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THE IMPLICATIONS OF CONVEX STRUCTURE ON THE FLUCTUATIONS OF EMPLOYMENT AND REAL WAGES

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Employing the stochastic growth model, this paper examines the implications of strict convex preferences and technology on the asymmetric movement of the labor market. With convex preferences and technology, hours increase at a decreasing rate with respect to the shock. Under the same conditions, wages increase either at a decreasing or at an increasing rate with respect to the shock, depending on the nature of labor supply function. So, this study suggests a structural explanation for the asymmetric economic fluctuations observed over the cycle.

I. INTRODUCTION

Employing the stochastic growth model with a random shocks to production technology, this paper examines the implications of strict convex preferences and technology on the structures of the decision variables. More specifically, this paper has the goal of analyzing the nonlinear structure of hours and wage functions, which is proven to be caused by convexity of the economy.

The analysis will allow us to examine questions of whether the movement of work effort and real wages is symmetric or asymmetric over the cycle. The concepts developed here may be useful in understanding the third moments of hours and real wage data. An understanding of the third moments is interesting because of other studies' findings of asymmetric unemployment or employment movements over cycles. The nonlinearities and asymmetries discussed here may help to explain some empirical findings in Neftci(1984), Falk(1986) and Delong and Summers(1986) concerning the apparent asymmetry of unemployment data, and also in Brock and Sayers(1988) regarding the nonlinearity of employment and unemployment data.[1] For example, Neftci(1984) finds that decreases in unemployment are gradual and slow, but increases are large and quick. This paper, therefore, pro-

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[1] See also Dynarski and Sheffrin(1986, 1987), Davis(1987), Frank and Stengos(1988) and Pfann (1991).

vides a theoretical explanation to these empirical observations, which is an alternative to Brock and Sayer(1988)'s hypothesis that the adjustment costs cause the asymmetry and nonlinearity.[2)]

The theoretical findings in this paper indicate that with strict convex preferences and technology, working hours and wage functions are systematically nonlinear with respect to technology shocks. The finding of nonlinearity is not particularly interesting since in most specifications of the economy, decision rules are nonlinear. The results in this paper, however, are of interest because such nonlinear structures can be deterministic and predictable, mainly due to the convex structure of the economy. When there is an upward sloping labor supply function, hours are a strictly increasing function at a decreasing rate of the technology shock, while the real wage is a strictly increasing function either at a decreasing or at an increasing rate. The latter depends on the rate at which marginal disutility of work increases (equivalently, the rate of decreasing work incentive) with respect to real wages. It is a simple reasoning to imagine that "strictly increasing at a decreasing (increasing) rate" implies an asymmetry of upward (downward) rigidity.[3)] Under the condition of convex technology, the results in this paper may be used to infer the structure of the labor supply function. In other words, the results are useful to test whether or not the marginal disutility of work increases at an increasing or at a decreasing rate.[4)] This paper is developed as follows; in section 2, a one-sector stochastic model of the economy is introduced and the preferences and technology structures are described. In section 3, the nonlinear properties of the labor market are examined. The section 4 explains the interpretations of the results in terms of asymmetry, and its implications. In section 4, the identification problem is also discussed. Finally, concluding remarks close this study in section 5.

II. MODEL AND ITS EQUILIBRIUM

Here, we introduce the model economy with one good, two inputs and a large number of identical workers and firms. Output is subject to a random shock.

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[2] One may need to be cautious in understanding a decrease in working hours as an increase in unemployment. See the discussion in section 4.

[3] To economize on terminology, we often omit "strictly". The meaning of rigidity will be explained below. Relating the findings in this paper to Blatt(1980)'s arguments, they imply that economists need not give up the Frisch-type business cycle models for the reason that asymmetries of some variables are observed. He argues that the observed asymmetry is not consistent with the Frisch-type business cycle theory.

[4] That is, the second derivative of an empirical wage function with respect to shock may provide some information about the nonlinear structure of labor supply function.

Preferences

Each worker is assumed to have a preference structure over a composite consumption good and leisure, which is described by a time-invariant utility function: $U[C, N]$; $R_2 \rightarrow R_1$ where R_i is i dimensional nonnegative real space, and C and N denote consumption and labor supply. Supply of working hours is limited by the maximum hours available to an agent, say, 1. The utility function is strictly concave, U_c, $-U_n$ are positive, and U_{cc}, U_{nn} are negative.[5] The separability of consumption and labor supply in preferences is convenient for later discussions; U_{cn}=0. The worker's problem is summarized as ;

(1) $V = Max E(t) \sum_{t=0}^{\infty} q(t) U\{(C(t), N(t)\}$ over $C(t)$ and $N(t)$ s.t

(2) $A(t+1) \leq N(t)W(t)+(1+R(t))A(t)-C(t)$ and $C(t)$, $N(t)$ and $A(t) \geq 0$.

