

## Recognizability and Liquidity of Assets\*

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*The recognizability of assets is embedded into a standard search model to determine liquidity returns. Assuming that money is universally recognizable but bond is not, two types of trades arise—one where both money and bond are accepted and the other where only money is accepted as a medium of exchange—depending on a seller's strategy of accepting or rejecting the bond of unrecognized quality and a buyer's strategy of carrying the counterfeit bond. Equilibrium restrictions imply that the liquidity differentials between money and bond tend to increase with the recognizability problem. Money commands higher liquidity than bond by providing additional liquidity service when sellers reject the bond of unrecognized quality as well as when they recognize counterfeit bond. The coexistence of money and bond requires a higher full (liquidity augmented) return for bond than money, implying a positive liquidity premium.*

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### I. Introduction

An asset's liquidity—the degree to which it facilitates exchange—has been regarded as being essential to the determination of asset price. Townsend (1987), Bansal and Coleman (1996), and Kiyotaki and Moore (2005, 2008) share the common implication that liquidity plays a significant role in accounting for the equity premium or the risk-free rate puzzle. Using a search-based model, Lagos (2010) shows that small liquidity differences have significant quantitative implications for

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asset prices in general, and the equity premium and risk-free rate puzzles in particular. Aruoba et al. (2007), Geromichalos et al. (2007), and Lagos and Rocheteau (2008) also use similar models to examine the implications of liquidity for asset prices and monetary policy. However, in these studies, either equal or differential liquidity between assets is exogenously assumed.

The goal of this paper is to derive endogenously the liquidity returns from holding assets, given the physical characteristics of multiple assets, such as money and bond. Specifically, in the spirit of Jevons (1875), King and Plosser (1986), Williamson and Wright (1994), and Banerjee and Maskin (1996), we note the recognizability of assets such that money is universally recognizable, whereas bond is not. Following Wallace's (1998) dictum, we incorporate the recognizability of assets explicitly into the standard search-based model of exchange such as Lagos and Wright (2005), and Lagos (2010) to determine the liquidity of assets as an equilibrium outcome.

Specifically, we assume that buyers have access to the bond counterfeit technology at no cost in the decentralized market. In a bilateral meeting, a seller can imperfectly recognize the quality of a buyer's bond. If a seller cannot recognize the quality of bond and she decides not to accept the bond of unrecognized quality, only money is accepted. If a seller cannot recognize the quality of bond but she decides to accept the bond of unrecognized quality, both money and bond are accepted. It is worth noting that these two types of meetings in the decentralized market arise endogenously depending on a seller's strategy of accepting or rejecting the bond of unrecognized quality and a buyer's strategy of carrying the counterfeit bond. This is in contrast to Lagos (2010) where an agent is placed in these two types of meetings due to some exogenous institutional restrictions. That is, Lagos (2010) studies the asset pricing implications of exogenously given liquidity differentials. The natural next step is then to explore the "deeper reasons" for these differentials as pointed out properly by Lagos (2010, p. 916).

We first show that there exist three types of equilibria where both money and bond are accepted as media of exchange. More importantly, the equilibrium restrictions imply that liquidity differentials between money and bond tend to increase with the recognizability problem. When the recognizability problem is not so severe, there only exists an equilibrium where all buyers bring the authentic bond and sellers always accept the bond of unrecognized quality. This implies that money and bond become equally liquid in the sense that buyers obtain the same expected marginal benefit from bringing additional money or bond to a bilateral trade. This also means that money would not be essential if the sufficient liquidity service were provided by the ample supply of bond, as shown in Geromichalos et al. (2007), and Lagos and Rocheteau (2008).

As it becomes more difficult to recognize the quality of bond, there also arises an equilibrium where some buyers bring the counterfeit bond to the decentralized

market, but sellers either accept or randomize between accepting and rejecting the bond of unrecognized quality. In an equilibrium where sellers accept the unrecognized bond, money commands higher liquidity than bond in the sense that, unlike bond, money provides the additional liquidity service when the counterfeit bond is recognized. In an equilibrium where sellers randomize the acceptance of the unrecognized bond, further liquidity service of money arises when sellers reject the bond of unrecognized quality as well as when they recognize the counterfeit bond.

This implies that, in the presence of the recognizability problem with bond, the coexistence of money and bond requires higher full (liquidity augmented) return for bond than money, implying a positive liquidity premium. This also means that, unlike the equilibrium with equal liquidity, money is essential due to its additional liquidity service. When the recognizability problem is sufficiently severe, there only exists an equilibrium where some buyers bring the counterfeit bond and sellers randomize between accepting and rejecting the bond of unrecognized quality.

These results are complementary to Rocheteau (2008) which links the partial liquidity of a real asset to the properties of its dividend process. Specifically, in the presence of asymmetric information on the dividend of the real asset, its liquidity is shown to decrease as the dispersion of the dividends across states increases. The imperfect substitutability between fiat money and the real asset also yields a rate-of-return differential, implying a positive liquidity premium. Thus, the relationship between the liquidity of a real asset and its physical characteristics appears to be robust regardless of the specific characteristics under consideration, recognizability or payoffs.

Moreover, our results generalize the recent studies on liquidity, asset prices and monetary policy (e.g., Geromichalos et al. 2007, Lagos 2008, and Lagos and Rocheteau 2008) in the sense that the liquidity differentials between assets and the essentiality of money are endogenously determined in the environment where money and bond differ in the inherent properties such as recognizability. Lester et al. (2008) also consider a similar situation but focus only on the class of equilibria where any unrecognized asset is never accepted.

The rest of this paper is organized as follows. Section 2 describes the model economy, followed by the equilibrium characterization in Section 3. Section 4 discusses the different types of equilibria with equal and differential liquidity between money and bond, including the relationship between recognizability and liquidity. Section 5 summarizes the paper with a few concluding remarks, followed by Appendix which contains the proofs of the propositions in the paper.

## II. Model

The basic setup is Lagos and Wright (2005), and Lagos (2010) with the explicit consideration of the asset recognizability. There is a non-atomic unit measure set of infinitely-lived agents. Time is indexed by  $t \in \mathbb{Z}_+$  and in each period  $t$  there are two markets, the decentralized and the centralized markets that open sequentially. There are three perishable and perfectly divisible consumption goods at each period: coconuts, apples and general good. People cannot make any binding intertemporal commitments and their trading histories are private, and hence all the trades should be on the spot. Each agent maximizes the discounted expected utility with the discount factor  $\beta \in (0, 1)$  between the decentralized market and the centralized market.<sup>1</sup>

In the model, there are two durable assets, money and a set of *Lucas trees*. The stock of perfectly divisible money is exogenously given by  $\bar{M} > 0$ , and the number of trees is also fixed and equal to the number of agents. Each tree yields  $d_t$  units of apples as dividends after the decentralized trades but before the current trades in the centralized markets where  $d_t$  follows an i.i.d. process realized at the beginning of each period. Each tree has outstanding one durable and perfectly divisible consol bond that entitles the ownership of a share of tree as well as the dividends. In the spirit of Jevons (1875), King and Plosser (1986), Williamson and Wright (1994), and Banerjee and Maskin (1996), we assume that the quality of money is perfectly recognizable, whereas the quality of bond is not.

