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GOVERNMENT SUBSIDY FOR QUALITY IMPROVEMENT

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I examine the existence of a subgame perfect equilibrium in a duopoly where the products are differentiated in quality, and derive the conditions under which an interior solution for the optimal qualities is obtained. The effect of government subsidy for quality improvement is investigated. I find that, in equilibrium, the subsidization of the high quality good may cause the quality of both product to fall, while the subsidization of the low quality good unambiguously improves the quality of both products.

I. INTRODUCTION

In this paper, I examine the existence of a subgame perfect equilibrium in a duopoly where the products are differentiated in quality, and derive the conditions under which an interior solution for the optimal qualities is obtained. The effect of government subsidy for quality improvement is investigated. I find that, in equilibrium, the subsidization of the high quality good may cause the quality of both products to fall, while the subsidization of the low quality good unambiguously improves the quality of both products.

The model presented in this paper is a two-stage non-cooperative game of two firms, where consumers are ranked by their willingness to pay for quality. Firms choose qualities in the first stage and set prices in the second stage. According to Shaked and Sutton (1982), under the assumption of zero production costs, each firm benefits from raising quality, because a higher quality attracts more consumers, ceteris paribus. However, the firm faces a trade-off: if it chooses a product quality close to its competitor, it will gain in market share but will face more intense price competition. Therefore, each firm benefits by differentiating its product. As a consequence, the high quality firm always produces at the upper bound, while the optimal choice of the low quality firm may vary between the lower

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bound and an internal solution, depending on the extent of heterogeneity among the consumers.

However, in order to investigate the impact of policy on quality levels, we need to have an interior equilibrium. First, introducing a positive unit variable production cost, which is increasing in quality and independent of output level, increases the propensity of firms to choose interior solutions. Furthermore, if unit variable costs are assumed to rise rapidly with increases in quality, i.e., the cost curve is convex, then increasing the quality differential will diminish the firm's relative cost advantage, which is defined as the ratio of the cost difference to quality difference. Thus, if the exogeneously determined range of permissible qualities is sufficiently wide, the convexity of the unit variable cost curve guarantees the existence of an interior quality equilibrium. Second, where the response of each firm is less than its rival's initial movement, the uniqueness and stability of the equilibrium is ensured. Finally, a minimum level of heterogeneity in consumer tastes is necessary in order to justify the presence of two firms in the industry. When consumers are very similar, each firm can attract all the consumers in the market by lowering its price slightly. Price competition will continue until the firm which has a relative cost disadvantage is eliminated. However, beyond a critical level of heterogeneity, price competition becomes unprofitable because a firm has to lower its price greatly in order to capture the entire market.

After establishing the existence of a unique internal equilibrium, we extend the model in order to consider the quality implications of product subsidies. In particular, a subsidy targeted at the high quality firm reduces the likelihood that it will be eliminated from the industry through price competition. This increases the firm's incentive to lower the quality and price of its product in order to capture a greater market share. The low quality firm, which experiences an increased risk of being eliminated, lowers its own quality further in order to avoid the increased price competition. If interactions between the firms are considered, the implications of the subsidy for levels of quality become ambiguous.

In contrast, a subsidy targeted at the firm producing the low quality product increases the competitiveness of the producer, who responds by improving the product in order to gain a larger share of the market. The optimal response of the high quality producer in this case is to improve its own product in order to avoid intensifying price competition. Therefore, the subsidy has the effect of raising the quality of the products produced by both firms.

In Section 2, I derive the conditions for a duopolistic non-cooperative price equilibrium, and then find the conditions under which an interior solution for the optimal qualities is obtained. In Section 3, I analyze the effect of government subsidies. Section 4 concludes the paper.

II. MODEL

The present analysis is based on a two-stage non-cooperative game. In the first stage, each firm selects the quality level of its product, while, in the second stage, having observed the choices, each firm sets its price.

2.1 Price Equilibrium

To seek a subgame perfect equilibrium, we first analyze the second stage and work backwards. Consider two firms producing distinct, substitute goods. We label their respective products by an index $k = 1,2$ where firm k sells product k at price p_k.

Assume that each consumer is different in quality valuation, say t, which is uniformly distributed. That is, the density equals unity on some support $0 < a \leq t \leq b$. A consumer of type t derives the following (indirect) utility from buying one unit of product k of quality u_k :

$$U(u_k;t) = tu_k - p_k \tag{1}$$

Consumers will select the product for which utility (1) is higher, assuming that they buy exactly one unit of the differentiated commodity.

We now derive from (1) the consumer who is just indifferent between product 1 and 2. Its valuation t^* is obtained from

$$t^*u_1 - p_1 = t^*u_2 - p_2$$

where $u_2 > u_1$; hence

$$t^* = (p_2 - p_1)/(u_2 - u_1).$$

Thus the (necessary and sufficient) condition for two firms to share the market is $a < t^* < b$; that is,

$$a(u_2 - u_1) < p_2 - p_1 < b(u_2 - u_1). \tag{2}$$

This indicates that consumers with values of t higher than t^* strictly prefer good 2 at price p_2 to good 1 at price p_1 and the converse is true for persons with $t < t^*$. Hence consumers are partitioned into segments corresponding to the market share of each firm.

Let $c(u)$ represent the level of unit variable cost as a function of quality. It is assumed to be independent of the level of output. We also assume that $c(u)$ is continuously differentiable, and that the two firms share an identical unit variable cost function.

