THE KOREAN ECONOMIC REVIEW
Volume 10, Number 1 · 2, Winter 1994

THE URBAN-RURAL COST-OF-LIVING DIFFERENTIALS IN KANGWONDO

BYUNG HYUN KIM*

1. INTRODUCTION

In the dualistic developing countries such as Korea, we are particularly interested in comparing welfare levels in the modern (urban) sector with those found in the traditional (rural) sector. This has important implications for understanding the process of economic development through the modern sector enlargement, and the desirability of that process from a social welfare point of view.

Casual observations suggest that the cost-of-living is substantially greater in urban areas than in rural areas of developing economies, with implications for sectoral welfare comparisons. Accordingly, the search for consistent measures for comparing welfare levels in different circumstances is a long-standing concern. In this study, we examine a neglected determinant of relative welfare levels in a dual economy : the cost-of-living.

However, there are two problems to be overcome in making the cost-of-living comparisons. First, housing is a highly heterogeneous good and so observed housing rents can be a poor price index. For example, the considerably higher expenditures on housing in urban areas relative to rural areas undoubtedly reflect higher consumption levels of certain housing attributes as well as higher prices for those attributes. Observed housing expenditures thus reflect, at least in part, income differences. Second, even if we can devise a satisfactory price index for housing, there may well be significant substitution possibilities with other goods. The few studies for identifying the price effects of housing demand suggest that the compensated own-price elasticity of housing demand is far from negligible.[1] Thus, the difference in housing expenditures between urban and rural areas generally overestimate the underlying differential in the cost of a given level of utility to consumers. A similar comment can be made about food price comparisons.

The first problem is well known in applied work, and a solution based on the construction of hedonic price indices for housing has been widely used. The second problem is well appreciated theoretically, and the true cost-of-living indices for intertemporal welfare comparisons have been estimated, which have not to our

* Department of Economics, Hallym University.

1) See the surveys by Mayo(1981), and Malpezzi and Mayo(1987).

knowledge been applied in empirical work on spatial welfare comparisons, due to the existence of a number of empirical problems in identifying the price effects for housing demand.[2]

This study offers a joint solution to both these problems and uses it to make cost-of-living(welfare) comparisons between urban and rural areas of Kangwondo. The following section outlines a theoretical approach to the true hedonic cost-of-living index. In Section 3, we specify the true hedonic cost-of-living index in explicit functional form. Section 4 presents empirical results for the demand function and the true hedonic cost-of-living differentials. Section 5 offers summary and some conclusions.

2. CONSUMER PRICE INDICES

2.1 Meaning of Price Indices

In the context of consumers, economic index numbers attempt to construct a single ratio that measures one of two things. The first, the cost-of-living index, measures the relative costs of reaching a given standard of living under two different situations, while the second, the real consumption or utility index, compares two different standards of living in some appropriate units. The most convenient scale with which to measure welfare is the expenditure necessary, at constant prices, to maintain the various welfare levels being considered. These concepts, which use money to measure changes in welfare, can only be applied to situations where money and welfare are uniquely linked. This will not be the case where goods that are important for consumers' well-being are not purchased through the market; examples are health care, public parks, clean air, a noise-free environment, or some kinds of education. Hence, the index numbers we have to consider here are limited to the masurement of prices and quantities that arise in the market, as are most countries' consumer prices indices.

Our general approach makes the theory of index numbers very straightforward. The concept of the cost function of which we make use, first appeared in recognizable form in the literature on cost-of-living index numbers in the pioneering contributions by Konüs price index, or true cost-of-living index, is defined and the famous inequalities with the Paasche and Laspeyres indices explained.

Cost-of-living index numbers are devices for reducing the comparison between two complete price vectors such as p^1 and p^0 to a single scalar. If the two vectors are proportional to one another, so that, for example, p^1 is 5 percent greater than p^0, then we have no difficulty in saying that prices at 1 are 5 percent higher than prices at 0. However, when relative prices change, some standard of comparsion is required.

2) The price effects on housing demand have been mainly identified by two methods: (i) that proposed by Muth(1971) based on (more readily observed) land and other input prices, and subsuming a housing production function into the demand model, and (ii) that using hedonic price indices as the housing price variables, following Straszheim(1973).

Almost by definition, an index of the cost of living uses a measure of the standard of living as reference. One such measure would be some reference commodity bundle q^R, say. This technique, that of the fixed "shopping basket," yields a price index

$$P(p^1, p^0; q^R) = p^1 q^R / p^0 q^R \tag{1}$$

Clearly, (1) expresses the price level corresponding to p^1 relative to that at p^0 as the relative costs of buying the fixed basket q^R at the two sets of prices. However, a single bundle is an unnecessarily restrictive interpretation of what is meant by a constant standard of living, and the obvious alternative is to take a specific indifference curve as the reference concept that is to be held constant. On this interpretation, the cost-of-living index is the ratio of the minimum expenditures necessary to reach the reference indifference curve at the two sets of prices. Hence, if u^R is the label of the indifference curve taken as reference, the true cost-of-living index number is given by

$$P(p^1, p^0; u^R) = c(u^R, p^1) / c(u^R, p^0) \tag{2}$$

If the index numbers(1) or (2) are to be used to compare a large number of different prices on a consistent basis, some convenient "representative" q^R or u^R can be chosen. If, however, only two different price situations are being compared, the natural choices for reference are q^0, u^0 or q^1 and u^1.

