

RENEGOTIATION AND COALITION-PROOF NASH EQUILIBRIA IN A DYNAMIC GAME OF INFORMATION TRADING

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This paper investigates dynamic interactions in a game of information transmission. This information can be seen as a cost reducing manufacturing technique of output. A finite horizon dynamic game is examined. Two refinement concepts are compared: Pareto perfect Nash equilibrium and perfectly coalition proof Nash equilibrium. The perfectly coalition proof Nash equilibrium eliminates unstable subgame perfect equilibria, while the set of Pareto perfect Nash equilibrium outcomes is identical to the set of subgame perfect equilibrium outcomes. Finally, this paper contrasts the dynamic game with the static notion of resale proof outcomes and shows that the essential difference is due to the adoption of a marginal benefit with a cost of delay in selling the information.

1. INTRODUCTION

Economists are interested in understanding the dynamic interactions among competing agents. A standard approach to modeling history dependence in oligopolies is the use of repeated games. In the dynamic behavior of oligopolies, interdependence and repeated play make it possible to co-ordinate rather than to compete. Tacit collusion in infinitely repeated games allows firms to attain a collusive outcome without an explicit agreement.

A limitation of this analysis, however, is that the repeated game has many equilibria; then the models are unable to precisely predict oligopoly behavior. To tighten the predictions, refinements are required. It is also possible to investigate collusion by focusing directly on the limited cooperation available to oligopolists. This means considering solution concepts which take into account joint behavior.

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The present paper investigates dynamic interactions in a game of information transmission. The game may be thought of as a stylized representation of the dispersion of a cost-reducing technique among competitors. We model this as a purchase by an uninformed firm from an informed firm of a technique which then can be resold to further firms.

In this situation, this paper examines a dynamic trading game,¹ in which an (initially) informed agent sells information to many uninformed agents, assuming that the information may be freely resold without transferring costs. Since the information is used during the trading process, the marginal value between being informed and not being informed is considered as an important feature. In this case, the delay cost is also an important feature in the process of negotiations (see, for example, Rubinstein [18]); the cost of delay (either positive discount rates or foregone opportunities) is conceptually more attractive than the costless case. Then, we focus on the case where there is a marginal benefit with a cost of delay.

The basic idea of the game is as follows: At every period of the game, an informed agent can choose to make any other agent informed. In return, the other agent will pay the information provider for the privilege. The game ends when no one chooses to make anyone else informed.

Since the set of the informed agents varies at every period, this paper investigates how such information is traded in dynamic game rather than in repeated game. The payoff of agent depends upon the set of informed agents. Throughout the paper, the value of information is assumed to decrease as the number of information holders increase.

For the refinement of equilibria, we compare two solution concepts: Pareto-perfect Nash equilibrium and perfectly coalition-proof Nash equilibrium. In a finite horizon dynamic game, we show that perfectly coalition-proof Nash equilibrium eliminates unstable equilibria which are attained from the subgame perfection, while the set of Pareto-perfect Nash equilibrium outcomes is identical to the set of subgame perfect equilibrium outcomes. Then, perfectly coalition-proof Nash equilibrium provides natural restrictions on the set of subgame perfect equilibria. These equilibrium outcomes are shown to be resale-proof in the information trading game.

This theoretic framework can be applied to a variety of economic problems — among them, dynamic oligopoly behavior and joint venturer in transferring 'know-how'.

〈Literature Review〉

Economists have spent considerable energy on the examination of N-person non-cooperative games, assuming pre-play communication and non-binding commitments. The difficulties involve allowing equilibrium paths supported by incredible threats, off the equilibrium path. This leads to the refinement concept

of subgame perfection defined by Selten [19], thereby eliminating incredible threats. However, the application of subgame perfection may give rise to many perfect equilibrium paths. Some paths might be unstable in the case where players have an incentive to renegotiate for a more attractive (Pareto perfect) equilibrium, in which the original threat is no longer credible. This leads to the recent studies on renegotiation-proofness which appear in papers by Farrell-Maskin [11], Bernheim-Ray [6], Asheim [2], Blume [7,8], Benoit-Krishna [4], Pearce [16], Abreu-Pearce [1], Asilis-Kahn-Mookherjee [3], and Ray [17].

Farrell-Maskin [11] investigate how far ex post efficient renegotiation shrinks the class of subgame perfect equilibria, and how far it limits ex ante efficiency, introducing two equilibrium concepts: weakly renegotiation proof equilibrium and strongly renegotiation proof equilibrium. Bernheim-Ray [6] explore the notion of collective dynamic consistency in repeated games, thereby examining several alternative solution concepts. In two papers above which define an equilibrium, Bernheim-Ray's definition of internal consistency coincides with Farrell-Maskin's definition of weakly renegotiation proof equilibrium; strongly renegotiation proof is also similar to strong consistency. However, the relative strong renegotiation proofness of Farrell-Maskin yields different results in specific games.

Asheim [2] extends the concept of Pareto-perfect equilibrium to infinite horizon games, using Greenberg [12]'s theory of social situations. Abreu-Pearce [1] show the conceptual foundations for the respective definitions in terms of stationary stable sets of credible deviations. Benoit-Krishna [4] examine renegotiation in finitely repeated games. In the limit, as the number of repetitions increases, the set of renegotiation-proof payoffs is either a singleton or Pareto-efficient, with the latter case being generic.

Bernheim-Peleg-Whinston [5] provide a coalitional approach to refining the set of Nash equilibria. They introduce a concept of efficient self-enforcing agreement, called a coalition-proof Nash equilibrium. A coalition-proof equilibrium is immune to stable deviations by groups of individuals, where a stable deviation is one which is coalition-proof in the subgame holding all other players' actions fixed. They also propose the notion of perfectly coalition-proof Nash equilibrium in the consistent dynamic situation of agreements: This equilibrium concept can refine the set of subgame perfect equilibrium in the case where communication and threats are allowed throughout the game. The recent studies on coalition-proofness with infinite horizon games appear in papers by DeMarzo [10] and Kahn-Mookherjee [13].