Here $E(t)$ is the expectation operator conditional on all available information at time t, q(t) = the rate of time preference, $A(t)$ = asset, $W(t)$ = the real wage and $A(0)$ is given. The conditions describing the optimal labor supply decision are the first order condition of (1) with respect to leisure and consumption (after arranging) : $U_n[C(t), N(t)] / U_c[C(t), N(t)] = -W_t$ and (2).

Technology

Every agent in this economy has access to an instantaneous production technology subject to a random shock :

(3) $Y(t) = Z(t)\ F[N(t), K(t)]$,

where $Y(t)$ is the flow of output produced at time t, requiring input services of labor $N(t)$ and capital $K(t)$. The production function $F[\ \ ]$ is assumed to satisfy the marginal productivity and strict concavity conditions. Technology is assumed to be constant returns to scale and stationary over time. The technology shock $Z(t)$ is assumed to be a first-order Markov stochastic process :

(4) $Z(t)\ [1-bL] = e(t)$

(Here, b is a constant real number and L is the lag operator.) Agents can observe a realized shock $Z(t)$ at the beginning of time t. The innovation $e(t)$ has a distribution function (d.f) $G(e)$ with mean μ and variance σ. The d.f $G(e)$ is assumed to have a finite positive support so that $Z(t)$ is always positive. Further,

[5] Hereafter, subscripts denote the derivative of a function.

the mean of $Z(t)$ is assumed to be equal to 1 by imposing the condition that $1-b = \mu$.

Market Equilibrium

The economy is closed with a budget constraint and law of transition for capital stock:

$$(5) \quad C(t)+K(t+1)[1-\delta L] = C(t)+I(t) \leq Y(t)$$

where δ denotes the constant capital depreciation rate. In solving for market equilibrium, it is well known that if there is a social planner maximizing (1) subject to (3), (4), and (5), he can solve the problem using dynamic programing. Among the distributions of decision variables to be explained in equilibrium, those of wages and quantity of labor are discussed here. The persistence and serial correlation of working hours and wages are generated by the correlated technology shocks, and/or possibly, by the capital accumulation structure.

III. THE CHARATERISTICS OF HOURS AND REAL WAGE FUNCTIONS

To simplify the argument without loss of generality, a simple environment will be employed to derive the main arguments. Then, those results will be extended to the more general case.

A Simple Labor Market

Here, the rate of time preference is constrained to be equal to the real rate of return. This assumption, with no unearned income at time $t=0$, has the effect of eliminating an intertemporal substitution effect associated with changes in the real interest rate. Specifically, a worker has a zero time effect since interest and time preference effects cancel out. Alternatively, one may simply assume that the real interest rate is constant.[6]

To begin, the utility maximization problem can be solved to yield a simple labor supply function :

$$(6) \quad N^s(t) = H[W(t), \Omega]$$

where Ω is a vector of preference and budget parameters and $N^s(t)$ denotes the labor supply. The Ω may include series of future real wages, for which no attention is given in the analysis of this section. Note that in the usual equilibrium

[6] As shown in next subsection, however, the main results are robust to the introduction of constant rate of time preference and/or non-zero unearned income with which changes in the real interest rates will generate intertemporal substitution.

model of labor market, the labor supply decision includes the real interest rate $R(t)$ as a variable, $N^s(t) = H[W(t), R(t), \Omega]$. Here, the above labor supply function, (6), can be used because there is no time effect under the assumptions about the preferences and budget structure.[7] The parametric representation of the derivatives, H_w and H_{ww}, is defined from the structure of preferences and budget constraint. While H_w and H_{ww} may be positive or negative, depending on the values of preference and budget constraint parameters (affected by the conflict between income and substitution effects), the general presumptions are that H_w is positive, and H_{ww} is nonpositive.[8] H_w is called the marginal work incentive, or the uncompensated labor supply effect of wage changes. Nonpositive H_{ww}, given positive H_w, means that worker has a sufficiently decreasing substitution effect and/or increasing income effect for rising wages, so that the marginal work incentive H_w decreases for increasing wages.[9]

On the other side of labor market, the firm's optimal condition requires that the marginal productivity of labor must be equal to the price of labor

(7) $Z(t)F_n[N(t), K(t)] = W(t)$

This also can be converted into the labor demand function ($=N^d(t)$) by referring to the technology constraint, conditional on given technology parameters. Since there is a level of wage equilibrating labor demand and supply, i.e., satisfying (6) and (7) simultaneously, one can substitute (6) into (7), or vice versa. First, to see

[7] So, (6) is nothing but a simple Marshallian labor supply function derived with the above assumptions. One may also work with the marginal utility of wealth constant or Frisch labor supply function with other assumptions and some modification of arguments.(See Macurdy(1981) or Browning, Deaton and Irish(1983)). In that case, the discussion in this subsection corresponds to the case of fixed marginal utility of wealth while that in next subsection corresponds to the case of varying marginal utility of wealth. For the former case, the marginal utility of wealth (or sometimes called price of utility) can be fixed in the perfect foresight equilibrium or in the case where the new information leaves the price of utility constant in the rational expectation equilibrium under uncertainty. And the $R(t)$ in next subsection can be considered to be the price of utility.

