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Handset Subsidy Regulation, Replacement of Handsets, and Quality Investments*

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The regulatory agency's intervention in carriers' handset subsidization is based on an argument that carriers tend to provide excessive handset subsidies, and thus have a low incentive either to invest in quality enhancement or to compete in service charges. This paper examines this issue rigorously. It argues that in the case of subsidization for intra-service handset replacement, the handset subsidy regulation can achieve efficiency. Surprisingly, however, the paper demonstrates that in the environment of subsidization for inter-service handset replacement, the handset subsidy regulation is irrelevant in that it has no effect on consumer's replacement decision and carriers' incentive for quality enhancing investments. This result implies that the effective charge regulation rather than the handset subsidy regulation can attain efficiency.

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I. Introduction

From the perspective of mobile carriers, handset subsidization is one of the major means to attract consumers. In the growing phase of the mobile service market, they say that handset subsidies have played an important role in expanding the mobile service market because they can reduce consumer's pecuniary burden to purchase a mobile device, which is quite expensive. Handset subsidies have also been regarded as a useful mean for consumers to switch to either better quality devices or handsets satisfying their tastes.

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Recently, however, negative perspectives of handset subsidies have been pointed out in various dimensions. They state that handset subsidies are the major factor for consumers to replace their handsets too frequently. Entner (2011) examines the relationship between the handset subsidy size and the replacement cycle of 14 countries[1] and shows that even though the handset replacement cycle, on average, has edged up since 2007, the cycle is shorter as the handset subsidy size is larger.[2] It is also commonly mentioned that handset subsidies are a main factor for increasing consumer's total payments for using mobile services. For example, in the U.K. and Korea, as a consumer chooses a more expensive calling plan, he receives a higher handset subsidy from a carrier to which he subscribes.[3] Putting the expensiveness of the device aside, this kind of handset subsidy structure linked to a calling plan may lead consumers to pick a relatively expensive calling plan, thereby increasing their total payments.[4]

Whereas handset subsidization by carriers has contributed to a dramatic expansion of the mobile telecommunication market, the regulatory agency does not seem to have had a favorable view on handset subsidies. Its view is that for competing with each other, the carriers resort excessively to handset subsidization rather than service quality enhancement or provision of cheaper calling plans; hence, they have a low incentive to invest in quality improvement or to lower their service rates, which in turn hampers the long-term development of the mobile telecommunication market and deteriorates consumer welfare. Based on this perspective, in Korea, the regulatory agency, Korea Communications Commission (KCC), has been regulating the handset subsidy size since the late 1990s up to now.[5] For example, from 2003 to 2008, handset subsidization had been banned by the law. In 2010, KCC made the Guideline for Marketing Expenses for the purpose of inducing carriers to transfer their marketing expenses into R&D investment. In particular, KCC set 270,000 KRW (equivalent to approximately 250 USD) as the cap for handset subsidy size that a carrier can provide per handset. Furthermore, for three years consecutively from 2010, KCC charged fines and gave orders of business suspension to carriers violating the guidelines.

Although policy issues on handset subsidies have been highlighted, they, unfortunately, do not hinge on a rigorous analysis. Even academic researches

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[1] They are Brazil, Canada, Finland, France, Germany, India, Israel, Japan, Korea, Mexico, South Africa, the United Kingdom and the United States.

[2] As Park and Ahn (2004) mentioned, a short handset replacement cycle would be a factor of drain in national wealth for countries that, like Korea, depend heavily both on imports of parts as well as on overseas specific technologies requiring royalty payments.

[3] On the contrary, in the U.S., the handset subsidy size is the same, regardless of the calling plans.

[4] This is problematic because consumers usually do not use all the calling time provided by the expensive calling plan.

[5] Refer to Kim, Byun, and Park (2004) for an analysis of handset subsidy-related policies in Korea from 1997 to 2001.

related to handset subsidies have been limited to the effect of handset subsidies on consumer's behavior or to the appropriateness of handset subsidy size. Albon and York (2008) discussed the impact of handset subsidies on consumer's behavior. They argued that handset subsidies encourage migration to a new service rather than attracting first-time subscribers. Using the Korean data, Lee, Lee, and Lee (2011) estimated the size of handset subsidy that can be allowed for mobile carriers by emphasizing that network externality should be considered for the estimation of the size of handset subsidy because it is a main characteristic of the mobile service demand. They proposed that the cap of the handset subsidy size currently imposed by KCC should be higher for internalizing the network externality. Daoud (2004) confirms with some qualitative case studies that the handset subsidies are effective to increase service subscription when the market is a rapid growth phase. Barros (2006) shows that subsidy is more intensive in new model handset in the Portuguese market. Tallberg, Hämmäinen, Töyli, Kamppari and Kivi (2007) argue that handset bundling regulation is a risk tool for steering the market from examining the Finnish market. Kim (2013) shows that the carriers can create a positive profit even with the subsidy competition as long as service differentiation is maintained.

This paper examines the aforementioned policy issues on handset subsidies by utilizing a theoretical approach. In particular, it investigates whether the regulatory agency's view on handset subsidies is convincing, because the underlying objective of handset subsidy regulation is to discourage the excessive replacement of handsets and to convert the exorbitant amount of resources used for handset subsidies into those for quality enhancing investment. Precisely, this paper studies the impact of handset subsidy regulation both on the replacement of handset and on incentives for quality enhancing investments. To do so, the paper adopts Hotelling's linear city model in which the horizontally differentiated handset market is exogenously given and two mobile carriers compete in the mobile service market.

The paper considers two cases with different settings. First, it examines the effect of handset subsidy regulation on the carriers' service charges in a circumstance in which the carriers release new style handsets without launching a new service. When a consumer switches to a new style handset, he is subsidized by a carrier. All consumers are assumed to use the same service (say, 3G service), regardless of whether they have the old or new style handsets. We refer to this case as *subsidization for intra-service handset replacement*. Under no regulation, the carriers are involved in service charge competition before releasing the new style handset, and upon the introduction of new style handsets, they are engaged in handset subsidization competition. We find that the competition in handset subsidization leads to an excessive handset subsidization. This implies that the *effective charge* (= service charge net of handset subsidy size) is lower to the new style handset purchaser than to the old style holders. This results in consumers' *over-replacement* of handsets. In the regulation regime, however, the regulatory agency as a social planner can set the handset subsidy size so as to have the effective charge equally applied to all consumers, regardless of whether they have the old or new style handsets. This leads to the consumers' efficient handset replacement. This implies that regulating handset subsidization is desirable in order to achieve efficiency in this case.

This paper also analyzes the effect of handset subsidy regulation on the carriers' incentive for quality enhancing investments in the environment in which the carriers launch a new service of which the quality level depends on the amount of investments undertaken by the carriers; then, they compete with the service charges of the new service as well as subsidizing consumers who switch to the new style handsets architected for the new service. In order to capture the issue of consumer's handset replacement or switching to a new service, the model incorporates the initial situation of the mobile service market, in which all consumers use an outdated service, say 3G service, before a new service, say 4G service, is introduced. We refer this case as the *subsidization for inter-service handset replacement with investments for service quality*.