These refinements are very attractive concepts which model co-ordination among members of coalitions of economic agents, from the view point of incentive compatibility. Since individual agents can generally attain a higher payoff through non-cooperative behavior,² a coalition of agents has an incentive to co-ordinate only if a coalition formation makes all members better off.

In the information trading game, in order to examine the incentives of agents,

Nakayama—Quintas—Muto [15] propose a concept of resale proofness; it is characterized a set of informed individuals none of whom derives a benefit from selling the information to additional agents. The concept is essentially static.

Muto [14] describes this resale—proof notion in the game form of information trading, thereby showing an equilibrium outcome which is perfectly coalition proof Nash equilibrium. Although the game examined is dynamic, the situation is again static: resale is assumed to occur instantaneously and all benefits of the information accrue after the selling game is completed. This creates a tension in the model; if there is an endpoint to the trading period, a monopolist can do better by delaying sales until there is no opportunity for resale.

It seems that Muto's formulation misses the essence of the dynamic dilemma facing an informed agent. Viewed from today, it is not a matter of indifference to an individual to wait until tomorrow to sell, because this choice involves a tradeoff of capital values foregone for production benefits received in the interim. When these considerations are admitted into the model, we are able to obtain a more general formulation and clearer description of the dynamic problem.

The remainder of the paper proceeds as follows. Section 2 describes the basic idea of the game, notation, and definitions. In section 3, the basic two—period N —person game is described for a better understanding of the finite dynamic game. In section 4, first, the equilibria of the game are discussed through the concept of subgame perfection. Next, the following solution concepts are compared for the refinement: Pareto—perfect Nash equilibrium and perfectly coalition—proof Nash equilibrium. Finally, the relation between a resale—proof set and an equilibrium is examined. Some conclusions are given in section 5.

2. NOTATION AND DEFINITIONS

At the beginning of game, there is a single agent who is initially informed and he offers a price for the privilege of information. Uninformed agents then either accept or reject the offer at the given price. If the offer is accepted then the information is transferred at the offered price. In the next period, the process continues; now all the informed agents can offer the information to the remaining uninformed agents at new prices. The game continues T rounds where T is a finite number. In each round, in addition to purchases or sales, all informed agents gain a benefit from the use of the information. Throughout the paper, we make the following assumption.

Assumption 1. The value of information is nonincreasing in the number of information holders.

Let $N = \{1, 2, \dots, n\}$ be the set of all agents and $i \in N$ denotes a typical

agent. Also let Z be a nonempty set of informed agents. The per-period benefit to informed agents is denoted by the nonincreasing function $u(|Z|)$ where Z is the set of informed agents in that period. It is useful to consider the family of dynamic games.

$$\Gamma_t(Z) \text{ for } 1 \leq t \leq T \text{ and } Z \subseteq N$$

which indicates a t period game with Z informed agents in the initial period.

A one-shot game $\Gamma(Z)(=\Gamma_1(Z))$ can be described as follows. The strategy space of informed agent i is $I=\mathbb{R}_+$. For an uninformed agent j ($i \neq j$), the strategy space is $F_Z=\{f_j: \mathbb{R}_+^I \rightarrow Z \cup \{0\}\}$. An initially informed agent makes an offer, choosing a price. An uninformed agent observes the prices offered and chooses an individual from whom to purchase. (The uninformed agent can choose to purchase from no one; this is represented by the choice 0). In this case, if an uninformed agent accepts the offer then he pays the offered price and becomes informed. If Z agents are informed at the end of the one-shot game then each informed agent receives the payoff $u(|Z|)$.

In a multi-period game, the agent's payoff is the present discounted value of sales he makes plus the present discounted value of the per-period benefit of the information over the periods starting with the period in which he becomes informed. All agents have a common discount factor $\delta \in (0,1]$.

Let $S_Z=I \times F_Z^{N-Z}$ be the strategy space for the one shot game $\Gamma(Z)$, with typical element s . Each strategy generates a new set of informed agents; let

$$T_Z(\cdot): S_Z \rightarrow 2^N$$

represent the transition function. Note $T_Z(s) \supseteq Z$ for every $s \in S_Z$. In other words, we have $Z_t \subset Z_{t+1}$ by the transition rule, where the set Z_t is the set of informed agents at time t .

Let the initial set of informed agents be Z_0 . A history of length l is a sequence (s_0, s_1, \dots, s_l) , where for $t=0$, $s_0 \in S_{Z_0}$ and for all $t=1, 2, \dots, l$, $Z_t = T_{Z_{t-1}}(s_{t-1})$ for $s_{t-1} \in S_{Z_{t-1}}$.

The set of all histories can be defined by $H = \bigcup_{l=0}^T H_l$, where H_l is the set of all histories of length l . For a history h of length l , let $Z(h)$ denote Z_l , the set of agents who are informed at the end of that history. In a dynamic game, an agent i 's strategy σ_i is the choice of a strategy in the one shot game $\Gamma(Z(h))$ for each history. Let Σ_i denote a set of strategies for the agent i and also let $\Sigma = \prod_{i \in N} \Sigma_i$. In this case, a strategy profile is $\sigma(h): H \rightarrow S_{Z(h)}$. Then, we let $u(\sigma)$ denote the payoff to the i th agent under the strategy profile σ .

For a subgame following history $h \in H$, we let $\phi(h)$ be the set of Nash equilibria of the subgame. For a strategy σ , also let σ^h be the induced strategy in the subgame following history h . If $\hat{\sigma}$ is strictly preferred by all players in the game to σ , then we write $\hat{\sigma} \succ \sigma$.

