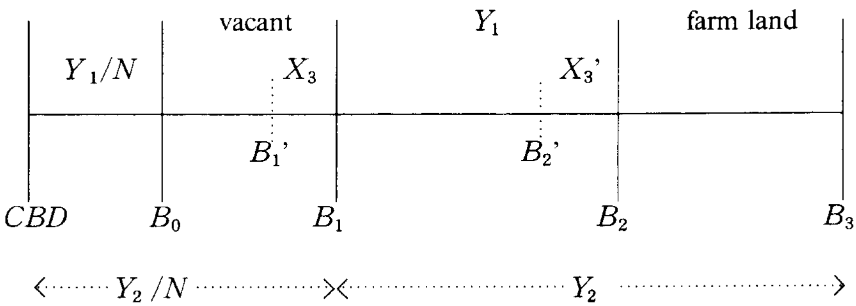
$$\frac{NH_2(1-cY_2/N-W_2)}{(1+r)}=(Y_m+T_tW_1)Y_1+\frac{(T_m+T_tW_2)}{(1+r)} \tag{11}$$

where r is discount rate. In equation (11) the left hand side represents the present value of industrial rent and the right hand side shows the sum of the present value of residential rents on the boundary B_1 in [Figure 1]. Since we have four unknowns and four equations, we can solve for the endogenous variables in terms of exogenous variables.

III. THE EFFECTS OF THE LAND VALUE TAX

For a system of cities the land value tax decreases the amount of vacant land in the first period by encouraging premature development of vacant land as residential land. As we assume that in each period population is exogenous and every household consumes a fixed amount of residential land and there exists fixed coefficient of production between labor and land, such a decrease of vacant land will result in less efficient leapfrog, industrial development in the second period beyond the outer edge of the residential zone.

In [Figure 2] X_3 represents the conversion of vacant land into residential land in the first period following the imposition of the land value tax. This decrease in vacant land results in leapfrog development, X_3' for industrial use in the second period. X_3 must be equal to X_3' as population is fixed exogenously and the industrial land per worker is fixed. Before the imposition of the tax the outer edge of the residential zone in the first period was B_2 and land rents on this boundary were zero. When the land value tax is imposed, the outer edge moves toward the *CBD* by the conversion of vacant land into residential land, and residential land rents on the boundary X_3', $(T_t W_1 + T_m)X_3$' are no longer zero. In the second period the outer edge of the residential zone does not change because of a fixed population. So, the residential land-rent functions don't change when the land value tax is imposed. However, industrial land rents in the second period will rise because firms will be located beyond the outer edge of the residential zone by leapfrog development and firms located near by the outer edge will bid up for inter city's industrial land to save transportation costs. The increase in industrial land rents in the second period will decrease the wage rate in this period.



[Figure 2] Pattern of Urban Land Use for a System of Cities Where the Land Value Tax is Imposed.

When the land value tax is imposed, we have the same equations as in the nondistorted equilibrium except for the change in the boundary condition between the industrial and the residential zones. When we start from the nondistorted equilibrium(no tax on land), we have four unknowns and four equations since $X_3 = X_3' = 0$.[6] As we assume that the local government imposes taxes and returns the proceeds lump-sum, the sum of net lump-sum rental income after paying the land value tax and lump-sum tax return is exactly equal to the nondistorted lump-sum rental income. The land value tax does not change the equations of the budget constraint but this tax is still distortionary as it changes the optimal timing of urban development. When the

[6] If we start from the positive land value tax(the distorted equilibrium), we have five unknowns including X_3 in [Figure 2] because of leapfrog development beyond the residential zone and we have one more boundary condition between the industrial zone and the residential zone in the second period, B_2 in [Figure 2]. Also, the land value tax is capitalized into the price of land.

land value tax is imposed, the boundary condition becomes[7)]

$$\frac{N(1-cY_2/N-W_2)H_2}{(1+r)}=\frac{(T_tW_1+T_m)Y_1}{(1-t)}+\frac{(T_tW_2+T_m)Y_2}{(1+r)} \tag{11)$'$}$$

where t is the tax rate of the land value tax.

To investigate how the land value tax affects the wage rates and leisure we totally differentiate equations (8), (9), (10), and (11)′ and set $t=0$ as the initial value for the tax.

$$[\frac{b}{(1-b)}W_1+\frac{1}{2}(1+W_1)]dL_1(B_1)=[\frac{1}{2}H_1-\frac{b}{(1-b)}L_1(B_1)-\frac{1}{2}T_t\,Y_1]dW_1 \quad (12)$$

$$[\frac{b}{(1-b)}W_2+1-\frac{1}{2}cY_2/N]dL_2(B_1)=-[\frac{b}{(1-b)}L_2(B_1)+\frac{1}{2}T_tY_2]dW_2 \quad (13)$$

$$-NH_1\,dW_1-N(1-cY_1/N-W_1)dL_1(B_1)=0 \quad (14)$$

$$\begin{aligned}N(1-cY_2/N-W_2)dL_2(B_1)=&-(NH_2+T_tY_2)dW_2-(1+r)T_t\,Y_1\,dW_1\\&-(1+r)(T_m+T_tW_1)Y_1\,dt\end{aligned} \quad (15)$$