The Laspeyres-Konüs cost-of-living index is defined as $P(p^1, p^0; u^0)$ and the Paasche-Konüs cost-of-living index is defined as $P(p^1, p^0; u^1)$. It turns out that the Laspeyres-Konüs index $P(p^1, p^0; u^0)$ is related to the Laspeyres price index $P(p^1, p^0; q^0)$ while the Paasche-Konüs index $P(p^1, p^0; q^1)$ as follows:

$$P(p^1, p^0; u^0) \leq P(p^1, p^0; q^0) \tag{3}$$

and

$$P(p^1, p^0; u^1) \geq P(p^1, p^0; q^1) \tag{4}$$

Note that these inequalities, which date back to konüs(1924), do not imply that the true index lies somewhere between the Paasche and the Laspeyres. In general, there is no unique true index and the Laspeyres-Konüs index that has the Laspeyres index as an upper limit is a different number from the Paasche-Konüs index that is no less than the Paasche index. Indeed, it is even possible for the Paasche index to exceed the Laspeyres!

The dependence of the true cost-of-living index(2) on u^R means that, even for a single individual obeying the axioms of changes in prices will affect the cost of living differently for different individuals, even if they have identical tastes, if their total expenditure levels differ. It is only under very special circumstances that these differences do not arise. It is easily shown that if preferences are homothetic, so that all indifference curves are the same shape and expenditure patterns do not vary with outlay, the cost function is proportional to utility, that is, $c(u, p) = ub(p)$ for some function $b(p)$. If we substitute this into (2), the utility based price index is simply $b(p^1)/b(p^0)$, which is independent of u. It is also clear that this is the only case in which this happens, so that homotheticity is both necessary and sufficient for the existence of the price index. This is also the only case in which it is always true that the true index lies between the Paasche and the Laspeyres. In general, homo-theticity is unlikely to be true in reality, so that it will always be necessary to allow for the effects of welfare or utility on cost-of-living indices.

2.2 Measurement of price Indices

The true cost-of-living index numbers $P(p^1, p^0; u^0)$ and $P(p^1, p^0; u^1)$ can be calculated straightforwardly using (2) if we know the cost function $c(u, p)$. The closest we are likely to approach this is through the estimation of a complete system of demand equations. However, such an exercise requires a great deal of data if generality is to be preserved and, even if these are available, the results do not always conform to theoretical preconceptions. It is thus natural to inquire whether it is possible to do with less information. Calculation of true indices $P(p^1, p^0; u^R)$ involves two separate issues. The first relates to the dependence of the index on u^R which, as we saw above, results from the nonhomotheticity of preferences. Changes in prices affect different households differently if they have different patterns of expenditure. To deal with this, we need information on Engel curves and on how family composition affects consumption. The second issue relates to substitution effects consequent on relative price changes. The differences between the Laspeyres index, the Paasche index, and the corresponding true indices are caused by subsitution effects, and these are generally present whether or not preferences are homothetic.

It can be assumed that, at a minimum, there will be information on p^0, q^0, p^1, and q^1, the prices and quantities in the two situations. Since both Paasche and Laspeyres indices require only this, they can always be easily evaluated and they are both used freely in practice. They will be exactly equal to their corresponding true indices if there is no substitution between commodities, that is, when the cost function takes the form

$$c(u, p) = \sum_k a_k(u) p_k \tag{5}$$

for quantities $a_k(u)$, which causes right-angle indifference curves. Even if (5) does not hold, the Laspeyres will offer a first-order approximation to the true index. Taking the first two terms of a Taylor expansion of $c(u^0, p^1)$ around u^0, p^0 yields

$$c(u^0, p^1) = c(u^0, p^0) + \sum q_k^0 (p_k^1 - p_k^0) + 1/2 \sum\sum s_{kj}^0 (p_k^1 - p_k^0)(p_j^1 - p_j^0) \tag{6}$$

since $\partial c(u^0, p^0)/\partial p_i = q_i^0$. Hence, if p^1 is fairly close to p^0, or if p^1 is almost proportional to p^0 (recall $\sum s_{kj}^0 p_j^0 = 0$), or if substitution is limited, the last term will be small so that.

$$P(p^1, p^0; u^0) = c(u^0, p^1)/c(u^0, p^0) \simeq p^1q^0/p^0q^0 = P(p^1, p^0; q^0) \tag{7}$$

Similarly, the Paasche index can be shown to give a first-order apporoximation to its corresponding true index. these approximations offer considerable encouragement to the use of Paasche and Laspeyres indices although it is important to be aware of cases where substitution is likely to be significant, for example, in comparisons between different countries.

There have been frequent attempts to improve on the Paasche and Laspeyres formulas without extending the information required. Following the work by Diewert (1976), perhaps the most useful of these is the Tôrnqvist(1936) price index defined by

$$\log P(p^1, p^0; T) = \sum_k 1/2(w_k^1 + w_k^0)\log(p_k^1/p_k^0) \tag{8}$$

where w^1 and w^0 are the budget shares in the two situations. Diewert shows that if the logarithm of the cost function is a quadratic form in the logarithms of prices and utility, then the Tôrnqvist index is attractive since the quadratic specification can provide a second-order differential approximation to an arbitrary, linearly homogeneous, twice continuously differentiable cost function, i.e., it is a flexible functional form.[3] However, without knowing the parameters of the cost function, we lack more specific information about the reference indifference curve(such as what budget level and price vector corresponds to it), and the result is of no help in constructing a constant utility to it), and the result is of no help in constructing a constant utility cost-of-living index series with more than two elements. A chained series of pairwise Tôrnqvist indices can always be constructed, but this has a different reference indifference curve for every link in the chain.