In the opening of the decentralized market, half of the agents are endowed with  $\varepsilon^h = (1 + \varepsilon)A$  units of coconuts and the remaining half with  $\varepsilon^l = (1 - \varepsilon)A$  units of coconuts, where  $\varepsilon \in (0, 1)$  and  $A > 0$ . We call the former as type- $h$  agents and the latter as type- $l$  agents. The realization of the stochastic individual endowments is i.i.d. across periods and agents. Note that the trades of coconuts for assets are motivated by these heterogenous endowments among agents. An agent gets utility  $v(q)$  from consuming  $q$  units of coconuts where  $v''(q) < 0 < v'(q)$  and  $v'(\varepsilon^l)$  is sufficiently large.

After the realization of the endowment shock, type- $l$  agents (as potential buyers) can access the bond counterfeit technology at no cost and they should decide whether to counterfeit bond or not. Those who do not counterfeit the bond move to the decentralized market with money and authentic bond, whereas those who counterfeit the bond move to the market with money and counterfeit bond. We assume for simplicity that potential buyers do not carry some combinations of authentic and counterfeit bonds to the decentralized market. It is worth noting that

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<sup>1</sup> The discounting is not performed between the centralized market and the next decentralized market. As in Rocheteau and Wright (2003, 2005), all that matters is the total discounting between one period and the next.

in Lester et al. (2008), buyers can counterfeit bond at any time but here buyers should make an ex-ante decision on whether to counterfeit bonds before moving to bilateral meetings.

In the following bilateral meetings, each agent is randomly matched with another agent. Trades can occur in the single-coincidence meetings between a type- $h$  agent (a potential seller) and a type- $l$  agent (a potential buyer). People in a meeting can observe each other's endowment and portfolio. Further, a type- $h$  agent receives a common-knowledge signal regarding the quality of bond held by the type- $l$  agent. With probability  $\theta_1 \in (0, 1)$ , the signal is informative and the quality of bond is revealed to a type- $h$  agent; with probability  $\theta_2 = 1 - \theta_1$ , the signal is uninformative and the quality of bond is not recognized. Hence, in an uninformative meeting, a type- $h$  agent has to decide whether to accept the bond of unrecognized quality. Immediately after the bilateral trade, the counterfeit bond turns into a worthless red paper.

In the centralized market, the bearer of the authentic bond collects dividends (i.e., apples from a tree). An agent gets utility  $u(c)$  from consuming  $c$  units of apples where  $u''(c) < 0 < u'(c)$  and  $u'(0) = \infty$ . All agents can also produce, trade, and consume general good. Each agent can produce one unit of general good using one unit of labor which incurs one unit of disutility. An agent gets utility  $U(y)$  from consuming  $y$  units of general good where  $U''(y) < 0 < U'(y)$  and  $U'(0) = \infty$ .

### III. Equilibrium

We first introduce some notations. Let  $\mathbf{a}_t = (b_t, m_t) \in \mathbb{R}_+^2$  denote a portfolio where  $b_t$  and  $m_t$  denote respectively bond and money holdings at the beginning of  $t$ . We will drop the time subscript  $t$  hereinafter and index the next period variable by  $+1$ , if there is no risk of confusion. Let  $\phi^g, \phi^b$  and  $\phi^m$  denote respectively the unit price of general good, bond (ex-dividend) and money in terms of apples. Also, let  $V(\mathbf{a}, d)$  denote the value function for an agent who enters the decentralized market with  $\mathbf{a} = (b, m)$  and dividend  $d$ , and  $W(\mathbf{a}, d)$  denote the value function when she enters the centralized market. In what follows, we formulate an equilibrium in the recursive manner and work backward from the centralized market to the decentralized market.

#### 3.1. Centralized Market

In the centralized market, agents produce general good, consume apples and general good, and adjust their portfolios. Hence, the problem for a representative agent entering the centralized market with  $\mathbf{a} = (b, m)$  is

$$\begin{aligned}
W(\mathbf{a}, d) &= \max_{\{y, c, h, \mathbf{a}_{+1}\}} \{U(y) + u(c) - h + \mathbb{E}V(\mathbf{a}_{+1}, d_{+1})\} \\
\text{s.t. } \phi^g y + c &= \phi^g h + (\phi^b + d)b + \phi^m m - \phi^b b_{+1} - \phi^m m_{+1}, \\
c \geq 0, \mathbf{a}_{+1} &\geq 0, y \geq 0, h \in [0, \bar{h}],
\end{aligned}$$

where  $h$  is the labor devoted to production of general good. We assume an interior solution for  $c$ ,  $y$  and  $h$ .<sup>2</sup> Let  $\phi = (\phi^b, \phi^m)$ , and  $\Phi = (\Phi^b, \Phi^m)$  with  $\Phi^b \equiv (\phi^b + d)/\phi^g$  and  $\Phi^m \equiv \phi^m/\phi^g$ . Substituting  $h$  from the budget constraint, we have

$$W(\mathbf{a}, d) = \Phi \mathbf{a} + \max_{\{y, c, \mathbf{a}_{+1}\}} \left\{ U(y) - y + u(c) - \frac{1}{\phi^g} (c + \phi \mathbf{a}_{+1}) + \mathbb{E}V(\mathbf{a}_{+1}, d_{+1}) \right\}.$$

The first-order and envelope conditions are

$$\begin{cases} U'(y) = 1 \\ u'(c) = 1/\phi^g \end{cases} \quad (1)$$

$$\begin{cases} \phi^b/\phi^g \geq \partial V/\partial b_{+1}, & "=" \text{ if } b_{+1} > 0 \\ \phi^m/\phi^g \geq \partial V/\partial m_{+1}, & "=" \text{ if } m_{+1} > 0 \end{cases} \quad (2)$$

$$\begin{cases} \partial W(\mathbf{a}, d)/\partial b = \Phi^b \\ \partial W(\mathbf{a}, d)/\partial m = \Phi^m. \end{cases} \quad (3)$$

Conditions (1) and (2) imply that  $(y, c)$  and  $\mathbf{a}_{+1}$  do not depend on the current asset holdings  $\mathbf{a} = (b, m)$ . The latter allows us to restrict attention to the case where the portfolio distribution is degenerated at the beginning of each period. The condition (3) suggests that  $W(\mathbf{a}, d)$  is linear in  $\mathbf{a} = (b, m)$ .

