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HOLDING OPTIONS IN SEARCH AND MATCHING*

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This paper investigates the implications of adding a recall option in a dynamic model of search and matching. The effects of a holding option on market efficiency are examined by comparing the markets with and without a holding option. When preferences are homogeneous, a holding option tends to decrease market efficiency when the population is small, and to increase it when the population is large. When preferences are heterogeneous, a holding option tends to increase market efficiency when population is small, and to decrease it when the population is large. It is also shown that the existence of a holding option is an equilibrium. The results show the basic features of a reservation system. They also indicate that, in modeling a non-steady state search-and-match situation, the assumption of no recall can lead to the wrong conclusion about market efficiency.
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I. INTRODUCTION

This paper examines a market where (i) transactions are conducted in pairwise meetings; (ii) individuals must search for a partner; and (iii) the number of potential partners is fixed. In particular, we analyze the effect of a holding option on market efficiency and examine whether the existence of a holding option can be an equilibrium.

A holding option is an option allowing the individuals to have the ability to recall. A typical example of a holding option is a reservation system in a market for summer-house rentals: In that system, when an agent reserves a house, the reservation can be cancelled before a specific date in the future; until that date, the reserved house is off the market and the agent can continue searching for a better house.

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Search is costly and not all the pairings have the same value. For simplicity, assume only two possible evaluations: good and bad. When the market opens, agents search for houses to rent. The matching process is a Poisson process. When a meeting results in a good match - e.g., the match when the agent's evaluation of the house is "good"- a lease is signed and both parties leave the market. When a bad match occurs, they may sign a lease and leave the market, or the agent may continue searching for a good house with or without putting the bad house on hold. We consider two kinds of preferences: (i) two agents agree about which of two houses is better (homogeneous preferences); (ii) two agents may or may not agree about which of two houses is better (heterogeneous preferences).

When holding is possible, an agent can avoid an outcome in which he settles for a match that is worse than the current match, while continuing to search for a better match. We investigate the effects of holding options through the comparison of markets with and without a holding option.

Let us consider the case of homogeneous preferences first. A fixed fraction of items is good and searchers have identical preferences. Searchers continue searching until the expected value of search is equal to the search cost; so the total surplus to be realized is the same whether or not holding is possible. Hence, in evaluating market efficiency, only total search cost matters. Therefore, socially optimal search would require that every searcher make a transaction with the first partner he meets. However, when the expected payoff from one more search is larger than the value of a bad match, a searcher either rejects the bad match or uses the holding option to keep it available. Such selective search leads to market inefficiency. The ability to place an object on hold leads to two changes in behavior compared to a model without a holding option. First, individuals have greater incentives to continue searching. Since all selective search is inefficient, this effect suggests that the holding option can reduce social welfare. Second, individuals with an item on hold need not continue their search when it is no longer profitable to search for a high value item. This effect suggests that a holding option could increase the social welfare when the population is large. We show that when the population is so small that there is no selective search in the market without a holding option, the holding option lowers the net surplus (total surplus to be realized less total search cost) in the market. Also when the population is large, under some conditions, a holding option can increase the net surplus (because the savings of search cost (gained from the ability to recall a bad match) more than compensate for the added search cost). Moreover, because offering a holding option increases the probability of a transaction for the bad-quality partner, the existence of a holding option is an equilibrium in a game where individuals can decide whether or not to offer a holding option.

In the case of heterogeneous preferences, a bad match to one searcher can be a good match to another searcher. We assume that the two sides of the market is symmetric. Since every searcher in the market enjoys the same probability of finding a good match from search, the holding option should treat the parties in a match symmetrically: (i) both parties in a match on hold continue searching for a good match; and (ii) the transaction of the match is completed only if both parties are available (so the availability of a fallback partner is not certain in the future). A searcher creates a positive externality for other searchers (an additional searcher raises the probability of a good match of all his potential partners). Because of this positive externality, surplus maximization calls for a longer selective-search than occurs in the equilibrium whether or not holding is possible. The existence of a holding option makes searchers more selective. Hence, when the population is so small that there is no selective-search in the market without a holding option, the holding option increases the net surplus in the market. However, when holding is possible, at the end of the selective-search phase, the concern of a searcher with a bad match on hold is the number of partnerless searchers (because all the individuals with a bad match on hold will leave the market). So they may leave the market too early, even though searching for a good match would be profitable if all the searchers - those with and those without a bad match on hold - participated in searching. Because of this negative externality, a holding option can decrease the market efficiency when population is large.

I also study a game in which individuals can decide whether or not to offer a holding option: the only pure-strategy equilibrium of this game specifies that individuals provide holding options, because offering a holding option increases the probability of making a bad transaction (when it is preferable to no transaction at all).

The results described in this paper show why a holding option exists in the seasonal markets with fixed supply and demand. Also, they show that a holding option can create market inefficiency. Moreover, they indicate that, in modeling a non-steady state search-and-match situation, an assumption of no recall can lead to the wrong conclusion about market efficiency.