An alternative approach to approximation is via the construction of Divisia indices. Instead of comparing two discrete price situations, these indices work by analyzing the continuous effects of price changes on the cost of living. Denote the proportional rate of change of the price level by $d\log P(p;u)$; this is equal to $d\log c(u, p)$, so that

$$d\log P(p;u) = d\log c(u, p) = \sum w_k(u, p) d\log p_k \tag{9}$$

Hence, for any fixed utility level u, we can alway write

3) f is a flexible functional form if it can provide a second-order (differential) approximation to an arbitrary twice continuously differentiable function f^* at a point x^*. f differentially approximates f^* at x^* iff (1) $f(x^*)=f^*(x^*)$, (ii) $\nabla f(x^*)=\nabla f^*(x^*)$, and (iii) $\nabla^2 f(x^*)=\nabla^2 f^*(x^*)$, where both f and f^* are assumed to be twice continuously differentiable at x^*(and thus the two Hessian matrices in (iii) will be symmetric). Thus a general flexible functional form f must have at least $1+N+N(N+1)/2$ free parameters. If f and f^* are both linearly homogeneous, then $f^*(x^*)=x^{*T}\nabla f^*(x^*)$ and $\nabla^2 f^*(x^*)x^*=O_N$, and thus a flexible linearly homogeneous functional form f need have only $N+N(N-1)/2=N(N+1)/2$ free parameters. The term 'differential approximation' is in Lau(1974). Diewert(1976) shows that the quadratic specification is a flexible linearly homogeneous functional form.

$$\log P(p^1, p^0; u) = \int_{p^0}^{p^1} \Sigma w_k(u, p) d\log p_k \tag{10}$$

This suggests constructing a price index that replaces $w_k(u, p)$ in (9) and (10) by the actually observed budget shares w_k, and this index is the Divisia index comparing p^1, p^0. However, unless preferences are homothetic, the utility constant budget shares are not equal to the actual budget shares, and the variation of u over the integration means that, if prices were to change through time so as to return after some variation to an earlier value, then the calculated integral will not generally give $P(p^0, p^0) = 1$. In practice, neither the quantities nor the prices are continuously observable, so that (9) would have to be approximated by some formula containing finite changes; the Tôrnqvist index (8) is an obvious possibility, but there are many others. Such differential price indices can be "chained" to give an approximation to (10).

2.3 Difference of Price Indices between Households

The true index can vary between households of differing standards of living. This is a phenomenon that exists independently of the degree of substitution between goods and can be seen, for example, by considering the Tôrnqvist index (8). If we assume that $\log(p^1_k/p^0_k)$ is the same for all households, differences in the index will exist if the average budget shares $1/2(w^1_k + w^0_k)$ vary from household to household. But with nonhomothetic preferences, richer households will have larger budget shares for luxuries and smaller budget shares for necessities, so that if changes in prices involve relative price changes between luxuries and necessities, the cost-of-living index will differ systematically between poor and rich households.

We can also illustrate these effects using true indices calculated from an assumed cost funcion the parameters of which are econometrically estimated. Consider the class of price-independent generalized linear preferences(PIGL) by Deaton and Muellbauer(1980), which will be explained in more detail in Section III. The logarithmic form of this class(PIGLOG) has the cost function

$$\log c(u, p) = (1-u)\log a(p) + u\log b(p) \tag{11}$$

where a(p) and b(p) are positive linearly homogeneous concave functions of prices. This gives a particularly simple form for the Laspeyres-Konüs cost-of-living index,

$$\log P(p^1, p^0; u^0) = (1-u^0)\log[a(p^1)/a(p^0)] + u^0\log[b(p^1)/b(p^0)] \tag{12}$$

where u^0 is equal to $\log[x^0/a(p^0)]/\log[b(p^0)/a(p^0)]$. Hence, across different households facing the same prices, u^0 is a linear function of $\log x^0$. Equation (12), which is of course specific to the PIGLOG model, shows very clearly how the price index varies with the standard of living of the household. Rich households have cost-of-living indices closer to $b(p^1)/b(p^0)$, while $a(p^1)/a(p^0)$ is more relevant for the less affluent.

2.4 The True Hedonic Cost-of-Living Index[4)]

The true hedonic cost-of-living index is considered as a special case of the true cost-of-living index by Konüs(1924). This subsection will discuss aspects of the theory relevant to our present interest.

The cost or expenditure function for a household with characteristics denoted by the vector z^R can be expressed as follows:

$$c=c(u^R, p^R, z^R) \tag{13}$$

which is the minimum cost necessary for a household with the characteristics vector z^R to achieve the reference utility u^R when facing the reference price vector p^R. We have been interested in comparing the cost of living between two regions (urban and rural) when households face two different price vectors (urban and rural).

The true hedonic cost-of-living index of our interest is given by

$$P(p, p^R; u^R, z^R)=c(u^R, p, z^R)/c(u^R, p^R, z^R) \tag{14}$$

which is the minimum cost of achieving the reference utility indexed by u^R when the household with the characteristics vector z^R faces one region price vector p(urban region in our case) relative to the minimum cost of achieving the same utility level when he faces the other region price vector p^R(rural region in our case). In general, it is shown that this index varies according to the value of p as well as the values taken by reference variables(u^R, p^R, and z^R). However, only for homothetic preferences, the cost function $c(u^R, p^R, z^R)$ is proportional to utility, that is, $c(u^R, p^R, z^R)=u^Re(p^R, z^R)$ for some function $e(p^R, z^R)$. If we substitute this into (14), the true hedonic cost-of-living index is simply $e(p, z^R)/e(p^R, z^R)$, which is independent of the reference utility level u^R. In general, the homotheticity assumption of preferences is unlikely to be possible in reality. Thus, it will always be necessary to allow for the effects of utility on this index.

For the purpose of calculating the true hedonic index which depends on the utility level, we need the inversion reference income level y^R in equation (14). According to the duality relation that exists between utility and cost functions,[5)] the reference income level y^R in the indirect utility function corresponds to the reference utility level u^R in the cost function. Thus, the true hedonic cost-of-living index (14) can also be written as a function of the reference income level y^R instead of the un-

4) Hedonic price theory is originally due to Court(1941), and formalized and sophisticated by Houthakker(1952), Stone (1956), Becker(1965), Lancaster(1966), Lipsey and Rosenbluth(1971), Ohta and Griliches(1972), and Rosen(1974). Hedonic price theory has been widely used to construct true price indices of industrial capital goods and household durable goods which are usually subject to quality change.