### 3.2. Decentralized Market

The following two types of the bilateral meetings arise in the decentralized market: one is a meeting where both money and bond are traded for coconuts, and the other is a meeting where only money is traded for coconuts. If a seller can recognize the quality of bond or if she cannot recognize the quality of bond but decides to accept it, both assets can be accepted. If a seller cannot recognize the quality of bond and she decides not to accept the bond of unrecognized quality, then only money is accepted.<sup>3</sup>

<sup>2</sup> An interior solution for  $c$  and  $y$  is guaranteed under the standard assumption on  $u(c)$  and  $U(y)$ . It would be also true for  $h$  under the assumption as in Lagos and Wright (2005).

<sup>3</sup> Although we assume that there is no direct cost in counterfeiting bonds, a potential buyer ends up

Notice that these two types of meetings in the decentralized market are determined endogenously depending on a seller's strategy of accepting or rejecting the bond of unrecognized quality and a buyer's strategy of carrying the counterfeit bond. This is in contrast to Lagos (2010) where an agent is placed in these two types of meetings due to some exogenous institutional restrictions.

Now, one needs to take a stand on how terms of trade are determined in the decentralized market. We here adopt the proportional solution where the bargaining power of a buyer is given by  $\eta \in (0,1)$ .<sup>4</sup> Further, to use standard bargaining results, we simply assume that in an uninformative meeting, a type- $h$  agent decides whether to accept the bond of unrecognized quality by taking into account the terms of trade in an informative meeting.<sup>5</sup> Then, there are two sets of terms of trade:  $(q^{bm}, p^{bm})$  for the meeting where both money and bond are accepted, and  $(q^m, p^m)$  for the meeting where only money is accepted. That is, a buyer hands over the portfolio  $p^i = (p_1^i, p_2^i)_{i=bm,m} \in \mathbb{R}_+^2$  to a seller in exchange for  $q^i \in \mathbb{R}_+^1$  units of coconuts where  $p_1^i$  and  $p_2^i$  denote respectively the amount of bond and money.

Let  $v_b(q) \equiv v(\varepsilon^l + q) - v(\varepsilon^l)$  and  $v_s(q) \equiv v(\varepsilon^h) - v(\varepsilon^h - q)$ . Then the surplus of a bilateral trade is  $[v_b(q) - v_s(q)]$  and  $(q^{bm}, p^{bm})$  in a bilateral meeting between a buyer with  $a = (b, m)$  and a seller with  $\tilde{a} = (\tilde{b}, \tilde{m})$  solves

$$(q^{bm}, p^{bm}) = \arg \max [v_b(q^{bm}) - \beta \Phi p^{bm}] \quad (4)$$

subject to  $v_b(q^{bm}) - \beta \Phi p^{bm} = [\eta / (1 - \eta)] [\beta \Phi p^{bm} - v_s(q^{bm})]$  and  $p^{bm} \leq a$ . After some calculation, this problem can be simplified to

$$q^{bm} = \arg \max \{ \eta [v_b(q^{bm}) - v_s(q^{bm})] \} \quad (5)$$

subject to  $(1 - \eta)v_b(q^{bm}) + \eta v_s(q^{bm}) \leq \beta \Phi a$ . If the constraint binds,  $q^{bm}$  is just the solution to

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bearing the cost of carrying the counterfeit bond by sacrificing some surplus in an informative single-coincidence meeting. For simplicity, the upper bound of this cost is considered here by assuming that a potential buyer does not carry some combinations of authentic and counterfeit bonds to the decentralized market.

<sup>4</sup> Unlike the Nash bargaining, the proportional bargaining ensures monotonicity that conveniently eliminates an incentive for buyers to hide their asset holdings. For more details, see Nosal and Rocheteau (2011, pp. 82-84).

<sup>5</sup> Alternatively, one could interpret this as follows: (i) a buyer in a pairwise meeting makes an offer  $(q, p)$  with the understanding that  $\pi = 0$  for any  $(q, p)$  such that  $(1 - \eta)[v_b(q) - v_s(q)] < [\beta \Phi p - v_s(q)]$ , where  $\pi$  is the probability that a seller accepts an unrecognized quality of bond. (ii) Then a seller accepts or rejects the offer she receives.

$$\beta \Phi \mathbf{p}^{bm} \equiv z(q^{bm}) = (1-\eta)v_b(q^{bm}) + \eta v_s(q^{bm}).$$

Similarly,  $(q^m, \mathbf{p}^m)$  solves

$$(q^m, \mathbf{p}^m) = \arg \max [v_b(q^m) - \beta \Phi \mathbf{p}^m] \quad (6)$$

subject to  $(1-\eta)v_b(q^m) + \eta v_s(q^m) \leq \beta \Phi^m m$  and  $\mathbf{p}^m \leq (0, m)$ .

Notice that in any equilibrium with  $\eta \in (0, 1)$ ,  $q^m \leq q^{bm} \leq \varepsilon A$  where the first inequality ( $q^m \leq q^{bm}$ ) follows from  $z'(q) > 0$ . These inequalities imply that sellers and buyers in single-coincidence meetings are respectively willing to accept and transfer bond in the following sense:

$$[v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm})] \leq \beta \Phi(\mathbf{p}^{bm} - \mathbf{p}^m) \leq [v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)] \quad (7)$$

where the first inequality in (7) represents a seller's participation constraint for accepting bond in a single-coincidence meeting and the second inequality represents a buyer's participation constraint for transferring bond.

Now, the linearity of  $W$  and the bargaining solutions imply that the value function for a seller (type- $h$  agent) entering the decentralized market with a degenerate portfolio  $\mathbf{a} = (b, m)$  should satisfy

$$\begin{aligned} V_h(\mathbf{a}, d) = & \frac{\theta_1}{2} [\sigma v_h^{bm} + (1-\sigma)v_h^m] + \frac{\theta_2}{2} \max_{\pi} \{ \pi [\sigma v_h^{bm} + (1-\sigma)\hat{v}_h^{bm}] + (1-\pi)v_h^m \} \\ & + \frac{1}{2} v(\varepsilon^h) + \beta W(\mathbf{a}, d). \end{aligned} \quad (8)$$

Here  $\pi \in [0, 1]$  denotes the probability with which a seller will accept the bond of unrecognized quality,  $\sigma \in [0, 1]$  denotes the fraction of buyers who bring the authentic bond,  $v_h^{bm} = v(\varepsilon^h - q^{bm}) + \beta \Phi \mathbf{p}^{bm}$ ,  $v_h^m = v(\varepsilon^h - q^m) + \beta \Phi \mathbf{p}^m$ , and  $\hat{v}_h^{bm} = v(\varepsilon^h - q^{bm}) + \beta \Phi \hat{\mathbf{p}}^{bm}$  with  $\hat{\mathbf{p}}^{bm} = (0, p_2^{bm})$ . The expected utility of a seller consists of the expected payoffs from the single-coincidence meetings where with probability  $\theta_1$  she recognizes the quality of bond and with probability  $\theta_2$  she does not recognize the quality of bond but accepts it with probability  $\pi$  or rejects it with probability  $(1-\pi)$ . In the meetings without single coincidence, a seller just consumes her coconut endowment  $\varepsilon^h$ .

