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THE IMPACTS OF A WAGE TAX ON URBAN STRUCTURE

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1. INTRODUCTION

Economists and policy makers have long recognized the apparent advantage of land value taxation and have argued that since the supply of land is perfectly inelastic the land value tax will be allocatively neutral and less distortionary relative to other taxes imposed by local governments. However, ever since Mieszkowski[5] and Grieson[3] have examined the allocative effects of a property tax on reproducible capital, a number of writers, Skouras[8], David Mills[6], Brian Bentick[1], Brueckner[2], and Lim[4] have demonstrated the nonneutrality of a tax on land value.

Once the nonneutrality of the land value tax is established it is important to develop a model that analyses the comparative, or relative efficiency of the land value tax and alternative sources of finance such as a wage tax. To carry out the analysis of the relative efficiency between the land value tax and the wage tax we have to investigate the impacts of these taxes on urban structure respectively. Lim[4] used a two-period model of urban development with perfect foresight to examine the impacts of the nonneutral land value tax on urban structure. When the land value tax is imposed, land owners who hold their land vacant for more profitable industrial use in the second period pay a tax in both the first and second periods even though they earn zero land rents in the first period. Thus, the vacant land relatively close to the residential zone will be developed residentially in the first period, This tax-induced increase in the supply of residential lands leads to a fall in residential rents and an increase in the labor supply and industrial output in the first period. It makes the city larger than it would have been without the land value tax in this period. In the second period. however, the city is smaller than it would have been without the land value tax since a smaller amount of land is available for industrial use as a result of the premature residential development in the first period.[1)]

In this paper, as a previous stage for the comparative analysis of the relative effi-

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1) Duck-Ho Lim, "The Nonneutrality of the land value tax : Impacts on Urban Structure," Journal of Urban Economics 32, 1992, pp. 186-194.

ciency between the nonneutral land value tax and the wage tax, the impacts of the wage tax on urban structure are analysed for a two-period model of urban growth in which the timing of urban development is endogenous. For the analytical simplicity I start from a nondistorted equilibrium.

2. THE MODEL

In a metropolitan area labor is combined with land in producing goods and services. The city lies on a featureless plane topographically and it is marked by a circle of radius X with a circumference of $2\Pi x$ and an area of Πx^2, and has the central business district(CBD). For analytical simplicity, however, we consider only one straight line from the CBD to the edge of the city.

There are two production activities, the production of a composite commodity and the production of housing srevices. The price of the composite commodity in period i, P_i, $i=1,2$, is fixed exogenously at the CBD. The production function for the composite commodity has fixed factor proportions, and one unit of the commodity is produced by v units of land and u units of labor. There is no technological progress between periods.[2] The composite commodity must be shipped through the CBD. In both periods the transportation cost for a unit of the composite good per unit of distance is c dollar.

Every worker provides one unit of labor, and workers are in perfectly elastic supply to the city at a constant real wage, W_i $i=1,2$ For simplicity we suppress the substitutability of consumption goods and assume that he or she consumes q units of the composite good and one unit of residential land, and spends on the transportation cost for traveling to the CBD for the sake of shopping and recreation in each period. The cost of transporting a worker per unit of distance is e doller. In contrast to some urban models, however, employment is not concentrated at the CBD. Instead, workers are located in an industrial zone of uniform density extending in a line leading from the CBD. For simplicity we assume that total travel cost, the cost of commuting to work along with a postwork visit to the CBD, does not depend on the work location. So, in determining the wage rate no allowance is made for differential commuting costs from the edge of the residential zone to different parts of the industrial zone.

By assumption the cost of transporting a unit of the composite good per unit of distance, c is so greater than e, the cost of transporting a worker per unit of distance that the industrial zone is located next to the CBD and the residential zone is located beyond the industrial zone. In a two-period model with an intertemporal allocation structure we assume an exogenous increase in the price of the composite good as the mechanism of city growth. So, $P_2>P_1$, where P_1, P_2 are the prices of the composite good in the first and second periods, respectively. Land developed as residential

2) If we allow for the change of the technology in production between periods, in the second period the amount of labor per unit of output will be reduced.