5) Duality theory implies that if consumers minimize costs of satisfying given preferences, and if product prices are exogenous, then the cost function satisfying the usual regularity conditions(i.e., the regularity conditions that are required to determine uniquely the corresponding utility function are that the cost function be increasing, linearly homogeneous, and quasiconcave in the product prices) contains sufficient information to describe completely the corresponding utility function, and *vice versa.*

measurable reference utility level u^R as follows:

$$P(p, p^R; y^R, z^R) = c(v(y^R, p^R, z^R), p, z^R)/c(v(y^R, p^R, z^R), p^R, z^R)$$
$$= f(y^R, p, p^R, z^R)/y^R \tag{15}$$

where v(·) denotes the indirect utility function. In equation (15), the numerator f (y^R, p, p^R, z^R) is then called the equivalent income function for evaluating the welfare of a household facing certain price vector p, but using y^R, p^R, and z^R as the references.[6] By inspection of (14) and (15) it can be readily shown that the true cost-of-living index increases as reference income(or utility) increases if and only if the marginal cost of utility(expressed as a proportion of expenditure) is greater at p than p^R.

3. SPECIFICATION OF THE TRUE HEDONIC COST-OF-LIVING INDEX.

3.1 The Equivalent Income Equation

The equivalent income function is defined as follows. Household preferences may be represented by either the direct or indirect utility function which are denoted, respectively, by

$$u = u(X^R) \tag{16}$$

and

$$v = v(y^R, p^R, z^R) \tag{17}$$

We wish to compare the levels of a household's welfare when it faces different consumption possibility sets. To do this we choose certain prices vector, denoted by p. For a given budget constraint(y^R, p^R, z^R), equivalent income is defined as that level of income which, at certain price vector, affords the same level of utility as can be attained under the given budget constraint. Formally,

$$v(y_E, p, z^R) = v(y^R, p^R, z^R) \tag{18}$$

Inverting the indirect utility function we obtain equivalent income in terms of the cost function

$$Y_E = c(v(y^R, p^R, z^R), p, z^R)$$
$$= f(y^R, p, p^R, z^R) \tag{19}$$

This definition of equivalent income has also been suggested by Varian(1980). It is very similar to the concept proposed by Mckenzie(1956) which was later christened "money metric utility" by Samuelson(1974), and which is defined by

$$m = c(u(x^R), p, z^R) = g(x^R, p, z^R) \tag{20}$$

Money metric utility implies, for the reference utility level, the minimum cost of

6) The equivalent income function is explained in more detail in Section 3.

reaching it at certain prices and this becomes money of utility itself. Money metric utility is a monotonic increasing function of $u(x^R)=u^R$ itself since $c(u, p)$ is increasing in u and p is constant, and can thus be taken as a utility indicator in its own right.

The properties of the euqivalent income function may be derived from the well-known properties of indirect utility and cost functions.[7] These imply that f is increasing in y^R and p and decreasing in p^R, is concave and homogeneous of degree one in p, and is continuous with first and second derivatives in all arguments.[8] Commodity demands are given by

$$x^R(y^R, p^R, z^R) = \partial f/\partial p \mid p=p^R = -(\partial f/\partial p^R)/(\partial f/\partial y^R) \tag{21}$$

Given a set of demand functions, the equivalent income function may be generated by solving the system of partial differential equations together with the boundary condition[9]

$$y^R = f(y^R, p^R, p^R, z^R, z^R) \tag{22}$$

As an example, consider the two-commodity Cobb-Douglas case with the indirect utility function without z^R

$$v(y^R, p_1^R, p_2^R) = (p_1^R)^a (p_2^R)^{1-a}/y^R \tag{23}$$

The equivalent income function is given by

$$Y_E = (p_1/p_1^R)^a (p_2/p_2^R)^{1-a} y^R \tag{24}$$

3.2 The Almost Ideal Demand System(AIDS)

Ever since Stone(1953) first estimated a system of demand equations derived explicitly from consumer theory, there has been a continuing search for alternative specifications and functional forms. Many models have been proposed, but perhaps the most important in current use, apart from the original linear expenditure system, are the Rotterdam model[see Deaton and Muellbauer(1980)]. These models have been extensively estimated and have, in addition, been used to test the homogeneity and symmetry restrictions of demand theory. Among them, the AIDS model is of comparable generality to the Rotterdam and translog models but has considerable advantages over both. Thus we are interested in the AIDS model for the purpose of the functional specification of the true hedonic cost-of-living index.

The AIDS model gives an arbitrary first-order approximation to any demand system; it satisfies the axioms of choice exactly; it aggregates perfectly over con-

---

7) See, for example, Deaton and Muellbauer(1980), and Diewert(1978).

8) Strictly speaking, first and second derivatives exist except possibly on a set of measure zero, and by increasing(decreasing) in $p(p^R)$ we mean nondecreasing(nonincreasing) in $p(p^R)$ and increasing(decreasing) in at least one element of $p(p^R)$.

9) Provided that the estimated demand system has a symmetric negative semidefinite matrix of price derivatives [Samuelson(1950), and Hurwicz and Uzawa(1971)] and the Lipschitz condition is satisfied.

sumers without invoking parallel linear Engel curves; it has a functional form which is consistent with known household-budget data; it is simple to estimate, largely avoiding the need for nonlinear estimation; and it can be used to test the restrictions of homogeneity and symmetry through linear restrictions on fixed parameters. Although many of these desirable properties are possessed by one or other of the Rotterdam or translog models, neither possesses all of them simultaneously.

