The Korean Economic Review
Volume 31, Number 1, Summer 2015, 25-56.

Aggregate Instability and Fiscal Policies: Balanced Budget Rules and Productive Public Spending

Hyun Park*

This article revisits the aggregate stability property of fiscal policies with balanced budget rules using a standard one-sector growth model. As in Schmitt-Grohe and Uribe (1997) aggregate instability is driven by self-filling beliefs in the presence of multiple equilibrium paths. However, I find the existence of an indeterminate balanced growth path with a continuum of competitive equilibrium paths when productive public spending on aggregate production is endogenously financed by a predetermined rate of income taxes under balanced budget rules. In this economy, local and global indeterminacy emerges when labor supply is highly elastic and a slope of aggregate (not necessarily individual) labor demand is positive by the productivity of public spending, independent of the usual indeterminacy conditions under fiscal policies such as types of distortionary taxes, sources of government revenue, composition of government expenditure, countercyclical taxation, and predetermined public spending.

JEL Classification: E32, E62, H21, O41
Keywords: Balanced Budget Rules, Endogenous Labor Supply, Fiscal Increasing Returns, Indeterminacy, Productive Public Spending

I. Introduction

The role of fiscal policies on aggregate stability is a constant theme of academic analysis as well as a controversial topic of policy debate.[1] In addition to traditional

Received: April 22, 2014. Revised: Oct. 22, 2014. Accepted: Jan. 22, 2015.

* Professor, Department of Economics, Kyung Hee University, 001 Hoeki-Dong, Dongdaemoon-Gu, Seoul 130-701, Korea Phone: +82–2–961–9375. Fax: +82–2–966–7426. E-mail: econhyun@khu.ac.kr. I thank Joshua Aizenman, John Conlon, Jang-Ting Guo, Tatsu Haruyama, Apostolis Philippopoulos, and Carl Walsh for comments and suggestions. I also thank seminars and conference participants for invaluable discussions in University of California at Santa Cruz, Kobe University, Nagoya University and Hokkaido University, and the 2014 Conference of the Chaos and Complexity Society. This research is supported by the 2013-2014 Sabbatical Grant of Kyung Hee University and the Korea Grant (2014S1A3A2044459) of National Research Foundation by Korean Government.

[1] A few examples include the attempt to reduce the federal deficit in the Gramm–Rudman–Hollings Deficit Reduction Act of 1985 and the Balanced Budget Act of 1997 in the United States and the

arguments against usual stabilization fiscal policies,[2] recent studies have shown that some fiscal policy may destabilize a competitive economy and that self-fulfilling fluctuations can be persistent when a government follows a balanced budget rule. Specifically, expected equilibrium can be indeterminate, thereby allowing the emergence of sunspot equilibria under common fiscal policies, including balanced budget rules. In their influential paper, Schmitt-Grohe and Uribe (1997) showed that endogenous cycles emerge under balanced budget rules when combined with predetermined government expenditure financed by distortionary income taxes. Recent studies also suggest that the structure of fiscal policy—that is, countercyclical taxation, predetermined public spending, sources of government revenue, types of distortionary taxes, composition of government expenditure, and allocative and consumptive public spending—may play a critical role in the existence of indeterminacy.[3]

In the macro-policy literature, the separability assumption of the effects of taxation and government expenditure is standard; however, in practice, the revenue and expenditure decisions are interdependent (e.g., the level of public spending is affected by the marginal cost of collecting taxes). Another standard assumption is that public spending is exogenous and nondistortionary. However, clearly, some fraction of government expenditure is on investment goods. Noticeably, following Shell (1967) and Arrow and Kurz (1970), dynamic growth models with fiscal policies incorporate the allocative element of government expenditure to enhance the productivity capacity in the economy (Barro, 1990; Glomm and Ravikumar, 1997).[4] The effect of productive public spending may generate "fiscal increasing returns" to lead the aggregate level of a competitive economy exhibiting increasing returns to scale although each individual agent faces diminishing returns to each input factor in its corresponding production process. This study postulates that an understanding of the effect of fiscal policies on aggregate instability, in tandem with the distortionary effect of various taxes on individuals' activities, is necessary for properly discerning public spending productivity.

This study reconsiders the issue of aggregate instability when public spending is endogenously determined under balanced budget constraints with a predetermined rate of capital and labor income taxation in a dynamic competitive economy. I introduce an allocative fiscal policy whereby public spending is productive.

---

adoption of the EU Growth and Stability Pact in Europe. Notably, every state in the United States, except for Vermont, has balanced budget restrictions in the state constitution or in registration.