Diamond and Maskin (1981) analyzed the non-steady state equilibrium in a model with heterogeneous preferences, similar to Section IV in this paper where holding is possible. However, they neither compare the markets with and without a holding option, nor consider whether the existence of a holding option can be an equilibrium. Instead, their paper focuses on the possibility of breach of contract and the effect of breach on market efficiency. In their paper, there is a damage rule that specifies what a searcher must pay to leave a bad match. In this paper, there is no damage rule, which leads to less selective-search. Mortensen (1982) and Diamond and Maskin (1979) examined the symmetric steady-state equilibrium in the search-and-match context. In Mortensen(1982), only unmatched searchers are permitted to search. These papers examined the steady state equilibrium, while we examine the non-steady state equilibrium. Roth and Xing

(1994) considered the timing of transaction and the inefficiency of matching institutions. In the markets examined in their paper, the quality of a match is uncertain until some time in the future, and the matching process is not random (and so one-to-many matching is possible). These two factors lead to unraveling of transaction time, and the offering of a short holding period to prevent the current partner from contacting other potential partners. In our paper, there is no uncertainty about the quality of a match (as soon as matched, the individuals know the quality of the match) and the matching process does not allow one-to-many matching. Without uncertainty about the quality of matches, search behavior is affected by the mere possibility of matching itself: in the heterogeneous preference case, there is an unraveling of transaction time (compared to the socially optimal transaction time), leading to market inefficiency; In the homogeneous preference case, the partner on hold in a bad match has an incentive to reduce the current partner's ability to contact other potential partners by offering a holding option with a deadline. But the effect of a deadline depends on the other matching possibilities. The holding options with deadlines are discussed in SectionIII-4.

This paper is organized as follows: The basic model is presented in Section II. Section III is about the case of the homogeneous preferences. Section IV is about the case of the heterogeneous preferences. Section V concludes the paper.

II. MODEL: HOMOGENEOUS PREFERENCES

There are two sides in the market. One side of the market searches. Let D denote a member of the non-searcher side of the market. Transactions will be concluded in pairwise meetings. Matches are either good or bad. The quality of a match is determined by the quality of the members on the non-searcher side of the market. Let H(L) denote a good (bad) partner. In a good (bad) match, the surplus to be divided is $2h$ $(2l)$ $(h > l)$. Once a searcher decides to make a transaction with a D he has found, both searcher and D leave the market with each party having h in a good match or l in a bad match. Let n_0 be the common number of individuals of the two sides, or equivalently, the number of possible pairs. And let n_{h0} (n_{l0}) be the initial number of H's (L's).

The matching technology is as follows (quadratic matching rate) :
(i) the probability that a searcher meets a D per unit of time, γ, is independent of the number of other searchers remaining in the market;
(ii) a searcher's probability of meeting some D rises linearly with the number of D's remaining in the market (i.e., the average instantaneous rate at which the searchers meet D's is proportional to the number of D's remaining in the market).
Searchers are perfectly informed about the distribution of D's they might meet. They search under continuous time with the flow cost of search, $c > 0$. There is

no discounting.

We are going to compare two markets:

- A market without a holding option: whenever a searcher meets a D, he can take one of two actions - (i) accept the D or (ii) reject the D.
- A market with a holding option: whenever a searcher finds a D, he decides on one of three actions - (i) accept the D; (ii) put the D on hold and continue searching, in which case the D on hold disappears from the market until the searcher makes a decision about the match[1], or (iii) reject the D and continue searching. (When holding is costless, it is always better to use the holding option).

Let S (R) denote a searcher who has nothing on hold (who has a bad match on hold). Let $n(t)$ ($N(t)$) denote the number of D's remaining in the market without (with) a holding option at time t. Let $n_h(t)$ and $n_l(t)$ ($N_h(t)$ and $N_l(t)$) be the number of H's and L's respectively in the market without (with) a holding option at time t. Let $V^j(t)$ denote the j's expected payoff from search at time t, $j = \text{R, S}$.

We are interested in Nash equilibrium paths. An equilibrium path specifies a behavior configuration at each instant of time and has the property that each individual finds the behavior specified for him optimal, given the specified behavior of others. We consider only the equilibrium paths where all individuals in the same position behave identically.

Ⅲ. ANALYSIS : HOMOGENEOUS PREFERENCES

Whether or not holding is possible, a searcher accepts an H whenever he finds it and hence, the number of H's decreases as search goes on. When there is a large number of H's in the market, it is profitable for the searchers to continue searching for an H while rejecting L or putting L on hold (selective-search phase). The difference between markets with and without a holding option is in this phase: During this phase, all the L's remain in the market when no holding is possible, while some of the L's are out of the market when holding is possible. Eventually, every searcher accepts any D he meets first (non-selective search phase). And all search ceases when the number of potential partners in the market is so small that the value of search cannot cover the search cost (no-search phase). In Sections 3.1 and 3.2, we will show the following:

Proposition 1

(1) Introducing a holding option lengthens the interval of selective search.