The result is surprising. Unlike the underlying policy aim, the handset subsidy regulation is *irrelevant* to the incentives for quality enhancing investments of new service. The reason for this lies in the fact that the effective charge (= service charge net of handset subsidy size) of new service matters for consumers who plan to switch to new services. Indeed, the carriers do consider, as a variable for competition, the effective charge rather than the service charge and the handset subsidy size individually. Specifically, even if the handset subsidy size is regulated, the carriers can compete with each other with service charges which consist of the effective charge. Thus, the handset subsidy regulation has no effect on the effective charge competition outcome whatsoever. This implies that the handset subsidy regulation cannot affect the incentive to invest for quality enhancement because regulation does not alter the anticipated returns to investments.

The different results obtained from analyzing the two cases rely crucially on whether the service charge applies to all consumers or only to those who switch to the new handsets. The carriers should determine the service charge, which is assigned to all the consumers, including those who stay with the old style handsets, and then determine the handset subsidy size, which applies only to the new style handset purchasers in the case of subsidization for intra-service handset replacement. The carriers consider a competition with service charges and handset subsidies, one by one, under no regulation. Because the handset subsidy regulation eliminates the handset subsidy competition, it affects the service charge competition. However, in the case of subsidization for inter-service handset replacement with investment for service quality, the service charge and the handset subsidy size for new service apply only to the new service subscribers. In other words, carriers compete for an effective charge in the new service market. This implies that the profit maximizing effective charge is not altered by the handset subsidy regulation because the carriers can manipulate the service charges by obtaining the profit maximizing effective charge although the handset subsidy size is regulated. Thus, the handset subsidy regulation is irrelevant in this case.[6]

The paper is organized as follows. Section 2 analyzes the effect of handset subsidy regulation in the case of subsidization for intra-service handset replacement. Section 3 discusses the effect of handset subsidy regulation in the case of subsidization for inter-service handset replacement with investments for service quality. Section 4 compares the results of the two cases and concludes.

II. Case 1: Subsidization for Intra-service Handset Replacement

2.1. The Model

There are two mobile carriers, A and B, who provide mobile services for consumers. They are assumed to incur no cost for serving the consumers throughout the paper. For receiving mobile service, the consumers should prepare a handset. In order to highlight the issue of carriers' handset subsidization for consumers and carriers, we assume that a handset market is exogenously given as follows. The horizontally differentiated handsets are supplied into the market not by the carriers, but by some other entities such as manufacturers. This implies that the carriers cannot exert any influence on the prices and the degree of horizontal differentiation of handsets, and the number of horizontally differentiated handsets introduced.[7] In order to describe the configuration of horizontal differentiation of handsets, we adopt the Hotelling's linear city model, in which a continuum of consumers is uniformly distributed on a line of length 1, i.e., [0,1].[8] The timeline is divided into two periods depending on whether or not the new style handsets are released. In each period, all consumers consume one unit of mobile service provided by either A or B and value it at v. The quality of mobile service does not

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[6] Okholm, Karlsen, Pedersen, and Tops (2008) discuss that regulating handset subsidization may have a bad effect on carriers' innovation.

[7] Although a handset of a manufacturer should be technically specialized for a certain carrier's mobile service, it is in general the manufacturer's own decision on the design, performance, and so on of its handsets. In addition, it is observed in many countries that the distribution of handsets is independent of the mobile carriers. Under the assumption that the handset market is exogenously given, we do not consider the handset manufacturers' strategic behaviors such as pricing and determining the degree of horizontal differentiation of handsets.

[8] Adopting the Hotelling's model is useful to derive the demand functions from the consumer's choice.

depend on whether the handsets are old or new, which implies intra-service handset replacement, if exists.

In the first period, only the old style handsets are available. The configuration of horizontally differentiated old style handsets is given such that the handset specialized for carrier A is located at 0, whereas that for carrier B is located at 1 in the linear city. The distance between a consumer and a location of the handset represents the deviation of the handset's characteristic from the consumer's ideal. A consumer located at x, where $x \in [0,1]$, has to cover distance x to the handset located at 0 or distance $(1-x)$ to the handset located at 1. In order to cover the distance, a consumer pays t as the unit cost of transportation. Carrier $i \in \{A,B\}$ charges p_i for the service rate, which lasts for two periods.[9] The carriers choose their service rates simultaneously. We assume that the price for the old style handsets is 0 for simplicity and that the carriers do not provide any handset subsidy in the first period. A consumer located at x has utility $v - p_A - tx$ if he is a carrier A subscriber, and $v - p_B - t(1-x)$ if he is a carrier B subscriber. Under the assumption that a consumer subscribes to at most one carrier, the demand function that carrier i faces in the first period is given by $q_i^1(p_i,\ p_j) = \frac{1}{2} + \frac{1}{2t}(p_j - p_i)$ for $|\,p_j - p_i\,| \le t$, where $i \ne j \in \{A,B\}$.

In the second period, the new style handsets are supplied. They are horizontally differentiated not only from the new style handsets of each other, but also from the old style ones. In order to capture this case, we model that a new style handset for carrier A is located at $\frac{1}{3}$, and that for carrier B is located at $\frac{2}{3}$.[10] Thus, the second period configuration of horizontally differentiated handsets is such that the old style handset for carrier i is at O_i, where $O_A = 0$ and $O_B = 1$, and such that the new style one of for carrier i is at N_i, where $N_A = \frac{1}{3}$ and $N_B = \frac{2}{3}$.[11] Let $m > 0$ denote the price of the new style handset, which is exogenously given as assumed.[12] The carriers provide the handset subsidies only for the consumers who purchase the new style handsets. Specifically, carrier i can offer a handset subsidy

[9] In reality, the carriers do not change their service rates for the specific mobile service although consumers' handsets are replaced. For example, the service rates of 2G service are applied equally to the 2G users although the 2G users have different types of handsets.

[10] We may consider the circular city model in a Salop fashion, where the old style and the new style handsets of carrier A are located at 0 and $\frac{1}{4}$, respectively, and those of carrier B are at $\frac{1}{2}$ and $\frac{3}{4}$, respectively. We do not adopt this type of circular model because it fails to capture the direct competition between the new style handsets for attracting consumers in the second period.

[11] Many articles adopt the Hotelling's model with the locations of firms given, for example, in duopoly, at the end points in the city (= assume maximum differentiation), and then investigate the price competition or firms' choices of other variables. This is to ensure the continuous demand functions. For the same reason, we set the locations of four types of handsets at 0, 1/3, 2/3, 1. As long as the four types of handsets are horizontally differentiated, that is, as long as they are not at the same locations, the result of this paper does not depend on the locations of handsets.

[12] For ensuring the positive demand and the size of handset not exceeding the handset price in equilibrium below, it is assumed that $m \in [\frac{34}{129}t, \frac{2}{3}t]$.

s_i for the consumer who used the old style handset in the first period and switches to the new style handset dedicated to carrier i's mobile service in the second period. The carriers pick their level of subsidy at the same time.