[2] See, for example, Barro (1979) and King, Plosser, and Rebelo (1988), who argued that a balanced budget rule amplifies business cycles by stimulating/reducing aggregate demand during booms/recessions through corresponding changes in tax rates and public expenditures.

[3] See Nourry, Seegmuller, and Venditti (2011), Giannitsarou (2007), Guo and Harrison (2004, 2008), and Pintus (2004), among many others.

[4] Although not discussed here, the productive role of government functions also include protecting property rights, enforcing contracts, correcting externalities, and regulating the economy. See Mueller (1989) for excellent surveys on government functions.

Productive public spending increases the total and individual factor productivity in aggregate production and endogenously stimulates short-run and long-run economic growth.[5] That is, I consider government fiscal policies that include balanced budget rules, productive public spending, and distortionary income taxes. Under these government policies, I explore the possibility of indeterminacy and aggregate instability in a growing competitive economy.

My analysis is based on the one-sector dynamic model à la Ramsey, augmented by an endogenous labor supply and fiscal policies. The economy consists of infinitely many identical households and firms and the government. The final output is produced by using private technology that exhibits the constant returns to scale in the two private inputs: capital and labor with given public capital services. The initial capital stocks are given, the labor is endogenously supplied and public capital services are financed by proportional income taxes. The government must satisfy the balanced budget rules. In what follows, I will describe each of these features of the dynamic competitive economy in details.

Although I maintain the basic framework of previous studies, to extend the aggregate instability result under balanced budget rules, my model departs from a common feature in the related literature—particularly Schmitt-Grohe and Uribe (1997)—in the following ways. First, the model incorporates productive public spending. In other words, public spending is neither exogenously wasteful nor nondistorted lump-sum transfers.[6] Second, I also assume that both capital and labor income tax rates are predetermined over time; therefore, under balanced budget constraints, government expenditures are not necessarily predetermined but rather endogenously change over time. As opposed to a countercyclical tax policy in Schmitt-Grohe and Uribe (1997), the alternative specification of balanced budget rules describes exogenous tax rates with endogenous procyclical expenditure in commonly used RBC literature.[7] Third, unlike in the usual case of external effects of production, which are not mitigated by markets (e.g., Benhabib and Farmer, 1994), the difference between the social productivity and the agent's perceived productivity of input factors is due to productive government expenditure. Forth, unlike the indeterminacy literature, for example, Turnovsky (2000) and Benhabib and Farmer (1999), I evaluate the robustness of indeterminacy with both empirically and theoretically plausible parameter values for the fundamentals, including values of the

---

[5] Some empirical studies found that fiscal spending is productive (Aschauer, 1989; Devarajan, Swaroop, and Zou, 1996; Kneller, Bleaney and Gemmell, 1999).

[6] Alternatively I could introduce consumptive and redistributive public spending with a utility function. A few studies using this setup have demonstrated a possibility of local indeterminacy. See, for example, Park and Philippopoulos (2004) explored second-best optimal policies; and Guo and Harrison (2008) focused on nongrowing economy with nonseparable felicity between private and public consumption. Cazzavillan (1996), and de Hek (1998) examined instability of a competitive equilibrium in growth models with public spending and leisure choices.

[7] See Cooly and Hansen (1992), Gali (1994), Greenwood and Huffman (1991) and many others.

productivity of public spending in close accord with production externalities to investigate their crucial roles in explaining a primary source of aggregate instability.[8]

The main finding of this study is that the slope of demand and supply in the aggregate labor market with distortive taxes and productive public spending under balanced budget rules is the primary source of existence of both local and global indeterminacy. Along with the condition on the relation between consumption and capital growth rates, local indeterminacy emerges when aggregate labor supply is elastic enough with respect to the weighted elasticity of the aggregate labor demand by the productivity of public spending. In the context of equilibrium, as in Schmitt-Grohe and Uribe (1997) and Benhabib and Farmer (1994), these indeterminacy conditions are equivalent to the upward sloping aggregate labor demand curve, which is steeper than the aggregate labor supply curve. However, in this economy, the aggregate labor demand curve can slope up, even though each individual labor demand curve slopes down, because public spending is productive and increases the marginal productivity of each individual's labor input. As in models of endogenous labor–leisure choices, this labor market equilibrium is comparable with the long-run growth in output (and, thus, consumption and capital), and the consumption–capital ratio is accompanied by an increase in equilibrium employment in a growing economy.[9]

The intuitive mechanism of the emergence of indeterminacy is following. When agent's optimistic expectation leads to higher investment and working hours, the government is forced to increase productive public expenditure and thus further increases as the final output increases given a predetermined income tax schedule under the balanced budget rules. This procyclical government expenditure, along with the positive equilibrium relation of wage-hours in the labor market, helps to fulfill the agent's initial optimistic expectations, thereby generating indeterminacy and aggregate instability. This property is different from the conjecture of Schmitt-Gorhe and Uribe (1997). Moreover, Guo and Harrison (2004) obtained the unique equilibrium and thus determinacy when government expenditure is unproductive so that an exogenous constant tax rate dampens higher anticipated rate of returns from higher investment and lowers wage rates of an equilibrium labor allocation.