The profit of each firm now becomes

$$\Pi_1(p_1;\ p_2,\ u_1,\ u_2) = \{\ p_1 - c(u_1)\ \}(t^* - a),\ \text{and} \tag{3}$$

$$\Pi_2(p_2;\ p_1,\ u_1,\ u_2) = \{\ p_2 - c(u_2)\ \}(b - t^*). \tag{4}$$

Firm 1 chooses its price to maximize Π_1, given its rival's price. From (2), we find that, when the price of product 2 is low, i.e., $c(u_2) < p_2 \leq c(u_1) + b(u_2 - u_1)$, firm 1 has no chance to expel its rival from the industry as long as it remains profitable, i.e., $p_1 > c(u_1)$. Thus its optimal price p_1^* is selected to satisfy $\partial\Pi_1/\partial p_1 = 0$. On the other hand, if $p_2 > c(u_1) + b(u_2 - u_1)$, firm 1 can eliminate its rival. However, the elimination may be less profitable than sharing the market with its rival. That is, when firm 2 charges $c(u_1) + b(u_2 - u_1) < p_2 < c(u_1) + (2b - a)(u_2 - u_1)$, it is more profitable for firm 1 to charge p_1^* and share the market. If its rival's price is $p_2 \geq c(u_1) + (2b - a)(u_2 - u_1)$, it will be optimal for firm 1 to charge $p_1^d = p_2 - b(u_2 - u_1)$ and win the entire market.[1] Therefore, we find that, if $c(u_2)$ is greater than or equal to $c(u_1) + (2b - a)(u_2 - u_1)$, firm 2 can not remain profitable in the industry. In other words,

$$2b - a > \frac{c(u_2) - c(u_1)}{u_2 - u_1} \tag{5}$$

must be satisfied in order for firm 2 to make a positive profit in the industry. From condition (5), we derive the reaction function of firm 1:

$$p_1^R = \begin{cases} p_1^* = \{p_2 - a(u_2 - u_1) + c(u_1)\}/2 & \text{when } c(u_2) < p_2 < c(u_1) + (2b - a)(u_2 - u_1) \\ p_1^d = p_2 - b(u_2 - u_1) & \text{when } p_2 \geq c(u_1) + (2b - a)(u_2 - u_1) \end{cases}$$

Similarly, firm 2 chooses its price to maximize Π_2, given p_1. When its rival's price is $c(u_1) < p_1 \leq c(u_2) - a(u_2 - u_1)$, firm 2 has no chance to profitably occupy the whole market. Thus it chooses p_2^* to satisfy $\partial\Pi_2/\partial p_2 = 0$. On the other hand,

[1] Consider a tangent line Q_i at p_i^d to an imaginary profit function of firm i which is defined as if its market can increase without a bound as its price is lowered. If the slope of line Q_i is nonpositive, i.e., $p_2 \geq c(u_1) + (2b - a)(u_2 - u_1)$, it will be optimal for firm 1 to charge p_1^d and occupy the market. If the slope is positive, i.e., $p_2 < c(u_1) + (2b - a)(u_2 - u_1)$, the optimal price $p_1^* = \{p_2 - a(u_2 - u_1) + c(u_1)\}/2$, will be selected.

if $p_1 > c(u_2)-a(u_2-u_1)$, firm 2 can eliminate its rival. However, When firm 1 charges $c(u_2)-a(u_2-u_1) < p_1 < c(u_2)+(b-2a)(u_2-u_1)$, it is more profitable for firm 2 to charge p_2^* and share the market. If its rival's price is $p_1 \geq c(u_2)-(b-2a)(u_2-u_1)$, it will be optimal for firm 2 to charge $p_2^d = p_1+a(u_2-u_1)$ and force its opponent out of the market.[2] Thus we find that, if $c(u_1) \geq c(u_1)+(b-2a)(u_2-u_1)$, firm 1 will be expelled from the market. Therefore,

$$b-2a > -\frac{c(u_2)-c(u_1)}{u_2-u_1} \tag{6}$$

is the condition under which firm 1 can make a positive profit. From condition (6), we derive the reaction function of firm 2:

$$p_2^R = \begin{cases} p^*_2=\{p_1+b(u_2-u_1)+c(u_2)\}/2 & \text{when } c(u_1)<p_1<c(u_2)+(b-2a)(u_2-u_1) \\ p_2^d = p_1+a(u_2-u_1) & \text{when } p_1 \geq c(u_2)+(b-2a)(u_2-u_1) \end{cases}$$

From conditions (5) and (6), we obtain the equilibrium prices for firms 1 and 2:

$$p_1^E = \frac{1}{3}\{(b-2a)(u_2-u_1)+c(u_2)+2c(u_1)\}$$

$$p_2^E = \frac{1}{3}\{(2b-a)(u_2-u_1)+2c(u_2)+c(u_1)\}$$

Point E in Figure 1 is one such equilibrium. In other words, when conditions (5) and (6) are satisfied, there exists a unique price equilibrium (p_1^E, p_2^E), at which both firms have a positive market share. This equilibrium is globally stable.[3]

Assuming zero production costs, Shaked and Sutton (1982) find that the condition for two firms to share the market is $b-2a > 0$. That is, when consumers are very different, firm 2 has to lower its price greatly in order to occupy the

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[2] If the slope of Q_2 is nonpositive, i.e., $p_1 \geq c(u_2)+(b-2a)(u_2-u_1)$, p_2^d is profit maximizing. However, if the slope is positive, i.e., $p_1 < c(u_2)+(b-2a)(u_2-u_1)$, $p^*_2 = \{p_1+b(u_2-u_1)+c(u_2)\}/2$ will be chosen.