(2) Whether or not holding is possible, socially optimal search behavior requires a searcher to accept the first match he meets.

[1] Using a holding option is more than remembering the current partner's location - a holding option guarantees a future transaction for a searcher.

Proposition 1-(1) shows that the searcher in a market with a holding option is more selective than the searcher in a market without a holding option. The total surplus to be realized (potential surplus less surplus unrealized after all search) is the same whether or not holding is possible, because all search ceases when the expected value from search is equal to the search cost. Hence, from Proposition 1-(2), the efficiency of the markets with and without a holding option depends only on the total search cost. Section 3.3 compares the efficiency of the markets with and without a holding option.

3.1 Market without a holding option

Search behavior affects the numbers of H's, L's and S's: During the selective-search phase, there is no change in the number of L's while the number of H's declines as good matches are transacted. During the non-selective search phase, the numbers of H's and L's decrease steadily as the matched pairs leave the market.

As the number of potential partners decreases, the searcher's expected payoff from search decreases. Selective-search behavior stops when a large number of H's disappears from the market. All search ceases when the number of potential partners in the market is so small that the value of search cannot cover the search cost.

We will examine the time paths of variables and the terminal conditions for each phase.

Let $\overline{t}_1$ denote the time when the non-selective search starts. Let t^*_1 be the time when all search ceases.

3.1.1 No-search phase

All search ceases when one more search is not profitable.

$$\gamma n^*_h h + \gamma n^*_l l = c \tag{1}$$

3.1.2 Non-selective search phase

All the S's search until they find a D. So the numbers of H's and L's decline steadily. At any instant of time, the number of H's decreases by $\gamma n_h n$ (the probability to meet an H, γn_h, times the number of D's, n). Hence, the instantaneous changing rates of H's and L's are as follows:

$$\dot{n}_h = -\gamma n_h n \tag{2}$$

$$\dot{n}_l = -\gamma n_l n \tag{3}$$

$$\dot{n} = -\gamma n^2 \tag{4}$$

$$n = n_h + n_l$$

The expected payoff from search at time t is calculated as follows: S incurs the search cost $c\Delta t$ in a small amount of time, finds a D with the probability $\gamma n(t)\Delta t$, and with the probability $(1-\gamma n(t)\Delta t)$, he is matched with no D.

$$V^S(t) = -c\Delta th + \gamma n_h\Delta th + \gamma n_l\Delta tl + (1-\gamma n\Delta t)V^S(t+\Delta t).$$

Rearranging terms and letting Δt go to zero,

$$\dot{n}_h \frac{dV^S(n_h, n_l)}{dn_h} + \dot{n}_l \frac{dV^S(n_h, n_l)}{dn_l} = c - \gamma n_h h - \gamma n_l l + \gamma n V^s(t). \tag{5}$$

The time-paths of variables are as follows:

$$n(t) = \frac{\bar{n}}{\gamma(t-\bar{t}_1)\bar{n}+1}$$

$$n_h(t) = \frac{\bar{n}_h}{\gamma(t-\bar{t}_1)+1}$$

$$n_l(t) = \frac{n_{l0}}{\gamma(t-\bar{t}_1)+1}$$

From the time paths of n_h and n_l, $\frac{n_h}{n_l}$ does not change during this phase. Hence the average value of a match conditional on meeting a D remains constant at $v \equiv \frac{\bar{n}_h h + \bar{n}_l l}{\bar{n}}$. Hence (5) becomes

$$\dot{n}\frac{dV^S(n)}{dn} = c - \gamma nv + \gamma nV^S(n). \tag{6}$$

By using (1) as the terminal condition; i.e., $V^S(n^*)=0$ where, $n^* = n^*_h + n^*_l$, (6) becomes

$$nV^S(n) = -\frac{c}{\gamma}\ln\left(\frac{n}{n^*}\right) + v(n-n^*). \tag{7}$$

3.1.3 Selective-search phase

In the selective-search phase, all the L's remain in the market and only the number of H's decreases steadily:

$$\dot{n} = -\gamma n_h n = \dot{n}_h \tag{8}$$

$$\dot{n}_l = 0$$

$$V(n_h(t)) = -c\Delta t + \gamma n_h \Delta t h + (1 - \gamma n_h \Delta t) V(n_h(t + \Delta t)).$$

Hence

$$\dot{n} \frac{d\, V(n_h)}{d n_h} = c - \gamma n_h h + \gamma n_h V(n_h). \tag{9}$$

