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EVIDENCE FORGING COLLUSIONS IN HIERARCHICAL ORGANIZATIONS

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We study the problem of designing some optimal collusion free contracts in the simple three-tier principal/supervisor/agent hierarchical structures. We conider two types of information manipulation as a coalition between the supervisor and the agents: (I) Ignoring relevant information and (II) Creating false information. We show that the principal can design optimal collusion free contracts with some additional cost by putting proper incentive compatibility conditions and individual rationality conditions. We find that the optimal collusion contract is the prespecified allocation rule, so that the evaluation about the agent does not depend on the report by the supervisor, who is simultaneously "judge and party". In our model, it turns out that the supervisor has a degree of freedom to act either as an advocator for the principal or for the agent or neither, which is differ from the Tirole[1986]'s main results that the supervisor naturally acts as an advocator for the agent. We find that the role and the behavior of the supervisor within the hierarchical organization crucially depend upon not only the possibility of collusion but more importantly the nature of collusion, which is the nature of information manipulation.

I. INTRODUCTION

Organizations are often considered as networks of overlapping principal-agent relationships (See e.g., Williamson [1967b], Alchian and Demsetz[1972], Mirrlees[1976] and Calvo and Wellisz[1978]. As is well known, in principal-agent models, there is no room for collusive behavior between two parties, because they have strictly conflicting objectives.[1] However, many studies of the organizations and bureaucracies have shown that collusive behavior, implicit or explicit, does exit and is often prominent.[2] For example, the manager/auditor collusion has been studied in various con

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1) We have one principal-many agents type model (e.g. Demski and Sappington[1984], Holstrom[1982] and Nalebuff and Stiglitz[1983]) and recently many principals-one agent type model (e.g. Baron[1985], Bernheim and Whinston[1985], [1986], Cremer and Riordan[1987]). However, in there models, they did not explicitly consider the collusive behavior.

2) See part 2 of Tirole[1986]. He largely referred this point the studies of Crozier[1963] and Dalton[1959]. Also see Holmstrom and Tirole[1987] and Tirole[1990] for various aspects of collusive behavior within hierarchical organizations.

texts for a long time.[3] Therefore, the analysis of the hierarchical organizations cannot boil down to two-tier principal/agent structures.

Principal/agent theory has paid considerable attention to the incentive problems which arise in two-layer hierarchies (Baron-Myerson[1982], Maskin-Riley[1984] and Laffont-Tirole[1986]). There have been some literature, which incorporate a third layer, usually a supervisor, in order to mitigate the incentive problems from asymmetric information(Baron-Besanko[1984], Demski-Sappington[1987], and Baiman, Evans and Noel[1987]).

The research in this area has largely ignored the possibility of collussion between two different layers. Major exceptions to this tradition are Tirole[1986] and Antle [1984], who study the effect of collusive behavior within multilateral organizations. Specifically, they considered three-tier Principal/Supervisor/Agent hierarchy and characterized the coalition proof contracts when the agent and the supervisor can collude about their reports to the principal. However most of previous studies considered only hard infomation, which is verifiable information, we consider hard and soft information, which is not verifiable information. Since collusive behavior in this study means manipulation of relevant information, it needs to be extended to the soft information case. Sociological studies on collusive behavior have observed two types of manipulation of information[4]

(I) Ignoring or concealing relevant information.(only hard information case)

(II) Distorting the information or creating false information

Ignoring or concealing relevant information might considered as the more implicit form of collusion. For example, supervisors usually neglect to report the employee's minor use of materials and services for personal ends, as far as it no too serious. This is because reporting this information often leads to bad evaluation of the suprvisor himself and/or processing this information is costly. Hence, the supervisor usually ignores this observation, and implicitly colludes for the benefit of both parties. Distorting the information can be considered as a more explicit form of collusion. A well known example is the coalition between top managers and accountants. For example, if the manager's salary level depends on the tota sales for a year, they may try to manipulate the sales records, for intance, by adding the sales of the first month of this year to last year's sales level. Distorting information includes both changing the level of given parameters or records and creating false information. In this paper, we consider both type of manipulation, whereas most of Literature confined to type(I) behavior, Tirole interprets his results as showing that the supervisor naturally acts as an advocator for the agent. However, we will show that this result crucially hinges on the fact that he conseders only type(II) behavior.

3) See part 2 of Tirole[1986]. Also Antle[1982], [1984] and Williamson[1975] discuss this problem explcitly. I will discuss this problem also in part 2.

4) See Crozier[1963] and Dalton[1959] for the detail.

In this paper, we concentrate on collusion within the firm.[5] The principal is the owner or top manager of the firm. The principal needs to hire a supervisor for various reasons. For example, he needs the supervisor's productive activities such as coordination, organization, counseling and selecting the agent etc. Also, the principal wants to monitor the agent and sometimes has to observe the agent's private information. However, he may not have time for this kind of monitoring because either he has too many agents to monitor, or he wants to spent his time at other activities such as long term planning. Because of this, the principal needs a specialist for monitoring.

The rest of paper is organized in the following way. Part 2 describes the simple one principal/one agent model. The principal cannot dbserve either the agent's effort of his private information (for example, realization of some production parameter). By hiring the supervisor, the principal can get some information. Uncertainties, information and the supervisor's reporting technology is described and some institutional assumptions about the bargaining process within the coalition are introduced.

In part 3, we investigate the mechanism(contract)which can guarantee a collusion free outcome to the principal. The principal can design a contract which implement the collusion free allocation by imposing the proper constraints. We investigate the properties of coalition proof contracts and also find that the principal should pay some cost for this mechanism, That is, the suboptimality of the agent's effort in the coalition proof mechanism is more severe than in the overt contract case(the possibility of collusion is not allowed). Brief conclusions and some suggedtions for the future research is provided.

2. THE MODEL

We consider a firm as an example of the three tier principal/supervisor/agent relationship. The owner of a production process, the principal, wants to hire a worker, the agent, to perform some productive activity. As mentioned in the introduction. the principal also wants to hire a supervisor in order to monitor the agent. It is implicity assumed that the principal lacks the time and/or the special knowledge to efficiently monitor the agent. We start with the standard principal agent type model, and then we put more structures on this model in order to investigate the effect of coalition between the two parties, the sepervisor and the agents.

2.1 The Players

The agent(worker) chooses a level of effort $e \geq 0$, which together with the realization of an exogenous productivity parameter θ determines the profit X.

$$X = f(\theta,e) \text{ or simply } X = \theta + e$$

5) However, we can find three tier structure in other context. For example, voter/congressman or senator/government, voter(people)/department of defense/defense contractor. publisher of journal/referee/paper submitter and stockholder/manager/worker and so on. Tirole[1986] provided lots of interesting examples(see p183 of Tirole[1986]).

The function g(e) represents the agent's disutility of effort in monetary terms, where $g(\cdot)$ is increasing, strictly convex and satisties $g(0)=g'(0)=0$. The agent has an increasing, differentiable and strictly concave Von Neuman-Morgenstern utility function $U(\cdot)$. Hence, his expected utility is $EU(W-g(e))$, where W is the wage he receives from the principal. We assume that there exists ex-ante competition in the supply of agents, with reservation wage W_0 and reservation unility level $\bar{U}=EU(W_0)$. This gives the agent's participation constraint(equivalently the agent's individual rationality constraint. From now on, we will represent this AIR):

$$\text{(AIR)} \quad EU(W-g(e)) \geq \bar{U}$$

The supervisor also has a Von Neuman-Morgenstern utility function, $V(\cdot)$, which is increasing, differentiable and strictly concave. Let S be the wage which the supervisor receives from the principal. In this paper, we do not consider any kind of productive activity of the supervisor. Furthermore we assume that he exerts no effort in supervising the agent. In this model, supervision involves only information gathering and reporting.[6] Again, we assume that there exists ex-ante competitive supply of supervisors, with reservation wage S_0 and reservation utility $\bar{V} \equiv V(S_0)$. The supervisor's individual rationality condition is:[7]

$$\text{(SIR)} \quad EV(S) \geq \bar{V}$$

Later, when discuss coalitions, we will explain in more datail the supervisor's role and responsibility.

