

Patent Thicket, Secrecy, and Licensing

Illoong Kwon*

This paper considers a patent portfolio race where firms compete for complementary patents, called a patent thicket. When firms have an option to keep their innovation secret, this paper shows that there exists an equilibrium where firms' patent propensity is strictly between zero and one. In such an equilibrium, stronger patent protection reduces the firms' investment in innovation. Moreover, this result does not change even when a licensing contract is feasible.

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I. Introduction

Patent protection is generally regarded as the cornerstone of innovation, and has been greatly strengthened in the last few decades.¹ However, as new products increasingly depend on more complex and complementary technologies, there exist growing concerns that stronger patent protection may allow a single patent holder to prevent other firms from commercializing all their new products that rely on that patent, and discourage innovation as a consequence, called the *hold-up problem* (e.g., Hall and Ziedonis 2001, Parchomovsky and Wagner 2005, Shapiro 2001).² Consequently, there is a growing voice that the scope of patent protection should

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*Graduate School of Public Administration, Seoul National University, 599 Gwanak-ro, Gwanak-gu, Seoul, Korea, 151-742. Phone: 02-880-8551. Fax: 02-882-3998. E-mail: ilkwon@snu.ac.kr. I wish to thank two anonymous referees for their helpful comments and suggestions. Any remaining errors are mine.

¹ See, for example, Levin et al. (1987) and Merz and Pace (1994).

² For example, in 2004, NTP sued RIM, the maker of the popular BlackBerry hand-held wireless email device, for infringing on its patent on wireless communication technology. This case almost halted the entire BlackBerry service, even though RIM owned most other patents in hardware and software.

decrease.³

Note, however, that such hold-up problems can be solved or significantly diminished through licensing contracts, because the owners of complementary patents stand to lose their profits under the hold-up situation. Then, one can argue that with licensing contracts, stronger patent protection should encourage innovation even when firms compete for complementary patents, called a patent thicket. This paper shows that such an argument may not hold if firms do not always patent their innovation.

Intuitively, a patent offers two types of exclusivity. First, a patent prohibits other firms that have not succeeded in the same innovation from simply imitating the innovation, called defensive exclusion. Second, a patent prevents other firms that have also succeeded in the same innovation from using their own innovation, called offensive exclusion (Kwon 2011). Offensive exclusion reduces other firms' expected profits and generates negative externality, especially when firms compete for complementary patents. Then, firms tend to apply for patents with too high probabilities, and consequently reduce each other's profits. Therefore, if patent protection gets stronger and firms apply for patents with even higher probabilities, firms expect less profits and invest less.

Recent empirical research shows that firms rely more often on *secrecy* than on patents to appropriate the returns of their innovations (e.g., Levin et al. 1987, Cohen et al. 2000). Thus, the patent propensity (i.e., the ratio of innovations for which a patent application is made) is very small in many industries. For example, Arundel and Kabla (1998) and Cohen et al. (2000) found that the patent propensity among large European and US companies is, on average, less than 35%.

This paper shows that when firms compete for complementary patents with possible licensing contracts, if patent propensity is less than one, stronger patent protection can still discourage innovations. This result extends Kwon (2011) that shows similar results in a single patent race. Note that the extension is not trivial. The reason is that when firms compete for multiple complementary innovations, called a patent portfolio race, the equilibrium patent propensity can depend on the number of research projects in which a firm succeeded. Therefore, an equilibrium strategy can be quite complicated to analyze. This paper shows that, without a licensing contract, a simple symmetric equilibrium exists, and firms' patent propensity does not depend on the number of successful research projects. Moreover, Kwon (2011) does not consider the complementary innovations, and consequently, does not analyze the effect of a licensing contract. This paper shows that with a licensing contract, the simple equilibrium described above does not exist anymore.

A growing number of studies have emphasized the negative effect of the hold-up

³ See, for example, "The Patent War: Is it killing innovation?" at <http://www.extremetech.com/computing/101939-the-patent-war-is-it-killing-innovation>.

problem when firms compete for a portfolio of complementary patents, called a patent thicket (e.g., Bessen 2004, Hall and Ziedonis 2001, Shapiro 2001). However, most of these studies have assumed that firms will always apply for patents (i.e., patent propensity is one), and have not considered the option of secrecy. In other words, these studies do not reflect the stylized facts that the average patent propensity of firms in many countries is less than 35%.

Anton and Yao (2004), Horstmann et al. (1985), and Schneider (2008) study the strategic choice between patenting and secrecy, but focus on the signaling or information disclosure aspect of patents. Thus, they do not analyze the effect of stronger patent protection on research investment.

A series of studies on sequential or cumulative innovations have also shown the potentially negative effect of patent protection (e.g., Bessen and Maskin 2006, Scotchmer and Green 1990). Sequential innovations are important aspects of the innovation processes; meanwhile, in sequential innovations, new innovations build on and *replace* old ones. Therefore, in terms of creating value, new and old innovations are substitutes. In contrast, this paper considers complementary innovations where both innovations are necessary to create (or increase) the value.

The rest of the paper is organized as follows. Section 2 presents the basic model. In section 3, I first analyze the choice between a patent and secrecy given the successful innovation. Section 4 analyzes the firms' incentive for investment in innovation and the effect of stronger patent protection. I consider the effect of licensing contracts in section 5, and conclude in section 6.

II. The Model

For easier comparison, I follow the notations of Kwon (2011). Consider two symmetric firms (1 and 2) competing for two complementary research projects A and B. Once it succeeds in a project, each firm simultaneously decides whether to apply for a patent or to keep it secret, without observing whether the other firm has succeeded in a project or not. If both firms apply for a patent in the same project, each firm wins the patent with equal probability.

Each project can either succeed or fail. For firm i the probability of success in project k given the investment level x_{ik} is $p(x_{ik})$ where $i \in \{1, 2\}$ and $k \in \{A, B\}$. To ensure a symmetric interior solution, I assume that $p' > 0$, $p'' < 0$, $p'(0) = \infty$, $p'(\infty) = 0$, $p(0) = 0$, and $p'^2(x) + p(x)p''(x) < 0$ for all x . Note that in order to capture the inherent risk associated with research investment, I also assume that $p(x) < \frac{1}{2}$ for all x .

Note that the probability of success in a given project does not depend on the investment or the success of the other project or the other firm. That is, in contrast

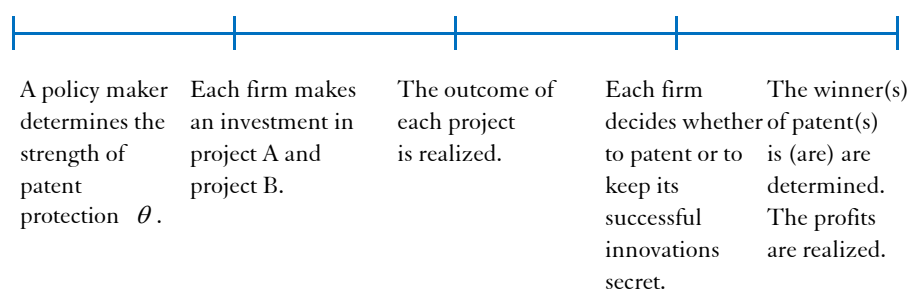
to Bessen and Maskin (2009), I rule out the spillover or the cumulative effects of innovation. Instead, I assume that the success of both projects A and B are essential to commercialize a new product. For example, project A may develop a new software, and project B may develop a new hardware. Then, the success or the investment in project A would not necessarily help the success of project B, and vice versa. For simplicity, assume that the success of a single project has no commercial value by itself, but that the combination of the successes in both projects can create a new product worth V , which is normalized to one.