In finite stage games Γ , Bernheim-Ray [6] define a Pareto-perfect Nash equilibrium, using Pareto-dominance refinement through recursive processes on the set of Nash equilibria. It is as follows:

Definition 2.1 (i) In a single stage game, a strategy profile σ is a Pareto-perfect Nash equilibrium if $\sigma \in \psi(h)$ and there does not exist another strategy profile $\tilde{\sigma} \in \psi(h)$ such that $\tilde{\sigma} > \sigma$.

(ii) For a finite stage game Γ , such that Pareto-perfect Nash equilibrium has been defined for all subgames, then, a strategy profile σ is a Pareto-perfect Nash equilibrium if and only if

(a) $\sigma \in \psi(h)$ and $\sigma|_{h'}$ is Pareto-perfect Nash equilibrium of h' for all h' following h .

(b) There does not exist another strategy profile $\tilde{\sigma} \in \psi(h)$ satisfying (a) such that $\tilde{\sigma} > \sigma$.

Let z be a (proper) subset of Z . Also let σ' denote the induced strategy in the game following z . Finally, for each z , let $\Gamma(z)$ denote the induced game on subgroup z .

For the finite stage game Γ , the notion of perfectly coalition-proof Nash equilibrium³ proposed by Bernheim-Peleg-Whinston [5] requires a dynamic consistency of deviations in terms of the number of agents and stages (the maximum number of nested proper subgames). The intuitive definition is recursive. It can be described as follows:

Definition 2.2 (i) For a single agent and single stage game Γ , a strategy profile σ is a perfectly coalition-proof Nash equilibrium if and only if σ maximizes $u_i(\sigma)$.

(ii) For z agents and t stage game $\Gamma(z)$ such that perfectly coalition-proof Nash equilibrium has been defined for all games, then, for any game $\Gamma_{t'}(z)$ where $t' \geq t$,

(a) A strategy profile σ is perfectly self-enforcing if, for all $z \in Z$, $\sigma|_z$ is a perfectly coalition-proof Nash equilibrium in the game $\Gamma_t(z)$ and if, for any proper subgame, the restricted σ is a perfectly coalition-proof Nash equilibrium in that subgame.

(b) A strategy profile σ is a perfectly coalition-proof Nash equilibrium if it is perfectly self-enforcing and if there does not exist another perfectly self-enforcing strategy profile $\tilde{\sigma}$ such that $u_i(\tilde{\sigma}) > u_i(\sigma)$ for all $i \in N$.

3. TWO-PERIOD N-PERSON GAME

In this section, we examine an information trading game in the case where there are two periods with N people, as a prelude to the general finite horizon dynamic game.⁴

Suppose that there is a single informed agent at the initial period 0. At the first period, the initially informed agent i offers a price p_i and then any uninformed agent either accepts or rejects the offered price. Let Z_1 be the number of agents who accept; these are the informed agents at the beginning of the second period. (Note that the informed people Z_1 includes a single agent who is initially informed.) In this case, we consider three possibilities:

1. $Z_1=N$, i.e., all uninformed agents accept.
2. $1<Z_1<N$, i.e., some uninformed agents accept.
3. $Z_1=1$, i.e., no uninformed agent accepts.

First, if all uninformed agents accept then the game ends and the payoff is

$$(1+\delta) u(N)+(N-1) p_i \text{ for the informed agent and} \\ (1+\delta) u(N)-p_i \text{ for every uninformed agent purchasing.}$$

Second, if some but not all agents accept in the first period then the Z_1 agents will compete in offering the information in the second period. Each informed agent i will offer a price \tilde{p}_i to sell the information. In this case, each remaining uninformed agent chooses whether to accept one of the informed agents' offers. If an uninformed agent purchases from an informed agent in period 2 then the present discounted value of the payoff for the uninformed agent is $\delta(u(Z_2)-\tilde{p}_i)$, where Z_2 is the number of informed agents at the end of the second period. If an uninformed agent buys from nobody then his payoff is zero.

In this subgame, Nash equilibria are of one of the following two forms: one is an equilibrium in which all informed agents offer prices so high that no purchase occurs. We will call this a "no-resale" equilibrium. The other possibility is Bertrand-like price competition among informed agents who will sell the good in period 2. In this case, the price will be zero. We will call this a "zero-price" equilibrium. Note that the other possible types of equilibria that may occur in the subgame belong to one of these two types of equilibria, i.e., no-resale and zero-price equilibria. To see that these are two possibilities, consider a situation in which a trade occurred at a positive price. Any informed agent who did not receive the sale would choose to reduce his price slightly below the price of the successful seller.

We now consider each possibility in turn. Let \bar{p} be the minimum \tilde{p} , Z_2-Z_1 uninformed agents purchase the information if

$$u(Z_2) \geq \bar{p}_i \geq u(Z_2+1).$$

The no-resale outcome is an equilibrium in the second period if

$$u(Z_1) \geq (Z_2-Z_1+1)u(Z_2) \text{ for all } Z_2 \text{ so that } Z_1 < Z_2 \leq N \quad (1)$$

where the lefthand side of (1) implies the informed agent's utility resulting from no-resale and the righthand side of (1) indicates the informed agent's payoff resulting from resale. Otherwise, an informed agent would deviate to offer the information to $Z_2 - Z_1$ uninformed agents at a price close to $u(Z_2)$ and this would be accepted.

For the whole game, if Z_1 sales occur in period 1 and no resales occur in period 2 then the payoff is

$(1+\delta)u(Z_1) + (Z_1 - 1)p_1$	for the initially informed agent who sold in period 1
$(1+\delta)u(Z_1) - p_1$	for the agents who purchased in period 1 and
zero	for the agents who do not purchase.