[3] To check the stability conditions for the price equilibrium, we have

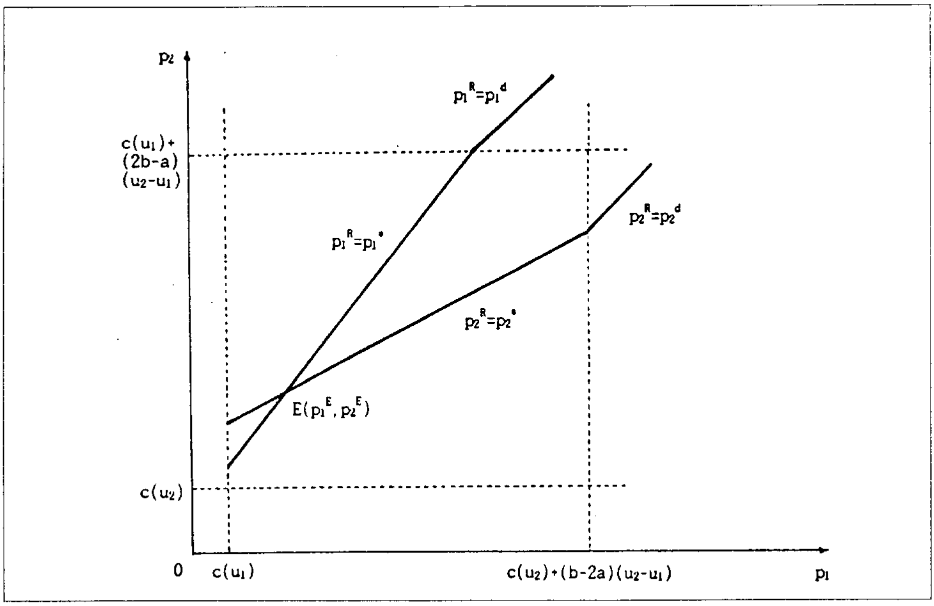
$\partial^2\Pi_1/\partial p_1^2=-2/(u_2-u_1) < 0$, $\partial^2\Pi_1/\partial p_2\partial p_1=1/(u_2-u_1) > 0$,

$\partial^2\Pi_2/\partial p_1\partial p_2=1/(u_2-u_1) > 0$, $\partial^2\Pi_1/\partial p_2^2=-2/(u_2-u_1) < 0$, and

$(\partial^2\Pi_1/\partial p_1^2)(\partial^2\Pi_2/\partial p_2^2)-(\partial^2\Pi_1/\partial p_2\partial p_1)(\partial^2\Pi_2/\partial p_1\partial p_2)=3/(u_2-u_1) > 0$

Therefore, all the conditions for global stability are satisfied.

Figure 1. Price Equilibrium



whole market. This threatens to reduce profits. However, when consumers are very similar, $b-2a \leq 0$, firm 2 can obtain a higher profit by lowering its price slightly, and so attracting all consumers. In this paper, however, we have introduced a positive unit variable cost, which increases with quality, so that firm 2 may not be able to take the whole market because it becomes to more difficult to profitably reduce its price sufficiently to eliminate its rival. Thus the condition $b-2a \leq 0$ does not guarantee that firm 2 will occupy the entire market, but a more strict condition

$$b-2a \leq -\frac{c(u_2)-c(u_1)}{u_2-u_1}$$

does. Therefore, condition (6) must hold in order to avoid the possibility that firm 2 occupies the market.

On the other hand, there is no way for the low quality firm to take the whole market in the Shaked and Sutton model. Whenever there is a threat of elimination by a low p_1, firm 2 can protect its market by lowering p_2 close to p_1. However, when a positive unit variable cost is introduced, firm 2 may no longer be able to lower its price close enough to p_1 to remain in the market. Condition (5) constrains the cost difference to be less than $(2b-2a)(u_2-u_1)$, and enables firm 2 to remain in the industry.

There are two types of heterogeneity which may affect the existence of a price equilibrium; namely, consumers' heterogeneity in quality valuation t, and the difference in quality of the products u_2-u_1, selected at the first stage of the game. When the quality differential becomes very small, $\{c(u_2)-c(u_1)\}/(u_2-u_1)$ approaches a certain value, i.e., the marginal cost in terms of quality, $c'(u)$, at the quality level where u_1 and u_2 converge. In fact, the value is independent of the quality difference, even though it depends on the quality level to which u_1 and u_2 approach. Thus, from conditions (5) and (6), we find that the existence of the price equilibrium is not significantly threatened by the similarities of the products as long as consumers' heterogeneity is sufficiently large.

On the other hand, when consumer heterogeneity is very small, even if the quality difference is great, conditions (5) and (6) are less likely to hold, so that we may end up with only one firm staying in the industry with a positive profit. In other words, the consumers' heterogeneity is indispensable to the maintenance of a duopoly, while product differentiation does not significantly threaten the existence of the price equilibrium.

2.2 Quality Equilibrium

We now turn to the stage in which each firm chooses the optimal quality level of its product. Substituting p_1^E and p_2^E into the profit functions (3) and (4) yields

$$\Pi_1^E = \frac{X^2}{9(u_2-u_1)} ,$$

$$\Pi_2^E = \frac{Y^2}{9(u_2-u_1)} ,$$

where $X = (b-2a)(u_2-u_1)+c(u_2)-c(u_1) > 0$ and $Y = (2b-a)(u_2-u_1)+c(u_1)-c(u_2) > 0$ from conditions (5) and (6).

Now we define an exogeneously given upper bound on quality $\bar{u}$ and a lower bound $\underline{u}$. We refer to $m(u_1, u_2) \equiv \{c(u_2)-c(u_1)\}/(u_2-u_1)$ as the relative cost disadvantage, the increase in cost resulting from a given increase in quality. We define u_1^m as the quality level satisfying $m(u_1^m, \bar{u}) = a-2b+2c'(\bar{u})$: That is, as we see in Figure 2, if u_1 is greater than u_1^m, it is optimal for firm 2 to choose the upper bound. Likewise, u_2^m is the quality level satisfying $m(\underline{u}, u_2^m) = b-2a+2c'(\underline{u})$, and therefore, if u_2 is lower that u_2^m, it is optimal for firm 1 to choose the lower bound. Next, we obtain the reaction functions under the assumption that $c(u)$ is convex: (See Appendix 1 for the derivation.)

$$R_1(u_1;u_2) = \begin{cases} u_1 - \underline{u} = 0 & \text{when } u_1 < u_2 \leq u_2^m \\ (2a-b)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_1)=0 & \text{when } u_2^m < u_2 < \bar{u} \end{cases}$$