[8] The specification of positive H_w and nonpositive H_{ww} is reasonable and convenient for the derivation of later results. The above specification may be changed to account for different cases. For example, among them, the case which one might want to discuss is the case of backward bending labor supply function. Appendix A discusses this case, with predictions about the behavior of hours and wages.

[9] A simple comparative static result for the labor supply function shows that with the separability of consumption and leisure in preferences, if H_w is positive, then H_{ww} is nonpositive. And H_{ww} is conditional on the third derivative of utility function with respect to consumption and leisure, which may reflect the degree of risk aversion. Here, we give no attention to higher than the first and second derivatives to emphasize the role of convexity since the first and second derivatives are sufficient to represent the convexity of the structure.

the structure of working hours with respect to technology shock, it is convenient to substitute (7) into (6) :

$$(8) \quad N(t) = H[Z(t)\, F_n(t)]$$

where $N^s(t) = N^d(t) = N(t)$ in equilibrium and parameter vectors are omitted. This is a policy function allocating labor, which develops in solving the social planning problem. With (8), we have first proposition.

Proposition 1 The equilibrium working hours are increasing at a decreasing rate with respect to the driving force.

Proof From (8), we have

$$(9) \quad N_z = [H_w(t)\, F_n(t)] \,/\, [1 - Z(t) H_w(t)\, F_{nn}(t)]$$

under the given conditions of economic structure, (9) is positive so that working hours is shown to be procyclical. To investigate the nonlinearity of working hours function, a differentiation of (9) gives (time notation is omitted)

$$(10) \quad N_{zz} = [2H_w F_{nn} N_z + H_{ww}\, W_z(F_n + N_z F_{nn})] \,/\, [1 - H_w Z F_{nn}]$$

with the positive W_z (see derivation (12) below), in determining the sign of (10), one need to show that the term $(F_n + N_z F_{nn} Z)$ is positive, which is proven in following Corollary 1.[10] Using the Corollary 1, the derivative (10) is negative. q.e.d..[11]

Corollary 1 $F_n + N_z F_{nn} Z$ is positive.[12]

Proof $F_n + N_z F_{nn} Z = F_n + [H_w F_n F_{nn} Z / (1 - H_w F_{nn} Z)]$. This expression can be rewritten as $F_n[1 + (H_w F_{ww} Z) / (1 - H_w F_{nn} Z)] = F_n[1/(1 - H_w F_{nn} Z)]$ which is positive. q.e.d..[13]

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[10] In (9) and (10), F_{kn} is assumed away for a simplicity. See the explanation below about this assumption.

[11] In this proof, the budget condition is not considered fully. In next subsection, however, the condition is considered by the role of ineterest rate.

[12] The interpretation of Corollary 1 is of interest. It means that $|N_z|$ is less than $|F_{nn}Z/F_n|$. Since $|F_{nn}Z/F_n|$ is N_z^d, it means that the actual equilibrium adjustment in working hours is less than the desired adjustment by the firm. This is true, intuitively, as long as the labor supply function is not perfectly inelastic.

[13] I appreciate George Neumann for simplifying this proof remarkably.

A larger value of H_{ww} makes the value of (10) larger and does not change the sign of (10). This point is emphasized because the role of H_{ww} is different in the case of wage function. Interpreting the first two derivatives, it is obvious from (9) and (10) that hours are strictly increasing at a decreasing rate with respect to the technology shock. This structure of working hours with respect to the shock may be called "concave".[14] [15]

Next, to examine the nonlinearity of the wage function, we substitute (6) into (7), yielding[16]

(11) $W(t) = Z(t)F_n[H(W(t))]$

Proposition 2 With (11), wage function is increasing at a decreasing rate or an increasing rate, which depends on the nature of labor supply function.

Proof From (11), one can have

(12) $W_z = [F_n(t)] \,/\, [1 - Z(t)F_{nn}(t)H_w(t)]$.

It is easy to verify that (12) is positive, meaning that real wage movement is procyclical. The differentiation of (12) yields ;

(13) $W_{zz} = [(2H_w + ZH_{ww}W_z)\, W_zF_{nn}] \,/\, [1 - ZF_{nn}H_w]$.