We consider the zero-price equilibrium occurs in the period 2 subgame. (This is always a subgame perfect equilibrium: given two sellers are offering zero price, no seller can successfully change to a different price.) In this subgame, all remaining individuals become informed. Thus the present discounted value of payoffs to agents in the two period game when $Z_1 - 1$ agents purchase in period 1 and everybody else purchases at price zero in period 2 are

$u(Z_1) + \delta u(N) + (Z_1 - 1)p_1$	for the initially informed agent
$u(Z_1) + \delta u(N) - p_1$	for an agent purchasing in period 1 and
$\delta u(N)$	for an agent purchasing in period 2.

Finally, suppose the initially informed agent makes no sales in period 1. Then, in period 2, he will choose a price \tilde{p}_1 to maximize his payoff. If he sells to $Z_2 - 1$ other individuals then he can sell for a price $\tilde{p}_1 = u(Z_2)$. Thus, his choice of \tilde{p}_1 is equivalent to a choice of Z_2 to maximize his payoff:

$$\max_{Z_2 \in \{1, 2, \dots, N\}} [u(1) + \delta Z_2 \cdot u(Z_2)].$$

All other individuals receive payoffs zero.

4. CHARACTERISTICS OF EQUILIBRIA

In section 4.1, we demonstrate that the notion of subgame perfection has multiple equilibria outcomes. Section 4.2 shows how the refinement concepts (Pareto-perfect Nash equilibrium and perfectly coalition-proof Nash equilibrium) reduce the number of such equilibria. The relation to resale-proofness with perfectly coalition-proof Nash equilibrium is examined in section 4.3.

4.1 Subgame Perfect Nash Equilibria

For the whole game, a subgame perfect Nash equilibrium can be examined

as follows. In this case, two cases are considered: when the zero-price equilibrium outcome occurs in every subgame and when the no-resale equilibrium outcome occurs in every subgame where equation (1) is satisfied.

We first consider the case where the zero-price equilibrium occurs. Assume that a price p_1 has been offered in the first period by a single agent who is initially informed. All other agents respond by accepting or rejecting the offer.

Given $Z_1 - 2$ other agents accept, the payoff to an agent who accepts in period 1 is

$$u(Z_1) + \delta u(N) - p_1$$

If an agent rejects in period 1 then his payoff is

$$\delta u(N)$$

provided some other agent accepts. (For then in the second period there will be a zero-price equilibrium.) It is zero if no other agent accepts. (For then the monopolist will change the monopoly price in the final period.) Thus, given p_1 , there is an equilibrium with Z_1 informed agents in the first period if

$$\begin{array}{ll} p_1 \leq u(N) & \text{for } Z_1 = N \\ u(Z_1 + 1) \leq p_1 \leq u(Z_1) & \text{for } Z_1 \in \{3, 4, \dots, N-1\} \\ u(3) \leq p_1 \leq u(2) + \delta u(N) & \text{for } Z_1 = 2 \\ u(2) + \delta u(N) \leq p_1 & \text{for } Z_1 = 1 \end{array}$$

Given these responses, agent 1 who is initially informed will choose the price p_1 to maximize his own payoff. The results can be described as follows:

Theorem 4.1 If the zero-price subgame equilibria occur for all subgames, then in equilibrium, the initially informed agent's payoff is the largest of

$$\begin{aligned} & \max_{Z_1 \in \{3, 4, \dots, N\}} [Z_1 \cdot u(Z_1) + \delta u(N)] \\ & 2[u(2) + \delta u(N)] \\ & \max_{Z_2 \in \{1, 2, \dots, N\}} [u(1) + \delta Z_2 \cdot u(Z_2)] \end{aligned}$$

The first expression is the maximum revenue which the initial seller can achieve, given he sells to $Z_1 - 1$ individuals in the first period in which $Z_1 > 2$. The second expression is the maximal revenue which the initial seller can obtain when selling to exactly one individual in the first period. The final expression is the revenue achieved by selling to no one in the first period and acting as a static monopolist in the final period.

Consider the case where the no-resale equilibrium occurs. Let Z be the set of $Z_1 \geq 2$ such that equation (1) is violated. For these values of Z_1 , the zero-

price outcome is inevitable for period 2. Given that $Z_1 - 2$ other players accept, the payoff to an agent who accepts in period 1 is

$$\begin{aligned} u(Z_1) + \delta u(N) - p_1 & \quad \text{if } Z_1 \in Z \\ u(Z_1) + \delta u(Z_1) - p_1 & \quad \text{if } Z_1 \notin Z. \end{aligned}$$

If an agent rejects in period 1 where $Z_1 - 2$ others accept then his payoff is

$$\begin{aligned} \delta u(N) & \quad \text{if } Z_1 - 1 \in Z \\ 0 & \quad \text{if } Z_1 - 1 \notin Z. \end{aligned}$$

Therefore, Z_1 is an equilibrium for a price p_1 if

$$u(Z_1 + 1) + \delta[v(Z_1 + 1) - v_i(Z_1)] \leq p_1 \leq u(Z_1) + \delta[v(Z_1) - v_i(Z_1 - 1)]$$

where

$$v_i(Z_1 - 1) = \begin{cases} u(N) & \text{if } Z_1 - 1 \in Z \\ 0 & \text{if } Z_1 - 1 \notin Z \end{cases} \quad v_i(Z_1) = \begin{cases} u(N) & \text{if } Z_1 \in Z \\ u(Z_1) & \text{if } Z_1 \notin Z \end{cases}$$

in which $v_i(\cdot)$ and $v(\cdot)$ denote the value being uninformed and being informed respectively.