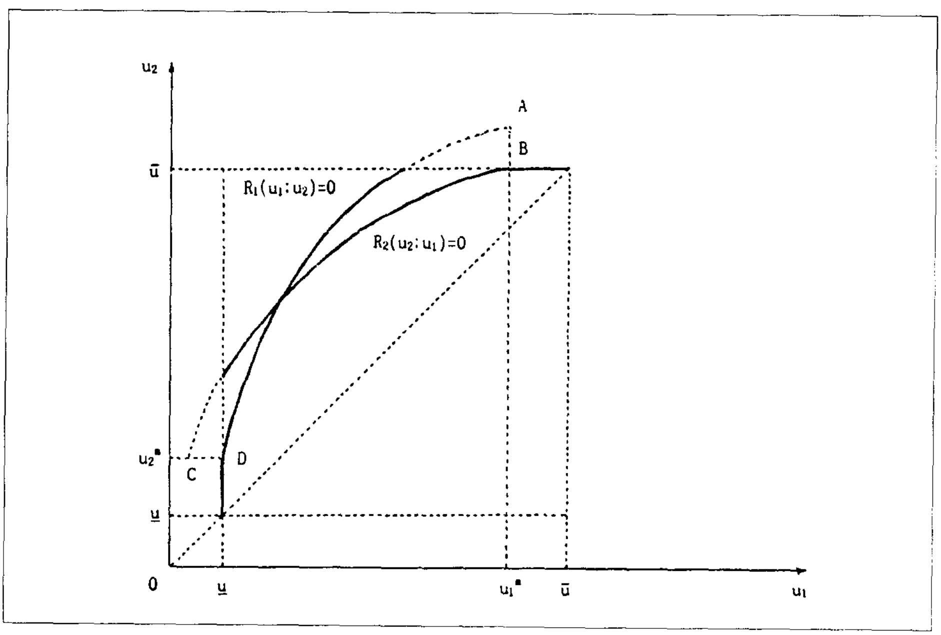
$$R_2(u_2;u_1) = \begin{cases} u_2 - \bar{u} = 0 & \text{when } u_1^m \leq u_1 < u_2 \\ (2b-a)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_2)=0 & \text{when } \underline{u} < u_1 < u_1^m \end{cases}$$

Thus

$$\underline{u} < u_1 < u_1^m \text{ and } u_2^m < u_2 < \bar{u} \tag{7}$$

are necessary conditions for an interior solution.

Figure 2. Quality Equilibrium



Proposition: Suppose condition (7) holds, and that $\underline{u}$ and $\bar{u}$ are selected to satisfy

$$\frac{3}{2}(b-a) > c'(\bar{u})-c'(u_1^m), \text{ and} \tag{8}$$

$$\frac{3}{2}(b-a) > c'(u_2^m)-c'(\underline{u}). \tag{9}$$

If $c(u)$ is convex and it satisfies

$$2c''(u_1) > \frac{c'(u_2)-c'(u_1)}{u_2-u_1}, \text{ and} \tag{10}$$

$$2c''(u_2) > \frac{c'(u_2)-c'(u_1)}{u_2-u_1}. \tag{11}$$

there exists a unique and globally stable interior solution (u_1^E, u_2^E).

The proof of this proposition is in Appendix 2. From Figure 2, we find that conditions (8) and (9) guarantee that the value of u_2 at point A is greater than $\bar{u}$, and the value of u_1 at point C is less than $\underline{u}$. Thus, there exists at least one intersection of the reaction curves where $\underline{u} < u_1 < u_1^m$ and $u_2^m < u_2 < \bar{u}$. Furthermore, conditions (10) and (11) imply that marginal cost in terms of quality, $c'(u)$, rises rapidly as quality improves. Thus the response of each firm is less than its rival's initial movement; that is, the slope of $R_1(u_1;u_2) = 0$ is greater than one and the slope of $R_2(u_2;u_1) = 0$ is less than one in the range denoted by (7). These conditions guarantee the uniqueness of the equilibrium. Under the second order conditions, global stability is also guaranteed.

We have examined a unique interior solution for the optimal qualities, under the assumption of convexity of $c(u)$. However, if $c(u)$ is concave, we can not expect an interior solution. Intuitively, when $c(u)$ is concave, as u_1 decreases, firm 2's disadvantage in relative costs, defined as $m(u_1, u_2)$, becomes greater. In order to minimize the impact, firm 2 will increase u_2 since $m(u_1, u_2)$ is decreasing in u_2. Firm 1 responds by lowering u_1 in order to achieve a greater cost advantage. This process continues until at least one firm chooses a bound of the available qualities. This is shown in Appendix 3.

Ⅲ. EFFECT OF A GOVERNMENT SUBSIDY

In this section, we will examine the effect of a government subsidy on the equilibrium qualities.[4] If the subsidy is offered proportionally to the unit variable cost of product 2, the production cost of firm 2 will be $(1-\delta)c(u_2)$, where $0 \leq \delta < 1$. When we will see how u_1^E and u_2^E change as δ increases from zero.

We have

[4] At the time that the identities of the high quality producer and the low quality producer are determined, they have no information about which product will be subsidized in the future. Also, once the industry is formed, a change from high quality production to low quality production (or vice versa) is assumed to incur a significantly high cost.

$$\frac{du_1^E}{d\delta} = \frac{|H_1|}{|R|} \text{ and } \frac{du_2^E}{d\delta} = \frac{|H_2|}{|R|},$$

where

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}, \quad H_1 = \begin{bmatrix} h_1 & R_{12} \\ h_2 & R_{22} \end{bmatrix}, \quad H_2 = \begin{bmatrix} R_{11} & h_1 \\ R_{21} & h_2 \end{bmatrix},$$

$$R_{11} = \partial R_1 / \partial u_1,\ R_{12} = \partial R_1 / \partial u_2,\ R_{21} = \partial R_2 / \partial u_1,\ R_{22} = \partial R_2 / \partial u_2,$$

$$h_1 = c(u_2) > 0, \text{ and}$$

$$h_2 = c(u_2) - 2(u_2 - u_1)c'(u_2).$$

Since we know that $R_{11} < 0$, $R_{11} > 0$, $R_{21} > 0$, $R_{22} < 0$, and $|R| > 0$[5] by conditions (10) and (11), the sign of $|H_1|$ and $|H_2|$ depends on h_2. When h_2 is negative, the effect on the optimal qualities is not clear. However, if h_2 is positive, $|H_1|$ and $|H_2|$ are both negative; that is, u_1^E and u_2^E will decrease as δ increases. Therefore, the government subsidy on the high quality product may result in the deterioration of the qualities of both commodities in the industry.