Given p_A, p_B, m, s_A, and s_B, all consumers in the second period face the decision problem of whether they should continue to use their old style handsets or switch to the new style handsets. If a consumer who is located at x keeps using the old style handset for mobile service of carrier $i \in \{A,B\}$, his net utility is $v - p_i - td_i^O$, where $d_i^O = |O_i - x|$, and $O_i \in \{0,1\}$. If he switches to the new style handset specialized for mobile service of carrier $j \in \{A,B\}$, his net utility turns out to be $v - p_j - (m - s_j) - td_j^N$, where $d_j^N = |N_j - x|$, and $N_j \in \{\frac{1}{3}, \frac{2}{3}\}$. In the second period, a consumer chooses a handset specialized for mobile service of carrier i, which gives him the maximal net utility. Assume that v is sufficiently large so that all consumers subscribe to the mobile service for two periods.[13] From consumer's choice, we can derive the demand function that carrier i faces in the second period. The derivation of demand function is described in Appendix 1, from which we obtain the demand functions that carrier i faces in the second period as follows.

$$q_i^O(s_i) = \frac{1}{6} + \frac{1}{2t}[-s_i + m],$$
$$q_i^N(p_i, p_j, s_i, s_j) = \frac{1}{3} + \frac{1}{2t}[-p_i + p_j + 2s_i - s_j - m],$$

where $q_i^O(s_i)$ and $q_i^N(p_i, p_j, s_i, s_j)$ are, respectively, the second period demand function of the old style handset and that of the new style handset for carrier $i \neq j \in \{A,B\}$. Note that $q_i^O(s_i)$ contains no service charge terms because the service charge is equally applied to the consumers of carrier i, regardless of whether those consumers have the old style or new style handsets, and because the old style handset users of carrier i do not consider whether to switch to the new style handset provided by carrier j. Note also that due to the fact that carrier i can attract consumers by increasing its own handset subsidy size not only from its own old style handset users but also from carrier j's new style handset purchasers, the second period demand for the new style handsets of carrier i is affected by $\frac{1}{2t} \cdot 2s_i$.

Under the assumption that the carriers do not discount their future profits, the profit function of carrier $i \neq j \in \{A,B\}$ is given by

$$\pi_i = p_i[q_i^1(p_i, p_j) + q_i^O(s_i)] + (p_i - s_i)q_i^N(p_i, p_j, s_i, s_j).$$

[13] Specifically, assuming $v > \frac{525}{426}t + \frac{69}{142}m$ insures that all consumers subscribe to the mobile service for two periods in equilibrium.

Since the carriers determine their service rates and subsidy levels sequentially, the subgame perfect equilibrium is adopted, and thus backward induction will be used for the analysis.

2.2. Benchmark

In order to deal with the handset replacement issue, as a benchmark, we derive the socially optimal demand sizes of the old style and the new style handsets in the second period. Suppose that the social planner or the regulatory agency can explicitly enforce the demand sizes of the handsets provided by each carrier in the second period. The social welfare (= sum of consumer surplus and the carriers' profits) function in the second period, W_2, is given by

$$W_2 = \int_0^{\bar{x}} (v - tx)dx + \int_{\bar{x}}^{\frac{1}{3}} [v - m - t\left(\frac{1}{3} - x\right)]dx + \int_{\frac{1}{3}}^{\bar{y}} [v - m - t\left(y - \frac{1}{3}\right)]dy$$
$$+ \int_{\bar{y}}^{\frac{2}{3}} [v - m - t\left(\frac{2}{3} - y\right)]dy + \int_{\frac{2}{3}}^{\bar{z}} [v - m - t\left(z - \frac{2}{3}\right)]dz + \int_{\bar{z}}^{1} [v - t(1 - z)]dz ,$$

where $\bar{x}$ and $\bar{z}$ denote the consumers who are just indifferent between keeping the old style handset and switching to the new style handset on $[0, \frac{1}{3})$ and $[\frac{2}{3}, 1]$, respectively, and $\bar{y}$ denotes the consumer who is just indifferent between switching to the handset designed for carrier A and to that for carrier B on $[\frac{1}{3}, \frac{2}{3})$.[14]

Note that $q_A^O = \bar{x}$, $q_A^N = \bar{y} - \bar{x}$, $q_B^N = \bar{z} - \bar{y}$, and $q_B^O = 1 - \bar{z}$. This implies that determining the welfare maximizing demand sizes of the old style and the new style handsets is equivalent to determining $\bar{x}$, $\bar{y}$ and $\bar{z}$. Letting $\frac{dW_2}{d\bar{r}} = 0$ results in the welfare maximizing demands as follows, where $\bar{r} \in \{\bar{x}, \bar{y}, \bar{z}\}$. That is, $q_i^{O*} = \frac{1}{6} + \frac{1}{2t}m$, $q_i^{N*} = \frac{1}{3} - \frac{1}{2t}m$ for carrier $i \in \{A, B\}$. Furthermore, note that for the welfare maximizing demands, it is necessary that $s_i^* = 0$. In other words, the welfare maximization can be achieved by banning any handset subsidization from carriers to consumers.

2.3. Equilibrium of Competition

Using backward induction, we begin to derive the second period equilibrium, i.e., the equilibrium handset subsidy size, given p_A and p_B. This is obtained by solving the maximization problem of carrier i's profit function with respect to s_i.

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[14] $\bar{x}$, $\bar{y}$, and $\bar{z}$ are derived by $\bar{x} = \frac{1}{6} + \frac{1}{2t}[-s_A + m]$, $\bar{y} = \frac{1}{2} + \frac{1}{2t}[-p_A + p_B + s_A - s_B]$, and $\bar{z} = \frac{5}{6} + \frac{1}{2t}[s_B - m]$ in Appendix 1.

That is,

$$\max_{s_i} \; p_i[q_i^1(p_i, p_j) + q_i^O(s_i)] + (p_i - s_i)q_i^N(p_i, p_j, s_i, s_j).$$

The best response function of s_i is given by

$$s_i = \frac{1}{4}s_j + \frac{1}{2}p_i - \frac{1}{4}p_j + \frac{3}{4}m - \frac{1}{6}t. \tag{1}$$

Equation (1) tells us that handset subsidies are strategic complements, which implies that the carriers' handset subsidies are positively related with each other. Indeed, carrier i will increase its handset subsidy size for not losing its customers when its competitor raises the handset subsidy size. The equilibrium handset subsidy size of carrier $i \neq j$ is obtained as

$$s_i^{**}(p_i, p_j) = \frac{1}{15}(7p_i - 2p_j) + m - \frac{2}{9}t. \tag{2}$$

It is easy to see that carrier i's equilibrium handset subsidy is positively related to its own service charges, but negatively affected by its competitor's service rates. Equation (2) leads to $p_i - s_i^{**}(p_i, p_j) = \frac{1}{15}(8p_i + 2p_j) - \frac{1}{3}m + \frac{2}{9}t$, which shows that the *effective charge* for the new style handset users (= service rate net of handset subsidy size) of a carrier is positively related to the competitor's service charges.