Local indeterminacy means that a continuum of equilibrium paths exists for any initial stock of capital, each of which exhibits a different growth rate during the transition to the same balanced growth path. The literature shows clearly that indeterminate balanced growth paths are associated with rational expectations in which a continuum of self-fulfilling equilibrium paths exist.[10] More precisely, an

[8] See, for example, Caballero and Lyons (1992), Baxter and King (1993) and Farmer and Guo (1996).

[9] This feature of the labor market is consistent with real business cycles models of externalities. See, for example, the survey in Benhabib and Farmer (1999).

[10] The consistent co-occurrence of local indeterminacy and sunspot equilibria is well studied in dynamic general equilibrium models (Azariadis, 1981; Shigoka, 1994; Woodford, 1986).

absolutely stable long-run growth path is associated with rational expectations in which many sets of self-fulfilling beliefs exist, each of which is consistent with a dynamic equilibrium that converges to the same growth rate along its asymptotic balanced growth path but not to the same level of consumption, labor–leisure, or capital in a growing economy.[11]

I also find the condition for the existence of global indeterminacy. More precisely, I demonstrate that global indeterminacy arises under a subset of the conditions for local indeterminacy. The presence of global indeterminacy is independent with a rate of time preferences and consumption and capital growth rates. In global indeterminacy with multiple balanced growth paths, the initial level of the capital stock is irrelevant in determining whether the economy is on one of the balanced growth paths: Even in the long-run, two initially identical economies grow at different growth rates. I then show that the existence of global indeterminacy is necessary to display a locally indeterminate balanced growth path with a continuum of transitional equilibrium paths. This joint indeterminacy result suggests an additional reason why an economically productive fiscal policy under balanced budget rules can destabilize a competitive economy in the absence of shocks to economic fundamentals.

Previous studies on indeterminacy have shown that the inclusion of an endogenous labor supply introduces a potential source for the existence of indeterminate balanced growth paths. For example, Benhabib and Farmer (1994) included increasing returns and market imperfection either through externalities or monopolistic competition in a one-sector growth model with endogenous labor supply. Noticeably, their necessary condition for indeterminacy, in the presence of externalities in the competitive equilibrium with no fiscal policies, is similar to the condition for the labor market for the existence of indeterminacy in my model with allocative fiscal policies. Again, in multisector growth models (based on the human capital accumulation model in Lucas (1988)) with endogenous labor–leisure choices, the elasticities of labor supply and human and physical capital continue to be an important sources of indeterminacy (e.g., Ladron-de-Guevara, Ortigueira, and Santos, 1999).[12] In contrast with this study in which indeterminacy arises exclusively from allocative government expenditure, indeterminacy in most previous studies relies on the presence of some sort of externalities as market failure, and none explicitly considers distortive fiscal policies such as productive public services under balanced budget rules, which play a critical role in generating the wedge

[11] In a nongrowing economy, local indeterminacy, however, implies that many self-fulfilling equilibrium paths converge to the same level of a steady state equilibrium.

[12] However, introducing leisure in a utility function may not necessarily ensure the existence of indeterminacy. Indeterminacy in models with leisure depends on the specification of leisure in a utility function. For example, Ortigueira (2000) showed a determinate balanced growth path when leisure activities are augmented by human capital accumulation.

between individual and aggregate wage in the labor market.[13]

The possibility of aggregate instability in a competitive economy under fiscal policies that include balanced budget constraints has been explored in the literature. However, this study is unique because I acknowledge and account for local and global indeterminacy as it arises under predetermined tax rates and non-predetermined public spending. Productive public spending contributes to fiscal increasing returns in the aggregate production function, which, combined with an endogenous labor supply, induces the underlying structure of indeterminate balanced growth paths. Unlike this analysis, previous instability results have relied mainly on government revenue from types of distortionary taxation (e.g., Utaka, 2003; Giannitsarou, 2007), sources of government revenue (e.g., Schmitt-Grohe and Uribe, 1997), composition of expenditure (e.g., Guo and Harrison, 2014), nondistortionary and predetermined public spending (e.g., Pintus, 2004)), and countercyclical taxation and predetermined public spending (e.g., Nourry, Seegmuller, and Venditti, 2011).