In deriving the system of demand equations, the starting point has generally been the specification of a function which is general enough to act as a second-order approximation to any arbitrary direct or indirect utility function or, more rarely, a cost function. The AIDS model follows this approach in terms of generality, but it start, not from some arbitrary preference ordering, but from a specific class of preferences, which by the theorems of Muellbauer(1975, 1976) permit exact aggregation over consumers : the representation of market demands as if they were the outcome of decisions by a rational representative consumer. These preferences, known as the PIGLOG class, are represented via the cost or expenditure function which defines the minimum expenditure necessary to attain a specific utility level at given prices. We denote this function c(u, p) for utility u and price vector p, and define the PIGLOG class by

$$\log c(u,p)=(1-u)\log\{a(p)\}+u\log\{b(p)\} \tag{25}$$[10]

u lies between 0(subsistence) and 1(bliss) so that the positive linearly homogeneous functions a(p) and b(p) can be regarded as the costs of subsistence and bliss, respectively.

Next we take specific functional forms for loga(p) and logb(p). For the resulting cost function to be a flexible functional form, it must possess enough parameters so that at any single point its derivatives $\partial c/\partial p_i$, $\partial c/\partial u$, $\partial^2 c/\partial p_i \partial p_j$, $\partial^2 c/\partial u \partial p_i$, and $\frac{\partial^2 c}{\partial u^2}$ can be set equal to those of an arbitrary cost function. We take

$$\log a(p)=\alpha_0+\sum_i \alpha_i \log p_i+1/2\sum_i\sum_j r_{ij} \log p_i \log p_j \tag{26}$$

and

$$\log b(p)=\log a(p)+\beta_0\prod_i p_i^{\beta_i} \tag{27}$$

so that the AID cost function, augmented to include household characteristics z, is written[11]

10) Equation (25) is the same as equation(11).

11) In theory, household characteristics can be introduced into the AIDS model by allowing any of its underlying parameters to be household-specific. Our method is the simplest way of introducing such effects. We experimented with more complicated multiplicative effects in the empirical work but were unable to obtain satisfactory results.

$$\log c(u, p, z)=\alpha_0+z\pi+\sum_i \alpha_i \log p_i+1/2\sum_i \sum_j r_{ij} \log p_i \log p_j+u\beta_0 \prod_i p_i^{\beta_i} \tag{28}$$

where α_i, β_i, γ_{ij}, and the vector π are parameters.

The demand functions can be derived directly from equation (28). It is a fundamental property of the cost function that its price derivatives are the quantities demanded : $\partial c(u, p, z)/\partial p_i=q_i$.
Multiplying both sides by $p_i/c(u, p, z)$ we find

$$\partial \log c(u, p, z)/\partial \log p_i=p_i q_i/c(u, p, z)=w_i \tag{29}$$

where w_i is the budget share of good i, which implies the price elasticity of the cost function with respect to good i as the compensated(Hicksian) demand function of good i in budget share form. Hence, logarithmic differentiation of (28) gives the budget shares as a function of utility and price :

$$w_i=\alpha_i+\sum_j r_{ij} \log p_j+\beta_i u\beta_0 \prod_i p_i^{\beta_i} \tag{30}$$

For a utility-maximizing consumer, total income y is equal to c(u, p, z) and this equality can be inverted to give the indirect utility function v(y, p, z) at the utility maximum. If we do this for (28) and substitute the result into (30) we have the budget shares as a function of y, p, and z;these are the set of uncompensated(Marshallian) demand functions or the AIDS demand functions in budget share form :

$$w_i=\alpha_i+\sum_j r_{ij} \log p_j+\beta_i \log(y/P) \tag{31}$$

where P is a price index defined by

$$\log P=\alpha_0+z\pi+\sum_i \alpha_i \log p_i+1/2\sum_i \sum_j r_{ij} \log p_i \log p_j \tag{32}$$

P is the minimum cost of zero utility and can thus be interpreted as the cost-of-subsistence for households.

The restrictions on the parameters of (28) imply restrictions on the parameters of the AIDS equation (31). We take these in three sets

$$\sum_i \alpha_i=1 \quad \sum_i r_{ij}=0 \quad \sum_i \beta_i=0 \tag{33}$$

$$\sum_j r_{ij}=0 \tag{34}$$

$$r_{ij}=r_{ji} \tag{35}$$

The restrictions (33)~(35) are required to make the model consistent with the theory of demand. The conditions (33) are the adding-up restrictions; as can easily be checked from (31), these ensure that $\sum w_i = 1$. Zero homogeneity in total income and prices of the demand functions requires restriction (34), which can be tested equation by equation, in addition to restrictions (33). Slutsky symmetry is satisfied by (31) if and only if the symmetry restriction (35) holds. The second-order condition(negativity condition) for the consumer's choice problem requires that the Slutsky matrix, s_{ij}, generated by this model is symmetric and negative semi-definite. As is true of other flexible functional forms, this condition can not be ensured by any restrictions on the parameters alone. It can however be checked for any given estimates by calculating the eigenvalues of the Slutsky matrix s_{ij}, say. In practice, it is easier to use not s_{ij} but $k_{ij} = p_i p_j s_{ij}/y$, the eigenvalues of which have the same signs as those of s_{ij} and which are given by

$$k_{ij} = r_{ij} + \beta_i \beta_j \log(y/P) - w_i \delta_{ij} + w_i w_j \tag{36}$$

where δ_{ij} is the Kronecker delta. Following Deaton and Muellbauer(1980), the negativity condition(concavity condition) requires that the eigenvalues of k_{ij} are solely negative, in addition to the restrictions (33)~(35) mentioned above. Note that apart from this negativity condition, all the restrictions are expressible as linear constraints involving only the parameters and so can be imposed globally by standard techniques.