Using equation (2), we can rewrite the second period demands functions of the old and the new style handsets as the functions of the service charges determined in the first period.[15] Then, the first period profit maximization problem of carrier i with respect to p_i can be written as

$$\max_{p_i} \; p_i[q_i^1(p_i, p_j) + q_i^O(p_i, p_j)] + (p_i - s_i^{**}(p_i, p_j))q_i^N(p_i, p_j).$$

The equilibrium service charge is obtained as $p_i^{**} = \frac{13}{71}m + \frac{244}{213}t$.[16] This implies that $s_t^{**} = \frac{28}{71}m + \frac{34}{213}t > 0$. The handset subsidy size provided by the carriers for the consumers who switch to the new style handsets is greater than the social optimum. This is due to the fact that in the competition of handset subsidy, the carriers subsidize excessively for attracting consumers in the second period. The equilibrium outcomes are summarized in Proposition 1.

[15] They are $q_i^O(p_i, p_j) = \frac{5}{18} + \frac{1}{30t}(-7p_i + 2p_j + 10m)$ and $q_i^N(p_i, p_j) = \frac{2}{9} + \frac{1}{30t}(p_i + 4p_j + 10m)$.

[16] The objective function is strictly concave and so the second order condition is satisfied.

Proposition 1. In the environment of subsidization for intra-service handset replacement,

$$q_i^{1**} = \frac{1}{2},\ q_i^{O**} = \frac{37}{426} + \frac{43}{142t}m < q_i^{O*},\ \text{ and }\ q_i^{N**} = \frac{88}{213} + \frac{43}{142t}m > q_i^{N*}.$$

Proposition 1 implies that in the second period, the equilibrium of competition results in *over-replacement* of handsets.

2.4. Handset Subsidy Regulation

In the handset subsidy regulation regime, the carriers cannot set the handset subsidy size at will in the second period; hence, they determine their service charges only in the first period. Given the regulated handset subsidy size $s = s_i = s_j$,[17] the profit maximization problem of carrier i with respect to p_i can be written as

$$\max_{p_i}\ p_i[q_i^1(p_i, p_j) + q_i^O(s)] + (p_i - s)q_i^N(p_i, p_j, s, s).$$

The equilibrium service charge for carrier i is obtained as $p_i^R = t + \frac{1}{2}s.$ Suppose that the social planner enforces $s = s_i^* = 0,$ which is the welfare maximizing handset subsidy size obtained in 2.2. Then the equilibrium service charge under this regulation turns out to be $p_i^R = t.$ It is easy to see that with $p_i^R = t,$ the second period demands in equilibrium replicate those under social optimum. This means that the handset subsidy regulation can achieve efficiency by setting $s^* = 0.$ That is, such a handset subsidy regulation can correct the over-replacement that is obtained under no regulation.

Proposition 2. In the case of subsidization for intra-service handset replacement, the handset subsidy regulation can achieve an efficient level of handset replacement by setting $s^* = 0.$

Note that $p_i^{**} = \frac{13}{71}m + \frac{244}{213}t > t = p_i^R,$ and that $p_i^{**} - s^{**} = -\frac{15}{71}m + \frac{210}{213}t < t = p_i^R.$ This implies that the competition for new style handset users is more intense than that for old style handset consumers when the carriers can compete with the handset subsidy. Interestingly, under no regulation, consumers in the first period are charged too high even if there is no handset subsidy competition in that period. This means that the carriers take advantage of the gains by charging the service rate too high or by sacrificing consumers with old style handsets in order to cope with

[17] We assume that the handset subsidy regulation is symmetric, which means that the regulated handset subsidy size is applied equally to carriers.

the handset subsidy competition. Put differently, the carriers offer lower effective charges to the new style handset users at the expense of old style handset consumers. Presumably, the handset subsidy competition is more problematic if only small group of consumers switch to the new style handsets. Thus, the handset subsidy regulation can be justified in that it induces the carriers to lower the service rate which is applied to all consumers, which results in enhancing the social welfare.

III. Case 2: Subsidization for Inter-service Handset Replacement with Investments for Service Quality

3.1. The Model

The model here is very close to that used in section 2. There is a continuum of consumers uniformly distributed on a line of length 1, i.e., [0,1]. There are two mobile carriers, A and B. The carriers incur no cost for serving consumers. In order to focus on the inter-service handset replacement and investment incentives, the model considers the case in which the carriers plan to launch the new mobile service, say 4G service, that requires investments for service quality, while all consumers initially subscribe to the old mobile service, say 3G service. As assumed in section 2, a handset market is exogenously given so that both the configuration of the horizontal differentiation of handsets and the prices of handsets are given.

Let us first illustrate the initial situation, which is exogenously given. All consumers initially subscribe to either of two carriers for 3G service. The 3G handsets are horizontally differentiated in that the 3G handsets for carrier A are located at 0, and those for carrier B are located at 1. Every consumer values 3G service equally at v. Each carrier charges p as a rate for 3G service. As assumed in section 2, we do not incorporate the price and handset subsidies of 3G handsets into the model because they are sunk already. We also assume that the initial 3G service market is equally divided by the two carriers in that consumers located on $[0,\frac{1}{2}]$ subscribe to carrier A and the other consumers located on $(\frac{1}{2},1]$ subscribe to carrier B. Thus, while using 3G service, a consumer located at x has utility $v-p-tx$ if he is a carrier A subscriber, and $v-p-t(1-x)$ if he is a carrier B subscriber. Without loss of generosity, we assume that $p=t$.[18] Because the 3G service market is fully covered initially, it holds that $v \geq \frac{3}{2}t$.

Now let us describe the environment in which the two carriers introduce the new 4G service. We assume that for subscribing to 4G service, the consumer must switch to the handset dedicated to 4G service. Similarly to the model of section 2, the

[18] This is the equilibrium of the standard static pricing game in the Hotelling's model when the firm's marginal cost is 0.

configuration of horizontal differentiation of handsets is given such that 3G handset for carrier i is at O_i, where $O_A = 0$ and $O_B = 1$, and such that 4G handset for carrier i is at N_i, where $N_A = \frac{1}{3}$ and $N_B = \frac{2}{3}$. Carrier $i \in (A, B\}$ sets p_i as the service charge for 4G service. Let m denote the price of 4G handset, which is exogenously given. Carrier i can also offer a handset subsidy s_i for the consumer who switches his subscription from 3G to 4G service.[19] Furthermore, carrier i can determine the quality level of their 4G service, which is denoted by $v_i \in [v, \infty)$. Note that the consumer values 4G service greater than 3G service. However, provision of the quality of 4G service is costly to the carriers, in that carrier i pays $C(v_i)$ for choosing v_i. The function $C(v_i)$ is increasing, convex, and satisfies that $\lim_{v_i \to v} C'(v_i) = 0$ and $\lim_{v_i \to v+m+\frac{1}{3}t} C'(v_i) = \infty$. This assumption implies that a higher quality of 4G service requires higher costs. This assumption also ensures that in equilibrium, 4G service of a carrier cannot seize the entire mobile service market.