To see how the reaction curves shift in the diagram, we find $\partial u_1/\partial\delta|_{R_1=0} = h_1/R_{11} < 0$ and $\partial u_2/\partial\delta|_{R_2=0} = h_2/R_{22}$ whose sign depends on h_2. As a consequence of an increase in δ, $R_1(u_1;u_2) = 0$ will shift to the left and $R_2(u_2;u_1) = 0$ will shift downward when h_2 is positive and upward when $h_2 = 0$ is negative. This is illustrated in Figures 3 and 4.

The intuition behind the quality movements is as follows: When the high quality good is subsidized, firm 1 becomes less competitive in the sense that it is more likely to be eliminated from the industry due to an intense price competition. Thus u_1^E will be lowered to avoid the price competition. For firm 2, the subsidy ameliorates its relative cost situation, so that product 2 becomes more competitive with product 1. In consequence, u_2^E will be lowered to take a greater market share. However, as u_2^E falls, the size of the subsidy is reduced. In other words, there are two opposite forces on u_2^E. Therefore, depending on their relative magnitude, u_2^E might be higher or lower. After the interactions between the two firms are considered, the final outcome of this subsidy is not guaranteed to be quality-improving.

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[5] Shown in Appendix 2.

Figure 3. Movement of Quality Equilibrium by a Government Subsidy for the High Quality Good(when $h_2 \geq 0$)

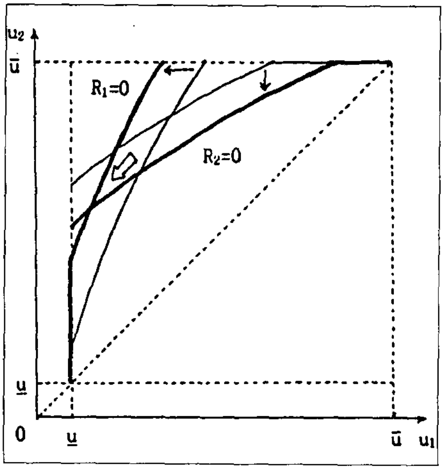
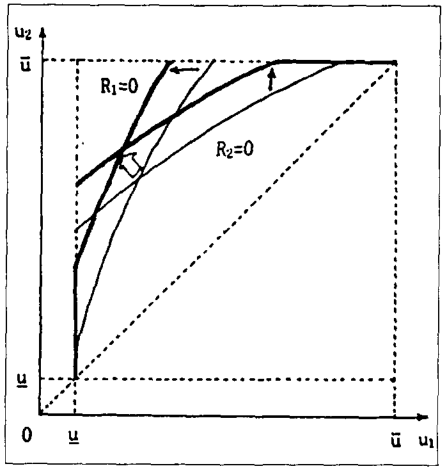


Figure 4. Movement of Quality Equilibrium by a Government Subsidy for the High Quality Good(when $h_2 < 0$)



Now, if the government gives a subsidy proportional to the unit variable cost of product 1, the production cost of firm 1 will be $(1-\gamma)c(u_1)$, where $0 \leq \gamma < 1$. Thus we examine how the equilibrium quality changes as γ increases from zero:

$$\frac{du_1^E}{d\gamma} = \frac{|F_1|}{|R|} \text{ and } \frac{du_2^E}{d\gamma} = \frac{|F_2|}{|R|},$$

where

$$F_1 = \begin{bmatrix} f_1 & R_{12} \\ f_2 & R_{22} \end{bmatrix}, \quad F_2 = \begin{bmatrix} R_{11} & f_1 \\ R_{21} & f_2 \end{bmatrix},$$

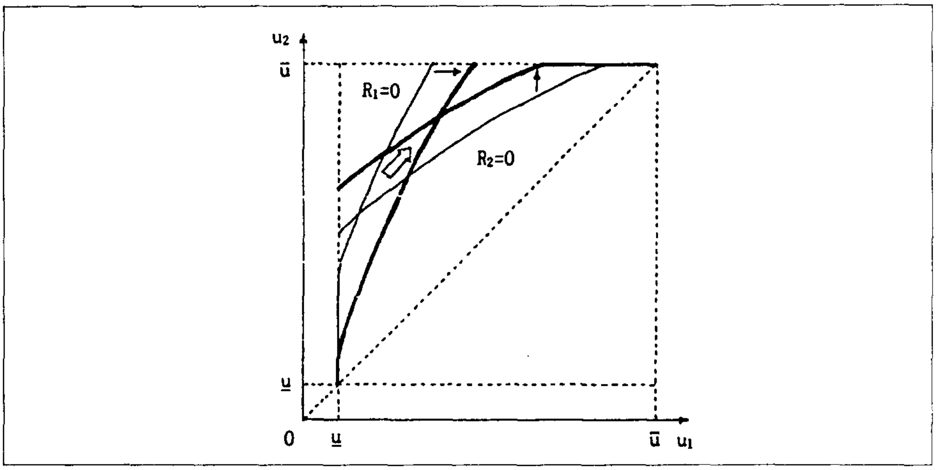
$$f_1 = -c(u_1) - 2(u_2 - u_1)c'(u_1) < 0, \text{ and}$$

$$f_2 = -c(u_1) < 0.$$

It follows that $|F_1| > 0$ and $|F_2| > 0$, so that the effects of r on u_1^E and u_2^E are both positive. That is, government subsidization of the low quality product gives each firm greater incentive to improve its product.