The time-paths of variables are as follows:

$$n(t) = \frac{n_{l0}}{\frac{n_0}{n_{h0}} e^{\gamma t n_{l0}} - 1} + n_{l0}$$

$$n_h(t) = \frac{n_{l0}}{\frac{n_0}{n_{h0}} e^{\gamma t n_{l0}} - 1}$$

$$n_l(t) = n_{l0}.$$

There exists $\bar{n} \equiv \bar{n}_h + n_{l0}$ at which, for the searcher who finds an L, further search is not profitable. This $\bar{n}$ is the solution to (7) when $V^s(n)$ is equal to l

$$\bar{n} l = -\frac{c}{\gamma} \ln\left(\frac{\bar{n}}{n^*}\right) + v(\bar{n} - n^*). \tag{10}$$

It can be checked that the solution for (10) is unique. With $vn^* = \frac{c}{\gamma}$ from (1), by solving (10),

$$\gamma \bar{n}_h (h - l) = c\left[1 + \ln\left(\frac{\bar{n}}{n^*}\right)\right] > c. \tag{11}$$

This condition is important because a searcher who could recall L (if holding were possible) would continue searching until the gain in surplus from a good match is equal to the search cost.

3.1.4 Net surplus

Now let us examine the net surplus to be realized after all search. The total surplus realized is equal to the original potential surplus minus the value of the remaining surplus

$$n_{h0}h + n_{l0}l - \frac{c}{\gamma}.$$

The total search cost is

$$c\,[\int_0^{\bar{t}_1} n(t)dt + \int_{\bar{t}_1}^{t^*_1} n(t)].$$

By using (4) and (8), with the change of variables, the total cost is

$$-\frac{c}{\gamma}\,[\int_{n_{h0}}^{\bar{n}_h} \frac{1}{n_h}\,dn_h + \int_{\bar{n}}^{n^*} \frac{1}{n}\,dn] = \frac{c}{\gamma}\ln\left(\frac{n_{h0}\bar{n}}{\bar{n}_h n^*}\right).$$

Hence the net surplus is

$$n_{h0}h + n_{l0}l - \frac{c}{\gamma} - \frac{c}{\gamma}\ln\left(\frac{n_{h0}\bar{n}}{\bar{n}_h n^*}\right).$$

It can be checked that (12) is increasing in $\bar{n}$ and the value of n^* maximizing (12) satisfies (1). Hence, socially optimal search requires no selective-search (Proposition 1-(2)). This result makes sense because, to maximize the net surplus, we need to maximize the value of transactions per unit of time. This outcome is achieved when every searcher accepts the first D he meets.

3.2 Market with a holding option

In this section, we examine the market with a holding option. Remember that in this market, the searcher can put a D on hold when he finds it.

Now in the selective-search phase, a searcher who finds an L puts the L on hold. If the searcher has not found an H by the end of the selective-search phase, he may recall L without a cost.

Let $\bar{t}_2$ be the time when non-selective search starts. Let t^*_2 be the time when all search ceases.

3.2.1 No search phase

All search stops when

$$\gamma N_h^* h + \gamma N_l^* l = c \tag{13}$$

3.2.2 Non-selective search phase

In this phase, only S 's search and there is no R. Hence, the time-paths of N_h, N_l, N, and $V^S(N_h, N_l)$ have the same forms (with different initial values) as those in the non-selective search phase in the market without a holding option.

3.2.3 Selective-search phase.

In the selective-search phase, both R and S search. Any H found will be accepted. Any L found will be put on hold and temporarily, disappears from the market; some of the L's on hold will return to the market after some amount of time.

The instantaneous change rates of H, L, R and S are as follows:

$$\dot{N}_h = -\gamma N_h (N + N_r) \tag{14}$$

$$\dot{N}_l = -\gamma N_l N + \gamma N_h N_r = -\dot{N}_r \tag{15}$$

$$\dot{N} = -\gamma N^2 \tag{16}$$

$$N = N_l + N_h, \quad N_r = n_{l0} - N_l$$

$$\dot{N}_h \frac{d\,V^R(N_h)}{dN_h} = c - \gamma N_h h + \gamma N_h\, V^R(N_h)$$

$$\dot{N}_h \frac{d\,V^S(N_h, N_l)}{dN_h} + \dot{N}_l \frac{d\,V^S(N_h, N_l)}{d\,N_l}$$

$$= c - \gamma N_h h - \gamma N_l V^R(N_h) + \gamma (N_h + N_l) V^S(N_h, N_l).$$

Because N_h declines steadily, it ultimately reaches $\overline{N}_h$ at which, for R, the gain in surplus from one more search for an H is zero. At this point, the search cost equals R's expected gross gain from search:

$$\gamma \overline{N}_h (h-l) = c \tag{17}$$

Note that, from (11) and (17), $\overline{N}_h < \overline{n}_h$(Proposition 1-(1)).
The time-paths are as follows:

$$N(t) = \frac{n_0}{\gamma t n_0 + 1}$$

$$N_h(t) = \frac{n_{l0}}{\frac{n_0}{n_{h0}} e^{\gamma t n_{l0}} - 1}$$

$$N_l(t) = N(t) - N_h(t)$$

$$N_r(t) = n_{l0} - N_l(t)$$

3.2.4 Net-surplus.

Now consider the net surplus realized after all search. All search stops when the value of search is equal to c. So the total surplus realized after all search is the same as in the market without a holding option. Hence, it suffices to examine the total cost. The total cost is

$$c\,[\int_0^{\overline{t}_2} (N(t) + N_r(t))dt + \int_{\overline{t}_2}^{t^*_2} N(t)dt].$$