The goal of this paper is to show the possible negative effect of patent protection. Therefore, following Bessen and Maskin (2009), I make the following conservative assumptions that should increase the value of patents. First, I assume that the firms can appropriate the full social value of the product. Thus, if only one firm can use the successful outcomes of both project A and B, it would earn the monopoly profit $V(=1)$. Thus, I rule out the monopoly inefficiency that would reduce the value of a patent. Second, if both firms can use the successful outcomes of both project A and B, each would earn the duopoly profit, $\frac{V}{2}(=\frac{1}{2})$. Even though the duopoly profit is typically less than $\frac{V}{2}$ due to the profit dissipation effect, I rule out such profit dissipation effect that would diminish the value of a patent.

Neither patent protection nor secrecy is perfect. Thus, the other firm can imitate and/or get around a patent with probability $1-\theta$, where θ represents the strength of patent protection. For example, a firm can imitate a patent of the other firm, and market an essentially identical product with a slightly different design. One can interpret θ as the probability that a court would prohibit the sales of such a product.⁴ Also, even if a firm keeps an innovation secret, the other firm can imitate the innovation with probability $1-\sigma$, where σ represents the strength of secrecy.

The timing of the game is summarized in Figure 1.

[Figure 1] Timing of the Model



⁴ For example, recent patent lawsuits by Apple have stopped the sale of Samsung's Galaxy Tab 10.1 in Germany and the Netherlands.

III. Patent and Secrecy

The goal of this paper is to show the possible negative effect of stronger patent protection on research investment. Thus, I will focus on the simplest possible symmetric equilibrium where such a negative effect can arise. Consequently, I cannot rule out the existence of other types of equilibria, especially the ones where the results of this paper do not hold. With this caveat, I will construct a *correlated* Nash equilibrium with the following properties:

“Firm j invests $x_{jA} = x_{jB} = x^*$ in projects A and B. If firm j succeeds in a project (or both projects), for each successful project, firm j applies for a patent with probability $\gamma_j = \gamma^*$ following a public signal.” ($j = 1, 2$)

I will characterize the equilibrium more specifically in proposition 3 below. For a symmetric equilibrium, it is sufficient to show that if firm j follows a strategy like above, firm $i (\neq j)$ has no incentive to deviate from the same strategy. I will show later that there exists no pure strategy symmetric Nash equilibrium in this game. Therefore, I will use the notion of correlated Nash equilibrium.

For the correlated equilibrium, players must coordinate their action following a random public signal. Such a public signal can include the results of the most recent patent litigations. Note that the scope of patent protection and the target of legal enforcement are uncertain. Thus, firms are likely to update their subjective belief on the strength of patent protection based on the recent outcome of patent litigation. For example, a patent holder's loss in litigation could provide a public signal to other firms not to apply for patents. Moreover, the outcome of patent litigation is random. In 2002 and 2004, only 24% of patent holders won in the patent litigation at the appellate level (Janicke and Ren 2006). Therefore, the outcome of recent patent litigations can potentially serve as a random public signal for a correlated Nash equilibrium.

To solve the model backwards, consider a firm's choice between patenting and secrecy after it succeeds in a project. Suppose that after succeeding in a project, firm i will either apply for a patent with probability γ_i or keep the innovation secret with probability $1 - \gamma_i$. That is, firm i 's patent propensity is γ_i . Note that for firm i , the patent propensity is the same for both project A and B, which I will show as a part of an equilibrium strategy. Therefore, for γ_i there is no need for a subscript for project A or B.

As noted above, I will focus on a symmetric equilibrium where $\gamma_i = \gamma_j = \gamma^*$ ($j \neq i$). The equilibrium patent propensity can depend on whether a firm has succeeded in one or both projects. But in this section, I will show that there exists an equilibrium where the equilibrium patent propensity does not depend on how

many projects a firm has succeeded in.

For now, assume that licensing contracts are not feasible. I will analyze the effect of licensing contracts in section 5.

To illustrate firm i 's choice between patenting and secrecy, suppose that firm j will always keep its innovations secret, regardless of its success in both projects or one project only. To find firm i 's best response, I need to consider two cases: (i) when firm i has succeeded in both projects, and (ii) when firm i has succeeded in one project only.

Case (i): In the case where firm i has succeeded in both projects, firm i has essentially three options. First, firm i can also keep its innovations secret. For simplicity, let us denote $p(x_{jk})$ by p_{jk} . Note that maximizing with respect to x_{jk} is equivalent to maximizing with respect to p_{jk} .

Then, firm i 's expected profit, denoted by V_{AB}^S , is as follows:

$$\begin{aligned} V_{AB}^S = & p_{jA}p_{jB} \frac{1}{2} + p_{jA}(1-p_{jB})[\sigma + (1-\sigma)\frac{1}{2}] + (1-p_{jA})p_{jB}[\sigma + (1-\sigma)\frac{1}{2}] \\ & + (1-p_{jA})(1-p_{jB})[(2\sigma - \sigma^2) + (1-\sigma)^2\frac{1}{2}]. \end{aligned} \quad (1)$$

That is, if firm j also succeeds in both innovations (with probability $p_{jA}p_{jB}$) and keeps them secret, both firms will make a duopoly profit of $\frac{1}{2}$. If firm j succeeds in project A only (with probability $p_{jA}(1-p_{jB})$), then firm i can guarantee itself the monopoly profit one as long as it can keep innovation B secret (with probability σ). The other cases are self-explanatory.