The principal is the owner of the production process, and offers a contract to the supervisor and the agent. His expected utility is $E(X-S-W)$. This assumption implies that the principal is risk neutral. In other words, he takes all the risk and the supervisor has no risk sharing role. Finally, all players are assumed to be expected utility maximizers.

2. Information Structure and timing of the model

The uncertainty in the model stems from the randomness of productivity parameter. The principal cannot observe the realization of θ. However, the agent, after he accepts the contract, can observe θ before he chooses his effrot level. The principal hires the supervisor in order to get some information about θ and asks the supervisor to report his observation. We assume, however, that the supervisor cannot always

6) This assumption is also made for the simplicity of the model. We can introduce the supervisor's effort for information gathering in two different directions. First, we can assume that if the supervisor put some effort level a^* or more, he can always observe the true θ, otherwise he oberves nothing. Second, we can consider the uncertainty structure such that the probability for the supervisor to observe θ depends on the supervisor's level of effort for gathering the information.

7) We know that the supervisor's opportunity cost of gathering the information is zero. This means the principal hires the supervisor for the other productive activity. However, it will be more realistic to assume that.

observe the productivity parameter θ, which can take only two values : $\underline{\theta}$ and $\bar{\theta}$ such that $\Delta\theta \equiv \bar{\theta} - \underline{\theta}$ is strictly positive. $\underline{\theta}$ represents the low productivity state and $\bar{\theta}$, the high productivity state. By combining the two levels of productivity parameter and the cases whether the supervisor can observe θ or not, we have 4 states of nature as follows (S and A mean supervisor and agent respectively):

state 1 : $\theta = \underline{\theta}$, observed by both A and S

state 2 : $\theta = \underline{\theta}$, observed only by A (S observes nothing)

state 3 : $\theta = \bar{\theta}$, observed only by A (S observes nothing)

state 4 : $\theta = \bar{\theta}$, observed by both A and S

We assume that state of nature i occurs with probability $P_i (\sum_{i=1}^{4} P_i = 1)$[8]

Thus, in states 2 and 3, the supervisor can not observe the realized productivity parameter. In other words, he can not distinguish whether the agent observes $\underline{\theta}$ or $\bar{\theta}$, The agent's observation set is

$$a \in \{\underline{\theta}, \bar{\theta}\}$$

Formally, we will say that the signals, that the supervisor receives about the productivity parameter s is $\underline{\theta}$ in state of nature 2 and 3 and $\bar{\theta}$ in state of nature 4 i.e., $s \in \{\underline{\theta}, \bar{\theta}, \varphi\}$ in state of nature 1, φ, where φ means no observation. The possible reports by the supervisor to the principal will also be $r \in \{\underline{\theta}, \bar{\theta}, \varphi\}$, If r is identical to s in each state of nature, then the supervisor truthfully conveys the information. Also, we assume that the agent knows whether the supervisor observes θ or not. By this, we assume the agent knows the state of nature, not only the level of productvity.

We summarize the decision process of the model as follows : the principal offers contracts to the supervisor and to the agent which are functions of observable variables, i. e., the profit X and the supervisor's report r, The principal moves first, offering the contract S(X, r) and W(X, r) to the supervisor and the agent respectively. After accepting these offers, the agent knows the realized state of nature and the supervisor observes his signal s, which might be some θ or φ . Then the agent chooses his optimal level of effort e and then the output $X = \theta + e$ is revealed. In this model, the principal can choose the timing of the supervisor's report. Finally, the payment S

8) We asume that probability distribution is a common knowledge for every player in this model. This model differs from the models of Green and Storky[1983] and Sappington[1980], [1983], all of which consider precontractual asymmetry of information.

and W are determined.[9]

time

Principal offers contracts S(X, r) W(X, r)	Agent learns i Supervisor learns s	Agent choose e Supervisor report r	Profit $X=\theta+e$ revealed	Transfers S(X, r) W(X. r) are mead

2.3 Collusion free contracts

In this part, we want to deal with the case without collusion as a reference point. For the purpose of comparison, we first consider the full information (first-best) allocation. For the time being, we ignore the information structure described earlier and we assume that principal can directly observe the productivity parameter as well as the effort exerted by the agent. Hence, there is no uncertainty in this case and the supervisor has no supervisory role. Then, the agent will exert the optimal level of effort e^* where the marginal disutility of effort is equal to the marginal contribution to the profit. In this simple model we get e^* such as $g'(e^*)=1$ for all θ. If we denote $g^* \equiv g(e^*)$ as a corresponding disutility of optimal effort, then the agent's wage in each state will be W_0+g^*, which is independent of the state of nature. In this case, the principle can enforce a first-best contract by offering the wage level W_0+g^*, in all states of nature, Then, the agent receives no more than his reservation utility level in any state. Finally, since the supervisor has no supervisory function, he gets reservation wage S_0 for all the state of nature.

Now, we return to the information structure described in the section 2. We assume that collusion between the supervisor and agent (from now on, We use collusion S/A or coalition S/A for this) in not allowed or not feasible. Then the supervisor does not have any incentive to misreport. Also, given reservation wage level S_0 in all states of nature, the supervisor obtains full insurance and is willing to participate. Therefore, the principal can have the supervisor's information by paying S_0 in all the states of nature. Actually, in this case, the three tier relationship will boil down to the usual principal-agent relationship, where the principal pays constant wage S_0 and inherits the supervisor's informaion. Then the principal can design an enforceable contract by making the contract based on the publicly observed variable X (output). Further-

9) This model is different from a pure moral hazard model. That is because the agent has perfect information and the output X is determind nonstochastically in this model. Therefore, the main problem of this model is asymmetry of information. If the principal could observe θ, then the principal would have full informaion when he observes output X, hence he could monitor the agent's effort perfectly.

more, he can confine his contract to the output level in each state of nature by the revelation principle.[10] In other words, the principal can offer the contract $W_i \equiv u(X_i)$ and $S_i \equiv s(X_i)$ where $X_i = \theta_i + e_i$, $i=1,2,3,4$(See Harris and Townsend[1981] and Myerson[1979], [1982]). Then, we find an optimal enforceable contract for the principal as a mathematical program (CF) where the principal's expected utility is maximized subject to the agent's individual rationality constraints and the self-selection constraints, as follows:

$$\text{(CF)} \quad \underset{\{W_i,\ e_i\}}{\text{Max}} \quad \sum_{i=1}^{4} P_i(\theta_i + e_i - W_i)$$

s. t.

$$\text{(AIR)} \quad \sum_{i=1}^{4} P_i U(W_i - g(e_i)) \geq \overline{U}$$

$$\text{(AIC1)} \quad W_3 - g(e_3) \geq W_2 - g(e_2 - \Delta\theta)$$

$$\text{(AIC2)} \quad W_2 - g(e_2) \geq W_3 - g(l_3 - \Delta\theta)$$

In this model, the agent has private information in the states 2 and 3. The agent's incentive compatibility constraint $W_3 - g(e_3) \geq W_2 g(e_2 - \Delta\theta)$means that the agent cannot claim that he is in the state 2 by producing X_2 and exerts the effort level in state 3. Without (AICl), we could have $W_3 - g(e_3) < W_2 - g(e_2 - \Delta\theta)$ and the agent would prefer to produce X_2 by exering only $e_2 - \Delta\theta$ in state 3. (Observe that since the actual parameter is $\overline{\theta}$, he can produce $X_2 = (e_2 - \Delta\theta) + \overline{\theta}$, which is equal to $e_2 + \underline{\theta}$). Hence, by enforcing this constraint, the agent in state 3 does not have any incentive to misreport his true state. In state 2, the agent may have an incentive to claim his true state is 3 by producing W_3. However, we can easily see that this constraint is not binding at an optimum, since the agent can only cheat the principal by misrepresenting from $\overline{\theta}$ to $\underline{\theta}$.