With (14), (15) and the change of variables, the total cost becomes

$$-\frac{c}{\gamma}\,[\int_{n_{h0}}^{\overline{N}_h} \frac{1}{N_h}\, dN_h + \int_{\overline{N}}^{N^*} \frac{1}{N}\, dN] = \frac{c}{\gamma} \ln\left(\frac{n_{h0}\overline{N}}{\overline{N}_h N^*} \right).$$

Hence the net surplus is

$$n_{h0} h + n_{l0} l - \frac{c}{\gamma} - \frac{c}{\gamma} \ln\left(\frac{n_{h0}\,\overline{N}}{\overline{N}_h N^*} \right). \tag{18}$$

Again, it can be confirmed that socially optimal search behavior requires no selective-search: if there is no selective-search, every searcher incurs search cost only until he is matched. But if there is an interval for selective-search, the searchers with a bad match continue to search and increase the total search cost[2] (Proposition 1-(2)).

3.3 Comparison: the effects of a holding option on market efficiency[3]

In this section, the following will be shown:
(i) When population is small, the market without a holding option is more efficient than the market with a holding option.
(ii) When population is large, the market with a holding option can be more efficient than the market without a holding option.

Since the holding option encourages individuals to be more selective, it is possible that the selective-search phase exists when holding is possible but not when no holding is possible:
from (11) and (17), $\overline{n}_h > \overline{N}_h$. Since, at the end of the selective-search phase, all the L's remain when holding is not possible while some of L's are taken when holding is possible, $\overline{n} > \overline{N}$. Hence, if, $n_0 \in A \equiv \{n_0 \in (\overline{N}, \overline{n}) \mid \overline{N}_h < n_{h0} < \overline{n}_h\}$, then the above claim is true.

In this case, the holding option decreases the net surplus because the socially maximized surplus is realized when holding is not possible but not when holding is possible.

Now let us consider the situation where there exists the selective-search phase both in the market with and without a holding option. First, let us derive a condition under which the holding option increases the net surplus. Comparing (12) and (18), the holding option increases the surplus if and only if $\dfrac{n_{h0}\overline{n}}{\overline{n}_h n^*} > \dfrac{n_{h0}\overline{N}}{\overline{N}_h N^*}$.

Note that

$$\frac{\overline{n}}{\overline{n}_h n^*} = \frac{\overline{n}}{\overline{n}_h} \frac{v}{\frac{c}{\gamma}} = \frac{\overline{n}_h h + \overline{n}_l\, l}{\overline{n}_h} \frac{1}{\frac{c}{\gamma}} \text{ (by using } vn^* = \frac{c}{\gamma}\text{)}$$

$$\frac{\overline{N}}{\overline{N}_h N^*} = \frac{\overline{N}_h h + \overline{N}_l l}{\overline{N}_h} \frac{1}{\frac{c}{\gamma}}.$$

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[2] By the same reasoning, no selective search is socially optimal when there are more than two valuations for a match.

[3] Proposition 1 and the results in Section 3.3 hold even when there are more than two valuations for matches. The main reason is that (i) as long as a fixed fraction of population is good, socially optimal search behavior requires no selective search; and (ii) the holding option has two opposite effects on total search cost.

The condition is

$$\frac{n_{l0}}{\bar{n}_h} > \frac{\bar{N}_l}{\bar{N}_h}\ (\bar{n}_l = n_{l0}); \tag{19}$$

i.e., at the end of the selective-search phase, there are relatively more L's remaining in the market when no holding is possible than when holding is possible.

This condition is intuitive: since putting an L on hold does not influence the search behavior of others during the selective-search phase, the inefficiency of the market with a holding option arises from the lengthened selective-search phase. But the holding option saves search cost (i.e., the searcher with an L on hold can costlessly go back to the L). Lower $\bar{N}_l$ implies that more searchers have taken advantage of the holding option, thus increasing the efficiency of the market with a holding option.

The condition (19) also implies that, between the markets with and without a holding option, the net surplus is greater in the market where there are relatively fewer L's remaining at the end of search because (i) during the non-selective search phase, $\frac{n_l}{n_h}$ and $\frac{N_l}{N_h}$ do not change; and (ii) whether or not holding is possible, search ends at the same value of match.

Let us find a case where a holding option increases market efficiency[4] The holding option affects market efficiency only through the total search cost. Hence, we need to find a case where the savings of search cost, gained from the ability to costlessly go back to the bad matches, more than compensates the added search cost when holding is possible. We will show that this can happen, under some conditions, when population is large:

(i) Let us consider $\frac{\bar{N}_l}{\bar{N}_h}$ first.