Given v_A, v_B, v, p_A, p_B, m, s_A, and s_B, all consumers after 4G service is introduced face the decision problem of whether they should continue to use 3G service with the old handset or switch to 4G service with the new handset. If a consumer who is located at x keeps using 3G service of carrier $i \in (A, B\}$, his net utility is $v - p - td_i^O$, where $d_i^O = |O_i - x|$, and $O_i \in \{0,1\}$. If he switches to new 4G service of carrier $j \in \{A, B\}$, his net utility turns out to be $v_j - p_j - (m - s_j) - td_j^N$, where $d_j^N = |N_j - x|$, and $N_j \in \{\frac{1}{3}, \frac{2}{3}\}$. A consumer chooses 3G or 4G service along with the handset specialized for the service, which gives him the maximal net utility. From consumer's choice, we can derive the demand function that carrier i faces. The way to derive the demand functions of 3G and 4G service that carrier i faces is similar to that modeled in section 2, which is described in Appendix 2. The demand functions that carrier $i \neq j \in \{A, B\}$ faces are given by

$$q_i^3(v_i, p_i, s_i) = \frac{1}{6} + \frac{1}{2t}[v - v_i - p + p_i + m - s_i], \tag{3}$$

$$q_i^4(v_i, v_j, p_i, p_j, s_i, s_j)$$
$$= \frac{1}{3} + \frac{1}{2t}[-v + 2v_i - v_j + p - 2(p_i + m - s_i) + (p_j + m - s_j)], \tag{4}$$

where $q_i^3(v_i, p_i, s_i)$ and $q_i^4(v_i, v_j, p_i, p_j, s_i, s_j)$ are, respectively, the demand function of 3G service and that of 4G service for carrier i. Equation (3) shows that the demand for 3G service of carrier i is affected by the quality level of its own 4G service and the consumer's total payment ($= p_i + m - s_i$) for using carrier i's 4G service. Interestingly, equation (4) tells us that the demand of 4G service of carrier i

[19] Assuming that the handset price is not too high, say $m < \frac{2}{3}t$, ensures the positive demands in equilibrium.

is obtained from the fact that 4G service of a carrier competes with its own 3G service and 4G service of the competing carrier. The profit function of carrier $i \neq j \in \{A,B\}$ is given by

$$\pi_i = pq_i^3(v_i, p_i, s_i) + (p_i - s_i)q_i^4(v_i, v_j, p_i, p_j, s_i, s_j) - C(v_i) \quad .$$

The sequence of events is as follows. At Date 1, carriers A and B choose the level of quality of 4G service, v_A, v_B, simultaneously. At Date 2, they determine their service rates of 4G service (p_A, p_B) and subsidy sizes (s_A, s_B) at the same time. This is due to the fact that consumers take into account not only the service rates but also the net payment for purchasing the new handset specialized for 4G service when making a decision to switch to 4G service.[20] Finally, at Date 3, consumers make their decision of whether to switch to 4G service provided by either carrier A or B, or to keep the initial 3G service. The carriers do not discount their expected profits. The subgame perfect equilibrium is adopted, and thus backward induction will be used for the analysis.

3.2. Benchmark

As a benchmark, the socially optimal quality level of 4G service will be found. Suppose that the social planner can explicitly enforce the level of quality of 4G service and the demand sizes of 3G and 4G service provided by each carrier. The social welfare (= sum of consumer surplus and the carriers' profits) function is given by

$$\begin{aligned} W = &\int_0^{\bar{x}} (v - tx)dx + \int_{\bar{x}}^{\frac{1}{3}} [v_A - m - t\left(\frac{1}{3} - x\right)]dx + \int_{\frac{1}{3}}^{\bar{y}} [v_A - m - t\left(y - \frac{1}{3}\right)]dy \\ &+ \int_{\bar{y}}^{\frac{2}{3}} [v_B - m - t\left(\frac{2}{3} - y\right)]dy + \int_{\frac{2}{3}}^{\bar{z}} [v_B - m - t\left(z - \frac{2}{3}\right)]dz + \int_{\bar{z}}^{1} [v - t(1-z)]dz \\ &- C(v_A) - C(v_B) , \end{aligned}$$

where $\bar{x}$ and $\bar{z}$ denote the consumers who are just indifferent between keeping 3G service of carrier $i \in \{A,B\}$ and switching to 4G service of carrier i with the new handset on $[0,\frac{1}{3})$ and $[\frac{2}{3},1]$, respectively, and $\bar{y}$ denotes the consumer who

[20] We can consider the model in which the carriers choose the service charges and the handset subsidy sizes sequentially. The result of this case is the same as that of the current model. This is because they do not consider the service charge and the handset subsidy level separately, but rather take them into account all together in competing with each other for the 4G service market, because consumers consider the net payment for purchasing the new handset as well as the service rate when switching to 4G service. This can be shown upon the reader's request.

is just indifferent between switching to 4G service of carrier A and to that of carrier B on $[\frac{1}{3},\frac{2}{3})$.[21]

Because technology and consumers preferences are symmetric, the optimal level of each carrier's 4G service is symmetric in that $v_A^* = v_B^* = v^* > v$, which satisfies

$$\frac{1}{3}+\frac{1}{2t}(v^* - v - m) = C'(v^*)\,. \tag{5}$$

In order to derive the welfare maximizing demand sizes of both 3G and 4G service of each carrier, note that $q_A^3 = \overline{x}$, $q_A^4 = \overline{y} - \overline{x}$, $q_B^4 = \overline{z} - \overline{y}$, and $q_B^3 = 1 - \overline{z}$. This implies that enforcing the welfare maximizing demand sizes is equivalent to determining $\overline{x}$, $\overline{y}$, and $\overline{z}$. Letting $\frac{dW}{d\overline{r}} = 0$ results in the welfare maximizing demands as follows, where $\overline{r} \in \{\overline{x}, \overline{y}, \overline{z}\}$. The welfare maximizing demand for 3G service for carrier $i \in \{A,B\}$ is $q_i^{3^*} = \frac{1}{6} - \frac{1}{2t}[v^* - v - m]$, and that for 4G service for carrier i is $q_i^{4^*} = \frac{1}{3} + \frac{1}{2t}[v^* - v - m]$, where v^* is the welfare maximizing quality level of 4G service provided by each carrier and satisfies (5). Note also that the welfare maximizing demands depend only on the quality difference between 3G and 4G service. Furthermore, note that for the welfare maximizing demands, it holds necessarily that $p_A - s_A = p_B - s_B = p = t$. In other words, for social optimum, the effective charge (= the service rates net of handset subsidy level) should be the same regardless of the kinds of service, i.e., 3G or 4G. This implies that for the social optimum, they matter the consumer's disutility from transportation costs as well as the quality difference of two kinds of services. For example, when $v^* - m > v$, it is socially desirable to have more consumers choose higher quality service although they bear some more transportation costs, which can be implemented by letting the effective charges of two services equal.