Given these restrictions, the AIDS demand functions are simply interpreted : in the absence of changes in relative prices and real income(y/P) the budget shares are constant and this is the natural starting point for predictions using the model. Changes in relative prices work through the terms r_{ij}; each r_{ij} represents 10^2 times the effect on the ith budget share of a 1 percent increase in the jth jth price with(y/P) held constant. Changes in real income operate through β_i coefficients; these add to zero and are positive for luxuries and negative for necessities.

3.3 Explicit Functional Form for the True Hedonic Cost-of-Living Index

So far, we have explained the equivalent income equation and the almost ideal demand system(AIDS) to specify the true hedonic cost-of-living index, equation(15), in explicit functional form.

Given the AIDS demand functions in budget share form, equation (31), the equivalent income function is given by[12]

$$\log y_E = \alpha_0 + z^R \pi + \sum_i \alpha_i \log p_i + 1/2 \sum_i \sum_j r_{ij} \log p_i \log p_j$$
$$+ \prod_i \left(p_i / p_j^R\right)^{\beta_i} \{\log y^R - \alpha_0 - z^R \pi - \sum_i \alpha \log p_i^R$$

12) The equivalent income function for the AIDS model is derived in King(1983).

$$-1/2\sum_i\sum_j r_{ij}\log p_i^R \log p_j^R\}$$

$$=\log P+\prod_i\left(p_i/p_j^R\right)^{\beta_i}\{\log y^R-\log P^R\} \tag{37}$$

The logarithmic form of the true hedonic cost-of-living index, equation (15), is as follows :

$$\log P(p, p^R; y^R, z^R)=\log f(y^R, p, p^R, z^R)-\log y^R$$
$$=\log y_E-\log y^R \tag{38}$$

Substituting equation (37) into equation (38), under the AIDS model, we have the following equation for the true hedonic index

$$\log P(p, p^R; y^R, z^R)=\log P-\log y^R+\prod_i\left(p_i/p_i^R\right)^{\beta_i}\{\log y^R-\log P^R\} \tag{39}$$

where logP is given by (32) when evaluated at the price vector p while $\log P^R$ is evaluated at the price p^R, and both apply to the reference household(with characteristics z^R), and where p_i is the ith element of p, while p_i^R is the ith element of p^R. Note that $P(p, p^R; y^R, z^R)$ is now increasing(decreasing) in y^R if $\prod_i\left(p_i/p_i^R\right)^{\beta_i}$ is greater(less) than unity.

The task of empirical work to follow is to estimate the AIDS demand functions in budget share form (31) under the testable demand theory restrictions, and use the estimated demand parameters so as to retrieve logP and $\log p^R$, and calculate the true hedonic cost-of-living index $P(p, p^R; y^R, z^R)$ through equation(39).

4. EMPIRICAL IMPLEMENTATION

4.1 Data

We need the price and expenditure data facing each household for food and housing in estimating the demand model. The expenditures on food and housing can be obtained from the 1991 Survey Data by household in urban and rural areas.[13] Calculation of the price index for food is based upon the Consumer's Price Index by commodity and region. However, housing is a highly heterogeneous good. Its single scalar measure of the quantity consumed is not readily available and the observed market price(rent) can be a poor price index. This problem is well known in empirical work, and a solution depends on the construction of hedonic price indices for

13) The 1991 Survey Data is a random sample of 980 households living in 14 areas(urban areas : Chunchon, Wonju, Kangneung, Sokcho, Donghae, Samchok, Taeback, rural areas : Hongchon, Hwachon, Yanggu, Pyungchang, Youngweol, Hwoengsung, Chulwon) of Kangwondo, which is used for constructing the hedonic price index for housing.

housing at various locations(location-specific hedonic price indices). We treat housing as a composite good. The hedonic price index facing each household for its housing is thus the price index reflecting the estimated cost for a fixed reference bundle of housing attributes.

In this study, we have considered the following dwelling and location attributes in both urban and rural areas in constructing hedonic price indices for housing : (i) land area, (ii) construction area, (iii) number of rooms, (iv) dummy variable for waterworks, (v) dummy variable for electricity facilities, (vi) dummy variable for urban /rural location, and (vii) dummy variables for type of tenure. The hedonic price function has been specified in a loglinear form of the observed market price(rent) of housing and a bundle of housing attributes mentioned above.

The household demographic composition variables used in this study include (i) household income, (ii) household size, (iii) number of children, (iv) dummy variable for homeowner, (v) dummy variable for self-employed, and (vi) dummy variable for region(urban/rural). The statistical figures for these variables are obtained from the 1991 Survey Data.

4.2 Estimation

In general, estimation can be carried out by substituting (32) in (31) to give

$$w_i=(\alpha_i-\beta_i\alpha_0)+\sum_j r_{ij}\log p_j+\beta_i\{\log y-z\pi$$

$$-\sum_i \alpha_i \log p_i-1/2\sum_i\sum_j r_{ij}\log p_i \log p_j\} \tag{40}$$

and estimating this non-linear system of equations by maximum likelihood methods with and without the restrictions (34) and (35).[14] Equation (40) is not particularly difficult to estimate since the first-order conditions for likelihood maximization are linear in α and γ given β and *vice versa* so that "concentration" allows iteration on a subset of the parameters.[15] All the parameters in (40) are identified given sufficient variation in the independent variables.[16]

In order to complete the stochastic specification of the system of the AIDS demand functions in budget share form, a multivariate normal disturbance is added to the multivariate demand system. Since demand shares are bounded between zero and one, the multivariate normal distribution can only serve as an approximation to

14) Note that since the data add up by construction, (33) is not testable.

15) See, for example, Deaton(1975).