From the time path of $N_h(t)$

$$\bar{t}_2 = \frac{1}{\gamma n_{l0}} \ln\left(\frac{n_{h0}(n_{l0}+\bar{N}_h)}{n_0 \bar{N}_h}\right).$$

Hence at the time $\bar{t}_2$, $N \equiv N(\bar{t}_2)$ is

$$N = \frac{1}{\frac{1}{n_{l0}} \ln \frac{n_{h0}(n_{l0}+N_h)}{n_0} + \frac{1}{n_0}}.$$

[4] I thank Professor Joel Sobel for providing this example.

Let us pick $\frac{c}{\gamma}$ so that $\frac{\overline{N}_h}{n_{l0}}=1$. Then,

$$\frac{\overline{N}}{\overline{N}_h} = 1 + \frac{\overline{N}_l}{\overline{N}_h} = \frac{1}{\ln 2+\ln\left(\frac{n_{h0}}{n_{0}}\right)+\left(1-\frac{n_{h0}}{n_{l0}}\right)} \tag{20}$$

Note that (20) is decreasing (hence, $\frac{\overline{N}_h}{\overline{N}_l}$ is increasing) as n_{h0} increases. By letting n_{h0} go to infinity while keeping n_{l0} constant,

$$\frac{\overline{N}_h}{\overline{N}_l} = \frac{\ln 2}{1-\ln 2}.$$

(ii) Now, consider $\frac{\overline{n}_l}{\overline{n}_h}$.

From (10) and (17),

$$\frac{\overline{n}_h}{\overline{N}_h} = 1+\ln\left(\frac{\overline{n}}{n^*}\right)$$

$$= 1+\ln\left[\frac{\overline{n}_h}{\overline{N}_h}\frac{1}{h-1}\left(h+\frac{\overline{N}_h}{\overline{n}_h}l\right)\right] \quad (\because \gamma\,\overline{N}_h(h-l)=c \text{ and } \overline{N}_h=n_{l0})$$

$$= 1+\ln\left(\frac{h+\frac{\overline{N}_h}{\overline{n}_h}l}{h-l}\right)+\ln\left(\frac{\overline{n}_h}{\overline{N}_h}\right)$$

$$< 1+\ln\left(\frac{h+l}{h-l}\right)+\ln\left(\frac{\overline{n}_h}{\overline{N}_h}\right) \qquad (\because n_h > N_h).$$

Hence, when, $\frac{n_{l0}}{\overline{N}_h}=1$,

$$\frac{\overline{n}_h}{n_{l0}} - \ln\left(\frac{\overline{n}_h}{n_{l0}}\right) < 1+\ln\left(\frac{h+l}{h-l}\right).$$

(iii) Thus, from (i) and (ii), when $n_{l0} = \overline{N}_h$ and n_{h0} is sufficiently large, if,

$$1+\ln\left(\frac{h+l}{h-l}\right)<\frac{\ln 2}{1-\ln 2},$$

then the holding option increases the net surplus. The above condition is satisfied when h is sufficiently large relative to l (hence, $\overline{N}_h=n_{l0}$ must be small).

The intuition behind this result is as follows:

(a) Since h-l is large, and n_{h0} is large while n_{l0} is small, meeting an L has negligible effect on the searcher's expected payoff from search. Hence, the process mainly consists of the selective-search phase.

(b) Since the population is large with n_{l0} being small, most of L's are put on hold when holding is possible.

(c) From (a) and (b), the savings of search cost induced by the holding option more than compensates the added search cost when holding is possible[5].

3.4 Existence of a holding option: holding option with deadlines

In Sections 3.1 through 3.3, the availability of a holding option is exogenously fixed and when holding is possible, a searcher can put a bad match on hold as long as he wants to.

Since more search increases the probability that a searcher will find a good match, an L has an incentive to reduce the current partner's ability to contact other potential partners. Or the L may not have an incentive to offer a holding option if doing so would cause the current partner to decide to make a transaction with her. Moreover, making the selective-search phase shorter while allowing searchers to take advantage of the holding option could increase the surplus in the market with a holding option (because shorter selective-search phase reduces the total search cost). These arguments suggest that (a) a holding option with a deadline could be an equilibrium behavior when D's can decide whether or not to offer a holding option; and (b) it also could increase net surplus.

In this section, we examine whether the existence of a holding option with or without deadlines can be an equilibrium.

Suppose that, at the beginning of the market (time zero), each L decides whether or not to offer a holding option and if so, the deadline of the option (since H's transact whenever matched, only L's need consider offering a holding option). We are going to consider only the equilibrium under which every L (R) takes the same action. Let t_D represent the deadline of the holding option.