3.3. Equilibrium of Competition

Using backward induction, we begin by analyzing carriers' decisions on service rate and handset subsidy size at Date 2, given the quality level of 4G service of each carrier that was determined earlier. The profit maximization problem for carrier $i \neq j \in \{A,B\}$ is $\max_{p_i,s_i}\ pq_i^3(v_i,p_i,s_i) + (p_i - s_i)q_i^4(v_i,v_j,p_i,p_j,s_i,s_j) - C(v_i)$. Since service rate and handset subsidy level are simultaneously determined, we can take the derivative of carrier i's profit function with respect to the effective charge, $p_i - s_i$. The carrier i's reaction function is obtained by

[21] We obtain $\overline{x}$, $\overline{y}$, and $\overline{z}$ by $\overline{x} = \frac{1}{6} + \frac{1}{2t}[v - v_A - p + p_A - s_A + m]$, $\overline{y} = \frac{1}{2} + \frac{1}{2t}[v_A - v_B - p_A + p_B + s_A - s_B]$, and $\overline{z} = \frac{5}{6} + \frac{1}{2t}[-v + v_B + p - p_B + s_B - m]$, respectively, in Appendix 2.

$$p_i - s_i = \frac{1}{4}(p_j - s_j) + \frac{1}{4}(2v_i - v_j - v + 2p - m) + \frac{1}{6}t \tag{6}$$

The effective charges are strategic complements as observed usually in the price competition game. Note from equation (6) that the effective charge of carrier i is positively affected by its own service quality level, but negatively related to the competitor's service quality level. This means that carrier j's raising service quality undercuts carrier i's effective charge. The equilibrium effective charge of carrier i is

$$(p_i - s_i)^{**}(v_i, v_j) = \frac{1}{15}(7v_i - 2v_j - 5v) + \frac{1}{3}(2p - m) + \frac{2}{9}t\,. \tag{7}$$

Now we derive the equilibrium condition for carriers' quality levels from the quality competition. Substitute equation (7) into the carrier i's profit function. Then the profit function of carrier i can be rewritten as a function of v_i and v_j by $\pi_i(v_i, v_j) = pq_i^3(v_i, v_j) + (p_i - s_i)^{**}(v_i, v_j)q_i^4(v_i, v_j) - C(v_i)$.[22] Letting $\frac{d\pi_i(v_i, v_j)}{dv_i} = 0$ and $p = t$, and focusing on the symmetric equilibrium, $v_A^{**} = v_B^{**} = v^{**}$, yield the following condition for quality choice.[23]

$$\frac{89}{270} + \frac{14}{45t}(v^{**} - v - m) = C'(v^{**})\,. \tag{8}$$

Compare equation (8) with equation (5). We can see that in the quality competition, the carriers make an *underinvestment*, which is $v^* > v^{**}$. This is due to the fact that as Ma and Burgess (1993) pointed out, the price undercutting effect is detrimental to the incentives for providing quality. Equation (7) explains that the effective charge of carrier i in equilibrium is negatively affected by rival carrier j's quality enhancement investment. More specifically, a carrier's incentive for quality enhancing investments is dampened by the rival's price-undercutting reaction in the subsequent decision on the service rate and handset subsidy level, thereby leading to an underinvestment.

Proposition 3. In the case of subsidization for inter-service handset replacement with investments for service quality, the effective charge, $(p_i - s_i)^{**}$, and the quality level, v^{**}, in the subgame perfect equilibrium are given by

[22] For reference, $q_i^3(v_i, v_j) = \frac{5}{18} + \frac{1}{30t}(-8v_i - 2v_j + 10v - 5p + 10m)$ and $q_i^4(v_i, v_j) = \frac{2}{9} + \frac{1}{30t}(14v_i - 4v_j - 10v + 5p - 10m)$.

[23] For $\pi_i(v_i, v_j)$ to be concave, we need $C''(v_i) > \frac{98}{225t}$ for all $v_i > v$.

$$(p_i - s_i)^{**} = \frac{1}{3}(v^{**} - v - m) + \frac{8}{9}t \text{ , and}$$

$$\frac{89}{270} + \frac{14}{45t}(v^{**} - v - m) = C'(v^{**}) .$$

The corresponding amounts of subscription of 3G service and 4G service in equilibrium are summarized as follows.

$$q_i^3(v^{**}) = \frac{1}{9} - \frac{1}{3t}[v^{**} - v - m] ,$$

$$q_i^4(v^{**}) = \frac{7}{18} + \frac{1}{3t}[v^{**} - v - m] .$$

The assumption of $\lim_{v_i \to v+m+\frac{1}{3}t} C'(v_i) = \infty$ implies $v^{**} < v + m + \frac{1}{3}t$. So $(p_i - s_i)^{**} < p = t$, $q_i^3(v^{**}) > 0$, and $q_i^4(v^{**}) < \frac{1}{2}$. As found in section 2, even in the case in which quality choice is associated, the effective charge for 4G service is less than that for 3G. Combined with the fact that $v^{**} > v$, consumers switching to 4G service enjoy better quality and lower effective charge than that sticking to 3G service.

Note that (i) $q_i^3(v^*) < q_i^3(v^{**})$ and $q_i^4(v^*) > q_i^4(v^{**})$ if $9v^* > 6v^{**} + 3v + 3m + t$, and that (ii) $q_i^3(v^*) \geq q_i^3(v^{**})$ and $q_i^4(v^*) \leq q_i^4(v^{**})$, otherwise. It is interesting that whether the consumers switch to 4G service too much or too little compared to the social optimum is crucially affected by the difference between the optimal quality level and the equilibrium quality level under competition. Specifically, when the equilibrium quality level under competition is sufficiently small compared to the socially optimal level, switching to 4G service or the handset replacement is too inert or *under-replacement*, even though the carriers give handset subsidies to the new handset purchasers such that the effective charge of 4G service is lower than that of 3G service. More importantly, although the equilibrium quality level under competition is lower than the optimal quality level, when the difference between the two quality levels is sufficiently small, switching to 4G service or the handset replacement is excessively active or *over-replacement*. This is due to the fact that the effective charge of 4G is smaller than the 3G service rate.

3.4. Handset Subsidy Regulation

Under the handset subsidy regulation, the carriers determine the quality level and the 4G service charge sequentially, but not the handset subsidy level, because the social planner controls the latter. This implies that there are both quality competition and service charge competition. From the carrier's profit maximization, the equilibrium of service charge competition is, for carrier i, given by

$$(p_i - s_i)^{**}(v_i, v_j) = \frac{1}{15}(7v_i - 2v_j - 5v) + \frac{1}{3}(2p - m) + \frac{2}{9}t \tag{9}$$

Note that equation (9) is equivalent to equation (7). In other words, the carrier's decision for pricing 4G service under the handset subsidy regulation is equivalent to that under no regulation. This is due to the fact that from the consumer's perspective, the effective charge matters rather than the individual level of service charge and of the handset subsidy. Thus, although handset subsidies are regulated, the carrier's profit maximizing effective charge is not affected because the carrier can manipulate the service charge for its profit maximization. Furthermore, this means that the handset subsidy regulation is *irrelevant* in that it cannot affect the quality level and the consumers' handset replacement behavior at all.