Lemma 1 : The solution of (CF) has the following features :

a) The supervisor gets S_0 for all state of nature

b) $e_4 = e_3 = e_1 = e^* > e_2$

c) $W_3 - g(e_3) > W_4 - g(e_4) = W_1 - g(e_1) > W_2 - g(e_2)$

Hence, $W_3 > W_4 = W_1 > W_2$

d) AIC1 is binding and AIC2 is not.

We consider this case for comparision with the discussion of the coalition case.[11] Interpretation of these results clarifies characteristics of this model. In states of na-

10) The principal can choose direct mechanim without any loss of generality under which the agent is supposed to declare his private observation and then some allocation (here, W_i and recommadation about e_i) is effected following the prespecified allocation rule as a function of declaration of the agent.

11) Since this is a typical mechanism design problem, the proof of this lemma is not provided in this paper. However, we can find the froof for this lemma in Tirole[1986].

ture 1 and 4, the principal can perfectly monitor the agent as he knows the true θ and can observe the output. Therefore, the agent cannot shirk, hence $e_1=e_2=e^*$, and W_1 must be equal to W_4. In the state 3, by offering high wage level W_3, the principal can give the agent incentive to reveal his true information θ and in the state 2, offering low wage level W_2, make it less attractive for the agent to shirk in the good state of productivity. Since the principal can know the true parameter in the state 3, he can then successfully monitor the agent's effort, thus the agent exerts e^*. Hence the principal can guarantee to himself the outcome in which the agent cannot shirk in state 3. However, low wage level in state 2 cannot gives the agent enough incentive to exert the effort level e^*. Thus, the cost of truthful revelation is some suboptimal level of effort in state 2 i.2, $e_2=<e^*$.

3. COALITION PROOF CONTRACTS

As, we have seen in the previous sections, the possibility of collusion is the key reason why the analysis of organizations and/or hierarchies cannot boil down to the two tier principal-agent relationship. In the simple model that we introduced in the previous section, if we assume that either the main contract does not forbid any side contracts between the supervisor and the agent or that the side payment is not observable by the supervisor, we can easily see that the allocations in without collusion would not be sustainable. For example, the supervisor gets the same wage in stage 3 and 4, however, the agent gets higher utility at state 3. This gives the agent some incentives to bribe the supervisor to conceal his information. Also in states 2, the agent is willing to pay the supervisor for reporting θ, as an excuse for bad performance. In this section, we add some more structures and assumptions in order to discuss the S/A coalition problem. Then, we formally derive the mathematical program which yields the optimal collusion-free allocation to the principal.

3.1 Assumptions and the Structures of the S/A Coalition

In this model, agents collude by manipulating information. Under the S/A coalition, the supervisor may report false information, in one of the following two ways :

(I) by concealing relevant information

(II) by reporting a false level of the parameter θ.
(misrepresentation of evidence)

Most of literature deals only with type(I) misrepresentation (hard information case), since concealing information might be more plausible behavior in terms of verifiability of the report. However, we frequently observe colluding parties trying to manipulate evidence. In other words, once they agree to collude, they sometimes become aggressive enough to produce false evidence. For example, consider the stockholder/top manager/worker hierarchy. It is well known that manager/worker coalitions may try to manipulate the level of profit and/or the level of the sales when the manager's reward depends on the level of profit and/or the level of sales.

Thus, when we consider both types of information manipulation through S/A coalition, we formally represents the supervisor's possible reports to the principal as follows :

$$\text{if } s=\underline{\theta}, \gamma\in\{\underline{\theta}, \overline{\theta}, \varphi\}$$
$$\text{if } s=\varphi, \gamma\in\{\varphi, \underline{\theta}, \overline{\theta}\}$$
$$\text{if } s=\theta, \gamma\in\{\overline{\theta}, \underline{\theta}, \varphi\}$$

In this model, only the supervisor is supposed to report the productivity parameter to the principal[12]. In many cases, the agent may not be in position to produce proper information by himself. For example. if the production process is highly sophisticated, then even though the agent knows the contents of information that the principal needs, he cannot transmit this information convincingly. In other words, sometimes the agent may not be able to put the evidence in a form that the principal can understand or use. Alternatively the agent may not have time to accumulate evidence by himself. Obviously, the agent can also convey information properly in some cases.

We need another institutional assumption in order to ensure that coalition formation is not too complex. We assume that the supervisor's report is public information both to the principal and to the agent. Without this assumption, the agent cannot be sure about whether the supervisor reports what they agree to send or not. By this, we can assume that if anyone unilaterally deviates from the coalition, then the other one can call this and both will be severely punished by the principal. Within this structure, we can assume that the supervisor and the agent try to sign a Pareto Optimal side contract and each of the colluding parties can guarantee itself a strictly greater payoff that he would receive without side-contract.[13]

With these assumptions about the S/A coalition, the decision process of the model is follows : The principal offers contract {S, W} and recommendation about e_i to the supervisor and agent respectively. Then given this main contract, the supervisor and the agent try to sign a side contract before the uncertainty is revealed. Specifically, given main contract, they assent to do the following : if some state of nature 'i' occurs, then we would misrepresent out real state of nature as state j by manipulating the supervisor's report r and the output X according to the agent's message, a, and

12) This institutional assumption reflects the observation that the reporting system of hirarchical organization itself is often hierarchical. Obviously, the principal can ask the agent to report either. However, our analysis of coalitions does not change even if the principal ask the agent to send a message as well, because S/A coalition can always coordinate the message to be sent to the principal.

13) This is, of course, very weak restirction. By this, we ignore all the problems associated with bargaining within the coalition. However, in reality, the problem of allocating the surplus from the coalition may hinder the formation of coalition itself.

decide the side payments σ_i according to given main contract W_i, W_i and S_i, S_i. We denote this side-contract $\sigma_i \equiv \sigma(X, r)$ as a monetary side payment from the agent to the supervisor. Then the rest of decision process of the model is same with the previous section. Finally, the supervisor and the agent will receive $S_i+\sigma_i$ and $W_i-\sigma_i$ respectively. However, this side-contract has serious difficulity : the supervisor cannot distinguish the state of nature 2 from the state of nature 3 and vice versa. In state 2 and 3, they have the problem of coalition formation under asymmetric information whereas, in state 1 and 4, both of the colluding parties share the same information. In the next section, we will discuss the problems of coalition formation under asymmetric information.

3.2 Derivation of Optimazation Problem

The principal maximizes his expected payoff under some constaints including the usual participation constraints, the agent's incentive compatibility constraints and the constraints which can prevent the formation of S/A coalitions. Followins Tirole (1986), we call this last constraint the coalition incentive compatibility constraints (From now on, we represent this constraint as CIC).

In the states of nature 1 and 4, the agent and the supervisor have the same information. So, they can easily form a coalition by coordinating the supervisor's report to the principal and the output level. Therefore, the principal can easily find constraints, which ensure that the supervisor and the agent have no incentive to collude. In the states of nature 1 and 4, we have following CIC's :

$$\text{(CIC1)} \quad S_1+W_1-g(e_1) \geq S_2+W_2-g(e_2)$$

$$\text{(CIC2)} \quad S_1+W_1-g(e_1) \geq S_3+W_3-g(e_3+\Delta\theta)$$

$$\text{(CIC3)} \quad S_1+W_1-g(e_1) \geq S_4+W_4-g(e_4+\Delta\theta)$$

$$\text{(CIC4)} \quad S_4+W_4-g(e_4) \geq S_3+W_3-g(e_3)$$

$$\text{(CIC5)} \quad S_4+W_4-g(e_4) \geq S_2+W_2-g(e_2-\Delta\theta)$$

$$\text{(CIC6)} \quad S_4+W_4-g(e_4) \geq S_1+W_1-g(e_1-\Delta\theta)$$

The meaning of these constraints is clear. If the allocation specified in the main contract does not meet(CIC1) i.e., $S_2+W_2-g(e_2) \geq S_1+W_1-g(e_1)$, then, the S/A coalition tries to increase their wage bill by concealing the true information θ, thus, reporting nothing (r$=\varphi$) in state 1. Similary, if (CIC2) is not met, i.e., $S_3+W_3-g(e_3+\Delta\theta) > S_1+W_1-g(e_1)$, then S/A coalition conceals the true information θ and, furthermore, agrees to disguise the real θ by producing X_3 in state 1. Observe that if $S_3+W_3-g(e_3+\Delta\theta) > S_1+W_1-g(e_1)$, then we know $W_3-g(e_3+\Delta\theta) > W_1-g(e_1)$, and $S_3 > S_1$ since we assumed that each of colluding parties can guarantee itself a greater payoff than before. Then the agent prefers to produce X_3 by exerting the effort level $e_3+\Delta\theta$ in order to get W_3, since $X_3=e_3+\bar{\theta}=e_3+\Delta\theta+\theta$. Similarly, without the constraint(CIC

3), we can have $S_4+W_4-g(e_4+\Delta\theta) > S_1+W_1-g(e_1)$, This means that the supervisor and the agent can increase their total wage bill by reporting false level $r=\overline{\theta}$, instead of reporting true one $r=\underline{\theta}$. This gives them incentive to collude. Hence we need (CIC3). The same arguement can be applied for (CIC4), (CIC5) and (CIC6) at state 4.