16) In many examples the practical identification of α_0 is likely to be problematical. This parameter is only identified from the α_is in (40) by the presence of these latter inside the term in braces, originally in the formula for logP, equation (32). However, in situations where individual prices are closely collinear, logP is unlikely to be very sensitive to its weights so that changes in the intercept term in (40) due to variations in α_0 can be offset in the αs with minimal effect on logP. This can be overcome in practice by assigning a value to α_0 *a priori*. Since the parameter can be interpreted as

the true distribution of demand shares. In Rossi(1984), an alternative logistic-normal stochastic specification for the system of demand share equations is developed. In this study, we employ the additive normal specification. By definition, $\sum w_i = 1$. This implies that the multivariate normal distribution of all the demand shares is singular. For estimation purposes, any one of demand share equations has to be dropped.

We will base the computation on an iterative maximum likeihood procedure.[17] The systems estimator has been recommended as an alternative to direct estimation of each single equation by OLS on the grounds that it is a more efficient method of estimation. This may be an important consideration in the context of the second-order approximation we use here due to the possibility of multicollinearity problems[Christensen and Greene(1976)]. However, while it is clear that the systems estimator would be preferred under the standard error term assumptions that are usually made, it is not clear that it will outperform the single equation method of estimation when both methods are subject to functional misspecifications caused by the use of second-order approximations[White(1980)].

4.3 Empirical Results

We estimate the demand model using data inclusive on three nondurable groups of consumers' expenditure, namely, food, housing, and the other goods.[18] In this study, we employ the additive normal specification. By definition, $w_1+w_2+w_3=1$. This implies that the trivariate normal distribution of w_1, w_2, and w_3, is singular. For estimation purposes, any one of demand share functions has to be dropped. We arbitrarily choose the demand share function for the other goods(w_3) and jointly estimate the food demand share function(w_1) and the housing demand share function (w_2) by maximum likelihood methods with and without the restrictions implied by symmetry and homogeneity.

(1) The Demand Function

Table 1 reports the main parameter estimates and their t-values obtained from estimating the demand model (40) with the testable restrictions on the parameters, (34) and (35), as well as (33) which are automatically and costlessly stisfied.[19] The estimates of β classify food as a necessity($\beta_1<0$) while housing and the other goods are luxuries(β_2 and $\beta_3>0$). The γ coefficients are significantly different from zero, showing that all t-values are absolutely larger than 2. Even so, none of the variables con-

---

the outlay required for a minimal standard of living when prices are unity(usually in reference prices), choosing a plausible value is not difficult.

17) Deaton and Muellbauer(1980) advocates a price index approximation to (4) which makes the model linear in parameters whithout cross-equation restrictions and so estimable by OLS. However, as they point out, for satisfactory results this requires that individual prices are reasonably collinear. This is unlikely to hold in a household level cross-section estimation, though it is more plausible in a time-series application.

18) The other goods indicate all the goods except for food and housing in consumers' expenditure.

19) For brevity, the unrestricted demand parameter estimates and the parameters on household demographic and other characteristics are omitted but are available on request.

Table 1. The Demand Parameter Estimates

Parameters	Estimates	t-Values
α_0	314.542	53.312
α_1	6.283	24.764
α_2	-20.125	-32.135
α_3	14.842	19.565
β_1	-0.291	-1.211
β_2	0.042	2.616
β_3	0.249	1.785
γ_{11}	-1.081	-7.429
$\gamma_{12}(=\gamma_{21})$	1.070	3.192
γ_{22}	-1.076	-5.669
$\gamma_{13}(=\gamma_{31})$	0.011	2.574
$\gamma_{23}(=\gamma_{32})$	0.006	2.048
γ_{33}	-0.017	-4.016

Note : subscripts 1, 2, and 3 indicate food, housing, and the other goods, respectively.

sidered have any detectable effect on the value share for the other goods. Similarly, the prices of the other goods have little or no effect anywhere, while the prices of food and housing appear with considerable regularity in their relationship.

The two-good joint demand model performs well from the point of view of demand theory restrictions. In order to test the validity of the homogeneity condition, which can be tested equation by equation, we use the F-ratios obtained by comparing the residual sums of squares of the unrestricted equation and the equation with homogeneity restrictions($\sum_j r_{ij}=0$). The homogeneity condition is convincingly rejected. The symmetry condition, unlike homogeneity, can not be tested on an equation-by-equation basis and we must rely on a large-sample likelihood-ratio test for the system as a whole. The symmetry condition($r_{ij}=r_{ji}$) performed well. To test the concavity condition(negative semi-definite Slutsky matrix), the eigenvalues of k_{ij} were calculated at all data points in the sample and found to be negative for 76% of sample households.

Table 2 gives the implied demand elasticities at mean points. The income elasticities show that all three goods are normal goods, although food appears to be income inelastic. Note the general price inelasticity of demand. The cross-price effects between food and housing exist, and indicate that two goods are compensated substitutes, though the cross-price elasticities are fairly small.

(2) The True Cost-of-Living Differentials

The benchmark for comparing the cost-of-living differentials between urban and rural areas is a typical rural household of average size facing the mean prices of three goods(food, housing, and the other goods) for all rural households. We can then con-

Table 2. The Demand Elasticities

Elasticities / Goods	Income	Price			
		Uncompensated		Compensated	
		Own	Cross	Own	Cross
Food	0.811	-0.567	-0.027	-0.449	0.081
Housing	1.329	-0.858	0.060	-0.636	0.145
The other Goods	1.652	-0.361	-0.042	-0.305	-0.140

Note : all the elasticities were calculated at mean points.