Similar to (10), the value of (13) is determined by preference and technology properties. Note that, unlike (10), (13) has the possibility of being positive, implying that real wages increase at an increasing rate with respect to the shock. Specifically, one can see that the sign of (13) depends on the rate at which the marginal work incentive ($=H_w$) changes for varying wages. The rate is represented by the value of H_{ww}: W_{zz} is positive if $|H_{ww}|$ is greater than $|2H_wZW_z|$, and it is negative if $|H_{ww}|$ is less than $|2H_w \,/\, ZW_z|$. The former case can hap-

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[14] Note that the derivative conditions used here is not free from measurement units. Thus, the size of derivatives may vary with measurement units. One can have the elasticity of hours and wage, and also the elasticity version of the second derivatives with respect to shock by applying the standard formula. The elasticity version of the second derivative of hours and wage with respect to shocks are $N_{zz} Z/N_z$ and $W_{zz}Z/W_z$ respectively. Here, we continue to use the derivatives for convenience.

[15] For the sake of simplicity, we use "concave" to mean a function that is strictly increasing at a decreasing rate, and "convex" to denote a function that is strictly increasing at an increasing rate. We call a strictly increasing and concave function as concave, and strictly increasing and convex function as convex.

[16] The expression (11) defines a relative price supporting the policy function of (8).

pen when H_{ww} is a fairly large negative value. It means that the marginal disutility of work efforts increases quickly (or marginal work incentive decreases quickly). q.e.d..

The effects of convex preferences and technology on the structure of wage and employment functions can be examined separately by the followiing observation.

Observation The diminishing marginal returns and increasing marginal disutility of work effort, associated with the convex technology and preferences, are cooperative in making the hours function concave, but are not so in shaping the nonlinearity of wage function. In the case of wage function, if the latter is sufficiently strong, the wage function may be convex. Otherwise, it is concave. Specially, when marginal disutility of work is constant (H_{ww}=0), then both hours and wage functions are concave.(See the proofs of Propositions 1 and 2).[17]

A General Labor Market

In this subsection, the above analysis is extended to the case where the rate of time preference is constant and/or worker has a positive non-labor income. Then, the labor supply decision is affected by the real interest rate: $N^s(t) = H[W(t), R(t), \Omega]$, with the assumed derivative conditions of nonnegative H_r and nonpositive
H_{rr}.[18] With the structure described above, we have propositions corresponding to propositions 1 and 2.

Proposition 1* Hours function is an increasing function of driving force at a decreasing rate.

Proof First, the equilibrium condition (8) is written as follows:

[17] In the above derivations, F_{nk} has been assumed way. If F_{nk} is of a significant positive value, then, it can change the derivatives and arguments in the above. This event can happen when the positive value of F_{kn} dominates the diminishing marginal productivities, so, the negative values of F_{nn}. However, it seems plausible to presume that the value of F_{kn} is fairly small enough to be ignored. More fundamentally, it should be kept in mind that the theoretical results in this section is static, but not dynamic. Specifically, the role of (expected) future shock which is affected by the current shock is absent. As explained before, this point may not be so wrong in the perfect foresight equilibrium.

[18] The positive H_r is often found when preferences are time-separable. For example, see Alogoskoufis(1987). The nonpositive H_{rr} is corresponding specification to the nonpositive H_{ww} in the comparative statics I.

(8)* $N(t) = H[ZF_n(t), R(t)]$. This condition gives us[19]

(9)* $N(z) = [H_w F_n + H_r R_z] / [1 - H_w ZF_{nn}]$.

The expression (9)* is positive, implying a procyclical working hours. The second derivative of the hours function with respect to the shock is given by

(10)* $N_{zz} = [H_{ww} W_z(F_n + ZF_{nn}N_z) + 2H_w F_{nn}N_z + H_{rr}R_z^2] / [1 - H_w ZF_{nn}]$

First, one can examine the case of the linear labor supply function. If H_{ww} and H_{rr} are zero, (10)* is negative, implying a concave hours function. When H_{ww} is not zero, we need one (sufficient, not necessary) condition for the hours function to be concave: $|F_n|$ is greater than $|ZF_{nn}N_z|$. This is the same condition as proved in Corollary 1.[20] Now it is necessary to specify the structural relationship between the real interest rate and technology shocks. We assume that R_z is positive and $R_{zz} = 0$.[21] (This same condition about R_z and R_{zz} is employed in the next derivation and interpretation of wage function). Then, it is easy to see that (10)* is negative. q.e.d..[22]

Proposition 2 The wage function is an increasing function of technology shock at an increasing rate or a decreasing rate, depending on the nature of labor supply function.