In the states of nature 2 and 3, however, there is an informational asymmetry between the supervisor and the agent, only the agent can distinguish state 2 from state 3 whereas the supervisor cannot. Because of this asymmetry problem, we cannot directly dervie the coalition incentive compatibility conditions, We can imagine all the possible contents of side-contracts which might be signed between the supervisor and the agent, given any possible allocation(contract)W_i, S_i and e_i The actual side-contract depends on the main contract offered by the principal. At this point, however, we should consider all the possible cases of side contracts given any main contracts(allocations). This is because we want to design a coalition proof contract from the principal's view point.

Since the supervisor cannot distinguish the state 2 form the state 3, we can easily imagine that the side-contract will specify the coordinated report and the production level in the state 2 and the state 3 simultaneously. For example, given some main contract allocation, S/A coalition agrees to report $\underline{\theta}$ or $\overline{\theta}$ whatever the real state is(i. e. both state 2 and state 3). Therefore the collusions game between the supervisor and the agent will be very complicated when the colluding parties have asymmetric information problem. In a different medel, Felli[1993] observed that informational asymmetries may prevent colluding parties from realizing gains from trade.

The existence and/or the uniqueness of the equilibrium of the collusion game between the supervisor and the agent may depend on the various institutional assumptions about the bargaining process within coalition. For example, as pointed out in Felli[1993], if the agent is assumed to show his intention to form a coalition, he has to release some signals about his own private information during the bargaining process. If the supervisor has authority i.e., a delegated discretionary power to exploit this leaked informations to the detriment of the agent, the agent might refuse to participate in the collusive bargaining process and, thus, collusion will be cheaply prevented.

If the supervisor is assumed to make take-it-or-leave-it offer to the agent, there is no information leakage through the bargaining process for side contracts. In this case, we can consider some coalitional mechanism, in which the agent has no incective to misrepresent his own state and both colluding parties get more through the collusion than before.[14] However, since we don't have a established bargaining theory to make strong prediction within the coalition, we still have implementation

14) Cremer[1986] proposed and analyzed this kind of coalitional mechanism under the nonbayesian context. Also Maskin and Tirole[1990] adopted similar argurment to propose a mechanism design with an information principal.

problem of this kind of coalitional mechanism[15])

In this study, we introduce the concept of ex-post coalitional incentive compatability constraints in the following sence : The principal considers final allocations, which are the net wages to the supervisor and the agent after side payments through coalition. Then he can impose constraints such that this final total wage bill net of disutility in one state of nature must be greater than that of any other state of nature(Note that even if the principal considers the final allocation, it does not imply that the S/A coalition really emerged. This only happened in the principal's logical process in order to calculate his optimal contract). Then there is no room for any S/A coalition to get the higher total wage bill net of disutility by misrepresinting their true state of nature.

Thus, we consider the feasible allocation set by imposing the following ex-post constraints

$$
\begin{array}{ll}
(\mathrm{CIC1}') & S_1+W_1-g(e_1) = S_2+W_2-g(e_2) \\
(\mathrm{CIC2}') & S_3+W_3-g(e_3) = S_4+W_4-g(e_4) \\
(\mathrm{CIC3}') & S_3+W_3-g(e_3) \geq S_2+W_2-g(e_2-\Delta\theta) \\
(\mathrm{CIC4}') & S_4+W_4-g(e_4) \geq S_1+W_1-g(e_1-\Delta\theta) \\
(\mathrm{CIC5}') & S_2+W_2-g(e_2) \geq S_3+W_3-g(e_3+\Delta\theta) \\
(\mathrm{CIC6}') & S_1+W_1-g(e_1) \geq S_4+W_4-g(e_4+\Delta\theta)
\end{array}
$$

These inequalities are derived from self selection constraints of the total wage bill net of disutility in each state of nature. (The details are provided in the proof of proposition 2 i. e., Appendix (B)). Observe that the equality constraints (CIC1′) and (CIC2′) come from the inequalities in both direction respectively. We know that the allocations which satisfy these ex-post constraints (CIC1′)-(CIC6′) will also satisfy all the ex-ante coalition incentive constrains (CIC1) to (CIC6), specified in the states of nature 1 and 4.

3.3 Analysis of Collusion Proof Contracts

We can finally derive the optimization problem that can guarantee coalition proof allocation to the principal. That is, we want to solve the following program(C) :

$$
(\mathrm{C}) \quad \underset{\{W_i,\ e_i\}}{\mathrm{Max}} \quad \sum_{i=1}^{4} P_i(\theta_i+e_i-S_i-W_i)
$$

s. t. (SIR), (AIR), (AIC1), (CIC1′)−(CIC5′)

15) Actually, the implementation problem of a side contract between two parties is an open issue in the literature on collusion. A long term relationship or a reputational arguement is sometimes quoted to justify the enforceability of the side contract. See Felli[1993] for more detail discussion.

The solution to this problem is descrebed in the following proposition. (The proof of this proposition is provided in appendix (A))

Proposition 1 : The solution to (C′) has the following features :

a) $S_4 > S_3 \geq S_2 > S_1$
b) $e_4 = e_3 = \dot{e} > e_1 \geq e_2$
c) $W_3 - g(e_3) > W_4 - g(e_4) > W_1 - g(e_1) > W_2 - g(e_2)$
   Hence, $W_3 > W_4 > W_1 > W_2$
d) (AIC1) and (CIC4′) are binding while (CIC5′) is not.
e) $0 \leq S_3 - S_2 < W_2 - g(e_2) - \{W_3 - g(e_3 + \Delta\theta)\}$
f) $S_4 + W_4 - g(e_4) = S_3 + W_3 - g(e_3) > S_2 + W_2 - g(e_2) = S_1 + W_1 - g(e_1)$

There are two constaints that we ignore in this problem (C) : AIC 2 and CIC 6′. However we can see these constraints will indeed be automatically satisfied by the solution of problem (C). [16]

Now, we need interpretation of the results of proosition 1. First, observe that (f) of proposition 1 shows that the total wage bill of the supervisor and the agent net of disutility from effort depends only on the true level of productivity parameters. However, the principal can get the optimal coalition proof allocation by assigning different wages for the supervisor and for the agent respectively. For instance, the agent in the state 3 tries to shirk as the supervisor cannot provide the evidence. Hence the agent must be paid a higher wage in state 3 than in state 4 i.e., $W_3 > W_4$. On the other hand, the principal makes S_4 higher than S_3 in order for the agent not to bribe the supervisor to conceal this information $\bar{\theta}$. Furthermore, by making S_4 the highest, the principal tries to make the supervisor report $\bar{\theta}$ whenever he observe it.