Note that this means it is possible to have multiple deterministic equilibria for some levels of prices, and no deterministic equilibria for other levels of prices. Then, the theorem is somewhat weaker:

Theorem 4.2 There exists an equilibrium where the initially informed agent receives the larger of the following payoffs:

$$\begin{aligned} \max_{Z_1 \in \{2, 3, \dots, N\}} [Z_1(u(Z_1) + \delta v_i(Z_1)) - \delta(Z_1 - 1) v_i(Z_1 - 1)] \\ \max_{Z_2 \in \{1, 2, \dots, N\}} [u(1) + \delta Z_2 \cdot u(Z_2)] \end{aligned}$$

This describes an equilibrium in which the no-resale equilibrium is chosen in any subgame where it exists. We can generalize this as follows: let \tilde{Z} be any superset of Z , not including $Z_1 = 1$. Then, substituting \tilde{Z} for Z in the definition of v_u and v_i describes an equilibrium in which the zero-price equilibrium is chosen in any subgame where $Z_1 \in \tilde{Z}$.

Theorem 4.1 and 4.2 show that if equation (1) holds then there can be two different equilibrium payoffs for the initially informed agent 1 in this game. One equilibrium payoff assumes that in the subgame with Z_1 informed agents, nobody will sell. The other equilibrium payoff assumes that they will sell in the subgame with Z_1 informed agents. In some cases, they are indifferent between the two equilibria; in other case, some of them will prefer the no-resale

equilibrium. For a better understanding of the above game, let us consider the following example:

Example 1. $N = \{1,2,3\}$. $u(1) = 11$, $u(2) = 8$ and $u(3) = 4$. Agent 1 is assumed to be initially informed. Assume that $\delta=3/4$. Note that this is a case in which equation (1) holds for $Z_1=2$. In the subgame where the informed agent holds back to sell in the second period, the equilibrium second period price is 8 and exactly one uninformed player accepts; then the equilibrium payoff is (23, 0, 0).

If the zero-price equilibrium occurs in subgames where $Z_1=2$ then the payoffs of agent 2 and 3 are as follows.

Table 1.

		Agent 3	
		a	r
Agent 2	a	$(7-p_1, 7-p_1)$	$(11-p_1, 3)$
	r	$(3, 11-p_1)$	$(0, 0)$

where 'a' and 'r' denote the 'acceptance' and the 'rejection' of agent 1's first period price p_1 respectively.^{5,6}

The payoff of agent 1 as a function of his first period choice p_1 is

$$\begin{array}{ll} 23 & \text{if } 11 < p_1 \\ 11 + p_1 & \text{if } 4 \leq p_1 < 11 \\ 7 + 2p_1 & \text{if } p_1 \leq 4. \end{array}$$

There are two types of equilibria in the subgame following $p_1=11$. One equilibrium is (a,r) (or (r,a)) in which one agent purchases in the first period and both informed agents offer the information to the uninformed agent at zero prices in the second period. The equilibrium payoff is (22, 0, 3) (or (22, 3, 0)). The other equilibrium is (r, r) in which agent 1 waits and sells at $p_1=8$ in the second period to a single agent. Then, for the whole game, this equilibrium is the subgame perfect Nash equilibrium, generating the equilibrium payoff (23, 0, 0).

If the no-resale equilibrium occurs in both subgames then the payoff matrix of agent 2 and 3 is as follows.

Table 2.

		Agent 3	
		a	r
Agent 2	a	$(7-p_1, 7-p_1)$	$(14-p_1, 0)$
	r	$(0, 14-p_1)$	$(0, 0)$

In this case, agent 1's payoff as a function of his first period choice p_1 is

$$\begin{array}{ll} 23 & \text{if } 14 < p_1 \\ 14 + p_1 & \text{if } 7 \leq p_1 < 14 \\ 7 + 2p_1 & \text{if } p_1 \leq 7. \end{array}$$

If $p_1 = 14$ then there are two types of equilibria in the subgame which follows. One equilibrium is (a, r) (or (r, a)) in which a single informed agent purchases at $p_1 = 14$ in the first period and neither sells to the remaining agent in the second period. Thus, for the whole game, this equilibrium is the subgame perfect Nash equilibrium, generating the equilibrium payoff $(28, 0, 0)$. The other equilibrium is (r, r) which is identical to the zero-price equilibrium case.

To summarize, there are two types of equilibria in this example. In the first type, agent 1 sells to one agent in the first period and no resales are made in the second period. In the second type, agent 1 expects that if he sells early then the good will be resold. Therefore, he waits until the final period and sells to a single agent. Both of these are subgame perfect, although agent 1 strictly prefers the early sale (no-resale) equilibrium.

4.2 Refinements of Subgame Perfect Nash Equilibria

For a refinement of subgame perfect equilibria in the dynamic game, we use the concepts of Pareto-perfect Nash equilibrium and perfectly coalition-proof Nash equilibrium.

First, we look for Pareto-perfect Nash equilibrium. We begin by examining the equilibria in the second period. If equation (1) holds then the important subgame is the subgame where $1 < Z_i < N$ because this has two equilibria, a zero-price equilibrium and a no-resale equilibrium as described above. Since neither Pareto dominates the other, both are Pareto-perfect Nash equilibria for the subgame. If equation (1) does not hold, then there is a unique zero-price equilibrium. Pareto-perfect Nash equilibria for the whole game are similar to the subgame perfect Nash equilibria. (Note that if a weak Pareto dominance is used in Pareto-perfect Nash equilibrium, it generates a unique no-resale equilibrium.)

Next, we look for perfectly coalition-proof Nash equilibrium. We begin with an examination of any second period subgame where equation (1) holds. In this case, the Z_i informed agents prefer the no-resale equilibrium. It is a unique perfectly coalition-proof Nash equilibrium because no proper subset of agents can make higher payoffs from any deviation, given the other's action as fixed. (Note that the zero-price equilibrium payoff which an agent purchases in period 2 is $\partial u(N)$, while the no-resale equilibrium payoff is zero for the agents who do not purchase in period 2.) Then we have the following conclusion:

Theorem 4.3 In any perfectly coalition-proof Nash equilibrium, the no-resale equilibrium occurs in any second period subgame where Z_1 satisfies the following equation (1):

$$u(Z_1) \geq (Z_2 - Z_1 + 1)u(Z_2) \text{ for all } Z_2 \text{ so that } Z_1 < Z_2 \leq N$$

For many players, the analysis becomes extremely involved. For a better understanding, we examine a three-agent example:

Example 2. We repeat Example 1 in which there are multiple subgame perfect Nash equilibria. Note that equation (1) holds for $Z_1=2$ in this example.