Proof We have

(11)* $W(t) = Z(t)F_n [H(W(t), R(t))]$

From (11)*, we can have (12)*.

[19] Since the cross derivative of production function is assumed away as in the preceding subsection, the role of real interest rate in the labor demand may be ignored here. Note that even when the real interest rate can not be ignored in the labor demand, the argument in this section is still valid. In that case, the argument may be strengthened or weakened.

[20] In proving this condition algebraically, one condition is required: the intertemporal substitution for change in the real interest rate should not dominate that for change in real wages. This appears to be a reasonable condition.

[21] The direction of real interest rate movement is not predicted theoretically. The question should be answered empirically. For example, the positive R_z is observed in a simulation work of Kydland and Prescott(1982). And the assumption of constant R_z is convenient to focus on the nonlinearity of labor market. Note that this assumption has been already used in deriving (9)*. This assumption is not crucial, but can be relaxed.

[22] Note that if H_r (thus H_{rr}) is zero, (11)* and (12)* are equal to (11) and (12) respectively.

$$(12)^* \; W_z = [F_n + H_r R_z Z F_{nn}] \,/\, [1 - Z F_{nn} H_w]$$

(12)* implies that wages may not be procyclical if H_r and(or) R_z are (is) of (a) significant values. This is a very intuitive result. Based on empirical evidences, it is not too wrong to assume away the case of sgnificant values of H_r and R_z. So, (12)* is positive.

The nonlinearity of the real wages is given by

$$(13)^* \; W_{zz} = F_{nn}[W_z\,(2H_w + ZH_{ww}W_z) + (2H_r R_r + H_{rr}R_z^2)] \,/\, [1 - ZF_{nn}H_w].$$

The derivation of (13)* is a little complicated, but it can be given a consistent interpretation. First of all, one can focus on the implications of the convex technology, by presuming a constant marginal work incentive with respect to real wages and interest rate : $H_{ww} = 0 = H_{rr}$. Then, the wage function is concave since (13)* is positive and (13)* is negative. This is the same result as in the previous section. Alternatively, when there exists significantly increasing marginal disutility of work effort, the wage function may be convex since (13)* is positive if $|H_{ww}|$ and $|H_{rr}|$ are sufficiently large. q.e.d..[23]

Ⅳ. INTERPRETATION, IMPLICATIONS AND IDENFICATION

Interpretation and Implications

One may give an interpretation of the above results in terms of asymmetric movement in hours and wages. Concavity (convexity) means a smaller (larger) increase relative to a decrease for an equal change in the independent variable. Thus, the concavity of hours function may imply an asymmetry or "upward" rigidity of hours: large decreases during downturns and small increases during upturns over cycles. Similarly, convex wage function can mean "downward" rigidity : small decreases during downturns and large increases during upturns. It is remarkable that while wages and hours are allowed to be perfectly flexible, they may show a "rigidity" in the "second" derivative sense.[24] The above interpretations of nonlinearity in terms of asymmetric movement of hours and wages provide plausible explanations to some observed behavior in other studies. It has been pointed out that unemployment data show a type of asymmetric movement, specially, sharp or large increases and gradual or small decreases : see Neftci

[23] Again, if H_r (thus H_{rr}) is zero, (12)* and (13)* are equal to (12) and (13) respectively.

[24] These asymmetries implied by the nonlinear hours and wage functions may be more relevant than nonlinearities themselves in understanding the observed hours and real wage data since we have only discrete observations.

(1984), Falk(1986) and Davis(1987).[25] One can explain this asymmetry under the presumption that movements of working hours are the converse of unemployment:decreasing working hour means rising unemployment, and vice versa.[26] Under this interpretation, asymmetric movement of unemployment data is easily generated in this model. The opposite interpretation of concave employment in terms of unemployment is just the observed movement in the above cited studies: relatively large (or quick) increases and small (or gradual) decreases in unemployment. In another context, Brock and Sayers(1988) find nonlinearity in both employment and unemployment time series data. For nonlinearity, the above concave structure of hours with respect to shock is a sufficient condition.

Being concerned with econometric modelling, the results in this paper raise question about studies where the linearity of endogenous variables with respect to exogenous variables is assumed. One obvious point is that decisions and predictions based on the linear model may be inefficient because they are not using information fully.[27] The possible problems caused by the specification of linear model when the true structure is nonlinear are discussed in Neftci(1984, 1982) and Brock and Sayer(1988). The former points out the problem in applying the asymptotic theory of linear model to nonlinear structure. The latter finds that the nonlinearity left in the residuals of the best fit linear model is significant (strong dependence in the residuals of the best fit linear model for industrial production, employment and unemployment). The results in section 3 imply that linear structures (specially, in the labor market) can be justified only when the productivity of labor is zero, and the economy has a perfectly inelastic labor supply function.