Contrary to this, when the agent has private information $\underline{\theta}$ in state 2, he wants this information transmitted to the supervisor as an excuse for his bad performance, because the supervisor can not provide the evidence for this. Hence, in this case, the agent suffers the lowest level of utility since this information $\underline{\theta}$ actually is valuable only to the agent as a proof of low output level. When the agent privately observes $\bar{\theta}$, he has no room for shirking. This is because $S_2 + W_2 - g(e_2) > S_3 + W_3 - g(e_3 + \Delta\theta) = S_4 + W_4 - g(e_4 + \Delta\theta)$ from the fact that (CIC5′) is not binding. Hence the principal sets S_2 (also S_3 and S_4) higher than S_1 in order for the agent not to bribe the supervisor to report a good excuse for his bad performance.

In this model, we do not consider the supervisor's effort in observing the produc

16) The result (e) of the proposition directly shows that (AIC2) meets with strict inequality. Since (CIC4′) is binding, we have $S_4 + W_4 - g(e_4) \geq S_1 + W_1 - g(e_1 - \Delta\theta)$. Then, we have $S_1 + W_1 - g(e_1) + \{g(e_1) - g(e_1 - \Delta\theta)\} = S_4 + W_4 - g(e_4 + \Delta\theta) + \{g(e_4 + \Delta\theta) - g(e_4)\}$. By the strict convexity of g(e) and the fact $e_4 > e_1$, we know $g(e_4 + \Delta\theta) - g(e_4) > g(e_1) - g(e_1 - \Delta\theta)$. This implies that $S_1 + W_1 - g(e_1) > S_4 + W_4 - g(e_4 + \Delta\theta)$. which means that (CIC6′) meets with strict equality.

tivity parameter, which means the supervisor's wage level is irrelevant to the information cost[17] The essence of the problem in this model is the private information of the agent which is unobservable by the supervisor. Part (c) of proposition 1 shows that the agent enjoys the highest net utility when he has a private information $\bar{\theta}$ which is valuable to the principal. Contrary to this, the agent suffers the lowest net utility when has a private information $\underline{\theta}$, which is not valuable to the principal.

Finally, by comparing the contract described in propostion 1 with that of lemma 1, we can find the cost for preventing all the possibile coalitions, First, under the possibility of collusion, the supervisor's information is more costly to obtain. Without the collusion possibility, the principal can pay constant reservation wage S_0 in each state. However, under the possibility of side contract, the principal must pay a risk premium to the supervisor, since the supervisor is risk-averse and S_i in proposition 1 is not constant. Second, another source of cost preventing the possible collusion is some suboptimal level of effort in state of nature 1 as well as state 2. The principal needs to make W_1 and W_2 lower than W_3 and W_4 (same with the supervisor's wage) in order to make it less attractive for agent to shirk in good state of productivity and this low wage cannot give enough incentive for the agent to exert e^* in the state 1 and 2.

We know the principal indeed guarantees himself the coaltion proof contract. However when the principal figures out the optimal contract, he already considers all the possible type of S/A coalition. Therefore, even if the principal allows them to collude, the final outcome shall satisfy all the conditions of proposition 1. In other words, if the principal offers the coatract described in proposition 1, there in no state of nature in which the total wage bill net of disutility of effort (after side contract) can be increased by changing the reports or the effort level.[18] Thus, we restate this in

17) Tirole's interpretation of his equality $S_4 > S_3 > S_2 = S_1$ is misleading. This result comes mainly from his assumption on the coalitional behavior of the supervisor within the coalition. The reason for high $S(S_3 > S_2 = S_1)$ cannot be an information cost, since the supervisor pays nothing for gathering the information. In our model, the supervisor does not act as an advocator for the agent. He acts as an advocator for the agent only when it is good for himself, Tiorle's this interpretation also comes form his assumption on the supervisor's reporting technology.

18) We should mention the supervisor's incentive problem not colluding with the agent given the main allocation described in proposition 1. In other words, the supervisor may have some incentive to unilaterally deviate from the S/A coalition and to misrepresent the true state of nature. Given contract a) of proposition 1, $S_2 > S_4 \geq S_3 > S_1$ the supervisor has incentive to change his report in the following three ways. :

(i) $r = \theta \Rightarrow r = \varphi$ (ii) $r = \underline{\theta} \Rightarrow r = \bar{\theta}$

(iii−1) $r = \varphi$ (but actual $\theta = \bar{\theta}$) $\Rightarrow r = \underline{\theta}$
(iii−3) $r = \bar{\theta}$ (but actual $\theta = \bar{\theta}$) $\Rightarrow r = \theta$

First, consider the case (i). Since r is public information, the agent can observe the supervisor's misepresentation, which reduces the agent's payoff. Hence, even if the agent is not supposed to speak, he voluntarily tells that the supervisor conceals true information intentionally. Second. consider case (ii) and (iii−1). The principal can prevent this kind of misrepresentations by making the timing of report after the output X is revealed. This is because $X = \theta + e_i$ and $X = \bar{\theta} + e_i$ can never be compatible with the report $\bar{\theta}$. Of course, we assume that the supervisor is severly punished if any shirking behavior is known to the principal. Finally, since the supervisor actually cannot distinguish (iii−2) from (iii−1), even if he he has some possibility to successfully increase his payoff with case (iii−2), he will not try to report θ alone when he observes nothing.

the following proposition. The proof is provided in appendix(B).

Proposition 2 : The principal can grarantee himself the final allocation $\{S, W, e_i\}$ *described in proposition 1, even if the agent and the supervisor are allowed to form collusions.*

For the rest of this section, we try to investigate several special cases of model in order to clarify the intuition of the results. This will also allow us to compare the implications of this model with Tirole's[1986]. First, let us consider two extreme cases of the supervisor's preference.

Proposition 3 : If the supervisor is risk neutral, the optimal contract is the same as the no collusion case described in lemma 1.

Proof is provided in the appendix (C). The intuition behind this proposition is obvious. Since the supervisor is risk neutral, the principal can sell the ownership of the production process to the surpervisor at the price of the expected profit minus the supervisor's reservation wage. Then the relationship between the risk neutral supervisor and the risk averse agent become a typical prinipal-agent relationship. Hence there is no room for coalition.

Proposition 4 : If the supervisor is infinitely risk averse, he receives a fixed wage. S_{i}*. Then the principal has one of the folloowing three types of information to monitor the agent :*

$$\{s_j=\bar{\theta}\}\forall, \ \{s_j=\underline{\theta}\}\forall_j \text{and } \{s_j=\varphi\}\forall_j$$

Proof is provided in appendix (F). Since the supervisor is infinitely risk averse, he only cares about the certain wage no matter what the level of wage is. As is shown in the proof, if S_i is constant then the allocations described in part b) and c) of proposition 1 change to

$$(\text{b}') \ W_3-g(e_3)=W_4-g(e_4) > W_1-g(e_1)=W_2-g(e_2)$$

$$(\text{c}') \ \ e_3=e_4=e^* > e_1=e_2$$

In other words, given constant S, the principal does not try to distinguish state 1 from state 2 and state 3 from state 4 respectively. This makes the role of the supervisor trivial since the distinction of the state 1 from 2 and the state 2 from 4 is possible only from the existence of the supervisor. Moreover, b′) and c′) show that the principal wants to distinguish the states 1 and 2 from the states 3 and 4. However, since the supervisor cannot distinguish state 2 from state 3, his report cannot have the following structures : $\{s_1=s_2=\varphi, \ s_3=s_4=\bar{\theta}\}$, $\{s_1=s_2=\underline{\theta}, \ s_3=s_4=\bar{\theta}\}$ or $\{s_1=s_2=\underline{\theta}, \ s_3=s_4=\varphi\}$.

Thus we get the information structure described in the proposition 3. Intuition is clear. Since the supervisor extremely prefers a constant wage, the principal expects that he always tries to report the same signal in all the states of nature. We consider the supervisor as an advocator of the principal if he always reports θ and an advocator for the agent if he report θ respectively. Reporting φ for all the states of nature is considered to represent the neutral position of the supervisor. In our model, unlike Tirole's[1986], the supervisor does not necessairly at an advocator for the agent.