Remember that $\overline{t}_1$ $(\overline{t}_2)$ is the time when the selective-search phase in the mar-

[5] Proposition 1 and the results in Section 3.3 hold, even when there exists a holding cost. The reason is that the searcher's incentive after paying the holding cost is the same as that in the market with no holding cost.

ket without (with) a holding option ends; i.e., the time when $n_h(\overline{t}_1)=\overline{n}_h(N_h(\overline{t}_2)=\overline{N}_h)$. We assume that $n_0 > \overline{n}$[6].

Lemma 1

There is no equilibrium without a holding option.

Proof
Suppose that (i) no L other than one L (let's call her L_d) allows a holding option; and (ii) the deadline of L_d's holding option is $\overline{t}_1$. Then L_d has more chance to get transaction until time $\overline{t}_1$ than other L's. Hence, L_d's deviation is profitable. Q.E.D.

Hence a holding option must exist in an equilibrium. The following lemma suggests the range of deadlines we will examine.

Lemma 2

(1) If, $t_D < \overline{t}_1$ then every R's rejection of the L's on hold at time t_D can be an equilibrium.
(2) When the deadline is $t_D \geq \overline{t}_1$, it cannot be an equilibrium for every R to reject L at time t_D.

Proof
(1) After the rejection of L's, the situation is the same as in the selective-search phase in the market without a holding option. Hence, R's behavior is optimal.
(2) Consider a deviation by one R (let us call him R_d). His deviation is the acceptance of L when all the other R's reject L at the time of t_D. Since one individual's behavior has negligible effect on the market behavior, $N_h(t_D)\leq \overline{n}_h$ and $N_l(t_D)=n_{l0}$ at time t_D. Then the non-selective search starts at time t_D. And the time-paths of the numbers of remaining H's and L's have the same forms (with different initial values) as those in the non-selective search phase in the market without a holding option. Computation shows the following: given t such that $t-t_D=t-t_1$(i.e., after the same amount of time has passed since the beginning of

[6] When, $\overline{N} < n_0 < \overline{n}$, it can be an equilibrium that no L offers a holding option: first, note that if there is no holding option, all search is non-selective search. Hence, by offering a holding option, L loses some chance of transaction.

the non-selective search phases), $n>N$, $n_h>N_h$, and $n_l<N_l$. So, given time t such that $t-t_D=t-\bar{t}_1$, (a) the value of match ($N_h h+N_l l$ or $\frac{N_h h+N_l l}{N}$) from one more search is less than that in the market without a holding option; (b) the probability of no match from one more search is greater than that in the market without a holding option; and (c) the searcher searches less than does the searcher in the market without a holding option. Hence, the expected payoff of the searcher (who just rejected an L) is less than l. Thus, R_d's deviation is profitable. Q.E.D.

From Lemma 2, if an equilibrium exists, the only time it is different from that in the market without a holding option is when the deadline is greater than $\bar{t}_1$.

Proposition 2

(1) For every $t_D \geq \bar{t}_2$, there is an equilibrium in which every L offers a holding option with the deadline t_D.

(2) There is a deadline $t_D<\bar{t}_2$ with which the existence of a holding option is an equilibrium.

Proof

(1) Since $t_D \geq \bar{t}_2$, the holding option with this deadline has the same effect as the holding option without a deadline. Suppose that only one L (let us call her L_d) does not allow the holding option. Because one individual's behavior has negligible effect on market behavior, L_d's behavior is not profitable: (a) during the selective-search phase, no transaction occurs with any L including L_d; (b) L_d loses the chance to make a transaction with some searcher at the end of the selective-search phase. Hence, L_d's behavior is not profitable. The same reasoning shows that one L's setting a deadline less than $\bar{t}_2$ is not profitable.

(2) Now let us find one equilibrium such that $t_D<\bar{t}_2$. At time t_D, on the assumption that all R's accept the L's on hold, we can calculate the S's expected payoff from the continuation of non-selective search. And if the calculated expected payoff is less than l, then all the individuals' behavior (including L's) described in the behavioral assumption is optimal:

(a) Note that the S's expected payoff from the continuation of non-selective search at time zero is greater than l.

(b) At time $t_D=\bar{t}_2$, the S's expected payoff is less than l (because all L's on hold are accepted at time $\bar{t}_2$).

(c) From the time path of N_l and $N_h(t)$ during the selective-search phase,

$$\frac{\dot{N}}{N} > \frac{\dot{N}_h}{N_h}. \text{ So } \frac{d\left(\frac{N_h(t)}{N}\right)}{dt} = \frac{\dot{N}_h}{N} - \frac{N_h \dot{N}}{N^2} = \frac{N_h}{N}\left(\frac{\dot{N}_h}{N_h} - \frac{\dot{N}}{N}\right) < 0.$$

Hence, during the selective-search phase, $\frac{N_h(t)}{N(t)}$ is decreasing over time and hence $\frac{N_l(t)}{N(t)}$ is increasing over time. So, as t_D increases, the value of match, $\frac{N_h(t_D)h + N_l(t_D)l}{N(t_D)}$, is decreasing. Moreover, $N(t_D)$ is decreasing as t_D increases. Therefore, the S's expected payoff from continuing non-selective search is decreasing as t_D increases. Thus, between $\bar{t}_1$ and $\bar{t}_2$, there must exist a t_D with which the existence of a holding option is an equilibrium. Q.E.D.