The value function for a buyer (type- $l$  agent) entering the decentralized market with the authentic bond should satisfy

$$V_l^g(\mathbf{a}, d) = \frac{\theta_1}{2} v_l^{bm} + \frac{\theta_2}{2} [\Pi v_l^{bm} + (1-\Pi)v_l^m] + \frac{1}{2} v(\varepsilon^l) + \beta W(\mathbf{a}, d) \quad (9)$$



where  $\Pi \in [0,1]$  denotes the buyer's belief that a seller will accept the bond of unrecognized quality,  $v_i^{bm} = v(\varepsilon^l + q^{bm}) - \beta \Phi \mathbf{p}^{bm}$ , and  $v_i^m = v(\varepsilon^l + q^m) - \beta \Phi \mathbf{p}^m$ . The expected utility of a buyer carrying the authentic bond consists of the expected payoffs from the single-coincidence meetings where with probability  $\theta_1$  the authentic bond is recognized by her trading partner and with probability  $\theta_2$  it is not recognized but accepted with probability  $\Pi$  or rejected with probability  $(1 - \Pi)$ . In the meetings without single coincidence, she just consumes her coconut endowment  $\varepsilon^l$ . Similarly, the value function for a buyer entering the decentralized market with the counterfeit bond should satisfy

$$V_i^f(\mathbf{a}, d) = \frac{\theta_1}{2} v_i^m + \frac{\theta_2}{2} [\Pi \hat{v}_i^{bm} + (1 - \Pi) v_i^m] + \frac{1}{2} v(\varepsilon^l) + \beta W(\mathbf{a}, d) \quad (10)$$

where  $\hat{v}_i^{bm} = v(\varepsilon^l + q^{bm}) - \beta \Phi \hat{\mathbf{p}}^{bm}$ . From (8), (9) and (10), the expected value of an agent entering the decentralized market with portfolio  $\mathbf{a} = (b, m)$ , before knowing the endowment shock, can be written as

$$V(\mathbf{a}, d) = \frac{1}{2} [V_h(\mathbf{a}, d) + \sigma V_l^g(\mathbf{a}, d) + (1 - \sigma) V_l^f(\mathbf{a}, d)]. \quad (11)$$

**Definition 1** An equilibrium is a set of allocations  $\{c_t, y_t, h_t, \mathbf{a}_{t+1}\}_{t=0}^\infty$ , a set of prices  $\{\phi_t^g, \phi_t^b, \phi_t^m\}_{t=0}^\infty$ , bilateral terms of trade  $\{(q_t^i, p_t^i)_{i=b,m}\}_{t=0}^\infty$ , and the values of  $(\sigma, \Pi)$  such that

1.  $\{c_t, y_t, h_t, \mathbf{a}_{t+1}\}_{t=0}^\infty$  solve the individual maximization problem in the centralized market as summarized by (1) through (3);
2.  $\{(q_t^i, p_t^i)_{i=b,m}\}_{t=0}^\infty$  are given by (4) and (6);
3.  $\{\phi_t^g, \phi_t^b, \phi_t^m\}_{t=0}^\infty$  clear the centralized market:  $c_t = d_t$ ,  $b_t = 1$  and  $m_t = \bar{M}$  for all  $t \in \mathbb{Z}_+$ ; and
4. for the value functions  $V_h(\mathbf{a}_t, d_t)$ ,  $V_l^g(\mathbf{a}_t, d_t)$ ,  $V_l^f(\mathbf{a}_t, d_t)$ , given by (8), (9) and (10) respectively,  $(\sigma, \Pi)$  satisfies (i)  $V_l^g(\mathbf{a}_t, d_t) - V_l^f(\mathbf{a}_t, d_t) > 0$  implies  $\sigma = 1$ ,  $V_l^g(\mathbf{a}_t, d_t) - V_l^f(\mathbf{a}_t, d_t) < 0$  implies  $\sigma = 0$ , and  $\sigma \in (0, 1)$  implies  $V_l^g(\mathbf{a}_t, d_t) - V_l^f(\mathbf{a}_t, d_t) = 0$ ; (ii) given  $\sigma$ ,  $\pi = \Pi$  must solve the maximization problem in (8).

There are potentially 9 types of equilibria depending on  $\sigma \in \{0, \Theta, 1\}$  and  $\Pi(\pi) \in \{0, \Theta, 1\}$  where  $\Theta$  denotes the open interval  $(0, 1)$ . Note that the agents with low coconut endowment (i.e., buyers) should choose whether to counterfeit bond  $\sigma \in [0, 1]$ , while the agents with high coconut endowment (i.e., sellers) should choose whether to accept the bond of unrecognized quality  $\Pi(\pi) \in [0, 1]$ . In doing so, they take as given the strategies of others. Here,  $\pi = \Theta$  stands for the

case in which a seller is indifferent between accepting and rejecting the bond of unrecognized quality, so that she randomizes between them with an arbitrary probability. Similarly,  $\sigma = \Theta$  stands for the case in which a buyer is indifferent between carrying the counterfeit and the authentic bond so that  $V_i^g(a, d) = V_i^f(a, d)$

From (1), (2) and (11), potential liquidity services of money and bond imply  $(\phi^m / \phi^g) \geq \beta \mathbb{E}(\phi_{+1}^m / \phi_{+1}^g)$  and  $(\phi^b / \phi^g) \geq \beta \mathbb{E}[(\phi_{+1}^b + d_{+1}) / \phi_{+1}^g]$  in any equilibrium. In order to focus on the role of recognizability in the liquidity differentials between money and bond, we restrict attention to equilibria with  $(\phi^m / \phi^g) > \beta \mathbb{E}(\phi_{+1}^m / \phi_{+1}^g)$  and  $(\phi^b / \phi^g) > \beta \mathbb{E}[(\phi_{+1}^b + d_{+1}) / \phi_{+1}^g]$  where bond as well as money provides liquidity service and hence  $q^m < q^{bm}$  and  $b > 0$ .

**Proposition 1** *There exist three types of equilibria where both money and bond are accepted as media of exchange:  $(\sigma, \Pi) = (1, 1)$ ,  $(\sigma, \Pi) = (\Theta, 1)$ , and  $(\sigma, \Pi) = (\Theta, \Theta)$ .*

**Proof.** See Appendix. ■

There exists no equilibrium with  $\Pi = 0$  where no bond of unrecognized quality is accepted. Intuitively, with  $0 < \sigma < 1$ , some sellers have to accept the bond of unrecognized quality in order for buyers to bring the counterfeit bond to the decentralized market. When  $\sigma = 1$  so that no buyer carries the counterfeit bond, rejecting any bond of unrecognized quality would not be a best response. Similarly, with  $\Pi = 0$ ,  $\sigma = 0$  would not be optimal because the authentic bond is still accepted as long as a seller can recognize the quality of bond with a positive probability  $\theta_1 > 0$ . However, if  $\theta_1 = 0$ , there exists an  $(\sigma, \Pi) = (0, 0)$  equilibrium where only money is accepted as a medium of exchange in a bilateral trade.