Proposition 5 : *Even if the agent inherits the supervisor's reporting technology in state 1 and 4, we need the supervisor, except when the supervisor is infinitely risk averse and he acts as an advocator for the agent.*

We can imagine situations where the agent inherits the supervisor's reporting technolgy. In other words, the agent is supposed to report the principal in all the states of nature and in the state 1 and 4 he can convey his information in a verifiable way. Then obviously the agent always tries to report $\underline{\theta}$ since this gives not only some room for shirking when the actual θ is $\bar{\theta}$ but also gives an excuse for low output level when the actual θ is θ. This information structure (reporting $\underline{\theta}$ in all states of nature) is exactly same as the infinitly risk averse supervisor acts as an advocator for the agent.

4. CONCLUSION

In this paper we studied the problem of designing some optimal collusion free contract under simple three-tier principal/supervisor/agent hierarchical structure. Unlike most of literatures on collusion, we consider the soft information, which is unverifiable, as well as hard information. We show the principal can design an optimal collusion free contract by putting proper incentive compatiblity conditions and individual rationality conditions. Of course, the prindipal must pay some additional cost for this machanism. The most important feature of this optimal collusion free contract is that the allocation rule (in our model the specification of S, W, e) is prespecified so that the evaluation about the agent does not depend on the report by the supervisor who is so-called "simultaneously judge and party". Moreover, we find that the role and the behavior of the supervisor within the hierarchical organization crucially depend not only on the possibility of collusion but alsso the nature of possible collusion (i. e. it depends whether creating false information is possible or not).

The analysis in this paper is very restictive, of course. Actual organizations have more complex hierarchies than one principal/one supervisor/one agent structure. A supervisor will monitor several agents or an agent may have more than one supervisor. Also we might have several layers of supervisors. Furthermore, in this model, we ruled out the supervisor's productive role. Considering the suporvisor's effort might be an interesting future work. Finally, we did not consider the dynamic aspects of coalitions. Actually, the long term relationship between players has been considered to improve the performance of the organization. For example, as is shown in the repeated moral hazard literature, repeated relationship alleviates incentive problem (see Radner[1986]) and sometimes, help to accumulate specific assets and to reduce transaction costs (see Williamson [1975]). However, if we consider the effect of collusive behavior within the organization, the long term relationship, as is pointed out in Tirole [1986] and [1990], may not always be a blessing, since the long term relationship will also strengthen the bounds within the coalitions.

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APPENDIX (A)

Proof of proposition 1 (coalition case)

Since we two equality constraints, we put

$$S_2+W_2-g(e_2)=S_1+W_1-g(e_1)=\varphi_1$$
$$S_4+W_4-g(e_4)=S_3+W_3-g(e_3)=\varphi_4$$

Then, we can rewrite the program (C) as follows :

$$\underset{\{S,e,\varphi_1,\varphi_4\}}{\text{MAX}} \quad p_1(\theta_1+e_1-g_1(e_1)-\varphi_1)+p_2(\theta_2+e_2-g_2(e_2)-\varphi_1)+p_3(\theta_3+e_3-g(e_3)-\varphi_4)-p_4(\theta_4+e_4-g(e_4)-\varphi_4)$$

s.t.

(SIR) $\sum_{i=1}^{4} P_iV(S_i) \geq \bar{V}$

(AIR) $P_1U(\varphi_1-S_1)+P_2U(\varphi_1-S_2)+P_3U(\varphi_4-S_3)+P_4U(\varphi_4-S_4) \geq \bar{U}$

(AIC) $\varphi_4S_3\{\varphi_1-S_2g+(e_2)-g(e_2-\Delta\theta)\} \geq 0$

(CIS3′) $\varphi_4-\{\varphi_1+g(e_2)-g(e_2-\Delta\theta)\} \geq 0$

(CIS4′) $\varphi_4-\{\varphi_1+g(e_1)-g(e_1-\Delta\theta)\} \geq 0$

(CIS5′) $\varphi_1-\{\varphi_4+g(e_3)-g(e_3+\Delta\theta)\} \geq 0$

Lagrangian fuction is :

$$\begin{aligned}L^c=&P_1(\theta_1+e_1-g(e_1)-\varphi_1)+P_2(\theta_2+e_2-g(e_2)-\varphi_1)\\&+P_3(\theta_3+e_3-g(e_3)-\varphi_4)+P_4(\theta_4+e_4-g(e_4)-\varphi_4)+\rho(\sum_{i=1}^{4} P_iV(S_i)-\bar{V})\\&+\mu\{P_1U(\varphi_1-S_1\}+P_2U(\varphi_1-S_2)+P_3U(\varphi_4-S_3)+P_4U(\varphi_4-S_4)-\bar{U}\}\\&+\gamma\{\varphi_4-S_3-\varphi_1+S_2-g(e_2)\}+g(e_2-\Delta\theta)\}+\pi\{\varphi_4-\varphi_1-g(e_2)+g(e_2-\Delta\theta)\}\\&+\alpha\{\varphi_4-\varphi_1-g(e_1)\}+g(e_1-\Delta\theta)\}+\beta\{\varphi_1-\varphi_4-g(e_3)+g(e_3+\Delta\theta)\}\end{aligned}$$

(F.O.C)

$$\frac{dL}{dS_1}=0 \Longrightarrow \rho V'(S_1)=\mu U'(\varphi_1-S_1) \tag{1}$$

$$\frac{dL}{dS_2}=0 \Longrightarrow \rho V'(S_2)=\mu U'(\varphi_1-S_2)-\frac{\gamma}{P_2} \tag{2}$$

$$\frac{dL}{dS_3}=0 \Longrightarrow \rho V'(S_3)=\mu U'(\varphi_4-S_3)+\frac{\gamma}{P_3} \tag{3}$$

$$\frac{dL}{dS_4}=0 \Longrightarrow \rho V'(S_4)=\mu U'(\varphi_4-S_4) \tag{4}$$

From (1) and (4), we have

$$\frac{V'(S_1)}{V'(S_4)}=\frac{U'(\varphi_1-S_1)}{U'(\varphi_4-S_4))} \Longrightarrow \{S_1 \geq S_4 \text{ iff } \varphi_1-s_1 \geq \varphi_4-s_4\} \tag{5}$$

$$\frac{dL}{d\varphi_1}=0 \Longrightarrow -P_1+P_2+P_1\mu U'(\varphi_1-S_1)+P_2\mu U'(\varphi_1-S_1)-\gamma-\pi-\alpha+\beta=0$$

From (1) and (2), we have

$$\rho(P_1V'(S_1)+P_2V'(S_2))=P_1+P_2+\pi+\alpha-\beta \tag{6}$$

similarly, $\frac{dL}{d\varphi_4}=0$ gives

$$\rho(P_3V'(S_3)+P_4V'(S_4))=P_3+P_4-\pi-\alpha+\beta \tag{7}$$

$$\frac{dL}{de_1}=0 \Longrightarrow P_1(1-g'(e_1))=\alpha(g'(e_1)-g'(e_1-\Delta\theta))$$

By the strict convexity of $g(\cdot)$, $g'(e_1)-g'(e_1-\Delta\theta)$ is always positive, Hence, we have

$$\begin{array}{ll} e_1=e^* & \text{if } \alpha=0 \\ e_1<e^* & \text{if } \alpha>0 \end{array} \tag{8}$$

$$\frac{dL}{de_2}=0 \Longrightarrow P_2(1-g'(e_2))=(\gamma+\pi)(g'(e_2)-g'(e_2-\Delta\theta))$$

By the strict convexity of $g(\cdot)$, $g'(e_2)-g'(e_2-\Delta\theta)$ is always positive, Hence, we have

$$\begin{array}{ll} e_2=e^* & \text{if } \gamma+\pi=0 \\ e_1<e^* & \text{if } \gamma+\pi>0 \end{array} \tag{9}$$

$$\frac{dL}{de_3}=0 \Longrightarrow P_3(1-g'(e_3))=\beta(g'(e_3)-g'(e_3+\Delta\theta)$$