Second, firm i can apply for patents for both innovations. Then, its expected profit, denoted by V_{AB}^P , is as follows:

$$V_{AB}^P = (2\theta - \theta^2) + (1-\theta)^2 \frac{1}{2}. \quad (2)$$

Firm j is not applying for patents; thus, firm i will certainly win the patent for both innovations. Therefore, unless firm j invents around both patents [with probability $(1-\theta)^2$], firm i will make the monopoly profit one.⁵

Third, firm i can keep one innovation (e.g., project A) secret, and apply for a patent for the other innovation (e.g., project B). In this case, firm i 's expected profit, denoted by V_{AB}^{SP} , is:

⁵ In reality, these events would occur sequentially, and the choice of filing a lawsuit against the other firm is an interesting strategic choice to analyze. However, the focus of this paper is the effect on the initial research investment (x_{ik}). Thus, I consider the reduced form model for simplicity.

$$\begin{aligned}
V_{AB}^{SP} &= p_{jA}p_{jB}(\theta + (1-\theta)\frac{1}{2}) + p_{jA}(1-p_{jB})(\theta + (1-\theta)\frac{1}{2}) \\
&\quad + (1-p_{jA})p_{jB}((\theta + \sigma - \theta\sigma) + (1-\theta)(1-\sigma)\frac{1}{2}) \\
&\quad + (1-p_{jA})(1-p_{jB})((\theta + \sigma - \theta\sigma) + (1-\theta)(1-\sigma)\frac{1}{2}). \tag{3}
\end{aligned}$$

Firm j is keeping its innovations secret; thus, firm i will certainly win the patent for project B . Therefore, as long as firm i can prevent imitation of its patent (with probability θ), it can make the monopoly profit one. Note that if firm j succeeds in project B only, firm j can compete only if it can imitate both of firm i 's innovations (with probability $(1-\theta)(1-\sigma)$).

Assuming that $x_{jA} = x_{jB} = x_j$ or $p_{jA} = p_{jB} = p_j$ it is straightforward to show that if $\sigma \leq \frac{\theta}{1-p_j}$, then $V_{AB}^P \geq V_{AB}^{SP} \geq V_{AB}^S$, where equality holds if $\sigma = \frac{\theta}{1-p_j}$. Also, if $\sigma > \frac{\theta}{1-p_j}$, then $V_{AB}^P < V_{AB}^{SP} < V_{AB}^S$. In other words, if $\sigma \geq \frac{\theta}{1-p_j}$, then firm i will also keep both innovations secret.

Case (ii): In the case where firm i has succeeded in only one innovation, say A , if firm i files for a patent, its expected profit, denoted by V_A^P , is:

$$\begin{aligned}
V_A^P &= p_{jA}p_{jB}(\theta(1-\sigma) + \frac{1}{2}(1-\theta)(1-\sigma)) \\
&\quad + (1-p_{jA})p_{jB}(\theta(1-\sigma) + (1-\theta)(1-\sigma)\frac{1}{2}). \tag{4}
\end{aligned}$$

Recall that I am assuming that firm j will always keep its innovations secret. Therefore, firm i will certainly win the patent in this case.

If firm i keeps the innovation (A) secret, its expected profit, denoted by V_A^S , is:

$$V_A^S = p_{jA}p_{jB}(1-\sigma)\frac{1}{2} + (1-p_{jA})p_{jB}(\sigma(1-\sigma) + (1-\sigma)^2\frac{1}{2}). \tag{5}$$

Then, assuming $p_{jA} = p_{jB} = p_j$, it is straightforward to show that $V_A^S \geq V_A^P$ if and only if $\sigma \geq \frac{\theta}{1-p_j}$.

Therefore, from cases (i) and (ii), if firm j always keeps its innovation(s) secret and if $\sigma \geq \frac{\theta}{1-p_j}$, then firm i will also keep its innovations secret, regardless of whether firm i has succeeded in one or both projects. I can also show in the following proposition that if $\sigma < \frac{\theta}{1-p_j}$, both firms will always apply for patents.

Proposition 1 (i) Suppose that firm j always keeps its innovation secret. If $p_{jA} = p_{jB} = p_j$ and $\sigma > \frac{\theta}{1-p_j}$, then it is always optimal for firm i to keep its innovation secret. ($i, j = 1, 2, i \neq j$)

(ii) Suppose that firm j always applies for a patent(s). If $p_{jA} = p_{jB} = p_j$ and $\sigma < \frac{\theta}{1-p_j}$, then it is always optimal for firm i to apply for patents. ($i, j = 1, 2, i \neq j$)

(iii) Suppose that firm j either always keeps its innovation secret or always applies for a patent(s). If $p_{jA} = p_{jB} = p_j$ and $\sigma = \frac{\theta}{1-p_j}$, then firm i is indifferent between secrecy and patenting ($i, j = 1, 2, i \neq j$).

Proof. The proof for (i) follows from the discussion above. See Appendix for the proof of (ii) and (iii). ■

Not surprisingly, firms apply for patents if the relative strength of patent protection ($\frac{\theta}{\sigma}$) is large enough. Note that the probability of success by the other firm, p_j , also determines the firm i 's decision between patenting and secrecy. Intuitively, if the other firm j is likely to succeed and to apply for a patent, firm j can hold-up firm i 's innovation. Therefore, firm i is also likely to apply for a patent to prevent such a hold-up problem.

Note that firm i 's choice between patenting and secrecy entirely depends on firm j 's investment decision x_{jA} and x_{jB} (or p_{jA} and p_{jB}) and the institutional parameters σ and θ , but does not depend on its own investment level x_{iA} and x_{iB} because they are already sunk.

It is worth emphasizing that as long as $p_{jA} = p_{jB} = p_j$ firm i 's optimal choice between patenting and secrecy does not depend on the number of projects in which it succeeded. This result is important as it extends that of Kwon (2011) to multiple patent races, and greatly simplifies the analysis. This result is also consistent with the equilibrium strategy proposed above. However, as I will show later, this result does not hold if licensing contracts are feasible.

IV. Patent Protection and Innovation

For the full characterization of an equilibrium, first define

$$\underline{\sigma}(\theta) \equiv \frac{\theta}{1-p(x^P(\theta))} \quad \text{and} \quad \bar{\sigma}(\theta) \equiv \frac{\theta}{1-p(x^S(\bar{\sigma}))}, \quad (6)$$

where $x^P(\theta)$ is the symmetric equilibrium investment level, assuming that firms

always apply for patents.⁶ Likewise, $x^S(\sigma)$ is the symmetric equilibrium investment level assuming that firms always keep their innovations secret.⁷ Note that $\bar{\sigma}(\theta)$ is defined from an implicit function $\bar{\sigma} \equiv \frac{\theta}{1-p(x^S(\bar{\sigma}))}$.

Then, from proposition 1, one can conjecture that if $\sigma \leq \underline{\sigma}(\theta)$, there would exist a symmetric equilibrium where both firms apply for patents with probability one regardless of whether they have succeeded in one or both innovations. Similarly, if $\sigma \geq \bar{\sigma}(\theta)$ then there would exist a symmetric equilibrium where both firms keep their innovation(s) secret with probability one regardless of whether they have succeeded in one or both innovations. To prove this conjecture, I first establish the following results.

Proposition 2 (i) $x^P(\theta) < x^S(\underline{\sigma}(\theta))$.

(ii) $\bar{\sigma}(\theta)$ is unique.

(iii) $\underline{\sigma}(\theta) < \bar{\sigma}(\theta)$.

Proof. See Appendix. ■

Proposition 2 implies that when $\sigma = \underline{\sigma}(\theta)$, if σ increases just slightly (or if θ decreases slightly), ‘always keeping innovation(s) secret’ cannot be an equilibrium because σ would still be less than $\frac{\theta}{1-p(x^S(\sigma))}$. In other words, if $\underline{\sigma}(\theta) < \sigma < \bar{\sigma}(\theta)$, there exists no symmetric pure strategy Nash equilibrium. Instead, I show that there exists a *correlated Nash equilibrium*.