Proposition 4. In the case of subsidization for inter-service handset replacement with investments for service quality, the handset subsidy regulation has no impact on the quality level for carriers and the handset replacement for consumers. That is, in equilibrium, the effective charge, $(p_i - s_t)^{**}$, and the quality level, v^{**}, under handset subsidy regulation are given by

$$p_i - s_i^{**}(v^{**}) = \frac{1}{3}(v^{**} - v - m) + \frac{8}{9}t \text{, and}$$
$$\frac{89}{270} + \frac{14}{45t}(v^{**} - v - m) = C'(v^{**}) .$$

IV. Comparison and Conclusion

It is interesting that the handset subsidy regulation is effective in the case of subsidization for intra-service handset replacement, while it has no impact in the circumstance of subsidization for inter-service handset replacement with investment for service quality. In order to see why, recall that in the former case, while carriers subsidize consumers who switch to the new style handsets, they should also determine the service charge that applies to all consumers because the consumers subscribe to the same service regardless of whether they use the old style handset or the new one. This means that the carriers should first consider service charge competition and then the handset subsidy competition, separately. Regulating the handset subsidy level is equivalent to eliminating the handset subsidy competition. Hence, the service charge competition is affected by the handset subsidy regulation.

In the latter case, since the 3G service charge is given, the 4G service charge is applied only for consumers who switch to 4G service. This implies that the carriers compete for effective charge only for consumers who plan to switch to 4G service. Thus, carriers' determining the profit maximizing effective charge for 4G service is equivalent to choosing the service charge and the handset subsidy level one by one for their profit maximization. Accordingly, the handset subsidy regulation cannot affect the profit maximizing effective charge, because the carriers can manipulate their service charge for the profit maximizing effective charge. Furthermore, no impact of the handset subsidy regulation on the profit maximizing effective charge implies that the handset subsidy regulation has no effect on the incentives for quality enhancing investments, thereby failing to correct the underinvestment problem. In addition, the handset subsidy regulation has no impact on the consumers' behavior for switching to new service.

The regulatory agency's intervention in the carriers' handset subsidization is based on the view that the carriers are apt to provide handset subsidies excessively, and thus are reluctant either to invest in quality enhancement or to lower service charges. This paper examines this issue rigorously. It argues that setting the optimal handset subsidy size for regulation can achieve the efficiency in the environment of subsidization for intra-service handset replacement. Surprisingly, the paper also shows that if the handset subsidy regulation is targeted for quality enhancement, it has no effect on the effective charge competition: hence, consumers' replacement decision and carriers' quality decision are not affected in the case of subsidization for inter-service handset replacement with investment for service quality. This result implies that in order to achieve the efficiency, the regulation on the effective charge should be considered rather than that on the handset subsidy size only.

This paper does not explicitly incorporate the decision making of handset supplier(s). Since the handset manufactures determine design or quality of handsets, it would be the future interesting research topic how the degree of differentiation of handset[24] or quality level of handsets is affected by handset subsidy regulation.

[24] This situation is very interesting because a locating entity, a handset manufacture, is not the handset subsidy provider, a carrier, whereas the existing researches focus on the case in which firms choose both locating and pricing. Cheong (2013) shows that handset subsidy regulation has no impact on the competition of handset market and concludes that no one can claim that handset subsidy should be restricted for intensifying the handset competition.

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Appendix

1. Derivation of second period demand function in the case of subsidization for intra-service handset replacement

Depending on p_i, s_i, and m, it is not straightforward to characterize the second period consumer's handset replacement decision. Fortunately, due to the fact that both the carriers' cost structure and the consumers' utility function are symmetric, we can focus on the symmetric case in which $p_A = p_B$ and $s_A = s_B$. Note that in the symmetric environment, the mobile service market in the first period is equally divided by the two carriers, where consumers located on $[0,\frac{1}{2}]$ subscribe to carrier A and the other consumers located on $(\frac{1}{2},1]$ do so to carrier B.

All consumers in the second period face the decision problem of whether they should continue to use their old style handsets or switch to the new style handsets provided by either of the two carriers. Let $x, y,$ and z denote any consumer, respectively, on $[0,\frac{1}{3})$, $[\frac{1}{3},\frac{2}{3})$, and $[\frac{2}{3},1]$. First, in the following, we describe the utility levels, which can be obtained by the handset replacement decision of a consumer who is located at x on $[0,\frac{1}{3})$ and was a carrier A subscriber in the first period. If he keeps using the old style handset in the second period, then his second period utility is given by $v - p_A - tx$. If he switches to the new style handset designed for carrier A, then his utility will become $v - p_A - (m - s_A) - t(\frac{1}{3} - x)$. If he switches to the new style handset specialized for carrier B, then his utility will be $v - p_B - (m - s_B) - t(\frac{2}{3} - x)$.[25] The similar description can be applied for determining the utility levels of a consumer on the other intervals, $[\frac{1}{3},\frac{2}{3})$, and $[\frac{2}{3},1]$. Given the service rates and handset subsidy sizes, a consumer on each interval chooses the handset that gives him the highest utility level. The consumer's utility level depending on his decision is summarized in Table 1.

We can derive the second period demand function that each carrier faces in the symmetric environment as follows. Note that a consumer with index x on $[0,\frac{1}{3})$ prefers switching to the new style handset dedicated to carrier A rather than carrier B, if he decides to switch. We can then find $\overline{x}$, which is the index or location of the consumer who is just indifferent between keeping the old style handset and switching to the new style handset on $[0,\frac{1}{3})$. Thus, any consumer with index $x \leq \overline{x}$ will keep the old one, whereas any consumer with index $x \geq \overline{x}$ on $[0,\frac{1}{3})$ will switch to the new one specialized for carrier A. If there exists a consumer who switches to carrier A's new style handset on $[0,\frac{1}{3})$, then any consumer on $[\frac{1}{3},\frac{1}{2})$ will switch to the new style handset provided by either carrier

[25] We do not consider the case in which consumers switch to the old style handset designed for carrier B.