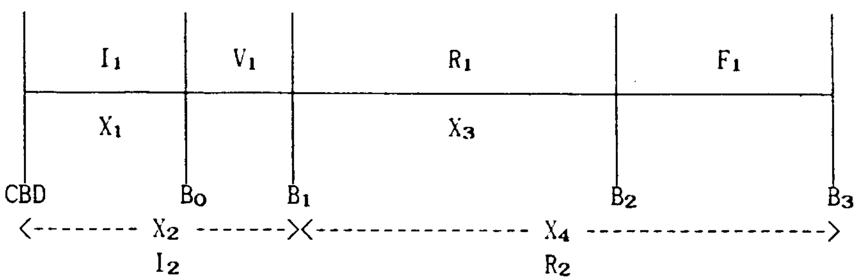
land in the first period remains residential in the second period as the cost of land conversion is assumed to be very high so that a pattern of leapfrog development occurs. Under this pattern of development some land relatively close to the industrial zone is held vacant during the first period and is developed as industrial land in the second period as shown in Fig. 1.

The lana shown as v1 in Fig. 1 could be developed as residential land in the first period and could earn residential land rents in both periods. But land owners keep the land idle in the first period, and wait for the more profitable industrial development in the second period. The trade-off is between two periods of residential land rents versus one period of higher industrial land rents. Leapfrogging of the industrial zone to the area beyond the residential zone is not permitted by the assumption that c is sufficiently large relative to e. Land owners are assumed to have perfect foresights. Thus, the decision on the amount of vacant land determines the sizes of the industrial and residential zones in the second period.

Finally, we assume that the agents who make land-use decisions in the model are nonresidents. The nonresidency means land rents do not reappear in the analysis as worker's income

The model consists of six unknowns, the wage rates in the two periods, W_1 and W_2 and four lengths of the industrial and residential zones in each period.

X_1 : the length of the industrial zone in the first period.
X_2 : the length of the industrial zone in the second period.
X_3 : the length of the residential zone in the first period.
X_4 : the length of the residential zone in the second period.



where, I_i $i=1,2$: industrial zone in each period.
R_i $i=1,2$: residential zone in each period.
V_1 : vacant land in the first period.
F_1 : farm Pattern of urban development in a two-period model

Fig. 1. Pattern of urban development in a two-period model

The six equations that determine these variables are as follows.

$$W_1 = P_1q + e(X_2+X_3) \tag{1}$$
$$W_2 = P_2q + e(X_2+X_4) \tag{2}$$

Equations(1) and (2) represent worker's budget constraints in the first and second periods for every resident at location B_2 and B_3, respectively. The locations B_2 and B_3 are on the outer edge of the residential zone in each period where land rents are zero as we assume that the opportunity cost of land in agriculture is zero. The wage rate $W_i(i=1,2)$ is the gross wage and is high enough to finance q units of the composite commodity, to finance the purchase of the services of one unit of residential land, and to pay for transportation costs. total expenditures on residential land rents plus transportation costs for workers are constant at all residential locations.

From the zero-profit condition for perfect competition in the composite commodity market we get a product exhaustion condition.

$$(1/v)(P_1 - cX_1 - uW_1) = 0 \tag{3}$$

Equation (3) means that in the first period land rents on the outer edge of the industrial zone, B_0 in the Fig. 1 are zero. If land rents were positive, land owners would develop the land industrially. The value of output is divided between the per-unit wage, uW_1 and firm's transportation costs, cX_1.

The fourth equilibrium condition is that land on the outer edge of the industrial zone in the second period, B_1 in Fig. I is equally profitable as industrial land in the second period and as residential land in the first and second periods. This condition is written as

$$\frac{(1/v)(P_2-cX_2-uW_2)}{(1+r)} = eX_3 + \frac{eX_4}{(1+r)} \tag{4}$$

where r is the discount rate. In equation (4) the left-hand side shows the present value of industrial land rents in the second period and the right-hand side measures residential land rents in the first and second periods, respectively. In calculating residential land rents we make use of the assumption that these rents are zero on the outer edge of the residential zone in each period.

The final two equations are derived by the fixed coefficient structure of production, the assumption that each unit of output is produced by v units of land and u units of labor. This implies that there is a proportional relationship in the sizes of the industrial and the residential zones in each time period.

$$X_3 = (u/v)X_1 \tag{5}$$
$$X_4 = (u/v)X_2 \tag{6}$$

Equations (5) and (6) state that the residential zone is u/v times the size of the industrial zone in each period.