First, we consider Pareto-perfect Nash equilibrium. In the subgames where $Z_1=2$, since neither Pareto dominates the other, Pareto-perfect Nash equilibrium generates the following equilibrium payoffs: both (22, 0, 3) (or (22, 3, 0)) under the zero-price equilibrium and (28, 0, 0) under the no-resale equilibrium.

For the whole game, Pareto-perfect Nash equilibrium generates the equilibrium payoffs, both (28, 0, 0) under the no-resale equilibrium and the equilibrium payoff (23, 0, 0) in the subgame where $Z_1=1$. In this case, Pareto-perfect Nash equilibria are identical to the subgame perfect Nash equilibria. (Note that if we use a weak Pareto dominance in the definition of Pareto-perfect Nash equilibrium then the payoff (23, 0, 0) will be eliminated.)

Next, we consider perfectly coalition-proof Nash equilibrium. In the subgames where $Z_1=2$, perfectly coalition-proof Nash equilibrium generates a unique no-resale equilibrium payoff (28, 0, 0) because no deviation occurs any more under that equilibrium. (A deviation occurs at the zero-price equilibrium).

For the whole game, the no-resale equilibrium payoff is generated by perfectly coalition-proof Nash equilibrium because agent 1's payoff under that equilibrium is greater than that under the subgame $Z_1=1$ that is $28 > 23$.

Example 2 shows that if equation (1) holds for some values of Z_1 then perfectly coalition-proof Nash equilibrium refines subgame perfect equilibria which are identical to Pareto-perfect Nash equilibria. This result shows the strength of perfectly coalition-proof Nash equilibrium.

4.3 Relation to Resale Proofness

We have examined a finite horizon (two-period) game in which there is a marginal benefit with a delay cost in selling the information. Such information is usable over time, the longer the buyer must wait to receive it the less use he can make of it.

In such a world, we have shown that some early sales may not be feasible

because of the possibility of renegotiation. Thus, renegotiation can reduce the number of equilibrium outcomes. Suppose on the other hand, we had assumed no delay cost in trading. For example, suppose the information would only be used beginning at a period which lay beyond the sequence of sale periods in the model. In such a world, trading can always be delayed so that the initial informed agent waits and sells at the final period. Then, renegotiation provides no limitation on multiple equilibria; every perfect equilibrium outcome is a renegotiation-proof outcome.

In this case, the dynamic structure of the model is a crucial element in the determination of outcomes. In a static approach for the information trading problem, Nakayama-Quintas-Muto [15] have introduced the notion of a resale-proof set, a set of informed agents none of whom derives a benefit from selling the information to additional agents. The purpose of this section is to relate our results to their formulation. They define a resale-proof set as follows:

Definition 4.1 If $Z=N$ then Z is resale-proof. When $Z \neq N$, Z is resale-proof if $u(Z) \geq (\bar{Z} - Z + 1) u(\bar{Z})$ for any resale-proof (\bar{Z}) where $Z < \bar{Z} \leq N$.

There are two difficulties with this concept. The first is that the value of information in any dynamic game with multiple equilibria depends on which equilibrium is assumed in any subgame. The second is that the value of information ought to be a marginal concept: the value is the difference between being informed and not being informed when $Z-1$ other people are informed. This is the amount that the initial seller can extract from all purchasers. Implicitly, in Muto's framework, the value of being uninformed is always zero. For us the value depends on whether the uninformed individuals will not receive the information a future period or whether they will receive it during a price war among the informed individuals.

We have a similar definition:

Definition 4.2 The number Z_i is one period resale proof if and only if

$$u(Z_i) \geq (\bar{Z}_i - Z_i + 1) u(\bar{Z}_i) \text{ for any } \bar{Z}_i \text{ where } Z_i < \bar{Z}_i \leq N.$$

In one period, for any \bar{Z}_i , $Z_i < \bar{Z}_i \leq N$, definition 4.2 shows that informed people Z_i will not sell the information to additional people $\bar{Z}_i - Z_i$ if the payoff of informed people Z_i resulting from no-resale is no less than that resulting from selling to additional people $\bar{Z}_i - Z_i$. In other words, there is no incentive for Z_i informed people who attempt to sell the information and make more profit possible. Then, definition 4.2 does not require that at the end of trading for any period, \bar{Z}_i is always assumed to be resale proof as shown in Nakayama-

Quintas-Muto [15] definition. We relax this assumption in definition 4.2 because this assumption is not reasonable in the dynamic structure of information trading game: it is possible for \tilde{Z}_1 to be no resale proof number. This leads to the following conclusion:

Theorem 4.4 Suppose Z_1 is one period resale proof. $Z_1 - 1$ is one period resale proof if $u(Z_1 - 1) \geq 2u(Z_1)$.