More specifically, if $\underline{\sigma}(\theta) < \sigma < \bar{\sigma}(\theta)$ suppose that with probability γ (with slight abuse of notation), *both* firms always apply for patent(s). And with probability $1-\gamma$, both firms always keep their innovation(s) secret. Let us denote the symmetric equilibrium investment level under this correlated strategy profile by $x^M(\gamma; \theta, \sigma)$. Note that γ must satisfy $0 < \gamma < 1$. Also, from Proposition 2(iii), if

$$\sigma = \frac{\theta}{1-p(x^M(\gamma; \theta, \sigma))}, \quad (7)$$

then each firm would be indifferent between patenting and secrecy. Thus, each firm would have no incentive to deviate from this strategy profile unilaterally.⁸

⁶ Note that $x^P(\theta)$ does not depend on σ because $x^P(\theta)$ is an equilibrium investment level conditional on that firms always apply for patents.

⁷ From the assumption $p'^2 + pp'' < 0$, it is straightforward to check that these symmetric equilibria exist.

⁸ In general, the indifference condition, such as (7), is not required for a correlated equilibrium. In other words, we are not imposing (7) as a necessary condition for all correlated equilibria. Instead, we are constructing a correlated equilibrium where the indifference condition (7) is satisfied when $0 < \gamma < 1$. More specifically, the indifference condition (7) is needed for the subgame described in

To show such a γ exists, each firm would maximize the following profit function:

$$\begin{aligned} \Pi_i^M(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \gamma, \theta, \sigma) = & \gamma \Pi_i^P(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \theta) \\ & + (1 - \gamma) \Pi_i^S(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \sigma), \end{aligned} \quad (8)$$

where $\Pi_i^P(x_{iA}, x_{iB}; \theta)$ and $\Pi_i^S(x_{iA}, x_{iB}; \sigma)$ are the expected profits when firms always apply for patents and when they always keep their innovation secret, respectively (specified in the proof of Proposition 2).

The first order condition for x_{jA} is

$$\begin{aligned} \frac{\partial \Pi_i^M(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \gamma, \theta, \sigma)}{\partial x_{iA}} = & \frac{\partial \Pi_i^S(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \sigma)}{\partial x_{iA}} \\ & + \gamma(\theta, \sigma) \left(\frac{\partial \Pi_i^P(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \theta)}{\partial x_{iA}} - \frac{\partial \Pi_i^S(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \sigma)}{\partial x_{iA}} \right) = 0. \end{aligned} \quad (9)$$

Given that $x_{jA} = x_{jB}$, the first order condition for x_{iB} is identical. Therefore, $x_{iA} = x_{iB} = x_i$ or $p_{iA} = p_{iB} = p_i$.

From (7) and the definition of $\underline{\sigma}$, if $\sigma > \underline{\sigma}$, then $x^M > x^P(\theta)$ and, at $x_{1A} = x_{1B} = x_{2A} = x_{2B} = x^M$, $\frac{\partial \Pi^P(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \theta)}{\partial x_{iA}} < 0$ from the second order condition for x^P . Also, from the proof of Proposition 2(ii) in the Appendix, if $\sigma < \bar{\sigma}$, then $\sigma < \frac{\theta}{1 - p(x^S(\sigma))}$. That is, if $\sigma < \bar{\sigma}$, then $x^M < x^S(\sigma)$ and, at $x_{1A} = x_{1B} = x_{2A} = x_{2B} = x^M$, $\frac{\partial \Pi^P(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \sigma)}{\partial x_{iA}} > 0$ from the second order condition for x^S . Therefore, if $\underline{\sigma} < \sigma < \bar{\sigma}$, there exists $0 < \gamma < 1$ such that x^M as defined by (7) satisfies the first order condition (9).

Then, I can characterize a symmetric equilibrium as follows:

Proposition 3 Given θ there exists a symmetric correlated equilibrium where

- (i) if $\sigma \leq \underline{\sigma}(\theta)$, then $x_{iA} = x_{iB} = x_{jA} = x_{jB} = x^P(\theta)$, and firms always apply for patent(s);
- (ii) if $\underline{\sigma}(\theta) < \sigma \leq \bar{\sigma}(\theta)$, then $x_{iA} = x_{iB} = x_{jA} = x_{jB} = x^M(\gamma; \theta, \sigma)$, and with probability γ , both firms always apply for patent(s), and with probability $1 - \gamma$, both firms always keep their innovation(s) secret, where γ is determined by (7); and
- (iii) if $\sigma > \bar{\sigma}(\theta)$, then $x_{iA} = x_{iB} = x_{jA} = x_{jB} = x^S(\sigma)$, and firms always keep their innovation(s) secret.

Proof. (i) Suppose that $\sigma \leq \underline{\sigma}$. Assume that firm j invests

Proposition 2 to be on the equilibrium path.

$x_{jA} = x_{jB} = x^P(\theta)$ and that firm j always apply for a patent(s). It is sufficient to show that firm i will follow the same strategy. Since $\sigma \leq \underline{\sigma}$ and $p(x_{jA}) = p(x_{jB}) = p(x^P(\theta))$, from (6), $\sigma \leq \frac{\theta}{1-p(x^P(\theta))}$. Then, from proposition 1, it is also always optimal for firm i to apply for patent(s). Since both firms always apply for patents, the first order condition for x_{iA} , (9), must be the same for x_{iB} , x_{jA} , and x_{jB} . Therefore, by the definition of $x^P(\theta)$, $x_{iA} = x_{iB} = x^P(\theta)$.

(ii) Suppose that $\underline{\sigma} < \sigma \leq \bar{\sigma}$. Assume that firm j invests $x_{jA} = x_{jB} = x^M(\gamma; \theta, \sigma)$ and that firm j applies for a patent(s) with probability γ following a public signal. Again, it is sufficient to show that firm i will follow the same strategy. Since $p(x_{jA}) = p(x_{jB}) = p(x^M(\gamma; \theta, \sigma))$, $\sigma = \frac{\theta}{1-p(x^M(\gamma; \theta, \sigma))}$ from (7). Then, from Proposition 1, firm i is indifferent between secrecy and patenting. Therefore, firm i is also willing to apply for a patent(s) with probability γ following the same public signal. Then, the first order condition for x_{iA} that is (9), must be the same for x_{iB} , x_{jA} , and x_{jB} . Therefore, $x_{iA} = x_{iB} = x_{jA} = x_{jB} = x^M(\gamma; \theta, \sigma)$.