From equation (10) we know that H_1 is positive as work hours and $(1-cY_1/N-W_1)=0$. So, from equation (14) it follows that $dW_1=0$. Substituting this value into equation (12) we obtain $dL_1(B_1)=0$. Even though the land value tax is nonneutral with respect to the timing of land development it does not affect the wage rate or leisure in the first period. This result is explained by the fact that the decreases of household's transportation costs, as the outer edge of the residential zone moves toward the CBD by the conversion of vacant land into residential land, is exactly offset by the increase in residential rents.

Solving equations (13) and (15) for dW_2, we obtain

$$dW_2=\frac{F}{(G-K)}dt \quad (16)$$

where, $F=(T_tW_1+T_m)Y_1/N(1-cY_2/N-W_2)$, $\quad F>0$

$$G=\frac{[bL_2(B_1)/(1-b)+T_tY_2/2]}{[bW_2/(1-b)+1-cY_2/2N]}, \quad G>0$$

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[7] Lim showed how the land value tax changes the boundary condition between the industrial and residential zones. See Duck-Ho Lim, "The Nonneutrality of the Land Value Tax: Impacts on Urban Structure," Journal of Urban Economics 32, 186-194(1992).

$$K=\frac{(NH_2+T_tY_2)}{N(1-cY_2/N-W_2)}, \qquad K>0$$

In equation (16) the numerator is positive (F>0) since $(1-cY_2/N-W_2)$ represents industrial land rents. But the denominator is negative $(G-K<0)$ if the restriction on b is satisfied as follows.

$$b<\frac{H_2}{[H_2+(1-cY_2/N-W_2)L_2(B_1)]} \tag{17}$$

We prove this as follows:

$$G-K=\frac{[bL_2(B_1)/(1-b)+T_tY_2/2]}{[bW_2/(1-b)+1-cY_2/2N]}-\frac{(H_2+T_tY_2/N)}{(1-cY_2/N-W_2)} \tag{18}$$

In equation (18) the denominator of the second term, $(1-cY_2/N-W_2)$ is very small value relative to W_2 as a large portion of firm's revenue is paid as wage income, and cY_2/N is also relatively small and the first term's denominator in equation (18) is greater than 1. So, if H_2 is greater than $[b/(1-b)\cdot(1-cY_2/N-W_2)L_2(B_1)]$, $(G-K)$ is negative. Rearranging this inequallity, we obtain the restriction on b as in equation (17). In equation (17) the right hand side is close to 1 since $(1-cY_2/N-W_2)$ is very small value. So, we conclude $(dW_2/dt)<0$, which means that the imposition of the land value tax decreases the wage rate in the second period. This result is explained by the leapfrog industrial development which increases industrial transportation costs in the second period.

Solving equations (13) and (15) for $dL_2(B_1)$, we obtain

$$dL_2(B_1)=-\frac{GF}{(G-K)}dt \tag{19}$$

From equation (16) we can determine $(dL_2(B_1)/dt)>0$, which means that the increase of the land value tax increases leisure in the second period. Conse-

[Table 1] Economic Effects of the Land Value Tax

Variables	First Period	Second Period
Nominal Wage Rate	constant	decrease
Leisure	constant	increase
Residential Rents	constant	constant
Industrial Rents	constant	increase
Output	constant	decrease

quently, the imposition of the land value tax decreases the wage rate and work effort in the second period and decreases output. Table 1 summarizes the economic effects of the land value tax.

Ⅳ. CONCLUDING REMARKS

In this paper I used a two-period model of urban development with perfect foresight to analyse the effects of the nonneutral land value tax on urban economy. I considered a system of cities to investigate whether the results from the perspective of a single open city are still valid at the national level that the supply of labor is relatively inelastic.

For a system of cities the land value tax encourages the development of vacant land for the residential use in the first period. Such a decrease of vacant land to be developed for industrial use will result in less efficient leapfrog, industrial development in the second period beyond the outer edge of the residential zone. So, the land value tax is nonneutral with respect to the timing of urban development. Even though the land value tax does not affect the wage rate or leisure in the first period, it decreases wage rate and increases leisure in the second period as firms will be located beyond the outer edge of the residential zone by leapfrog development that means industrial land rents in the second period will rise.