Now we will demonstrate this effect in Figure 5. When u_2 is given, the effect of r on u_1 is positive because $\partial u_1/\partial r|_{R_1=0}=0=f_1/R_{11} > 0$. Consequently, the reaction curve of firm 1 shifts to the right as r increases. Similarly, when u_1 is given, $u_2/\partial r|_{R_2=0}=0 = f_2/R_{22} > 0$, so that the reaction curve of firm 2 shifts upward as r increases. Therefore, the equilibrium quality levels (u_1^E, u_2^E) will both be raised.

Figure 5. Movement of Quality Equilibrium by a Government Subsidy for the Low Quality Good



Intuitively, subsidization lowers the unit variable cost of product 1 making relatively cheap to produce; hence the competitiveness of good 1 is improved. It is, therefore, optimal for firm 1 to raise u_1^E to capture a greater market share. Moreover, the additional cost due to the improvement of u_1^E is reduced by the government. In consequence, firm 1 has two incentives to increase the quality of its product. Meanwhile, the competitiveness of product 2 is weakened, so that it needs to be more differentiated from product 1 in order to avoid intense price competition. Furthermore, the interaction between the two firms accentuates the movements. Therefore, the subsidization of the low quality good raises both u_1^E and u_2^E.

Ⅳ. CONCLUDING REMARKS

In this paper, we have analyzed a two-stage non-cooperative game where duopolists decide sequentially upon quality and prices, and have derived the conditions under which a unique interior solution for optimal qualities exists. We have also found that government subsidization is unambiguously quality improving when the production of the low quality good is subsidized.

The conditions we have derived are based on the assumption that the consumers' quality valuations are uniformly distributed. However, if the distribution is more peaked around its center, like the normal distribution, firms will be more concerned with the taste of the median consumer and firms will thus be less likely to attempt to differentiate their products. This will tend to relax the conditions necessary in order to obtain interior solutions.

APPENDIX 1

Derivation of Reaction Functions

We will explore firm 1's optimal choice of u_1 given u_2. We know that Π_1^E increases at a decreasing rate as u_1 decreases. The maximum point of Π_1^E, u_1^*, is obtained from

$$\left.\frac{\partial \Pi_1^E}{\partial u_1}\right|_{at\ u_1^*} = \frac{X}{9(u_2-u_1^*)^2}\left[(2a-b)(u_2-u_1^*)+c(u_2)-c(u_1^*)-2(u_2-u_1^*)c'(u_1^*)\right]=0,$$

where $c'(u_1) = dc(u)/du$ at u_1. Here, Π_k^E is assumed to be defined even at points outside the range of permissible qualities. Next we check whether or not u_1^* is greater than $\underline{u}$: (i) If $\partial \Pi_1^E/\partial u_1$ at $\underline{u}$ is nonpositive, i.e.,

$$m(\underline{u}, u_2) \leq b-2a+2c'(\underline{u}), \tag{A-1}$$

firm 1 will choose $\underline{u}$. (ii) If $\partial \Pi_1^E/\partial u_1$ at $\underline{u}$ is positive, i.e.,

$$m(\underline{u}, u_2) > b-2a+2c'(\underline{u}), \tag{A-2}$$

firm will choose u_1^*.

Since $c(u)$ is assumed to be convex, $m(\underline{u}, u_2)$ is increasing in u_2, so that condition (A-1) holds for $u_1 < u_2 \leq u_2^m$. Thus, firm 1's best quality choice is $\underline{u}$. For $u_2^m < u_2 < \bar{u}$, condition (A-2) holds, so that firm 1 chooses u_1^m. Therefore, the reaction function of firm 1 can be represented by

$$R_1(u_1;u_1) = \begin{cases} u_1 - \underline{u} = 0 & \text{when } u_1 < u_2 \leq u_2^m \\ (2a-b)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_1) = 0 & \text{when } u_2^m < u_2 < \bar{u} \end{cases}$$

Similarly, the maximum point of Π_2^E is obtained from

$$\left.\frac{\partial \Pi_2^E}{\partial u_2}\right|_{at\ u_2^*} = \frac{Y}{9(u_2^*-u_1)^2}\,[(2b-a)(u_2^*-u_1+c(u_2^*)-c(u_1)-2(u_2^*-u_1)c'(u_2^*)]=0,$$

where $c'(u_2) = dc(u)/du$ at u_2. We now determine whether u_2^* is greater or less than $\bar{u}$: (i) If $\partial\Pi_2^E/\partial u_2$ at $\bar{u}$ is nonnegative, i.e.,

$$m(u_1, \bar{u}) \geq a-2b+2c'(\bar{u}), \tag{A-3}$$

firm 2 will choose $\bar{u}$. (ii) If $\partial\Pi_2^E/\partial u_2$ at $\bar{u}$ is negative, i.e.,

$$m(u_1, \bar{u}) < a-2b+2c'(\bar{u}), \tag{A-4}$$

firm 2 will choose u_2^*.