(iii) Suppose that $\sigma > \bar{\sigma}$. Assume that firm j invests $x_{jA} = x_{jB} = x^S(\sigma)$ and that firm j always keep its innovation(s) secret. It is sufficient to show that firm i will follow the same strategy. Since $\sigma > \bar{\sigma}$ and $p(x_{jA}) = p(x_{jB}) = p(x^S(\sigma))$, from (6), it is straightforward to show that $\sigma < \frac{\theta}{1-p(x^S(\sigma))}$. Then, from proposition 1, it is also always optimal for firm i to keep its innovation(s) secret. Since both firms always apply for patents, the first order condition for x_{iA} , (9), must be the same for x_{iB} , x_{jA} , and x_{jB} . Therefore, by the definition of $x^S(\sigma)$, $x_{iA} = x_{iB} = x^S(\sigma)$. ■

Proposition 3 shows that an equilibrium exists where the average patent propensity ($= \gamma$) is positive, but strictly less than one. As emphasized in the beginning, the stylized facts show that firms do not always patent their innovation, and that the average patent propensity is less than 35% in many countries. Therefore, the equilibrium where γ is strictly between zero and one is of particular interest. For such an equilibrium, I can state the following results.

Proposition 4 *When the equilibrium patent propensity is strictly between zero and one, strengthening patent protection (θ) reduces the equilibrium investment level.*

Proof. From (7), given σ , if θ increases, x^M must decrease. ■

Intuitively, if $\underline{\sigma} < \sigma \leq \bar{\sigma}$, firms' *ex-ante* expected profits are greater if they keep their innovations secret than when both apply for patents. However, once firms succeed in their innovations, each firm has a strong incentive to apply for patents to exclude the other firm from the product market, called offensive exclusion. If patent protection becomes stronger, firms would apply for patents even with higher probability despite the fact that the *ex-ante* profit is smaller when both apply for

patents than when they can commit to secrecy. Therefore, when patent protection becomes stronger and firms become more likely to apply for patents, firms invest less ex-ante. Note that this intuition holds only if the patent propensity is strictly less than one. If the patent propensity is one, as assumed by most previous studies, stronger patent protection would not be able to increase the patent propensity anymore.

V. Licensing Contract

Now suppose that licensing contracts are feasible. A licensing contract would arise in the following three cases; (i) each firm has one patent only, (ii) each firm has succeeded in one (and different) project only, and both firms have kept the innovations secret, and (iii) each firm has succeeded in one (and different) project only, and one firm has kept the innovation secret, and the other firm applied for a patent.

Assuming Nash bargaining, in the first two cases, the payoff of each firm through the licensing contract would be $\frac{1}{2}$. In case (iii), the payoff of the firm with a patent would be $\frac{1}{2} + \frac{1}{2}(\theta - \sigma)$, and the payoff of the firm with secrecy would be $\frac{1}{2} + \frac{1}{2}(\sigma - \theta)$.⁹

Let us denote firm i 's probability of applying for a patent when it succeeds in one project only by γ_i . Also, denote firm i 's probability of applying for patents when it succeeds in both projects by ρ_i . Recall that without licensing, section 4 shows that there exists a symmetric equilibrium where γ_i is always equal to ρ_i . However, when licensing contracts are feasible, it is straightforward to show that such an equilibrium where γ_i is always equal to ρ_i does not exist.

To focus on a symmetric equilibrium, suppose that $p_{jA} = p_{jB} = p_j$. Then, once firm i succeeds in an innovation(s), firm i chooses between secrecy and patenting as follows:

Proposition 5 (i) Suppose that $\gamma_j = \rho_j = 1$. If $\sigma < \frac{\theta - \frac{1}{2}p_j\theta^2}{1 - p_j}$, then firm i will also choose $\gamma_i = 1$ and $\rho_i = 1$.

(ii) Suppose that $\gamma_j = 0$ and $\rho_j = 1$. If $\frac{\theta - \frac{1}{2}p_j\theta^2}{1 - p_j} < \sigma < \frac{\theta}{1 - p_j}$, then firm i will also choose $\gamma_i = 0$ and $\rho_i = 1$.

(iii) Suppose that $\gamma_j = \rho_j = 0$. If $\sigma > \frac{\theta}{1 - p_j}$, then firm i will also choose $\gamma_i = \rho_i = 0$.

Proof. See Appendix. ■

⁹ These payoffs assume that firms cannot switch between patenting and secrecy after observing the other firm's choice.

Note that unlike the case without a licensing contract, the patent propensity depends on the number of projects in which a firm succeeded. Assuming $\gamma_1 = \gamma_2 = \gamma$ and $\rho_1 = \rho_2 = \rho$, let us denote the optimal investment level of each firm for given γ , ρ , and σ by $x(\gamma, \rho; \sigma)$. Define $\underline{\sigma}^L$, $\bar{\sigma}^L$, $\underline{\sigma}^H$, and $\bar{\sigma}^H$ such that

$$\begin{aligned}\underline{\sigma}^L &= \frac{\theta - \frac{1}{2} p(x(1, 1; \underline{\sigma}^L)) \theta^2}{1 - p(x(1, 1; \underline{\sigma}^L))}, \quad \bar{\sigma}^L = \frac{\theta - \frac{1}{2} p(x(1, 0; \bar{\sigma}^L)) \theta^2}{1 - p(x(1, 0; \bar{\sigma}^L))} \\ \underline{\sigma}^H &= \frac{\theta}{1 - p(x(1, 0; \underline{\sigma}^H))}, \quad \bar{\sigma}^H = \frac{\theta}{1 - p(x(0, 0; \bar{\sigma}^H))}.\end{aligned}\tag{10}$$

Let us denote the average γ_i and ρ_i in a symmetric equilibrium by γ^* and ρ^* . Then, I can characterize a symmetric equilibrium follows:

Proposition 6 (i) If $\sigma \leq \underline{\sigma}^L$, then $\gamma^* = \rho^* = 1$.

(ii) If $\underline{\sigma}^L < \sigma \leq \bar{\sigma}^L$, then $\rho^* = 1$. With probability γ^* , $\gamma_1 = \gamma_2 = 1$, and with probability $1 - \gamma^*$, $\gamma_1 = \gamma_2 = 0$.

(iii) If $\bar{\sigma}^L < \sigma \leq \underline{\sigma}^H$, then $\gamma^* = 0$ and $\rho^* = 1$.

(iv) If $\underline{\sigma}^H < \sigma \leq \bar{\sigma}^H$, then $\gamma^* = 0$ With probability ρ^* , $\rho_1 = \rho_2 = 1$, and with probability $1 - \rho^*$, $\rho_1 = \rho_2 = 0$.

(v) If $\sigma > \bar{\sigma}^H$, then $\gamma^* = \rho^* = 0$.

Given σ , γ^* and ρ^* in (ii) and (iv) are determined by the following implicit functions, respectively:

$$\begin{aligned}\sigma &= \frac{\theta - \frac{1}{2} p(x(\gamma^*, 1; \sigma)) \theta^2}{1 - p(x(\gamma^*, 1; \sigma))}, \\ \sigma &= \frac{\theta}{1 - p(x(0, \rho^*; \sigma))}.\end{aligned}\tag{11}$$

Proof. For each γ^* and ρ^* , the proof is essentially the same as that in Proposition 5, and is omitted. ■

Then, I can state the effect of stronger patent protection on equilibrium investment as follows:

Proposition 7 If $0 < \gamma^* < 1$ or $0 < \rho^* < 1$ then stronger patent protection reduces the equilibrium research investment level.