#### IV. Liquidity of Money and Bond

In the first type of equilibrium, where  $(\sigma, \Pi) = (1, 1)$ , both money and bond are equally liquid such that no counterfeit bond exists and both money and bond are equally acceptable even when the quality of bond is not recognized. On the other hand, the other two types of equilibria, where  $(\sigma, \Pi) = (\Theta, 1)$  and  $(\sigma, \Pi) = (\Theta, \Theta)$ , respectively, imply the differential liquidity between money and bond.

This is a generalization of the recent studies on the implications of liquidity for asset prices and monetary policy where either equal or differential liquidity is exogenously assumed. For instance, assuming that both money and real assets (e.g., capital or equity) in fixed supply are equally liquid, Geromichalos et al. (2007), and Lagos and Rocheteau (2008) show that money is essential as long as the supply of

real asset is sufficiently low. Motivated by legal or other institutional restrictions, Lagos (2010) assumes the liquidity differentials between bond and stock to examine the liquidity implications for asset prices.

An exception is Lester et al. (2008) in which the acceptability of real assets is determined endogenously by the acquisition of costly verification technology. However, they only consider the pure strategy of rejecting the real assets of unrecognized quality, which can be regarded as a limiting case of the  $(\sigma, \Pi) = (\Theta, \Theta)$  equilibrium.

#### 4.1. Equal Liquidity

In the equilibrium with  $(\sigma, \Pi) = (1, 1)$ ,  $\sigma = 1$  requires  $V_i^g(a, d) - V_i^f(a, d) > 0$  for  $\Pi(\pi) = 1$ . From (9) and (10), this condition implies the following restriction on the recognizability of bond:

$$\theta_1 > \frac{\beta \Phi(p^{bm} - p^m)}{v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)} \equiv \bar{\theta}_{1,1}. \quad (12)$$

Further, from (8) with  $\sigma = 1$ ,  $\pi = 1$  would be a best response if  $[v(\varepsilon^h - q^{bm}) + \beta \Phi p^{bm}] > [v(\varepsilon^h - q^m) + \beta \Phi p^m]$ . This inequality and (12) with  $\theta_1 < 1$  imply  $[v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm})] < \beta \Phi(p^{bm} - p^m) < [v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)]$ , which is consistent with the participation constraints in (7) with  $q^m < q^{bm}$  ( $b > 0$ ).

The value function (11) for a representative agent can be written as

$$V(a, d) = \frac{1}{4} [\bar{v}^{bm} + v(\varepsilon^l) + v(\varepsilon^h)] + \beta W(a, d) \quad (13)$$

where  $\bar{v}^{bm} \equiv v(\varepsilon^l + q^{bm}) + v(\varepsilon^h - q^{bm})$ . Then (1) and (2) imply that asset prices satisfy

$$\begin{cases} u'(d)\phi^b = \beta \mathbb{E}[u'(d_{+1})\mathcal{L}_{1,+1}(\phi_{+1}^b + d_{+1})] \\ u'(d)\phi^m = \beta \mathbb{E}[u'(d_{+1})\mathcal{L}_{1,+1}\phi_{+1}^m] \end{cases}$$

where

$$\mathcal{L}_{1,+1} \equiv 1 + \frac{1}{4} [\xi(q_{+1}^{bm})]$$

with  $\xi(q_{+1}^{bm}) \equiv [v'(\varepsilon^l + q_{+1}^{bm}) - v'(\varepsilon^h - q_{+1}^{bm})][1/z'(q_{+1}^{bm})]$ . Notice that the factor  $\mathcal{L}_{1,+1}$

can be interpreted as the liquidity returns from holding money or bond in the equilibrium where all the buyers bring the authentic bond ( $\sigma=1$ ) and sellers always accept the bond of unrecognized quality ( $\Pi=1$ ). That is, as in Lagos (2010),  $\mathcal{L}_{1,+1}$  represents the liquidity of money or bond in terms of the expected marginal benefit that the buyers obtain from bringing the value of an additional apple's worth of money or bond to the decentralized market.

Not surprisingly, there is no difference in the liquidity between money and bond as long as the recognizability problem with bond is not so severe in the sense that the quality of bond is recognized with a high probability given by (12). It implies that money would no longer be valued as liquidity if the ample supply of real asset (e.g., trees) provides sufficient liquidity service. This is consistent with Geromichalos et al. (2007), and Lagos and Rocheteau (2008) in which money is not essential when money and real asset are equally liquid and the fixed supply of real asset is sufficiently large.

#### 4.2. Differential Liquidity

In the  $(\sigma, \Pi) = (\Theta, 1)$  equilibrium, some buyers carry the counterfeit bond to the decentralized market, but sellers always accept bond even when they cannot recognize the quality of bond. First,  $\sigma \in (0, 1)$  requires  $V_l^s(a, d) - V_l^f(a, d) = 0$  for  $\pi(\Pi) = 1$ . From (9) and (10), this condition implies the following restriction on  $\theta_1$ :

$$\theta_1 = \frac{\beta \Phi(\mathbf{p}^{bm} - \hat{\mathbf{p}}^{bm})}{v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)} \equiv \bar{\theta}_{1,2}. \quad (15)$$

Further, from (8) with a given  $\sigma \in (0, 1)$ ,  $\Pi(\pi) = 1$  would be a best response if  $[v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm})] < \sigma \beta \Phi(\mathbf{p}^{bm} - \hat{\mathbf{p}}^{bm})$ . This inequality and (15) with  $\theta_1 < 1$  imply  $\{[v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm})] / \sigma\} < \beta \Phi(\mathbf{p}^{bm} - \mathbf{p}^m) < [v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)]$ , which is consistent with the participation constraints in (7) with  $q^m < q^{bm}$  ( $b > 0$ ).