Since $m(u_1, \bar{u})$ is increasing in u_1, from conditions (A-3) and (A-4), we find the reaction function of firm 2 as

$$R_2(u_2;u_1) = \begin{cases} u_2 - \bar{u} = 0 & \text{when } u_1^m \leq u_1 < u_2 \\ (2b-a)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_2) = 0 & \text{when } \underline{u} < u_1 < u_1^m \end{cases}$$

APPENDIX 2

Proof of Proposition

From each reaction function, we have

$$\begin{aligned} R_{12} &= \partial R_1/\partial u_2 = -m(u_1, u_2)+c'(u_2), \\ R_{21} &= \partial R_2/\partial u_1 = m(u_1, u_2)-c'(u_1), \\ R_{11} &= \partial R_1/\partial u_1 = m(u_1, u_2)-c'(u_1)-2(u_2-u_1)c''(u_1) \\ &= -R_{12}+c'(u_2)-c'(u_1)-2(u_2-u_1)c''(u_1), \text{ and} \\ R_{22} &= \partial R_2/\partial u_2 = -m(u_1, u_2)+c'(u_2)-2(u_2-u_1)c''(u_2) \\ &= -R_{21}+c'(u_2)-c'(u_1)-2(u_2-u_2)c''(u_2). \end{aligned}$$

Now, from conditions (10) and (11), we find

$$R_{11} < 0,\ R_{22} < 0,\ R_{12} > 0, \text{ and } R_{21} > 0.$$

Since

$$\left.\frac{du_2}{du_1}\right|_{R_1=0} = -\frac{R_{11}}{R_{12}} \quad \text{and} \quad \left.\frac{du_2}{du_1}\right|_{R_2=0} = -\frac{R_{21}}{R_{22}},$$

the slope of each reaction curve is positive. Furthermore,

$$(-R_{11}) - R_{12} = -\{c'(u_2) - c'(u_1)\} + 2(u_2 - u_1)c''(u_1), \text{ and}$$

$$(-R_{22}) - R_{21} = -\{c'(u_2) - c'(u_1)\} + 2(u_2 - u_1)c''(u_2).$$

Therefore, under conditions (10) and (11), the slope of firm 1's reaction curve is greater than one, and the slope of firm 2's reaction curve is less than one.

Upon inspection of Figure 2, it is clear that the value of u_2 at point A uniquely satisfies the condition $m(u_1^m, u_2) = b-2a+2c'(u_1^m)$, while $m(u_1^m, \bar{u}) = a-2b+2c'(\bar{u})$ is satisfied at point B. Since the function $m(u_1^m, u_2)$ is increasing in u_2, when $b-2a+2c'(u_1^m) > a-2b+2c'(\bar{u})$, i.e.,

$$\frac{3}{2}(b-a) > c'(\bar{u}) - c'(u_1^m),$$

the value of u_2 at point A must be greater than $\bar{u}$. Analogously, the value of u_1 at point C uniquely satisfies $m(u_2, u_2^m) = a-2b+2c'(u_2^m)$, while $m(\underline{u}, u_2^m) = b-2a+2c'(\underline{u})$ at point D. Since the function $m(u_1, u_2^m)$ is increasing in u_1, when $a-2b+2c'(u_2^m) < b-2a+2c'(\underline{u})$, i.e.,

$$\frac{3}{2}(b-a) > c'(u_2^m) - c'(\underline{u}),$$

the value of u_1 at point C is less than $\underline{u}$. Therefore, from Figure 2, we can see that, under conditions (10) and (11), we find that at least one intersection of the reaction curves exists where $\underline{u} < u_1 < u_2^m$ and $u_2^m < u_2 < \underline{u}$. Also, conditions (8) and (9) guarantee its uniqueness. And since the matrix

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$$

is negative definite within the ranges of $\underline{u} < u_1 < u_2^m$ and $u_2^m < u_2 < \bar{u}$, the equilibrium is globally stable.

APPENDIX 3

The Case of Concave Cost Curve

If c(u) is concave, the reaction functions are

$$R_1(u_1;u_2) = \begin{cases} (2a-b)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_1)=0 & \text{when } u_1 < u_2 < u_2^m \\ u_1-\underline{u} = 0 & \text{when } u_2^m \leq u_2 < \bar{u} \end{cases}$$

$$R_2(u_2;u_1) = \begin{cases} (2b-a)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_2)=0 & \text{when } u_1^m < u_1 < u_2 \\ u_2-\bar{u} = 0 & \text{when } \underline{u} < u_1 \leq u_1^m \end{cases}$$

In order for the reaction curves to intersect each other in the range of $u_1^m < u_1 < u_2$, $u_1 < u_2 < u_2^m$, $(2a-b)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_1)$ must be equal to $(2b-a)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_2)$; that is,

$$c'(u_2)-c'(u_1) = \frac{3}{2}(b-a).$$

Since this condition can not be satisfied under the concavity of $c(u)$, no interior solution can be obtained.

Also, when $u_1^m < u_1 < u_2$ and $u_1 < u_2 < u_2^m$, the slope of each reaction curve is

$$\left.\frac{du_2}{du_1}\right|_{R_1=0} = -\frac{R_{11}}{R_{12}} < 0\,, \text{ and } \left.\frac{du_2}{du_1}\right|_{R_2=0} = -\frac{R_{21}}{R_{22}} < 0\,.$$

Since, by the second order conditions and the concavity of $c(u)$, we find

$$R_{11} < 0,\ R_{22} < 0,\ R_{12} < 0, \text{ and } R_{21} < 0,$$

the slope of $R_1(u_1;u_2) = 0$ is less than -1, and the slope of $R_2(u_2;u_1) = 0$ is greater than -1. (i) When $u_1^m > \underline{u}$ and $u_2^m < \bar{u}$, i.e., $b-2a+2c'(\underline{u}) \geq m(\underline{u}, \bar{u}) \geq a-2b+2c'(\bar{u})$, the equilibrium qualities are $(\underline{u}, \bar{u})$, as we see in Figure A-1. In the area of $u_1^m < u_1 < u_2$ and $u_1 < u_2 < u_2^m$, the reaction curves do not intersect each other because the slope of $R_1(u_1;u_2) = 0$ is less than the slope of $R_2(u_2;u_1) = 0$. (ii) When $u_1^m > \underline{u}$ and