By the strict convexity of $g(\cdot)$, $g'(e_3)-g'(e_3+\Delta\theta)$ is always negative, Hence, we have

$$\begin{array}{ll} e_3=e^* & \text{if } \beta=0 \\ e_3>e^* & \text{if } \beta>0 \end{array} \tag{10}$$

$$\frac{dL}{de_4}=0 \Longrightarrow g'(e_4)=1$$

This gives

$$e_4=e^* \tag{11}$$

1. Show that (AIC) constraint is binding

Assume that (AIC) is not binding. Then we have $\gamma=0$ and

$$\varphi_4-S_3-\{\varphi_1-S_2+g(e_2)-g(e_2-\Delta\theta)\}>0 \tag{12}$$

Since $\gamma=0$, (2) and (3) imply that Borch's rule holds between the states 2 and 3:

$$\frac{V'(S_2)}{V'(S_3)}=\frac{U'(\varphi_1-S_2)}{U'(\varphi_4-S_3)} \tag{13}$$

From (12), since $g(e_2)-g(e_2-\Delta\theta)>0$, We know that $\varphi_4-S_3>\varphi_1-S_2$. Then, from (13) we have

$$S_3>S_2 \tag{14}$$

From (12) and (14), we have

$$\varphi_4-\{\varphi_1+g(e_2)-g(e_2-\Delta\theta)\}>0 \tag{15}$$

This implies that (CIC3′) is not binding, which implies $\varphi=0$. Since $\gamma=\varphi=0$, $e_2=e^*$ by (9). Then, we can say $e_2\geq e_1$. From (15) we have $\varphi_4-\varphi_1>g(e_2)-g(e_2-\Delta\theta)$. And $e_2>e_1$ implies $g(e_2)-g(e_2-\Delta\theta)\geq g(e_1)-g(e_1-\Delta\theta)$ by the strict convexity of $g(\cdot)$. These two relationships imply

$$\varphi_4-\varphi_1>g(e_1)-g(e_1-\Delta\theta) \tag{16}$$

This implies that (CIC4′) is not binding, which implies $\alpha=0$.

However, we can show that $\gamma=\varphi=\alpha=0$ is impossible. If $\gamma=\varphi=\alpha=0$, we have $e_1=e_2=e^*$ from (8) and (9) and $\frac{V'(S_1)}{V'(S_2)}=\frac{U'(\varphi_1-S_1)}{U'(\varphi_1-S_2)}$ from (1) respectively. Thus we have $S_1=S_2$. Similarily, (3) and (4) give $S_4=S_3$. Since $S_1=S_2$ and $S_1=S_3$, we have following relationship by (6) and (7):

$$\rho V'(S_1)=1-\frac{\beta}{P_1+P_2}<\rho V'(S_4)=1+\frac{\beta}{P_3+P_4} \tag{17}$$

By the concavity of $V(\cdot)$, (17) implies $S_1>S_4$. From (5), we have $\varphi_1-S_1>\varphi_4-S_4$, which implies $\varphi_1-\varphi_4>S_1-S_4>0$. Hence, we get $\varphi_1>\varphi_4$. This contradicts with $\varphi_4 - \varphi_1 > g(e_1) - g(e_1 - \Delta\theta) = g(e_2) - g(e_2 - \Delta\theta) > 0$, Hence, (AIC) is binding.

Furthermore, we can easily observe that γ cannot be 0. Assume $\gamma = 0$. Then, as we have shown eariler, we get equation (13). Even if (AIC) is binding (thus $(\varphi_4 - S_3) - (\varphi_1 - S_2) = g(e_2) - g(e_2 - \Delta\theta)$), we stil have $\varphi_4 - S_3 > \varphi_1 - S_2$, which gives $S_3 > S_2$. Then we can apply the same argument to derive the contradiction.

2. Show that constraint (CIC4′) is binding

Claim 1 : we cannot have $\gamma > 0$ and $\pi = \alpha = 0$

(proof)

Assume that we have $\gamma > o$ and $\pi = \alpha = 0$, Since (AIC) is binding we have $\varphi_4 - \varphi_1 - (g(e_2) - g(e_2 - \Delta\theta)) = S_3 - S_2$. However we know $\varphi_4 - \varphi_1 - (g(e_2) - g(e_2 - \Delta\theta)) \geq 0$ by (CIC3′). Hence we have $S_3 \geq S_2$. Observe that $\gamma > 0$ implies $S_2 > S_1$ and $S_4 > S_3$ from (1), (2), (3), and (4). Hence, we have $S_4 > S_3 \geq S_2 > S_1$, which implies $V'(S_4) < V'(S_3) \leq V'(S_2) < V'(S_1)$ by the strict concavity of $V(\cdot)$ Then we have

$$P_1 V'(S_1) + \frac{P_2 V'(S_2)}{P_3 V'(S_3) + P_4 V'(S_4)} > \frac{(P_1 + P_2) V'(S_2)}{(P_3 + P_4) V'(S_3)} > \frac{P_1 + P_2}{P_3 + P_4} \tag{18}$$

However, since $\pi = \alpha = 0$, (6) and (7) give

$$P_1 V'(S_1) + \frac{P_2 V'(S_2)}{P_3 V'(S_3) + P_4 V'(S_4)} = \frac{P_1 + P_2 - \beta}{P_3 + P_4 + \beta} \leq \frac{P_1 + P_2}{P_3 + P_4}$$

This contradicts with (17).

Claim 2 : if $\gamma > 0$ and (CIC3′) is binding, then (CIC4′) must be binding

(Proof)

Assume (CIC4′) is not binding. Then we have

$$\alpha = 0 \text{ and } \varphi_4 - \varphi_1 > g(e_1) - g(e_1 - \Delta\theta) \tag{19}$$

Observe that $\gamma > 0$ and $\alpha = 0$ imply $e_1 = e^* > e_2$ by (8) and (9). This gives $g(e_1) - g(e_1 - \Delta\theta) > g(e_2) - g(e_2 - \Delta\theta)$ by the strict convexity of $g(\cdot)$. However, since (CIC3′) is binding, we have $g(e_2) - g(e_2 - \Delta\theta) = \varphi_4 - \varphi_1$. By these two relationships, we get $g(e_1) - g(e_1 - \Delta\theta) > \varphi_4 - \varphi_1$. which contradicts with (19).

Finally, by claim 1 and 2, we can claim that(CIC4′) is always binding.

(Proof)

Assume (CIC4′) in not binding. If (CIC4′) is not binding, then we have $\alpha = 0$. However, by claim 1, we cannot have $\gamma > 0$ and $\pi = \alpha = 0$, thus π must be positive. Hence, (CIC3′) must be binding. However, if (CIC3′) is binding (CIC4′) must not be binding by claim 2. This is contradiction.

3. Show that constriant(SIC5′) is not binding

Assume that (CIC5′) is binding, Then we have $\varphi_4 - \varphi_1 = g(e_3 + \Delta\theta) - g(e_3)$. However, since (CIC4′) is binding, we have

$$g(e_3 + \Delta\theta) - g(e_3) = g(e_1) - g(e_1 - \Delta\theta) \tag{20}$$

However, since $\alpha \geq 0$ and $\beta \geq 0$, we have $e_3 \geq e_1$ by (8) and (10). Then by the strict convexity of $g(\cdot)$, we have

$$g(e_3 + \Delta\theta) - g(e_3) > g(e_3) - g(e_3 - \Delta\theta) \geq g(e_1) - g(e_1 - \Delta\theta)$$

This contradicts with (20).

We can show the rest of proposition 1 as follows : Since $\gamma > 0$, we have $S_4 > S_3$ and $S_2 > S_1$ from (1), (2), (3), and (4). Since (AIC) is binding, we have $W_3 - g(e_3) = W_2 - g(e_2 - \Delta\theta)$. Then by (CIC3′) we have $S_3 > S_2$(Note that we cannot determine whether (CIC3′) is binding or not). Hence we have $S_4 > S_3 \geq S_2 > S_1$.