The value function (11) for a representative agent can be written as

$$V(a, d) = \left( \frac{\theta_1 \sigma + \theta_2}{4} \right) \bar{v}^{bm} + \left( \frac{\theta_1 (1 - \sigma)}{4} \right) \bar{v}^m + \left( \frac{v(\varepsilon^l) + v(\varepsilon^h)}{4} \right) + \beta W(a, d) \quad (16)$$

with  $\bar{v}^m \equiv v(\varepsilon^l + q^m) + v(\varepsilon^h - q^m)$ . Then (1) and (2) imply that asset prices satisfy

$$\begin{cases} u'(d) \phi^b = \beta \mathbb{E}[u'(d_{+1}) \mathcal{L}_{2,+1}^b (\phi_{+1}^b + d_{+1})] \\ u'(d) \phi^m = \beta \mathbb{E}[u'(d_{+1}) \mathcal{L}_{2,+1}^m \phi_{+1}^m] \end{cases}$$

where

$$\begin{cases} \mathcal{L}_{2,+1}^b = 1 + (\frac{\theta_1 \sigma + \theta_2}{4}) \xi(q_{+1}^{bm}) \\ \mathcal{L}_{2,+1}^m = \mathcal{L}_{2,+1}^b + (\frac{\theta_1(1-\sigma)}{4}) \xi(q_{+1}^m) \end{cases}$$

with  $\xi(q_{+1}^m) \equiv [v'(\varepsilon^l + q_{+1}^m) - v'(\varepsilon^h - q_{+1}^m)] [1 / z'(q_{+1}^m)]$ .

Now, the factors  $\mathcal{L}_{2,+1}^b$  and  $\mathcal{L}_{2,+1}^m$  denote respectively the liquidity returns from holding bond and money in the equilibrium where some buyers bring the counterfeit bond to the decentralized market ( $\sigma = \Theta$ ), but sellers still accept the bond of unrecognized quality ( $\Pi = 1$ ). Notice that  $1 < \mathcal{L}_{2,+1}^b < \mathcal{L}_{2,+1}^m$  where the second inequality comes from the additional liquidity service provided by money when the counterfeit bond is recognized with probability  $\theta_1$ .

This also implies that, in the presence of the recognizability problem with bond, the coexistence of money and bond requires a higher full (liquidity augmented) return for bond than money, implying a positive liquidity premium. More specifically, since  $b_{+1} > 0$  and  $m_{+1} > 0$ , (2) should hold with equality. Simple manipulations of these equalities then imply  $(\mathbb{E}\phi_{+1}^m / \phi^m) = (\mathcal{L}_{2,+1}^b / \mathcal{L}_{2,+1}^m) [\mathbb{E}(\phi_{+1}^b + d_{+1}) / \phi^b]$ . Since  $\mathcal{L}_{2,+1}^b < \mathcal{L}_{2,+1}^m$ ,  $(\mathbb{E}\phi_{+1}^m / \phi^m) < [\mathbb{E}(\phi_{+1}^b + d_{+1}) / \phi^b]$  of which the difference represents the liquidity premium. Notice that, unlike the  $(\sigma, \Pi) = (1, 1)$  equilibrium with equal liquidity, money is essential in the  $(\Theta, 1)$  equilibrium due to its additional liquidity service in the presence of the recognizability problem with bond.

When  $(\sigma, \Pi) = (\Theta, \Theta)$ , some buyers carry the counterfeit bond to the decentralized market and some sellers accept the bond of unrecognized quality. Now,  $\sigma \in (0, 1)$  requires  $V_l^g(a, d) - V_l^f(a, d) = 0$  for a given  $\pi(\Pi) \in (0, 1)$ . From (9) and (10), this condition implies the following equilibrium value for  $\pi \in (0, 1)$ :

$$\pi = \frac{\theta_1 [v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m) - \beta \Phi(p^{bm} - \hat{p}^{bm})]}{(1 - \theta_1) \beta \Phi(p^{bm} - \hat{p}^{bm})}. \quad (17)$$

Then  $\pi < 1$  implies the restriction on  $\theta_1$  as follows:

$$\theta_1 < \frac{\beta \Phi(p^{bm} - \hat{p}^{bm})}{v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)} \equiv \bar{\theta}_{1,3}. \quad (18)$$

Further, for a given  $\sigma \in (0, 1)$ , any  $\pi \in (0, 1)$  would be a best response if a seller is indifferent between accepting and rejecting the unrecognized bond. This requires from (8) that  $\sigma[v(\varepsilon^h - q^{bm}) + \beta \Phi p^{bm}] + (1 - \sigma)[v(\varepsilon^h - q^{bm}) + \beta \Phi \hat{p}^{bm}] = v(\varepsilon^h - q^m) + \beta \Phi p^m$ , which can be rewritten as

$$\sigma = \frac{v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm})}{\beta \Phi(\mathbf{p}^{bm} - \hat{\mathbf{p}}^{bm})}. \quad (19)$$

Because  $\theta_1 < 1$  and  $\sigma < 1$ , (18) and (19) imply  $[v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm})] < \beta \Phi(\mathbf{p}^{bm} - \mathbf{p}^m) < [v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)]$ , which is consistent with the participation constraints in (7) with  $q^m < q^{bm}$  ( $b > 0$ ).<sup>6</sup>

The value function (11) for a representative agent can be written as

$$V(\mathbf{a}, d) = \left( \frac{\theta_1 \sigma + \theta_2 \pi}{4} \right) \bar{v}^{bm} + \left( \frac{\theta_1 (1 - \sigma) + \theta_2 (1 - \pi)}{4} \right) \bar{v}^m + \left( \frac{v(\varepsilon^l) + v(\varepsilon^h)}{4} \right) + \beta W(\mathbf{a}, d). \quad (20)$$

Now, (20) with (1) and (2) implies the following equilibrium asset prices:

$$\begin{cases} u'(d) \phi^b = \beta \mathbb{E}[u'(d_{+1}) \mathcal{L}_{3,+1}^b (\phi_{+1}^b + d_{+1})] \\ u'(d) \phi^m = \beta \mathbb{E}[u'(d_{+1}) \mathcal{L}_{3,+1}^m \phi_{+1}^m] \end{cases}$$

where

$$\begin{cases} \mathcal{L}_{3,+1}^b = 1 + \left( \frac{\theta_1 \sigma + \theta_2 \pi}{4} \right) \xi(q_{+1}^{bm}) \\ \mathcal{L}_{3,+1}^m = \mathcal{L}_{3,+1}^b + \left( \frac{\theta_1 (1 - \sigma) + \theta_2 (1 - \pi)}{4} \right) \xi(q_{+1}^m). \end{cases}$$

Notice that the liquidity factors  $\mathcal{L}_{3,+1}^b$  and  $\mathcal{L}_{3,+1}^m$  represent respectively the liquidity returns from holding bond and money in the equilibrium where some buyers bring the counterfeit bond to the decentralized market ( $\sigma = \Theta$ ) and sellers randomize between accepting and rejecting the bond of unrecognized quality ( $\Pi = \Theta$ ). These have the property of  $1 < \mathcal{L}_{3,+1}^b < \mathcal{L}_{3,+1}^m$  with the same qualitative interpretations as in the  $(\sigma, \Pi) = (\Theta, 1)$  equilibrium including a positive liquidity premium,  $(\mathbb{E} \phi_{+1}^m / \phi^m) < [\mathbb{E}(\phi_{+1}^b + d_{+1}) / \phi^b]$ .