Identification Problem

The above discussions of working hours and wage functions show that they are inherently nonlinear and asymmetric, basically because of the convex structure of the economy. However, as mentioned in Brock and Sayer(1988), it is also true that if the economy incurs convex adjustment costs over cycles, then, it may generate the nonlinearity or the asymmetry of the same kind. One can conjecture

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[25] Such an asymmetry is also found in Delong and Summer(1986) which use the skewness measurement, and in Dynarski and Sheffrin(1987) which also examine the asymmetry of consumption expenditure.

[26] In this discussion, one should keep in mind the following points. First, as a referee point out, the unemployment we are talking about should only include the layoff and/or quit, but not the new entrants to the labor market. Note also that the explanation is limited only to the flow part of unemployment stock. Another point is that the theory in this paper is only about the asymmetric amplitude of upward rigidity, but not about the other asymmetries of frequency or duration also examined in other literatures.

[27] Ashely and Patterson (1989) also show that the nonlinearity is an important factor to consider in developing a model.

that both structural explanations are theoretically equivalent, raising an identification problem. In other words, interpreting working hours and wages as outputs, the technology shock as an input, and the structural relationship as a system, the above two hypotheses are concerned about the system(mechanism), that is, the system may be nonlinear either because of convex structure of the economy, or because of convex adjustment costs. Identification problem does not end here. Besides these two reasons, there is another possible reason for the asymmetry, as briefly mentioned in Neftci(1984). This third case is concerned with the distribution of inputs (here, technology shocks). It may be the case that underlying inputs(shocks) are non-linear (non-Gaussian) or asymmetric while the system itself is linear or symmetric. That is, the shock may be with an asymmetric amplitude of large decreases and small increases.

The identification between the two structural reasons and the third reason of "nonlinear inputs" may be solved by an elementary fact in time series studies.[28] For an identification between the two structural explanations, however, there exists no satisfactory theory. It seems more difficult to distinguish two structural explanations of convex structure and adjustment cost since they are (at least theoretically) equivalent, as proved below in the proposition 3. While the adjustment cost may be in the preferences and/or technology, the proposition is concerned with the adjustment costs in the technology.[29] So, the production technology is

$$(14) \quad Y(t) = Z(t)\,F[N(t),\ N(t-1),\ \mathrm{k}(t)].$$

Proposition 3 With convex adjustment costs in technology, hours function is concave with respect to technology shock.(See Appendix for the proof)

The proposition 3 suggests that for the identification between the convex structure and convex adjustment costs, behavioral parameters in the decision rules are not useful, but structural parameters should be examined. One plausible method is to specify a complete model of preferences and technology and derive the first order conditions which can be estimated (generally by an instrumental variable

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[28] If the system is linear, the non-normalized spectral density function of the output series will equal the non-normalized spectral density function of the input series times the squared modulus of the transfer function. Also, there is another supplementary test for checking the non-linearity or non-Gaussianity of time-series data, by measuring the polyspectra: e.g., all polyspectra of higher order than second order will vanish when a series is Gaussian. It is common to examine the bispectrum (and higher spectrum) for assessing the nonlinearity or non-Gaussianity of time series data. It is known that the linear spectrum is not useful in the examination of asymmetry or nonlinearity. For a good summary, see Priestly(1988).

[29] The other cases where both preferences and technology incur adjustment costs, or the preferences alone incurs adjustment costs can be also easily examined. See Corollary 1.

procedure). Then, it is possible to test the parameters related to convex structure and convex adjustment costs in such an estimation procedure (for example, using test statistics developed by Gallant and Jorgenson(1979) in the case of nonlinear model).[30] Thus, one can test the relative importance of two factors as long as one can specify an econometrically identified model, the specification of which is not simple.

V. CONCLUSION

This paper shows that in the labor market of one sector model, when the economy is strictly convex, hours and wage functions are inherently nonlinear with respect to driving force of technology shock. Thus, we conjecture that hours and wages determined in the labor market may generate systematic and predictable nonlinearities and asymmetries over cycles. In particular, it is shown that given a convex technology, the labor supply function may be important in shaping the strucutre of those functions. The nonlinear structure found in this paper is used to explain some asymmetriy or nonlinearities observed in other places. The results of this paper suggest that the (absolute) elasticity of hours and real wages with respect to the shock is larger in the contraction than in the expansion. It may be a valuable observation in understanding the asymmetry of aggregate data in different market economies.(See Falk(1986)). For example, one can conjecture about the preference structure by looking at the asymmetry of wage and hours data, and then compare the conjecture with parameter values observed in other studies of preferences.[31]

The results in this paper suggest further works in several directions. First, one may attempt other possible methods, including simulation, to test the hypothesis. One can examine how the simulated hours and wages change (in the sense of nonlinearity) for different choice of parameters. Second, as already mentioned, more work to solve the identification problem is desirable to find correct sources for the nonlinear and asymmetric movements of hours and wage (possibly other economic time-series). Finally, note that discussions in this paper proceed under the assumption of no systematic preference shock relevant to decision making. If the measure of such preference shock is significant, then the above derived results might be given a different interpretation.