APPENDIX

The nonlinear land-rent functions are derived by making use of utility function and income equations. We solve the utility maximization problem for a worker who lives on the boundary between the industrial and the residential zones, B_1 and for a worker who resides on the outer edge of the residential zone in the first period, B_2. They have the same level of utility at each location. For the residential location B_1

$$\begin{aligned} \max\quad & U_1(B_1)=[q_1(B_1)]^b[L_1(B_1)]^{1-b} \\ \text{s.t.}\quad & W_1[TO_1-L_1(B_1)-T_tY_1/N]+TR_1/Y_1 \\ & =q_1(B_1)+R_1(B_1)+T_mY_2/N \end{aligned} \tag{20}$$

where TO_1 is total time allocated between leisure, work hours, and travel time in the first period. For the residential location B_2

$$\begin{aligned} \max\quad & U_1(B_2)=[q_1(B_2)]^b[L_1(B_2)]^{1-b} \\ \text{s.t.}\quad & W_1[TO_1-L_1(B_1)-T_t(Y_2/N+Y_1)]+TR_1/Y_1 \\ & =q_1(B_2)+T_m(Y_2/N+Y_1) \end{aligned} \tag{21}$$

They supply the same work hours:

$$[TO_1 - L_1(B_1) - T_t Y_2/N] = [TO_1 - L_1(B_2) - T_t(Y_2/N + Y_1)] \qquad (22)$$

Solving equation (22) for $L_1(B_2)$ to obtain

$$L_1(B_2) = L_1(B_1) - T_t Y_1 \qquad (23)$$

Since workers have the same level of utility, we substitute equation (23) into the utility function for B_2 and equate two utility functions to obtain

$$[1 - T_t Y_1/L_1(B_1)]^{(1-b)/b} = q_1(B_1)/q_1(B_2) \qquad (24)$$

Solving equation (21) for $q_1(B_2)$, we have

$$q_1(B_2) = M - T_m(Y_2/N + Y_1) \qquad (25)$$

where M is worker's income. Since they have the same income, we equate the right hand sides of equations (20) and (21) to calculate land rents.

$$R_1(B_1) = q_1(B_2) - q_1(B_1) + T_m Y_1 \qquad (26)$$

Solving equations (24) and (25) for $q_1(B_1)$, we have

$$q_1(B_1) = [1 - T_t Y_1/L_1(B_1)]^{(1-b)/b}[M - T_m(Y_1/N + Y_1)] \qquad (27)$$

Substituting equations (25) and (27) into (26), we obtain the nonlinear land-rent function.

$$R_1(B_1) = M - T_m Y_2/N - [1 - T_t Y_1/L_1(B_1)]^{(1-b)/b}[M - T_m(Y_2/N + Y_1)] \qquad (28)$$

For analytical simplicity let $b = 0.5$. Then the nonlinear land-rent function is shown by

$$R_1(B_1)' = T_t Y_1[M - T_m(Y_2/N + Y_1)]/L_1(B_1) + T_m Y_1 \qquad (28)'$$

On the other hand, the linear land-rent function which we assumed is shown by

$$R_1(B_1) = (T_t W_1 + T_m) Y_1 \qquad (29)$$

From equations (28)′ and (29) we know that two land rents exactly equal if the following expression is satisfied.

$$\begin{aligned} W_1L_1(B_1) &= M - T_m(Y_2/N + Y_1) \\ &= W_1H_1 + TR_1/Y_1 - T_m(T_2/N + Y_1) \end{aligned} \tag{30}$$

From the nondistorted equilibrium we can calculate the value of each endogenous variables. From equation (10) we obtain

$$W_1 = 1 - cY_1/N \tag{31}$$

Solving equation (8) for $L_1(B_1)$, we obtain

$$\begin{aligned} L_1(B_1) = [&(1+W_1)(TO_1 - T_tY_2/N) \\ &-(T_tW_1 + T_m)Y_1 - 2T_mY_2/N]/(1+3W_1) \end{aligned} \tag{32}$$

For the numerical analysis we assume that

$$c=0.1,\ 1/N=0.25,\ Y_1=1.0,\ Y_2=1.5,\ T_t=0.025,\ T_m=0.08,\ TO_1=24$$

Substituting these estimated values for the exogenous variables into equation (30), we find that the left hand side of equation (30) equals 11.729 and the right hand side equals 11.753. The difference between these two values is less than 0.03. Therefore, the approximated linear land-rents we have assumed are very close to the value of the actual nonlinear land-rents.

REFERENCES

BENTICK, B.L., A Tax on Land Value may not be Neutral, *National Tax Journal* 35, 113(1982).

BRUECKNER, J.K., A Modern Analysis of the Effects of Site Value Taxation, *National Tax Journal* 39, 49-57(1986).

GRIESON, R.E., The Economics of Property Taxes and Land Values: The Elasticity of Suppy of Structure, *Journal of Urban Economics* 1, 367-381(1974).

LIM, DUCK-HO, The Nonneutrality of the Land Value Tax: Impacts on Urban Structure, *Journal of Urban Economics* 32, 186-194(1992).

MIESZKOWSKI, P., The Property Tax: An Excise Tax or Profits Tax?, *Journal of Public Economics* 1, 73-96(1972).

MILLS, D.E., The Non-Neutrality of Land Taxation, *National tax Journal* 34, 125-129(1981).

SKOURAS, A., The Non-Neutrality of Land Taxation, *Public Finance* 33, 113-134 (1982).