Table 3. The True Cost-of-Living Differentials

(1) Reference Rural Income (Won/Month/ Household)	(2) Equivalent Urban Income (Won/Month/ Household)	(3) Cost-of-Living Indices (2)/(1)	(4) Cost-of-Living Differentials (%)
200,000	224,700	1.124	12.4
300,000	339,500	1.132	13.2
500,000	568,500	1.137	13.7
800,000	914,900	1.144	14.4
1,200,000	1,381,800	1.152	15.2
1,700,000	1,965,300	1.156	15.6
2,300,000	2,679,100	1.165	16.5
3,000,000	3,525,800	1.175	17.5

Note : cost-of-living differentials are obtained from the function, 100(cost-of-living indices-1).

sider, for various reference utility levels for such a rural household, the reference minimum costs(incomes) of reaching them at the referecne prices and these become our money measures of utility levels themselves. In common parlance, the reference utility level u^R might correspond to a "200,000 Won-month-household" or a "300,000 Won-month-household." The cost of living in urban areas is then calculated at the mean urban prices of three goods for each rural income level. Thus, for each reference rural income, we calculate the corresponding maximum utility level by inverting the rural cost function and then evaluate the urban cost function for that utility level. This is formally identical to calculating the equivalent income at urban prices of each rural income level using rural prices as the resference.

Table 3 gives the equivalent incomes in urban areas and the estimated true cost-of-living indices and differentials for various reference incomes in rural areas. With the mean prices and the estimates of β, the value of $\prod_i (p_i / p_j^R)^{\beta_i}$ is 1.01652 for urban areas. This implies that the marginal cost of utility(expressed as a proportion of expenditure) is greater in urban areas than in rural areas. Thus we find that the true cost-of-living indices and differentials are strictly increasing in reference incomes(or utilities), as can be seen from Table 3. This reflects the higher prices for housing and the other goods in urban areas and the empirical result that housing and the other goods are luxury goods(β_2 and $\beta_3 > 0$).[20]

5. CONCLUSIONS

We have offered the theoretical background and implemented a tractable empirical method for analyzing the true hedonic cost-of-living differentials between urban and rural areas. The traditional concept of a cost-of-living index is that it measures the relative costs of reaching a given standard of living under two different situations. The true hedonic cost-of-living index we have used is considered as a special case of the true cost-of-liveing index by Konüs, by reason that it includes the characteristics of a household in its cost function. For estimation purposes, we have specified the true hedonic cost-of-living index in explicit functional form making use of the almost ideal demand system(AIDS). In this study, we have applied maximum likelihood methods to a large household level data set for estimating the system of the AIDS demand functions in budget share form.

In empirical results, food is found to be a necessity, while housing and the other goods are luxury goods. The symmetry condition performed well. The validity of the homogeneity condition is convincingly rejected. The concavity condition was satisfied with 76% of sample households. All three goods(food, housing, and the other goods) generally appear to be price inelastic. The cross-price effects between food and housing exist, and indicate that two goods are compensated substitutes. The mean prices for housing and the other goods are higher in urban areas than in rural areas. We find that the marginal cost of utility is greater in urban areas, implying that the true hedonic cost-of-living differentials are strictly increasing with reference incomes.

20) Note that, although the mean prices for food are higher in urban areas than in rural areas, food is a necessity($\beta < 0$). If the mean prices for housing and the other goods were not higher in urban areas, then the true cost-of-living indices and differentials would decrease as reference incomes increase.

REFERENCES

A. Deaton, "Estimation of Own-and Cross-Price Elasticities from Household Survey Data," *Journal of Econometrics*, 36, pp. 7-30, 1987.

A. Deaton and J. Muellbauer, *Economics and Consumer Behavior*, Cambridge Univ. Press, Cambridge, 1980.

A. Deaton and J. Muellbauer, "An Almost Ideal Demand System," *American Economic Review*, 70, pp.312-326, 1980.

A. Deaton and J. Muellabuer, "On Measuring Child Costs : With Applications to Poor Countries," *Journal of Political Economy*, 94, pp.720-744, 1986.

W.E. Diewert, "The Economic Theory of Index Numbers : A Survey," in *Essays in the Theory and Measurement of Consumer Behavior(A. Deaton, Ed.)*, Cambridge Univ. Press, Cambridge, 1980.

R. Gillingham, "Place to Place Rent Comparisons," *Annals of Economic and Social Measurement*, 4, pp.153-178, 1975.

M.A. King, "Welfare Analysis of Tax Reforms Using Household Data," *Journal of Public Economics*, 21, pp.183-214, 1983.

A.A. Konüs, "The Problem of the True Index of the Cost-of-Living," *Econometrica*, 7, pp.10-29, 1939.

S.Malpezzi and S.K. Mayo, "The Demand for Housing in Developing Countries : Empirical Estimates from Household Data," *Economic Development and Cultural Change*, 35, pp.687-721, 1987.

S.K.Mayo, "Theory and Estimation in the Economics of Housing Demand," *Journal of Urban Economics*, 10, pp.95-116, 1981.

J.Muellbauer, "Prices and Inequality : The United Kingdom Experience," *Economic Journal*, 84, pp.32-55, 1974.

R.F. Muth, "The Derived Demand for Urban Residential Land," *Urban Studies*, 8, pp.243-254, 1971.

S. Osmani, *Economic Inequality and Group Welfare*, Oxford Univ. Press, Oxford, 1982.

A.K. Sen, "The Welfare Basis of Real Income Comparisons : A Survey," *Journal of Economic Literature*, 17, pp.1-45, 1979.

J.R.N. Stone, *The Measurement of Consumers' Expenditure and Behaviour in the United Kingdom*, 1920-1938, Vol.1, Cambridge, 1953.

M.R. Straszheim, "Estimation of the Demand for Urban Housing Services from Household Interview Data," *Review of Economics and Statistics*, 55, pp.1-8, 1973.