Proof. From (11), if θ increases, x must decrease (by increasing γ^* or ρ^*) in

order to satisfy the equations. ■

An important implication of these results is that a licensing contract may not eliminate the potentially negative effect of patent protection on innovation. In contrast, previous literature has largely focused on the hold-up problem and suggested more efficient licensing contracts among competitors as a potential solution (e.g., Shapiro 2001). However, Propositions 4 and 7 show that when the patent propensity is less than one, the negative effect of patent protection on research investment arises from the option of secrecy, not necessarily from the hold-up problem. Thus, the negative effect can persist even with an efficient cross-licensing contract.

A caveat is, though, that these results are based on the Nash bargaining solution. Even though the Nash bargaining solution is one of the standard solutions for licensing contracts especially with two symmetric players, there are other solution concepts for bargaining. Thus, it would be interesting for future research to check whether different bargaining solution concepts can change the results of this paper.

VI. Conclusion

Traditional models of a patent race assume that firms compete for a single (or independent) innovation(s), and that firms always apply for a patent once they succeed in a project. These models show that stronger patent protection always increases research investment and encourages innovation, while it creates a monopoly in the product market. Therefore, the literature on the optimal patent policy has largely focused on this trade-off between encouraging innovation and creating monopoly (e.g., Gilbert and Shapiro 1990, Klemperer 1990, Nordhaus 1969).

However, as new products depend on increasingly more complex and complementary technologies and patents, firms now compete for a portfolio of complementary innovations, not just a single innovation. For example, in 2011 Apple won the design-related patent infringement case against Samsung in Germany, which resulted in the ban on Samsung Galaxy Tab 10.1 sales. In response, Samsung has filed a patent lawsuit against Apple in Korea. These lawsuits show that even a single patent infringement can result in the ban on the sale of an entire product. Thus, on the one hand, firms would try to build up their patent portfolio, or patent thicket, to defend their product. On the other hand, such potential patent lawsuits would eventually reduce the R&D investment, called the hold-up problem.

In order to reduce such a hold-up problem, one could suggest a licensing contract

or a settlement. For example, in another patent lawsuit in the United States between Samsung and Apple, a US judge sent them to settlement talks. However, this paper shows that when the patent propensity is small, stronger patent protection can reduce research investment, even when a licensing contract is available.

In other words, when firms do not patent their innovations, a policy maker can be tempted to increase the strength of patent protection to encourage patent applications. Precisely in such a case, however, this paper shows that strengthening patent protection may reduce R&D investments, especially if innovations are complementary. Therefore, in an industry with a low patent propensity and with complementary innovations, it can be optimal to reduce the strength of patent protection or reduce the patent holders' winning rate in patent litigations, compared with other industries with a high patent propensity and with independent innovations.

While these results may seem counter-intuitive, they are consistent with recent empirical findings that stronger patent protection has no significant effect on innovation, especially in industries where innovations are complementary or cumulative (e.g., Bessen and Hunt 2004, Kortum and Lerner 1999, Sakakibara and Branstetter 2001).

Note, however, that this paper focuses on a particular symmetric equilibrium. Therefore, as emphasized above, the results of this paper may not hold in other types of equilibria, especially asymmetric ones. The full characterization of the equilibria remains as a topic for future research.

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Appendix

Proof of Proposition 1 (ii) and (iii) If firm i wins both patents, its expected profit, denoted by v_2 is

$$v_2 = (2\theta - \theta^2) + (1 - \theta)^2 \frac{1}{2}. \quad (\text{A.1})$$

That is, as long as the other firm does not invent around both patents (with probability $1 - (1 - \theta)^2 = 2\theta - \theta^2$), firm i is the only firm that can use both innovations, and will make the monopoly profit $V (= 1)$. However, if the other firm invents around both patents with probability $(1 - \theta)^2$ the two firms will compete in the product market, and each firm will make the duopoly profit $\frac{V}{2} (= \frac{1}{2})$.

If firm i holds one patent and firm j holds the other, firm i 's expected profit, denoted by v_1 is

$$v_1 = \theta(1 - \theta) + (1 - \theta)^2 \frac{1}{2}. \quad (\text{A.2})$$

That is, if firm i can protect its own patent but invent around the other firm's patent (with probability $\theta(1 - \theta)$), it can make the monopoly profit one in the product market. However, if both firms invent around the other firm's patent (with probability $(1 - \theta)^2$), each firm will make the duopoly profit $\frac{1}{2}$.

If firm j holds both patents, firm i 's expected profit, denoted by v_0 is

$$v_0 = (1 - \theta)^2 \frac{1}{2}. \quad (\text{A.3})$$

That is, firm i can make a positive profit $\frac{1}{2}$ only if it can invent around both patents.

Suppose that firm j will always apply for patents for its innovation(s) regardless of whether it succeeds in one or both projects.

First, consider the case where firm i has succeeded in both projects. If firm i applies for patents for both innovations, its expected profit, denoted by V_{AB}^P , is as follows:

$$\begin{aligned} V_{AB}^P = & p_{jA}p_{jB} \left(\frac{1}{4}v_2 + \frac{1}{2}v_1 + \frac{1}{4}v_0 \right) + (1 - p_{jA})p_{jB} \left(\frac{1}{2}v_2 + \frac{1}{2}v_1 \right) \\ & + p_{jA}(1 - p_{jB}) \left(\frac{1}{2}v_2 + \frac{1}{2}v_1 \right) + (1 - p_{jA})(1 - p_{jB})v_2, \end{aligned} \quad (\text{A.4})$$

where v_2 , v_1 , and v_0 are defined as (A.1)-(A.3).

If firm i keeps both innovations secret, its expected profit, denoted by V_{AB}^S , is as follows:

$$\begin{aligned} V_{AB}^S &= p_{jA}p_{jB}(1-\theta)^2\frac{1}{2} + (1-p_{jA})p_{jB}(\sigma(1-\theta) + (1-\sigma)(1-\theta)\frac{1}{2}) \\ &\quad + p_{jA}(1-p_{jB})(\sigma(1-\theta) + (1-\sigma)(1-\theta)\frac{1}{2}) \\ &\quad + (1-p_{jA})(1-p_{jB})((2\sigma - \sigma^2) + (1-\sigma)^2\frac{1}{2}). \end{aligned} \quad (\text{A.5})$$