Let one such deadline as in Proposition 2-(2) be t^*_D. Since this deadline is less than $\bar{t}_2$ the surplus under the holding option with t^*_D is greater than that under the holding option without a deadline (the selective-search phase is shortened).

IV. HETEROGENEOUS PREFERENCES[7]

In the heterogeneous preference case, two individuals form a good match with probability p, and a bad match with probability $1-p$. So the bad match for one individual can be a good match for other individuals. In this model, the total number of the individuals remaining in the market describes the state of the market. Since every individual has the same probability of making a good match from search, it is natural to treat the two sides of the market symmetrically. We will consider a model with two-sided search and a holding option such that when a match turns out to be bad, (a) both parties continue searching for a good match, but (b) the current bad match is transacted if both parties decide to carry out the transaction and if both of them are available. Since we assume a continuous time Poisson matching process, we can ignore the possibility that two partners who are searching will simultaneously find new partners.

A searcher's expected payoff from search increases with the number of searchers in the market, because an additional searcher raises the probability of a good match as well as the meeting probability of all his potential partners. Hence, a searcher creates a positive externality for other searchers. By this positive externality, as shown below, the socially optimal interval of selective search is longer than the equilibrium interval whether or not holding is possible:

Proposition 3

Whether or not holding is possible, the equilibrium interval of the selective search

[7] The formal proofs for the assertions in this section can be shown upon request.

is shorter than the socially optimal interval of the selective search.

Furthermore, we can show that (I) a holding option increases the net surplus when population is small, but (ii) a holding option can decrease the net surplus when the population is large, the difference of the values between the good match and the bad match is large, and the probability of a good match is small.

We also consider a game where each individual decides whether or not to offer a holding option with a transaction time i.e. at the offered transaction time, the match on hold will be transacted if both parties are available.

Proposition 4
The existence of a holding option is the only symmetric-pure strategy equilibrium.

Proposition 4 is related to the unraveling of transaction time in Roth and Xing (1994). In the markets examined there, all markets close early due to the individuals' competition to acquire potentially good partners. In the market in this paper, the individuals offer (accept) a holding option because it increases the probability of making a bad transaction when the bad transaction is acceptable. However, as shown in Proposition 3, this option causes the selective search ends too early.

V. CONCLUSION

We have examined how holding options affect search behavior and market efficiency:

(i) When preferences are homogeneous, a holding option tends to decrease market efficiency when the population is small, and to increase it when the population is large.

(ii) When preferences are heterogeneous, a holding option tends to increase market efficiency when population is small, and to decrease it when the population is large.

(iii) In a game where individuals can decide whether or not to offer a holding option, only the existence of a holding option is an equilibrium when population is large.

We do not wish to push the implications of our simple model too far. They show the basic features of the holding options, and the individuals' incentive to use the holding options that can lead to improved or worsened market performance. Moreover, they indicate that, in modeling a non-steady state search and matching situation, the assumption of no recall can lead to the wrong conclusion about market efficiency.

The holding option examined in this paper is specific; i.e., we ruled out the possibility that one individual can offer (have) a holding option to (from) many potential transaction partners. In the homogeneous preference case, the party placed on hold in a match has an incentive to offer a holding option to many searchers to reduce the possibility of being abandoned by the partners for whom she is on hold. The searchers have an incentive to put many potential transaction partners on hold. In this formulation, the interval of the selective search shrinks because, the bad matches placed on hold are no longer secure. This effect can increase the efficiency of the market with a holding option (compared to the market in Section 3.2). In the heterogeneous preference case, individuals have an incentive to offer a holding option to many searchers to secure their potential fallback partner. This overlapping holding option increases the probability of a bad transaction when a bad transaction is acceptable, which lengthens the selective-search phase. Hence, the efficiency of the market with a holding option will increase (compared to the market in Section IV).

This paper ignores the possibility of bargaining for the division of surplus. One difficulty in modeling bargaining in our paper is that the situation examined is in non-steady state. In a steady-state equilibrium, the inflow and outflow of individuals in the market are equal, so that the distribution of potential matches does not change. Hence the individuals' outside option does not change over time. In this case, we can reasonably adopt a strategy that describes the same bargaining tactics against the partners of the same quality regardless of time. (For example, see Wolinsky(1987).) However, in non-steady state analysis, the distribution of potential matches changes over time, causing the individuals' size of outside option to change over time. So probably we need to consider different bargaining tactics over time, even against the partner of the same quality. There could exist a selective-search phase: if the searcher's share of the surplus is too large in a bad match at the early phase of the market, the bad matches may not be transacted. However, to derive the precise results, further research on the bargaining with a changing outside option and on the non-steady state matching process is required.

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