$u_2^m \geq \bar{u}$, i.e., m($\underline{u}$, $\bar{u}$)$\geq$min $\{a-2b+2c'(\bar{u}),\ b-2a+2c'(\underline{u})\}$, and the u_2 value at point A is less than that at point B, i.e., $c'(\bar{u})-c'(u_1^m) \leq 3(b-a)/2$, the equilibrium is $(u_1^E, \bar{u})$, as in Figure A-2. (iii) When $u_1^m \leq \underline{u}$ and $u_2^m \langle \bar{u}$, i.e., m($\underline{u}$, $\bar{u}$) min $\{a-2b+2c'(\bar{u}),\ b-2a+2c'(\underline{u})\}$, and the u_1 value at point C is greater than that at point D, i.e., $c'(u_2^m)-c'(\underline{u}) \leq 3(b-a)/2$, the equilibrium is $(\underline{u}, u_2^E)$, as in Figure A−3. (iv) The case of $u_1^m \leq \underline{u}$ and $u_2^m \geq \bar{u}$, i.e., $b-2a+2c'(\underline{u}) \langle$ m($\underline{u}$, $\bar{u}$) $\langle\ a-2b+2c'(\bar{u})$, is not feasible because $b-2a+2c'(\underline{u}) \rangle\ a-2b+2c'(\bar{u})$.

Figure A-1

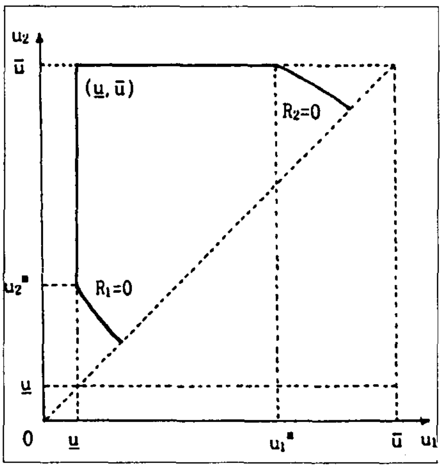


Figure A-2

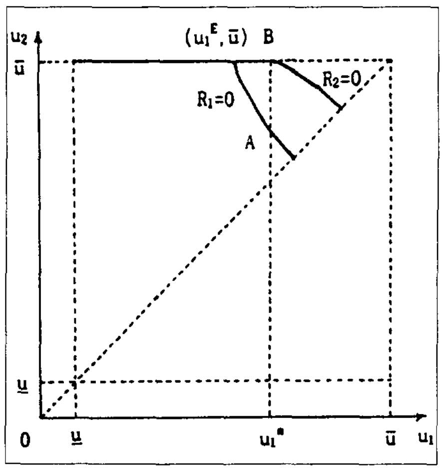
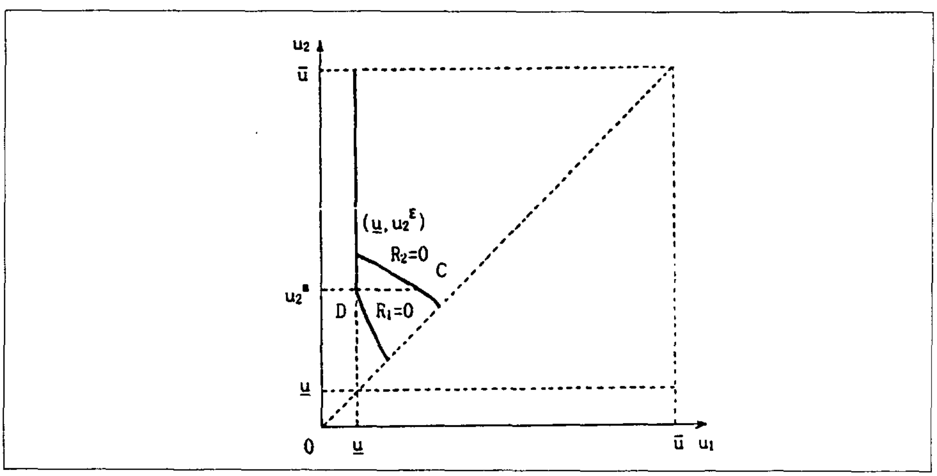


Figure A-3



REFERENCES

Jaskold Gabszewicz, J. and J.-F. Thisse (1979) "Price Competition, Quality and Income Disparities," *Journal of Economic Theory*, 20, 340-59.

Jaskold Gabszewics, J. and J.-F. Thisse (1982) "Product differentiation with Income Disparities: An Illustrative Model," *The Journal of Industrial Economics*, 31, 115-29.

Judd, K.L.(1985) "Credible Spatial Preemption," *Rand Journal of Economics*, 16, 153-66.

Martinez-Giralt, X.(1989) "On Brand Proliferation with Vertical Differentiation," *Economics Letters*, 30, 279-86.

Neven, D.(1986) "On Hotelling's Competition with Non-Uniform customer Distribution," *Economics Letters*, 21, 121-26.

Ordover, J.A. and R.D. Willig (1981) "An Economic Definition of Predation: Pricing and Product Innovation," *Yale Law Journal*, 91, 8-53.

Perloff, J. and S.C. Salop (1985) "Equilibrium with Product Differentiation," *Review of Economic Studies*, 52, 107-20.

Schmalensee, R.(1982) "Product Differentiation Advantage of Pioneering Brands," *American Economic Review*, 72, 349-65.

Selten, R.(1975) "Re-examination of the Perfectness Concept for Equilibrium Points in Extensive Games," *International Journal of Game Theory*, 4, 22-55.

Shaked, A. and J. Sutton (1982) "Relaxing Price Competition Through Product Differentiation," *Review of Economic Studies*, 49, 3-14.

Shaked, A. and J. Sutton (1983) "Natural Oligopolies," *Econometrica*, 51, 1469-84.

Shaked, A. and J. Sutton (1987) "Product Differentiation and Industrial Structure," *Journal of Industrial Economics*, 36, 131-46.

Sutton, J.(1986) "Vertical Product Differentiation: Some Basic Themes," *American Economic Association(papers and proceedings)*, 76, 393-8.

Waterson, M.(1989) "Models of Product Differentiation," *Bulletin of Economic Research*, 41, 1-27.