If firm i keeps one innovation (say A) secret, and applies for a patent for the other innovation (say B), its expected profit, denoted by V_{AB}^{SP} , is:

$$\begin{aligned} V_{AB}^{SP} &= p_{jA}p_{jB}(\frac{1}{2}(\theta(1-\theta) + (1-\theta)^2\frac{1}{2}) + \frac{1}{2}(1-\theta)^2\frac{1}{2}) \\ &\quad + p_{jA}(1-p_{jB})(\theta(1-\theta) + (1-\theta)^2\frac{1}{2}) \\ &\quad + (1-p_{jA})p_{jB}(\frac{1}{2}((\theta + \sigma - \theta\sigma) + (1-\theta)(1-\sigma)\frac{1}{2}) \\ &\quad + \frac{1}{2}((1-\theta)\sigma + (1-\theta)(1-\sigma)\frac{1}{2})) \\ &\quad + (1-p_{jA})(1-p_{jB})((\theta + \sigma - \theta\sigma) + (1-\theta)(1-\sigma)\frac{1}{2}). \end{aligned} \quad (\text{A.6})$$

Then, it is straightforward to show that if $\sigma \leq \frac{\theta}{1-p_j}$, then $V_{AB}^P \geq V_{AB}^{SP} \geq V_{AB}^S$ where the equality holds if $\sigma = \frac{\theta}{1-p_j}$. Thus, if $\sigma < \frac{\theta}{1-p_j}$, it is optimal for firm i to patent both innovations.

Second, now consider the case where firm i has succeeded in one innovation (say A) only. If firm i files for a patent, its expected profit, denoted by V_A^P , is:

$$\begin{aligned} V_A^P &= p_{jA}p_{jB}(\frac{1}{2}(\theta(1-\theta) + (1-\theta)^2\frac{1}{2}) + \frac{1}{2}(1-\theta)^2\frac{1}{2}) \\ &\quad + (1-p_{jA})p_{jB}(\theta(1-\theta) + (1-\theta)^2\frac{1}{2}). \end{aligned} \quad (\text{A.7})$$

If firm i keeps the innovation A secret, its expected profit, denoted by V_A^S , is:

$$V_A^S = p_{jA}p_{jB}(1-\theta)^2 \frac{1}{2} + (1-p_{jA})p_{jB}(\sigma(1-\theta) + (1-\sigma)(1-\theta)\frac{1}{2}). \quad (\text{A.8})$$

Again, it is straightforward to show that if $\sigma \leq \frac{\theta}{1-p_j}$, then $V_A^P \geq V_A^S$ where the equality holds if $\sigma = \frac{\theta}{1-p_j}$. Thus, if $\sigma < \frac{\theta}{1-p_j}$, it is optimal for firm i to patent the innovation.

Therefore, if firm j always apply for patent(s) and if $p_{jA} = p_{jB} = p_j$ and $\sigma < \frac{\theta}{1-p_j}$, then firm i will also always apply for patents.

Also, if $p_{jA} = p_{jB} = p_j$ and $\sigma = \frac{\theta}{1-p_j}$, then firm i will be indifferent between patenting and secrecy. ■

Proof of Proposition 2 For notations, suppose that both firm i and j will always apply for a patent. If firm i succeeds in project B only and applies for a patent, its expected profit, denoted by V_B^P , is

$$V_B^P = p_{jA}p_{jB}(\frac{1}{2}v_1 + \frac{1}{2}v_0) + p_{jA}(1-p_{jB})v_1. \quad (\text{A.9})$$

Finally, if firm i fails in both projects, its expected profit, denoted by V_0^P , is

$$V_0^P = p_{jA}p_{jB}v_0. \quad (\text{A.10})$$

Therefore, if both firm i and j always apply for a patent, firm i 's expected profit, denoted by $\Pi_i^P(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \theta)$, is

$$\begin{aligned} \Pi_i^P(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \theta) &= p_{iA}p_{iB}V_{AB}^P + p_{iA}(1-p_{iB})V_A^P \\ &\quad + (1-p_{iA})p_{iB}V_B^P + (1-p_{iA})(1-p_{iB})V_0^P - x_{iA} - x_{iB}, \end{aligned} \quad (\text{A.11})$$

where V_{AB}^P and V_A^P are defined by (A.6) and (A.7).

Now suppose that both firm i and j will always keep their innovations secret. If firm i succeeds in both projects, its expected profit, denoted by V_{AB}^S , is

$$\begin{aligned} V_{AB}^S &= p_{jA}p_{jB} \frac{1}{2} + p_{jA}(1-p_{jB})(\sigma + (1-\sigma)\frac{1}{2}) + (1-p_{jA})p_{jB}(\sigma + (1-\sigma)\frac{1}{2}) \\ &\quad + (1-p_{jA})(1-p_{jB})((2\sigma - \sigma^2) + (1-\sigma)^2 \frac{1}{2}). \end{aligned} \quad (\text{A.12})$$

That is, if firm j also succeeds in both projects, each firm makes a duopoly profit of $\frac{1}{2}$. If firm j succeeds in project $A(B)$ only, firm i can make a

monopoly profit as long as it prevents firm j from imitating innovation $B(A)$, with probability σ . Otherwise, firm i will make a duopoly profit of $\frac{1}{2}$. If firm j fails in both projects, firm i can make a monopoly profit one as long as firm j does not imitate both innovations, with probability $2\sigma - \sigma^2$. Otherwise, firm i makes a duopoly profit of $\frac{1}{2}$.

Similarly, we can determine firm i 's expected profits for the other cases as follows. If firm i succeeds in project A only, its expected profit, denoted by W_A^S , is

$$W_A^S = p_{jA}p_{jB}(1-\sigma)\frac{1}{2} + (1-p_{jA})p_{jB}(\sigma(1-\sigma) + (1-\sigma)^2\frac{1}{2}). \quad (\text{A.13})$$

If firm i succeeds in project B only, its expected profit, denoted by W_B^S , is

$$W_B^S = p_{jA}p_{jB}(1-\sigma)\frac{1}{2} + p_{jA}(1-p_{jB})(\sigma(1-\sigma) + (1-\sigma)^2\frac{1}{2}). \quad (\text{A.14})$$

Finally, if firm i fails in both projects, its expected profit, denoted by W_0^S , is

$$W_0^S = p_{jA}p_{jB}(1-\sigma)^2\frac{1}{2} \quad (\text{A.15})$$

Therefore, if both firm i and j always keep their innovations secret, firm i 's expected profit, denoted by $\Pi_i^S(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \sigma)$, is

$$\begin{aligned} \Pi_i^S(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \sigma) &= p_{iA}p_{iB}W_{AB}^S + p_{iA}(1-p_{iB})W_A^S \\ &\quad + (1-p_{iA})p_{iB}W_B^S + (1-p_{iA})(1-p_{iB})W_0^S - x_{iA} - x_{iB}, \end{aligned} \quad (\text{A.16})$$

where $W_{AB}^S, W_A^S, W_B^S, W_0^S$ are defined by (A.12), (A.13), (A.14), and (A.15), respectively.