We can eliminate X_3 and X_4 from the system by substituting equations (5) and (6) into equations (1), (2), and (4). This yields four equations in four unknowns, W_1, W_2, X_1, and X_2.

$$W_1 = P_1q + e(X_2 + (u/v)X_1) \tag{1'}$$
$$W_2 = P_2q + e(1 + u/v)X_2 \tag{2'}$$
$$P_1 - cX_1 - uW_1 = 0 \tag{3'}$$
$$\frac{P_2 - cX_2 - uW_2}{(1+r)} = euX_1 + \frac{euX_2}{(1+r)} \tag{4'}$$

Using equations (1′) and (3′) we eliminate W_1 to obtain

$$(1/u - q)P_1 = e((u/v)X_1 + X_2) + (c/u)X_1 \tag{5}$$

Equation (5) has a clear-cut interpretation. Sincel $1/u$ is the output per worker, the term $(1/u - q)P_1$ is always positive and is a measure of resources available for transportation costs. The first term in the right-hand side of equation (5) measures the cost of transporting the workers to the CBD who in the first period live on the outer edge of the residential zone, boundary B_2 in Fig. 1. The second term, $(c/u)X_1$ is the cost of transporting the composite good which is produced at distance X_1 from the CBD. We use equations(2′) and (4′) to eliminate W_2 to obtain

$$\frac{(1/u-q)P_2}{(1+r)} = e(X_1 + X_2/(1+r)) + \frac{(c/u+e(1+u/v))}{(1+r)}X_2 \tag{6}$$

Solving equations (5) and (6) for X_1 and X_2, we measure the size of the industrial zone in each period in terms of the exogenous variables as

$$X_1 = \frac{(1-qu)[(c/eu+(2+u/v))P_1 - P_2]}{(c/e+u(2+u/v)(c/u+eu/v)-(1+r)eu} \tag{7}$$

$$X_2 = \frac{(1-qu)[(c/eu+u/v)P_2-(1+r)P_1]}{(c/e+u(2+u/v))(c/u+eu/v)-(1+r)eu} \tag{8}$$

From equations (5) and (6) it follows that $(1-qu)>0$. In equation (7) and (8) the denominator is positive as the cost of transporting the composite good, c is greater than the cost of transporting worker, e, and $u/v>1$ as the amount of land required to produce a unit of the composite good is smaller than residential land required to house the labor used to produce the good. Thus, the numerator in equation (7) must be positive since X_1, the size of the industrial zone in the first period is positive. It follows that

$$P_2 < (c/eu+(2+u/v))P_1$$

As the city is growing, X_2, the industrial zone in the second period is larger than the industrial zone in the first period: $X_1 < X_2$. From equations (7) and (8) we obtain

$$X_1 < X_2 ; (1+ (2+r)/(1+c/eu+u/v))P_1 < P_2$$

Therefore, the range of p_2 is

$$(1+ (2+r)/(1+c/eu+u/v))P_1 < P_2 < (c/eu+(2+u/v))P_1 \qquad (9)$$

3. THE IMPACTS OF A WAGE TAX ON URBAN STRUCTURE

The imposition of a wage tax in a city where the supply of labor is perfectly elastic increases the cost of labor and discourages the production of the composite good in both the first and second periods. A wage tax decreases the real wage and induces some workers in the city to move out, and hence decreases land rents in this city. This emigration will lead to an increase of the gross wage rate and to decrease in output in each of the two periods. So, the demand for industrial land will decrease and industrial land rents will fall.

For a wage tax we can show quite generally that the tax decreases the amount of output and the size of the city in both periods. After the introduction of this tax the wage equations (1′) and (2′) become

$$(1-T)W_1 = P_1q + e(X_2 + (u/v)X_1) \qquad (10)$$
$$(1-T)W_2 = P_2q + e(1 + u/v)X_2 \qquad (11)$$

where T is the tax rate of a wage tax. Equations (10) and (11) state that the wage tax needs to be offset by a decrease in transportation costs. Such a decrease in transportation costs means that the residential zone will be moved closer to the CBD in the first period through the conversion of vacant land to residential land.