In two period, if \tilde{Z}_1 is one period resale proof then the present value of the information to a purchaser is

$$u(\tilde{Z}_1) (1 + \delta).$$

If \tilde{Z}_1 is not one period resale proof then the present value of the information is

$$u(\tilde{Z}_1) + \delta u(N).$$

In this case, we conjecture that if the number of informed people is not resale proof and not equal to one then all remaining people will receive the information at a zero price. However, a two period notion of resale proof must take into account what the second period value of information actually is. Let Z be the set of Z people which are not one period resale proof. Then, define the following: one period value of being uninformed is

$$v_1(Z - 1) = \begin{cases} u(N) & \text{if } Z - 1 \in Z \\ 0 & \text{if } Z - 1 \notin Z \end{cases}$$

And one period value of being informed is

$$v_1(Z) = \begin{cases} u(N) & \text{if } Z \in Z \\ u(Z) & \text{if } Z \notin Z \end{cases}$$

(Note that these are defined similarly as described in section 4.1.) Then we have the following two period resale proof definition:

Definition 4.3 The number Z_2 is two period resale proof if

$$(1 + \delta)u(Z_2) \geq (\tilde{Z}_2 - Z_2 + 1)u(\tilde{Z}_2) + \delta[v_1(\tilde{Z}_2) + (\tilde{Z}_2 - Z_2) \{v_1(\tilde{Z}_2) - v_1(\tilde{Z}_2 - 1)\}]$$

for any \tilde{Z}_2 where $Z_2 < \tilde{Z}_2 \leq N$.

In two period, unlike Muto [14], definition 4.3 considers the marginal concept of the value of information in the second period: the different value between being informed and not being informed when people become informed.

This leads to more reasonable specification of the dynamic information trading game.

From the above, the relation between a resale-proof set and an equilibrium can be described as follows:

Theorem 4.5 If 1 is two period resale proof and there is a number Z_2 such that $\delta Z_2 \cdot u(Z_2) > u(1)$ then there exists a unique perfectly coalition-proof Nash equilibrium and it has no sales in the first period.

Proof. Suppose $Z_2=1$ is two period resale proof. Then, from definition 4.3, we have the following conditions:

$$(1+\delta)u(1) \geq \tilde{Z}_2 \cdot u(\tilde{Z}_2) + \delta u(N) \quad \text{for } \tilde{Z}_2 \in Z, \tilde{Z}_2 - 1 \in Z \quad (\text{a-1})$$

$$(1+\delta)u(1) \geq \tilde{Z}_2 \cdot [u(\tilde{Z}_2) + \delta u(N)] \quad \text{for } \tilde{Z}_2 \in Z, \tilde{Z}_2 - 1 \in Z \quad (\text{a-2})$$

$$(1+\delta)u(1) \geq (1+\delta)\tilde{Z}_2 \cdot u(\tilde{Z}_2) - \delta(\tilde{Z}_2 - 1)u(N) \quad \text{for } \tilde{Z}_2 \notin Z, \tilde{Z}_2 - 1 \in Z \quad (\text{a-3})$$

$$(1+\delta)u(1) \geq (1+\delta)\tilde{Z}_2 \cdot u(\tilde{Z}_2) \quad \text{for } \tilde{Z}_2 \notin Z, \tilde{Z}_2 - 1 \notin Z \quad (\text{a-4})$$

Let Z_1^* and Z_2^* be a solution to the first and the second maximization problem respectively in theorem 4.2:

$$\begin{aligned} \max_{Z_1 \in \{2, 3, \dots, M\}} & [Z_1 \{u(Z_1) + \delta v_1(Z_1)\} - \delta(Z_1 - 1)v_1(Z_1 - 1)] \\ \max_{Z_2 \in \{1, 2, \dots, M\}} & [u(1) + \delta Z_2 \cdot u(Z_2)] \end{aligned}$$

Then, an equilibrium payoff for the initially informed seller is the largest of

$$Z_1^* u(Z_1^*) + \delta u(N) \quad \text{for } Z_1^* \in Z, Z_1^* - 1 \in Z \quad (\text{b-1})$$

$$Z_1^* [u(Z_1^*) + \delta u(N)] \quad \text{for } Z_1^* \in Z, Z_1^* - 1 \notin Z \quad (\text{b-2})$$

$$(1+\delta)Z_1^* u(Z_1^*) - \delta(Z_1^* - 1)u(N) \quad \text{for } Z_1^* \notin Z, Z_1^* - 1 \in Z \quad (\text{b-3})$$

$$(1+\delta)Z_1^* u(Z_1^*) \quad \text{for } Z_1^* \notin Z, Z_1^* - 1 \notin Z \quad (\text{b-4})$$

$$u(1) + \delta Z_2^* u(Z_2^*) \quad \text{under no sales in period 1.} \quad (\text{b-5})$$

Substituting Z_1^* into Z_2^* in (a-1) - (a-4), we find that the payoff $(1+\delta)u(1)$ resulting from no sale in period 1 dominates the others (b-1) - (b-4). Then, from (b-5), there is a unique perfectly coalition-proof Nash equilibrium where there is no sale in the first period if there is an optimal number Z_2^* such that

$$\delta Z_2^* u(Z_2^*) > u(1).$$

This indicates that the payoff for initially informed single seller resulting from no sale in the first period is the greatest of all others; then, this leads to a

unique perfectly coalition-proof Nash equilibrium for the whole game.

If 1 were strictly two period resale proof (by which we mean all the inequalities in the definition of two period resale proofness hold strictly), then the theorem 4.5 would hold without requiring the existence of Z_2 .

In the process of information trading, Muto [14] argues that the information will not diffuse when the set of informed agents is resale-proof; the initially informed agent shares the information with the uninformed agent so that a minimal resale-proof set is attained; the initially informed agent exploits the whole surplus obtained by the sharing. In Theorem 4.5, we have a similar result: Perfectly coalition-proof Nash equilibrium supports the minimal resale-proof number which satisfies the resale-proof condition in definition 4.3.

The difference is that the notion of resale-proof in games with delay costs depends on the equilibrium assumed for the future. We argue that not all resale-proof numbers are natural in games with delay costs because not all subgame perfect equilibria are natural. Perfectly coalition-proof Nash equilibrium provides natural restrictions on the number of subgame perfect equilibria.

Consider the following example for a better understanding of the relation between resale-proof numbers and equilibria.