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[30] The good examples are Shapiro(1986) or Pyndyck and Rotemberg(1983). Actually, both studies find that the adjustment cost for labor is not important in labor demand. Their finding, however, can not be applied directly for the identification problem in this paper because they ignore preferences.

[31] For example, in a simple case where the real interest rate is neutral in the labor supply decision, the covariance structure of real wages and hours can give some information about the labor supply function. In this case, this paper shows that working hour function will have the larger first derivative than wage function with respect to shock if H_w is greater than one.

APPENDIX

A. Nonlinearity with Backward Bending Labor Supply

With a backward bending labor supply, the discussions in the text are modified. First, note that with negative $H_w(=N^s_w)$, N_z and W_z will have different signs depending on the values of $|N^d_w|$ and $|N^s_w|$. One can verify that N_z is negative (positive) and W_z is negative (positive) if $|N^d_w|$ is greater (less) than $|N^s_w|$. Here, $|N^d_w|$ is derived as : $|N^d_w|=|1/ZF_{nn}|$ and $N^s_w = H_w$. One observation is that with a backward bending labor supply, either hours or wages can be countercyclical, but not both. This elementary fact is helpful in interpreting the empirical work. Secondly, for N_{zz} and W_{zz}, one can have a deterministic results only when $|N^d_w|$ is greater than $|N^s_w|$: N_{zz} and W_{zz} are nonpositive, as easily verified from (10) and (13) in the text.

Proposition A-1 With nonpositive H_w, only one of N_z and W_z should be negative with the other being positive, without regard to the values of $|N^d_w|$ and $|N^s_w|$. One can predict negative N_{zz} and W_{zz} with nonpositive N_z and nonnegative W_z, if $|N^d_w|$ is greater than $|N^s_w|$.

Applying the same argument to the case of the labor supply with interest rate as an argument, it is found that, with the above condition about $|N^s_w|$ and $|N^d_w|$, nonpositive N_z and undetermined N_{zz} and nonnegative W_z with negative (positive) W_{zz} if H_{ww} is very large (small).

B. Large Fluctuations of Hours Relative to Wage

A stylized fact in the labor market is large fluctuations of hours relative to wages. This fact can be partially explained by the nonlinear hours and wage functions. For this purpose, we distinguish two cases. First, the case of $H_{ww}=0$ is discussed: the constant marginal work incentive.

Proposition B-1 With $H_{ww}=0$, the volatile movements of hours relative to wages can be generated by right-skewed distribution (RSD) [or left-skewed distribution (LSD)] of shocks if hours is less [or more] concave than wage.

This observation can be verified by checking the degree of concavities and geometric shapes of two different concave functions: For example, for the less concave variable to have a larger variation, shock should be of RSD. While propositions in this section are very obvious intuitively, it is not easy but possible to prove. The more promising method may be a simulation work : measure the variances of hours and wages simulated in differently specified models. We have a similar observation for the case of negative H_{ww}. Here, an emphasis is on the case where there is a strong reduction in marginal work incentive for rising wage so as to make convex movements of wages (see (13) and (13)*) (because the other case is the same as the above observations). Note that the result does not depend on the derivative H_w.

Proposition B-2 With a value of $|H_{ww}|$ greater than $|2H_w / ZW_z|$, the shock of LSD is necessary to have the volatile hours and smooth wages.

C. Poof of proposition 3

Proof When production technology is (14), the social planer's problem produces the following first-order conditions

(15) $U_c(C(t), N(t)) - \Gamma(t) = 0$

(16) $U_n(t) + \Gamma(t)Z(t)F_n(t) + q\Gamma(t+1)Z(t+1)F_n(t+1) = 0.$

(17) $q\Gamma(t+1)Z(t+1)F_k(t+1) + (1-\delta) - \Gamma(t) = 0.$

(18) $Z(t)F[N(t), N(t-1), K(t)] + (1-\delta) - K(t) - K(t+1) - C(t) = 0.$

and the transversality condition, $\lim_{t \to \infty} q^t\Gamma(t)K(t+1) = 0.$

where $\Gamma(t)$ is the Lagrange multiplier.