Substituting equation (10) into equation (3′) to eliminate W_1, totally defferentiating, and setting $T=0$ as the initial value of the tax rate we obtain

$$-(c+eu^2/v)dX_1 - eu\,dX_2 = u(P_1q + (eu/v)X_1 + eX_2)dT \qquad (12)$$

Substituting equation (11) into equation (4′) into equation (4′) to eliminate W_3, totally differentiating the resulting expression and again setting $T=0$ we obtain

$$-eu\,dX_1 - \frac{c+eu(2+u/v)}{(1+r)}dX_2 = \frac{u(P_2q+e(1+u/v)X_2)}{(1+r)}dT \qquad (13)$$

Solving equations (12) and (13) for dX_1 and dX_2 to determine how a wage tax affects industrial output and city growth in each period we obtain

$$dX_1 = \frac{uqP_2-(c/e+u(2+u/v))(qP_1+(eu/v)X_1)-(c+eu)X_2}{(c/e+u(2+u/v))(c/u+eu/v)-(1+r)eu}dT \quad (14)$$

$$dX_2 = \frac{-uq((c/eu+u/v)P_2-(1+r)P_1)}{(c/e+u(2+u/v)(c/u+eu/v)-(1+r)eu}dT$$

$$-\frac{((c/eu+u/v)(1+u/v) - (1+r))euX_2 - (1+r)(eu^2/v)X_1}{(c/e+u(2+u/v))(c/u+eu/v)-(1+r)eu}dT \quad (15)$$

In equations (14) and (15) the denominator is positive, since the condition that c/v is greater than $(2+r)e$ is necessary for land speculation to take place;[3] the discount rate r is less than 1; $u/v>1$ by the assumption that the residential zone is larger than the industrial zone in each period. So, we determine $dX_1/dT<0$ if the numerator in equation (14) is negative. From equation (9) we know the range of P_2. Substituting the upper bound of equation (9) into P_2 in the numerator of equation (14), the numerator is shown to be negative

$$-(c/e+u(2+u/v))(eu/v)X_1 - (c+eu)X_2$$

Thus, $dX_1/dT<o$, which means that the imposition of a wage tax decreases the sizes of the industrial and residential zones in the first period. In equation (15) the numerator is also negative and $dX_2/dT\leq 0$. This implies that a wage tax also decreases industrial output and the size of the city in the second period. Table 1 summarizes the effects of a wage tax on urban development.

Table 1. Effects of a Wage Tax on Urban Development

Variables	First period	Second period
Nominal wage rate	increase	increase
Residential rents	decrease	decrease
Industrial rents	decrease	decrease
Output	decrease	decrease
City size	decrease	decrease

4. CONCLUDING REMARK

In a two-period model with an intertemporal allocation structure the nonneutral land value tax accelerates city growth by encouraging the residential development of vacant land in the first period. But this premature development decreases industrial output and city size in the second period as less land is available for industrial devel-

3) The necessary condition for land speculation to take place in a two-period model was proved by David Mills[7].

opment. A wage tax has quite different effects on urban development. The impact of a wage tax is to decrease the real wage and to induce some workers in the city to move out, and to decrease residential land rents in both the first and second periods. The imposition of a wage tax in a city where the supply of labor is perfectly elastic increases the nominal wage rate and discourages the production of the industrial good in both periods. The demand for industrial land will decrease and industrial land rents will fall in each period. Therefore the imposition of a wage tax decreases industrial output and city size in both of the two periods.

REFERENCES

B. L. Bentick, "A Tax on land value may not be neutral," *National Tax Journal*, 35, 113(1982)

J. K. Brueckner, "A modern analysis of the effects of site value taxation," *National Tax Journal*, 39, 49−57(1986).

R. E. Grieson, "The economics of property taxes and land values : The elasticity of supply of structure," *Journal of Urban Economics*, 1, 367−38(1974).

Duck-Ho Lim, "The nonneutrality of the land value tax : Impacts on urban structure," *Journal of Urban Economics*, 32, 186−194(1992).

P. Mieszkowski, The property tax : "An excise tax or profits tax?," *Journal of Public Economics*, 1, 73−96(1972).

D. E. Mills, "The nonneutrality of land taxation," *National Tax Journal*, 34, 125−129 (1981).

D. E. Mills, "Growth, speculation, and sprawl in a monocentric city," *Journal of Urban Economics*, 10, 201−226(1981).

A. Skouras, "The non-neutrality of land taxation," *Public Finance*, 33, 113−134(1982).