[Table 1] Consumer's Utility in the Second Period

Interval	Choice	Utility
$[0,\frac{1}{3})$	keeping the old handset of carrier A	$v-p_A-tx$
	switching to the new handset of carrier A	$v-p_A-(m-s_A)-t(\frac{1}{3}-x)$
	switching to the new handset of carrier B	$v-p_B-(m-s_B)-t(\frac{2}{3}-x)$
$[\frac{1}{3},\frac{1}{2})$	keeping the old handset of carrier A	$v-p_A-ty$
	switching to the new handset of carrier A	$v-p_A-(m-s_A)-t(y-\frac{1}{3})$
	switching to the new handset of carrier B	$v-p_B-(m-s_B)-t(\frac{2}{3}-y)$
$[\frac{1}{2},\frac{2}{3})$	keeping the old handset of carrier B	$v-p_B-t(1-y)$
	switching to the new handset of carrier A	$v-p_A-(m-s_A)-t(y-\frac{1}{3})$
	switching to the new handset of carrier B	$v-p_B-(m-s_B)-t(\frac{2}{3}-y)$
$[\frac{2}{3},1]$	keeping the old handset of carrier B	$v-p_B-t(1-z)$
	switching to the new handset of carrier A	$v-p_A-(m-s_A)-t(z-\frac{1}{3})$
	switching to the new handset of carrier B	$v-p_B-(m-s_B)-t(z-\frac{2}{3})$

A or carrier B. A similar analysis can be applied to the consumer's choice on $[\frac{1}{2},1]$. Specifically, if consumers on $[\frac{2}{3},1]$ wish to switch to the new style handset, then they will switch to that provided by carrier B instead of carrier A. We can find $\overline{z}$, which is the index or location of the consumer who is just indifferent between keeping the old style handset and switching to the new style handset on $[\frac{2}{3},1]$. Any consumer indexed by $z \geq \overline{z}$ will keep the old one, while any consumer with index $z < \overline{z}$ on $[\frac{2}{3},1]$ will switch to the new one provided by carrier B. Any consumer on $[\frac{1}{2},\frac{2}{3})$ will switch to the new style handset provided by either carrier A or carrier B, if there exists a consumer who switches to carrier B's new style handset on $[\frac{2}{3},1]$. Thus, any consumer $y \in [\frac{1}{3},\frac{2}{3})$ will choose between carrier A and carrier B for switching to the new style handset. We can also find index $\overline{y}$ of the consumer who is just indifferent between switching to the handset designed for carrier A and to that for carrier B. On $[\frac{1}{3},\frac{2}{3})$, any consumer with index $y < \overline{y}$ will switch to the handset dedicated for carrier A, while any consumer with index $y \geq \overline{y}$ will switch to that for carrier B.

In summary, $\overline{x}$, $\overline{y}$, and $\overline{z}$ are given by $\overline{x}=\frac{1}{6}+\frac{1}{2t}[-s_A+m]$, $\overline{y}=\frac{1}{2}+\frac{1}{2t}[-p_A+p_B+s_A-s_B]$, and $\overline{z}=\frac{5}{6}+\frac{1}{2t}[s_B-m]$. Note that $q_A^O(s_A)=\overline{x}$, $q_A^N(p_A,p_B,s_A,s_B)=\overline{y}-\overline{x}$, and $q_B^O(s_B)=1-\overline{z}$. Now, we can describe the demand functions that carrier i faces in the second period.

$$q_i^O(s_i)=\frac{1}{6}+\frac{1}{2t}[-s_i+m],$$

$$q_i^N(p_i,p_j,s_i,s_j)=\frac{1}{3}+\frac{1}{2t}[-p_i+p_j+2s_i-s_j-m],$$

where $q_i^O(s_i)$ and $q_i^N(p_i,p_j,s_i,s_j)$ are, respectively, the second period demand function of the old style handset and that of the new style handset for carrier $i \neq j \in \{A,B\}$.

2. Derivation of demand function in the case of subsidization for inter-service handset replacement with investment for service quality

Let x, y, and z denote any consumer, respectively, on $[0,\frac{1}{3})$, $[\frac{1}{3},\frac{2}{3})$, and $[\frac{2}{3},1]$. Consider, for example, the choice problem for a certain consumer with the index x on $[0,\frac{1}{3})$, who is initially a carrier A subscriber of 3G service. If he keeps using 3G service from carrier A, then his utility is given by $v-p-tx$. If he switches to 4G service provided by carrier A, then his utility will turn out to be $v_A-p_A-(m-s_A)-t(\frac{1}{3}-x)$. If he switches to 4G service provided by carrier B, then his utility will turn out to be $v_B-p_B-(m-s_B)-t(\frac{2}{3}-x)$. The utility of a consumer on each interval depends on his decision: this is summarized in Table 2.

[Table 2] Consumer's Utility

Interval	Choice	Utility
$[0,\frac{1}{3})$	keeping 3G service of carrier A	$v-p-tx$
	switching to 4G service of carrier A	$v_A-p_A-(m-s_A)-t(\frac{1}{3}-x)$
	switching to 4G service of carrier B	$v_B-p_B-(m-s_B)-t(\frac{2}{3}-x)$
$[\frac{1}{3},\frac{1}{2})$	keeping 3G service of carrier A	$v-p-ty$
	switching to 4G service of carrier A	$v_A-p_A-(m-s_A)-t(y-\frac{1}{3})$
	switching to 4G service of carrier B	$v_A-p_B-(m-s_B)-t(\frac{2}{3}-y)$
$[\frac{1}{2},\frac{2}{3})$	keeping 3G service of carrier B	$v-p-t(1-y)$
	switching to 4G service of carrier A	$v_A-p_A-(m-s_A)-t(y-\frac{1}{3})$
	switching to 4G service of carrier B	$v_B-p_B-(m-s_B)-t(\frac{2}{3}-y)$
$[\frac{2}{3},1]$	keeping 3G service to carrier B	$v-p-t(1-z)$
	switching to 4G service of carrier A	$v_A-p_A-(m-s_A)-t(z-\frac{1}{3})$
	switching to 4G service of carrier B	$v_B-p_B-(m-s_B)-t(z-\frac{2}{3})$

In order to derive the demand functions that each carrier faces, we adopt exactly the same way as used in section 2. Focusing on the symmetric case in which $v_A=v_B$, $p_A=p_B$, and $s_A=s_B$, we can obtain $\overline{x}$, $\overline{y}$, and $\overline{z}$. Note that $\overline{x}$ is the index of the consumer who is just indifferent between keeping 3G service of carrier A and switching to 4G service on $[0,\frac{1}{3})$. Further, $\overline{y}$ is the one who is just indifferent between switching to carrier A and to carrier B on $[\frac{1}{3},\frac{2}{3})$, and finally, $\overline{z}$ is the one who is just indifferent between keeping 3G service of carrier B and switching to 4G service on $[\frac{2}{3},1]$. $\overline{x}$, $\overline{y}$, and $\overline{z}$ are given by $\overline{x}=\frac{1}{6}+$

$\frac{1}{2t}[v-v_A-p+p_A-s_A+m]$, $\overline{y}=\frac{1}{2}+\frac{1}{2t}[v_A-v_B-p_A+p_B+s_A-s_B]$, and $\overline{z}=\frac{5}{6}+\frac{1}{2t}[-v+v_B+p-p_B+s_B-m]$, respectively. Now we can describe the demand functions that carrier $i \neq j \in \{A,B\}$ faces as follows.

$$q_i^3(v_i, p_i, s_i) = \frac{1}{6} + \frac{1}{2t}[v - v_i - p + p_i - s_i + m],$$
$$q_i^4(v_i, v_j, p_i, p_j, s_i, s_j)$$
$$= \frac{1}{3} + \frac{1}{2t}[-v + 2v_i - v_j + p - 2(p_i + m - s_i) + (p_j + m - s_j)],$$

where $q_i^3(v_i, p_i, s_i)$ and $q_i^4(v_i, v_j, p_i, p_j, s_i, s_j)$ are, respectively, the demand function of 3G service and that of 4G service for carrier i.