In exercising a comparative statics on these conditions, we impose a simplifying assumption that marginal utility of consumption is constant. Then, (15) is not relevant for the comparative statics, only (16), (17) and (18) are used for it. The comparative statics results are

(19) $N_z = \{-F_n(t)qZ(t+1)F_{kk}(t+1)\} / \{A\},$

Where $A = \{[U_{nn}(t) + Z(t)F_{nn}(t) + qZ(t+1)F_{nn}(t+1)]\ [qZ(t+1)F_{kk}(t+1)]\}$ which is positive. So, (19) is positive. And

(20) $N_{zz} = \{qZ(t+1)F_{kk}(t+1)[F_n(t) - F_{nn}(t)N_z]\} / \{A\}$ which is negative. This proves that hours function is concave with respect to technology shock. q.e.d..

Corollary 2 With convex adjustment cost or time-nonseparability in the preferences, hours function is concave with respect to technology shock.

Proof The proof of Corollary 2 is a simple exercise using that of proposition 3, with a modification.

REFERENCES

Alogoskoufis, G.S., "On Intertemporal Substitution and Aggregate Labor Supply", *Journal of Political Economy* 95, 1987.

Ashely, R. A. and Patterson, D. A., "Linear versus Nonlinear Macroeconomics: A statistical Test" *International Economic Review* vol 30. no 3. 1989.

Blatt, "On the Frisch-type model of Business Cycle" Oxford Economic Papers 1980.

Brock, W. A., and Sayers, C. L., "Is the Business Cycle characteried by Deterministic Chaos", *Journal of Monetary Economics* 22, 1988.

Browning, M. J., Deaton, A. S. and Irish, M., "A Profitable Approach to Labor Supply and Commodity Demand over Life-Cycle", *Econometrica* 53, 1983.

Davis, Steven J., "Fluctuations in the Pace of Labor Reallocation," Carnegie-Rochester Conference Series on Public Policy 27 (Autumn 1987).

Delong, Bradford and Summers, Lawrence, "Are Business Cycles Symmetrical?" The American Business Cycles. ed by Gordon, Robert. The University of Chicago Press, 1986.

Dynarski, Mark and Sheffrin, Steven., "Asymmetric Fluctuation in Consumption," *Economics Letters* 21, 1986.

Dynarski, Mark and Sheffrin, Steven., "Consumption and Unemployment," *Quarterly Journal of Economics* 1987.

Fark, Barry., "Further Evidence on the Asymmetric Behavior of Economic Time Series over business Cycle" *Journal of Political Economy* 94, 1986.

Frank, Murray Z. and Stengos, Thanasis. "Some Evidence Concerning Macroeconomic Chaos", *Journal of Monetary Economics* 1988.

Hall, Robert E. and Lilien,David M. "Cyclical Fluctuations in Labor Markets", Handbook of Labor Economics ed by Ashenfelter.O and Layard, Richard, 1986.

Hamermesh, D. S., "Labor Demand and the Structure of Adjustment Costs", American Economic Review 79, 1989.

Hercowitz, Zvi., "The real interest rate and Aggregate Supply," *Journal of Monetary Economics* 1986, 18.

Kannai, Y., "Concavifiability and Construction of Concave Utility Functions", *Journal of Mathematical Economics* 1977, 4.

Kydland, F and Prescott, E., "Time to Build and Aggregate Fluctuations", *Econometrica* 50, 1982.

Macurdy, T. E., "An Empirical Model of Labor Supply in a Life-Cycle Setting" *Journal of Political Economy* 89, 1981.

Neftci, S. N., "Optimal Prediction of Cyclical Downtown", *Journal of Economic Dynamics and Control.* 4, 1982.

Neftci, Salih.N., "Are Economic Time Series Asymmetric over the Business

Cycle" *Journal of Political Economy* 92,2, 1984.

Nickell, S., "Dynamic Models of Labor Demand", in Orley Ashenfelter and Richard Layard, eds., *Handbook of Labor Economics*, Amsterdam: North-Holland Press, 1986, ch9.

Pencavel, John., "Labor Supply of Man: A Survey", Handbook of Labor Economics in Orley Ashenfelter and Richard Layard, eds., *Handbook of Labor Economics*, 1986.

Pfann, Gerad A., "Employment and Business Cycle Asymmetries: A Data Based Study", Discussion Paper 39, Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis. Mar, 1991.

Pindyck, Roberts S. and Rotemberg, Julio J., "Dynamic Factor demands and the Effects of Energy Price Shock," *The American Economic Review*, vol.73, No.5, 1983.

Priestly, M. B., Non-inear and Non-tationary Time Series Analysis, Academic Press, 1988.

Shapiro, Matthew D., "The Dynamic Demand for Capital and Labor," *The Quarterly Journal of Economics*, 1986.

Wecker, W. E., "Asymmetric Time Series", *Journal of American Statistical Association*, Mar 1981, vol.76.