The difference is that money provides the additional liquidity service when sellers reject the bond of unrecognized quality as well as when they recognize the counterfeit bond. This also makes money essential as in the  $(\sigma, \Pi) = (\Theta, 1)$  equilibrium. Finally, it can be verified that when  $\Pi = 1$  the above liquidity factors,  $\mathcal{L}_{3,+1}^b$  and  $\mathcal{L}_{3,+1}^m$ , are reduced respectively to  $\mathcal{L}_{2,+1}^b$  and  $\mathcal{L}_{2,+1}^m$ , while they are

<sup>6</sup> Strictly speaking, (18) with  $\theta_1 < 1$  implies  $\beta \Phi(\mathbf{p}^{bm} - \hat{\mathbf{p}}^{bm}) \leq [v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)]$ . But (17) with  $\pi \in (0, 1)$  rules out the case of  $[v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)] = \beta \Phi(\mathbf{p}^{bm} - \hat{\mathbf{p}}^{bm})$ . Hence, we have  $\beta \Phi(\mathbf{p}^{bm} - \hat{\mathbf{p}}^{bm}) < [v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)]$ .

reduced to  $\mathcal{L}_{1,+1}^b = \mathcal{L}_{1,+1}^m = \mathcal{L}_{1,+1}$  when  $\sigma = \Pi = 1$ .

### 4.3. Recognizability and Liquidity

The equilibrium restrictions on the recognizability parameter  $\theta_1$  as given by (12), (15), and (18) imply that the equal liquidity equilibrium with  $(\sigma, \Pi) = (1, 1)$  exists for  $\theta_1 > \bar{\theta}_{1,1}$ , while the differential liquidity equilibrium with  $(\sigma, \Pi) = (\Theta, 1)$  exists for  $\theta_1 = \bar{\theta}_{1,2}$  and the one with  $(\sigma, \Pi) = (\Theta, \Theta)$  exists for  $\theta_1 < \bar{\theta}_{1,3}$ . More importantly, the equilibrium relationship between the recognizability parameter  $\theta_1$  and the liquidity differentials is obtained from the following result:

**Proposition 2**  $\bar{\theta}_{1,1} < \bar{\theta}_{1,2} < \bar{\theta}_{1,3}$  as long as buyers have sufficient bargaining power so that  $z''(q) > 0$ . This implies that (i) if  $\theta_1 \geq \bar{\theta}_{1,3}$ , only the equal liquidity equilibrium with  $(\sigma, \Pi) = (1, 1)$  exists; (ii) if  $\theta_1 \leq \bar{\theta}_{1,1}$ , only the differential liquidity equilibrium with  $(\sigma, \Pi) = (\Theta, \Theta)$  exists; (iii) if  $\theta_1 \in (\bar{\theta}_{1,1}, \bar{\theta}_{1,3})$ , at least both  $(1, 1)$  and  $(\Theta, \Theta)$  equilibria exist; and (iv) if  $\theta_1 = \bar{\theta}_{1,2}$ , all the three types of equilibria exist.

**Proof.** See Appendix. ▀

That is, as the recognizability problem becomes more severe, the liquidity differentials between money and bond tend to increase. When the recognizability problem is not so severe, there only exists an equilibrium with equally liquid money and bond. As it becomes more difficult to recognize the quality of bond, there also arises an equilibrium where money commands higher liquidity than bond. Finally, when the recognizability problem is sufficiently severe, there only exists an equilibrium where money commands higher liquidity than bond.

## V. Concluding Remarks

In order to understand the reasons for the liquidity differentials between assets, we have extended Lagos' (2010) search model of exchange by incorporating the recognizability of asset explicitly: i.e., the quality of money is universally recognizable, whereas the quality of bond is not. The two types of the single-coincidence meetings arise in the decentralized search market—one where both money and bond are accepted and the other where only money is accepted as a means of payment—depending on a seller's strategy of accepting or rejecting the bond of unrecognized quality and a buyer's strategy of carrying the counterfeit bond.

We have shown that the liquidity differentials between money and bond tend to increase with the recognizability problem. When the recognizability of bond is not

so serious, there only exists an equilibrium where all the buyers bring the authentic bond to the decentralized market and sellers always accept the bond of unrecognized quality, and hence money and bond become equally liquid. As the recognizability problem becomes more severe, there also arises an equilibrium where some buyers bring the counterfeit bond, but sellers either accept or randomize between accepting and rejecting the bond of unrecognized quality. Money commands higher liquidity by providing the additional liquidity service when sellers reject the bond of unrecognized quality as well as when they recognize the counterfeit bond. The coexistence of money and bond then requires a higher liquidity-augmented return for bond, implying a positive liquidity premium.

Finally, in the presence of an interest-bearing asset (e.g., bond), the essentiality of money is followed in an environment where the fixed supply of bond is assumed to be sufficiently small. A future research is to account for the coexistence puzzle by allowing for the endogenous supply of bond as an interest-bearing asset.

## Appendix: Proofs

**Proof of Proposition 1:** For the combinations of  $\sigma$  and  $\Pi(\pi)$ , which are marked as “N” in Table 1, there exists no equilibrium where both money and bond are accepted as media of exchange.

[Table 1] Candidate Equilibria

	$\sigma = 0$	$\sigma = \Theta$	$\sigma = 1$
$\Pi = 0$	N	N	N
$\Pi = \Theta$	N	Differential Liquidity	N
$\Pi = 1$	N	Differential Liquidity	Equal Liquidity

(1)  $\sigma = 0$  and  $\Pi(\pi) = 0$ :  $\sigma = 0$  requires  $V_l^g(a, d) - V_l^f(a, d) < 0$  for a given  $\pi(\Pi) = 0$ . From (9) and (10), this condition can be written as  $\beta \Phi(p^{bm} - p^m) > [\nu(\varepsilon^l + q^{bm}) - \nu(\varepsilon^l + q^m)]$ , which immediately implies  $b = 0$ .

(2)  $\sigma = 1$  and  $\Pi(\pi) = 0$ : From (8) with  $\sigma = 1$ ,  $\pi(\Pi) = 0$  would be a best response if  $[\nu(\varepsilon^h - q^m) - \nu(\varepsilon^h - q^{bm})] > \beta \Phi(p^{bm} - p^m)$ . This violates the seller's participation constraint for accepting bond, and hence  $b = 0$ .

(3)  $\sigma = \Theta$  and  $\Pi(\pi) = 0$ :  $\sigma \in (0, 1)$  requires  $V_l^g(a, d) - V_l^f(a, d) = 0$  for a given  $\pi(\Pi) = 0$ . From (9) and (10), this condition can be written as  $\nu(\varepsilon^l + q^{bm}) - \beta \Phi p^{bm} = \nu(\varepsilon^l + q^m) - \beta \Phi p^m$ . This equality holds if and only if  $q^{bm} = q^m$  and  $p^{bm} = p^m$  with  $b = 0$ .