Example 3. $N = \{1, 2, 3\}$, $u(1) = 15$, $u(2) = 8$, and $u(3) = 3$. Assume that $\delta = 1$. Note that equation (1) holds for $Z_1 = 2$ and there are multiple subgame equilibria.⁷

From definition 4.2, the resale proofness in this example is as follows: 3 is one period resale proof. 2 is one period resale proof because $8 > 2 \times 3$. But 1 is not one period resale proof because $15 > 2 \times 8$. Let Z be the set of Z people which are not one period resale proof. Then, $Z = \{1\}$. Note that Z coincides with the definition of Z in section 4.3.

From definition 4.3, 1 is two period resale proof because

$$30 = 2u(1) \geq 2\tilde{Z}_2 \cdot u(\tilde{Z}_2) - (\tilde{Z}_2 - 1)u(N) = 29 \quad (4.1)$$

where $\tilde{Z}_2 = 2$ for $\tilde{Z}_2 \in Z$, $\tilde{Z}_2 - 1 \in Z$, and

$$30 = 2u(1) \geq 2\tilde{Z}_2 \cdot u(\tilde{Z}_2) = 18 \quad (4.2)$$

where $\tilde{Z}_2 = 3$ for $\tilde{Z}_2 \in Z$, $\tilde{Z}_2 - 1 \in Z$. However, 2 and 3 are not two period resale proof because, from (4.2), $16 = 2u(2) \not\geq 2 \cdot 3u(3) = 18$ and $6 = 2u(3) \not\geq 2 \cdot 3u(3) = 18$.

Thus, perfectly coalition-proof Nash equilibrium in this example can be described in terms of resale proof sets. Let Z_1^1 be a solution to the first maximization problem in theorem 4.2. Let Z_1^2 be a solution to the second maximization problem. Thus, an equilibrium payoff for the initially informed seller is the largest of

$$2 Z_i^* u(Z_i^*) - (Z_i^* - 1)u(N) \quad \text{for } Z_i^* \in Z, Z_i^* - 1 \in Z \quad (4.3)$$

$$2 Z_i^* u(Z_i^*) \quad \text{for } Z_i^* \in Z, Z_i^* - 1 \in Z \quad (4.4)$$

$$u(1) + Z_i^* u(Z_i^*) \quad \text{under no sales in period 1.} \quad (4.5)$$

In this case, when $Z_i^* = 2$, the payoff for the initially informed seller is 29 from equation (4.3). When $Z_i^* = 3$, the initially informed seller's payoff is 18 from equation (4.4).

Since $\tilde{Z}_i = Z_i^*$, the payoff $2u(1) = 30$ resulting from no sale in period 1 dominates the others (4.3) – (4.4). It follows from (4.5) that there is a number $Z_i^* = 2$ such that $16 = Z_i^* u(Z_i^*) > u(1) = 15$. This leads to a unique perfectly coalition-proof Nash equilibrium where there is no sale in the first period.

5. CONCLUSIONS

We have examined the concepts of Pareto-perfect Nash equilibrium and perfectly coalition-proof Nash equilibrium in a dynamic game of information acquisition. For the analysis, we have focused on the case where there is a marginal benefit with delay cost under a history dependence which will affect the agent's strategy on the purchase of information, thereby determining actual self-enforcing agreements which lead to resale-proofness.

In many games, renegotiation-proof threats can force cooperation — players agree to follow a given equilibrium if there is no better one available. This leads to a possibility of smaller coalition formation of players as in Bernheim-Peleg-Whinston [5]. In the present paper, we have shown that the finite horizon (two-period) dynamic game in the case where there is a marginal benefit with delay costs leads to a unique stable equilibrium payoff which is generated by perfectly coalition-proof Nash equilibrium and then how it is resale-proof. Moreover, we argue that not all resale-proof sets are natural in games with delay costs because not all subgame perfect equilibria are natural. Therefore, perfectly coalition-proof Nash equilibrium provides natural restrictions on the set of subgame perfect equilibrium.

6. FOOTNOTES

1. Muto [14] has studied such issues. This paper's formulation differs in that the good is used during the trading process. As the discount rate approaches zero, this analysis reduces to his. Throughout the present paper, the informed agents are assumed to be taken first actions. Vincent [20] examines dynamic games in which uninformed agents first take actions.

2. Suppose there are two farmers that co-ordinate their activities. Both of them might work with less effort and so the resulting payoff might be less than under non-cooperative behavior, even though more efficient division of labor is possible.

3. The coalition-proof Nash equilibrium is similarly defined as the notion of perfectly coalition-proof Nash equilibrium if the stages are not considered in perfectly coalition-proof Nash equilibrium.

4. The general finite horizon game can be similarly extended from two-period game rule. In the finite horizon game, we conjecture that perfectly coalition-proof Nash equilibrium provides natural restrictions on the set of subgame perfect equilibrium. The further study on this issue including infinite horizon is required.

5. In this payoff matrix table, the payoffs are calculated from the payoff in the appropriate subgames: for example, if both accept, then the payoffs are derived from the subgame where $Z_1=3$. They are $7+2p_1$ for agent 1, $7-p_1$ for agent 2, and $7-p_1$ for agent 3.

6. Suppose $u(1)=11$, $u(2)=8$ and $u(3)=6$ so that $2u(2) < 3u(3)$. Note that $\delta=3/4$. If the zero-price equilibrium occurs then the payoff for agent 1 is

$$\begin{array}{ll} 23 & \text{if } 12.5 < p_1 \\ 12.5 + p_1 & \text{if } 6 \leq p_1 < 12.5 \\ 10.5 + 2p_1 & \text{if } p_1 \leq 6 \end{array}$$

In this case, there is a unique zero-price equilibrium which generates the payoff (25, 0, 4.5) in the subgame where $Z_1=2$.

7. These equilibria can be verified from applying to example 1.

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