(i) From (A.11) and (A.16), at $x_{1A} = x_{1B} = x_{2A} = x_{2B} = x^P$ and $\sigma = \underline{\sigma}$,

$$\begin{aligned} &\frac{\partial \Pi_i^P(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \theta)}{\partial x_{iA}} - \frac{\partial \Pi_i^S(x_{iA}, x_{iB}, x_{jA}, x_{jB}; \sigma)}{\partial x_{iA}} \\ &= \frac{1}{4}p^2p'\frac{\theta}{1-p}((4-5p+p^2)\theta - 4 + 2p) < 0. \end{aligned} \quad (\text{A.17})$$

The inequality is from $0 < p < \frac{1}{2}$ and $0 < \theta < 1$. Therefore, at $\sigma = \underline{\sigma}, x^P(\theta) < x^S(\underline{\sigma}(\theta))$.

(ii) Define $z(\sigma) = \sigma - \frac{\theta}{1-p(x^s(\sigma))}$. Note that $z(0) < 0$ and $z(1) > 0$ since $0 \leq p < \frac{1}{2}$ and $0 < \theta < 1$. Therefore, assuming that $x^s(\sigma)$ is continuous, from the intermediate value theorem, there exists a $\bar{\sigma} \in [1, 0]$ such that $z(\bar{\sigma}) = \bar{\sigma} - \frac{\theta}{1-p(x^s(\bar{\sigma}))} = 0$.

Since $z(0) < 0$ and $z(1) > 0$, in order to prove that $\bar{\sigma}$ is unique, it is sufficient to show that $z'(\bar{\sigma}) > 0$. From the definition of $z(\sigma)$,

$$z'(\bar{\sigma}) = 1 - \frac{\theta p' \frac{dx^s}{d\sigma}}{(1-p)^2} = 1 - \frac{\bar{\sigma} p' \frac{dx^s}{d\sigma}}{(1-p)}. \quad (\text{A.18})$$

The second equality is from the definition of $\bar{\sigma}$. Note that from (A.16) and the implicit function theorem,

$$(1-p) - \bar{\sigma} p' \frac{dx^s}{d\sigma} = (1-p) - \bar{\sigma} p' \frac{\frac{1}{2} p p' (p+4\sigma-8p\sigma+4p^2\sigma-2)}{\frac{1}{2} p'^2 X + \frac{1}{2} p p'' X + \frac{1}{2} p p' (-4p'p\sigma^2 + 2p'p + 4p'\sigma^2 - p'\sigma - 3p')} > 0, \quad (\text{A.19})$$

where $X = (-\frac{1}{4} p^2 \theta^2 + \frac{1}{2} p^2 + p\theta^2 - p\theta - p - \frac{1}{2} \theta^2 + \theta + \frac{1}{2})$. The inequality follows from the assumptions that $p < \frac{1}{2}$ and $p'^2 + p p'' < 0$. Therefore, $z'(\bar{\sigma}) > 0$.

(iii) From the definitions of $\underline{\sigma}$,

$$z(\underline{\sigma}) = \underline{\sigma} - \frac{\theta}{1-p(x^s(\underline{\sigma}))} = \frac{\theta}{1-p(x^s(\theta))} - \frac{\theta}{1-p(x^s(\underline{\sigma}))} < 0, \quad (\text{A.20})$$

where the inequality is from (i). Then, since $z(0) < 0$ and $z(1) > 0$, from the fact that $\bar{\sigma}$ is unique, we must have $\underline{\sigma} < \bar{\sigma}$. ■

Proof of Proposition 5 (i) Given $\gamma_j = 1$ and $\rho_j = 1$ suppose that firm i has succeeded in both innovations. If firm i applies for the patents for both innovations, its expected profit, denoted by V_{AB}^{PL} is the same as V_{AB}^P in (A.4), except that $v_1 = \frac{1}{2}$.

If firm i keeps both innovations secret, its expected profit, denoted by V_{AB}^{SL} with slight abuse of notation, is as follows:

$$\begin{aligned} V_{AB}^{SL} &= p_{jA} p_{jB} (1-\theta)^2 \frac{1}{2} + (1-p_{jA}) p_{jB} (\frac{1}{2} + \frac{1}{2}(\sigma-\theta)) \\ &\quad + p_{jA} (1-p_{jB}) (\frac{1}{2} + \frac{1}{2}(\sigma-\theta)) + (1-p_{jA}) (1-p_{jB}) ((2\sigma-\sigma^2) + (1-\sigma)^2 \frac{1}{2}). \end{aligned} \quad (\text{A.21})$$

If firm i keeps one innovation (say A) secret, and applies for a patent for the other innovation (say B), its expected profit, denoted by V_{AB}^{SPL} , is:

$$\begin{aligned} V_{AB}^{SPL} &= p_{jA}p_{jB}\left(\frac{1}{2}\left(\frac{1}{2}\right)+\frac{1}{2}(1-\theta)^2\frac{1}{2}\right)+p_{jA}(1-p_{jB})\left(\frac{1}{2}\right) \\ &+ (1-p_{jA})p_{jB}\left(\frac{1}{2}((\theta+\sigma-\theta\sigma)+(1-\theta)(1-\sigma))\frac{1}{2}\right)+\frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}(\sigma-\theta)\right) \\ &+ (1-p_{jA})(1-p_{jB})((\theta+\sigma-\theta\sigma)+(1-\theta)(1-\sigma))\frac{1}{2}. \end{aligned} \quad (A.22)$$

Then, it is straightforward to show that if $\sigma < \frac{\theta-\frac{1}{2}p_j\theta^2}{1-p_j}$, then $V_{AB}^P > V_{AB}^{SP}$ and $V_{AB}^P > V_{AB}^S$. Therefore, if $\sigma < \frac{\theta-\frac{1}{2}p_j\theta^2}{1-p_j}$, firm i will apply for patents for both innovations, that is, $\rho_i = 1$.

Now suppose that without loss of generality, firm i has succeeded in project A only. If firm i applies for a patent, its expected profit, denoted by V_A^{PL} , is:

$$V_A^{PL} = p_{jA}p_{jB}\left(\frac{1}{2}\frac{1}{2}+\frac{1}{2}(1-\theta)^2\frac{1}{2}\right)+(1-p_{jA})p_{jB}\frac{1}{2}. \quad (A.23)$$

If firm i keeps the innovation A secret, its expected profit, denoted by V_A^{AL} with slight abuse of notation, is:

$$V_A^{SL} = p_{jA}p_{jB}(1-\theta)^2\frac{1}{2}+(1-p_{jA})p_{jB}\left(\frac{1}{2}+\frac{1}{2}(\sigma-\theta)\right). \quad (A.24)$$

Then, it is straightforward to show that if $\sigma < \frac{\theta-\frac{1}{2}p_j\theta^2}{1-p_j}$, then $V_A^{PL} > V_A^{SL}$.

Therefore, if firm j is always applying for patents (that is, $\gamma_j = 1$ and $\rho_j = 1$), and if $\sigma < \frac{\theta-\frac{1}{2}p_j\theta^2}{1-p_j}$, then firm i will also always apply for patents regardless of whether it has succeeded in one or both innovations (that is, $\gamma_i = 1$ and $\rho_i = 1$)

(ii) and (iii) can be proved in the same way, and are omitted.