(4)  $\sigma = 0$  and  $\Pi(\pi) = \Theta$ : From (8) with  $\sigma = 0$ ,  $\pi(\Pi) = \Theta$  would be a best response if  $\nu(\varepsilon^h - q^{bm}) + \beta \Phi p^{bm} = \nu(\varepsilon^h - q^m) + \beta \Phi p^m$ . Again this equality holds if



and only if  $q^{bm} = q^m$  and  $p^{bm} = p^m = \hat{p}^{bm}$  with  $b = 0$ .

(5)  $\sigma = 1$  and  $\Pi(\pi) = \Theta$ : From (8) with  $\sigma = 1$ ,  $\pi(\Pi) = \Theta$  would be a best response if  $v(\varepsilon^h - q^{bm}) + \beta \Phi p^{bm} = v(\varepsilon^h - q^m) + \beta \Phi p^m$ . The exactly same argument in (3) gives the result of  $b = 0$ .

(6)  $\sigma = 0$  and  $\Pi(\pi) = 1$ : From (8) with  $\sigma = 0$ ,  $\pi(\Pi) = 1$  would be a best response if  $v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm}) < \beta \Phi (\hat{p}^{bm} - p^m)$ . This cannot hold because  $v(\varepsilon^h - q^m) > v(\varepsilon^h - q^{bm})$  with  $b > 0$  and  $\hat{p}^{bm} = p^m$ .

**Proof of Proposition 2:** It suffices to show that  $\bar{\theta}_{1,1} < \bar{\theta}_{1,2} < \bar{\theta}_{1,3}$ . The proof consists of several claims

**Claim 1** For a given  $q^{bm} > q^m$ ,  $\bar{\theta}_1$  is strictly increasing in  $q^m$ .

**Proof.** Since  $v(\cdot)$  is strictly concave and  $z''(q) > 0$  due to the assumption of sufficiently large  $\eta$ ,  $z'(q)/v'(\varepsilon^l + q)$  is strictly increasing in  $q$ . Then, for a given  $q^{bm} > q^m$ ,

$$\frac{z'(q^m)}{v'(\varepsilon^l + q^m)} < \frac{z'(q^{bm}) - z(q^m)}{v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)}$$

which gives the result. ■

**Claim 2**  $q_1^m < q_2^m < q_3^m$  where the subscript 1, 2, and 3 refer respectively to the equilibrium with  $(\sigma, \Pi) = (1, 1)$ ,  $(\sigma, \Pi) = (\Theta, 1)$ , and  $(\sigma, \Pi) = (\Theta, \Theta)$ .

**Proof.** In the equilibrium with  $(\sigma, \Pi) = (\Theta, 1)$ , the first-order condition with respect to  $m_{+1}$  can be written as

$$\beta \Phi_{+1}^m + \frac{\theta_1(1-\sigma)}{4} \xi(q_2^m) \beta \Phi_{+1}^m = \Phi^m \quad (21)$$

where  $\xi(q) \equiv [v'(\varepsilon^l + q) - v'(\varepsilon^h - q)](1/z'(q))$  denotes the value of an additional unit of money in a pairwise meeting. With the fixed supply of money,  $\Phi_{+1}^m = \Phi^m$  in equilibrium. Therefore, (21) can be rearranged as

$$\frac{\theta_1(1-\sigma)}{4} \xi(q_2^m) = \frac{1}{\beta} - 1. \quad (22)$$

The corresponding first-order condition in the equilibrium with  $(\sigma, \Pi) = (\Theta, \Theta)$  is

$$\frac{[\theta_1(1-\sigma)+\theta_2(1-\pi)]}{4}\xi(q_3^m)=\frac{1}{\beta}-1. \quad (23)$$

Noting that  $z''(q) > 0$  implies  $\xi'(q) < 0$ , the left-hand side of (22) and (23) is strictly decreasing, and  $\xi(0)$  is large enough because  $v'(\varepsilon^l)$  is sufficiently large. Hence, there exists a unique solution of  $q_2^m$  and  $q_3^m$  which satisfy (22) and (23), respectively. Now,  $\theta_1(1-\sigma) < [\theta_1(1-\sigma)+\theta_2(1-\pi)]$  implies  $q_2^m < q_3^m$ . Finally, in the equilibrium with  $(\sigma, \Pi) = (1, 1)$ , the smaller value of money holding brought to the decentralized market than in the  $(\sigma, \Pi) = (\Theta, 1)$  equilibrium implies  $q_1^m < q_2^m < q_3^m$ . ■

**Claim 3** There exists a unique  $q^{bm}$  such that  $q_1^{bm} = q_2^{bm} = q_3^{bm}$ .

**Proof.** Notice that  $q^{bm}$  is a function of real balance  $\kappa_{+1} = \hat{\phi}_{+1}^m m_{+1} + (\hat{\phi}_{+1}^b + \hat{d}_{+1}) b_{+1}$  where  $\hat{\phi}^m = \phi^m / \phi^g$ ,  $\hat{\phi}^b = \phi^b / \phi^g$ , and  $\hat{d} = d / \phi^g$ . When both money and bond are accepted, an agent chooses the portfolio  $a = (b_{+1}, m_{+1})$  that maximizes  $\{(1/4)[\bar{v}_{+1}^{bm} + v(\varepsilon^l) + v(\varepsilon^h)] + a_{+1}(\beta\hat{\phi}_{+1} - \hat{\phi}) + \beta\hat{d}_{+1}b_{+1}\}$  where  $\hat{\phi} = (\hat{\phi}^b, \hat{\phi}^m)$ . Since  $m_{+1}, b_{+1} > 0$ , (2) should hold with equality, and hence  $\hat{\phi}^b / \hat{\phi}^m = (\hat{\phi}_{+1}^b + \hat{d}_{+1}) / \hat{\phi}_{+1}^m$ . This suggests that the problem above can be rewritten as the choice problem of real balance ( $\kappa_{+1}$ ) that maximizes  $\{(1/4)[\bar{v}_{+1}^{bm}(\kappa_{+1}) + v(\varepsilon^l) + v(\varepsilon^h)] + \kappa_{+1}(\beta - 1)\}$ . The first-order condition is  $\xi[q_{+1}^{bm}(\kappa_{+1})] = 4\beta^{-1}(1 - \beta)$  in which the left hand side is strictly decreasing due to  $\xi'(q) < 0$  implied by  $z''(q) > 0$  and  $\xi(0)$  is large enough by the assumption on  $v'(\varepsilon^l)$ . Hence, there is a unique solution for  $\kappa_{+1}$  which then pins down a unique  $q_{+1}^{bm}$ . Since this is true in all the three types of equilibria,  $q_1^{bm} = q_2^{bm} = q_3^{bm}$ . ■

**Claim 4**  $\bar{\theta}_{1,1} < \bar{\theta}_{1,2} < \bar{\theta}_{1,3}$ .

**Proof.** Obvious consequence of the claim 1, 2 and 3. ■

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