Observe that $S_4 > S_3$ implies $W_3 - g(e_3) > W_4 - g(e_4)$ and $S_2 > S_1$ implies $W_1 - g(e_1) > W_2 - g(e_2)$ respectively. Now, since $S_4 > S_1$ by(5), we have $\varphi_4 - S_4 > \varphi_1 - S_1$, which is equivalent to $W_4 - g(e_4) > W_1 - g(e_1)$. Hence, we have $W_3 - g(e_3) > W_4 - g(e_4) > W_1 - g(e_1) > W_2 - g(e_2)$.

Since (CIC5′) is not binding, we have $\beta = 0$, which means $e_3 = e^*$. Since (CIC4′) is binding, we have $\varphi_4 - \varphi_1 = g(e_1) - g(e_1 - \Delta\theta)$. However, by (CIC3′), we have $\varphi_4 - \varphi_1 > g(e_2) - g(e_2 - \Delta\theta)$ These two relationships give $g(e_1) - g(e_1 - \Delta\theta) \geq g(e_2) - g(e_2 - \Delta\theta)$, which implies $e_1 \geq e_2$ by the strict convexity of $g(\cdot)$. Hence, we have $e_4 = e_3 = e^* > e_1 \geq e_2$.

Finally, the fact that (CIC5′) is not binding. (AIC) is binding, and constraint (CIC3′) can produce some limitation on $S_3 - S_2$, which is (e) of proposition 1.

Q. E. D

APPENDIX (B)

Proof of proposition 2

The solution of (C) gives following relationships among optimal allocations.

(a) AIC is binding $\Longrightarrow W_3-g(e_3) = W_2-g(e_2-\Delta\theta)$

(b) (CIC1′) and (CIC2′) $\Longrightarrow S_1+W_1-g(e_1)=S_2+W_2-g(e_2)$

$S_4+W_4-g(e_4)=S_3+W_3-g(e_3)$

(c) (CIC3′) $\Longrightarrow S_3+W_3-g(e_3)=S_2+W_2-g(e_2-\Delta\theta)$

(d) (CIC4′) is binding $\Longrightarrow S_4+W_4-g(e_4)=S_1+W_1-g(e_1-\Delta\theta)$

(e) (CIC5′) is not binding $\Longrightarrow S_2+W_2-g(e_2) \geq S_3+W_3-g(e_3+\Delta\theta)$

(f) $e_4=e_3=e^*>e_1\geq e_2$

Let assume the allocation $\{S, W, e\}$ is a final (after side contract) allocation. Then we are through if we can show the allocation with features of (a) to (f) satisfies the following twelve inequalities. This is because if the principal chooses the main contract $\{S_i, W_i, e\}$ satisfying all the twelve inequalities, then since we assumed that each colluding party can guarantee itself the payoff before the coalition, S/A coalition cannot find any incentive to misrepresent their true states of nature.

$$S_1+W_1-g(e_1) \geq S_2+W_2-g(e_2) \quad (1)$$
$$S_1+W_1-g(e_1) \geq S_3+W_3-g(e_3+\Delta\theta) \quad (2)$$
$$S_1+W_1-g(e_1) \geq S_4+W_4-g(e_4+\Delta\theta) \quad (3)$$
$$S_2+W_2-g(e_2) \geq S_1+W_1-g(e_1) \quad (4)$$
$$S_2+W_2-g(e_2) \geq S_3+W_3-g(e_3+\Delta\theta) \quad (5)$$
$$S_2+W_2-g(e_2) \geq S_4+W_4-g(e_4+\Delta\theta) \quad (6)$$
$$S_3+W_3-g(e_3) \geq S_1+W_1-g(e_1-\Delta\theta) \quad (7)$$
$$S_3+W_3-g(e_3) \geq S_2+W_2-g(e_2-\Delta\theta) \quad (8)$$
$$S_3+W_3-g(e_3) \geq S_4+W_4-g(e_4) \quad (9)$$
$$S_4+W_4-g(e_4) \geq S_1+W_1-g(e_1-\Delta\theta) \quad (10)$$
$$S_4+W_4-g(e_4) \geq S_2+W_2-g(e_2-\Delta\theta) \quad (11)$$
$$S_4+W_4-g(e_4) \geq S_3+W_3-g(e_3) \quad (12)$$

Observe that (1), (2), and (3) are equivalent to (4), (5), and (6) respectively and (7), (8), and (9) are equivalent to (10), (11), and (12) respectively by property (b). Thus we are through if we can show that the allocation with properties (a) to (f) satisfies inequalities (1), (2), (3), (7), (8), and (9).

First, (1) and (9) are met with equality by (b). Second, (2) is satisfied with strict inequality by (e). Also by (f), $e_3=e_4$ and by (b), $S_3+W_3-g(e_3)=S_4+W_4-g(e_4)$. These two imply $S_3+W_3-g(e_3+\Delta\theta)=S_4+W_4-g(e_4+\Delta\theta)$, which shows that (3) is equivalent to (2). Next (8) is equivalent to (c) 3 i.e., (CIC3′). Finally, (7) is met by the equality (d).

Q. E. D

APPENDIX (C)

Proof of proposition 3

Since the supervisor is risk neutral, $V'(S)$ is constant. By choosing ρ appropriately, the first order conditions of program (C) become equivalent to that of (CF). First, all the (CIC)′ s are not binding, hence $\pi=\alpha=\beta=0$, Second, we choose ρ such that $\rho V'(S)=1$, then we have :

$$\mu U'(W_1-g(e_1))=1 \qquad \text{from (1) of (C)}$$

$$\mu U'(W_2-g(e_2))=1+\frac{\gamma}{P_2} \qquad \text{from (2) of (C)}$$

$$\mu U'(W_3-g(e_3))=1+\frac{\gamma}{P_3} \qquad \text{from (3) of (C)}$$

$$\mu U'(W_4-g(e_4))=1 \qquad \text{from (4) of (C)}$$

$$P_2(1-g'(e_2))=\gamma(g'(e_2)-g'(e_2-\Delta\theta)) \qquad \text{from (9) of (C)}$$

These (F.O.C)′s are exactly that of program (CF).

Proof of proposition 4

Since the supervisor is infinitely risk averse, S_i in each state must be same. Otherwise, we should have unbounded level of wages for the agent. This is impossible, Hence, S_i is constant. Since S_i is constant, we have $W_3=W_4$ and $e_3=e_4$. Moreover, $S_2=S_3$ implies that CIC3′ is binding since we already know that AIC is binding. Then we have $S_3+W_3-g(e_3)=S_2+W_2-g(e_2-\Delta\theta)$ and $S_4+W_4-g(e_4)=S_1+W_1-g(e_1-\Delta\theta)$. This gives $S_2+W_2-g(e_2)=S_1+W_1-g(e_1-\Delta\theta)$ by (CIC2′). Since we have $S_2+W_2-g(e_2)=S_1+W_1-g(e_1)$ by (CIC1′), these two relationships give $g(e_1)-g(e_1-\Delta\theta)=g(e_2)-g(e_2-\Delta\theta)$. Thus we get $e_1=e_2$ by the strict convexity of $g(\cdot)$. This also gives $W_1=W_2$ since $S_1=S_2$ and $S_1+W_1-g(e_1) \geq S_2+W_2-g(e_2)$. Same is true between the state 3 and the state 4. In otherwords, since $e_3=e_4$, $S_3=S_4$ and $S_3+W_3-g(e_3) \geq S_4+W_4-g(e_4)$, we can have $W_3=W_4$. All these results imply that the principal does not try to distinguish state 2 from state 1 and state 4 from state 3 respectively. Observe that $W_3-g(e_3)=W_2-g(e_2-\Delta\theta) > W_2-g(e_2)$ and $W_4-g(e_4)=W_1-g(e_1-\Delta\theta) > W_1-g(e_1)$. This means that the principal wants to distinguish state 1 from state 4 and state 2 from state 3 respectively. However, since the supervisor cannot distinguish state 2 from state 3, his report will be confined to the following trivial structures : $\{s_i=\theta\}$, $\{s_j=\theta\}$ and $\{s_i=\varphi\}$ $\forall_{i,j}=1, 2, 3, 4$

Q. E. D