The remainder of the paper is as follows. Section 2 presents the economy with fiscal policy. Section 3 characterizes a dynamic competitive equilibrium and shows the conditions for global indeterminacy. Section 4 examines the possibility of local indeterminacy, and Section 5 presents the concluding remarks.

II. The Economy

In this section I set up a one-sector closed economy with a private sector and a government sector. The private sector consists of a representative household and a representative firm, both of which act competitively. The household consumes, supplies labor elastically, and rents out its assets to the firm. The firm produces output by choosing private capital and labor and taking advantage of public production services. Economic agents are endowed with perfect foresight. The government levies income taxes on the household's capital and labor income to finance productive public production services.[14] The government also balances its budget over time. There is no uncertainty, and time is continuous with an infinite time horizon. To make the results

[13] Exceptionally, Palivos, Yip, and Zhang (2003) also explored indeterminacy in endogenous growth model with productive public spending, but their production function and tax structure are slightly more restrictive than those in this investigation. While their primary interest is in fiscal policy as a selection device among multiple steady states, the present paper demonstrates endogenous fluctuations and aggregate business cycles under balanced budget constraints.

[14] Later in the section I show that the qualitative results do not change if I use output taxes on firms or consumption tax on households. In general, output taxes are less distortionary than capital or labor income taxes or consumption taxes. In contrast with previous results (e.g., Giannitsarou, 2007; Nishimura, Nourry, Seegmuller, and Venditti, 2012) in the literature on aggregate instability, introducing a different type of distortionary taxes is not essential for instability in this model.

directly comparable to those in the recent literature, I keep the model as simple as possible.

1. The Problem of the Representative Household

The household maximizes intertemporal utility:

$$\int_0^\infty u(c,l)e^{-\rho t}dt\,, \tag{1}$$

where c is private consumption, l is labor services, and the parameter $\rho > 0$ is the rate of time preference. The instantaneous utility function $u(c,l)$ is increasing in c and decreasing in l, is twice continuously differentiable and concave in (c,l), and satisfies a constant elasticity of intertemporal substitution in (c,l) and the Inada conditions in (c,l). For simplicity, let us assume that $u(c,l)$ is additively separable in c and l and takes the functional form:

$$u(c,l) = \log c - B\frac{1}{1+\gamma}l^{1+\gamma}\,, \tag{2}$$

where $B > 0$ and $\gamma \geq 0$.[15] That is, the elasticity of intertemporal substitution for c is 1, and γ is the reciprocal of the elasticity of labor supply.

The household saves in the form of assets, denoted by k, so that it receives interest income rk, where r is the market asset return. The household also supplies elastically its labor services l, so that wage income is wl, where w is the market wage rate. It also pays capital and labor income taxes at a constant rate τ_k, $0 \leq \tau_k < 1$, and τ_l, $0 \leq \tau_l < 1$, respectively. It receives net dividends π from ownership of firms. There is no tax on dividends and no asset depreciation. Thus, the household's budget constraint is

$$c + \dot{k} = (1-\tau_k)rk + (1-\tau_l)wl + \pi\,, \tag{3}$$

where a dot over a variable denotes a time derivative, and the initial stock of assets is

[15] This instantaneous utility function is commonly used for studying stability analysis in real business cycle models (e.g., Schmitt-Grohe and Uribe, 1997; Guo and Harrison, 2004). This function is also adopted in a growing economy (e.g., Benhabib and Perli, 1994; Milesi-Ferretti and Roubini, 1998). The indeterminacy result in this study can be extended to a nonseparable utility function, as in Bennett and Farmer (2000), for example, $u(x,l) = (1-\nu)^{-1}[x(1-l)^{\theta}]^{1-\nu}$, $\nu > 0$, $\nu \neq 1$, which supports a constant labor supply in a growing balanced growth path. Intending a direct comparison in previous studies, I maintain a separable form of preferences.

given.

The household acts competitively by taking prices and income rates as given. The necessary conditions are (3) and the usual first-order conditions:

$$\dot{c} = c[(1-\tau_k)r - \rho]; \tag{4}$$

$$l = \left[\frac{(1-\tau_l)w}{Bc}\right]^{\frac{1}{\gamma}}. \tag{5}$$

The necessary conditions (3), (4), and (5) are completed with the addition of the transversality condition: $\lim_{t\to\infty} e^{-\rho t} k/c = 0$. A unique solution exists given the assumed logarithmic utility function with endogenous leisure in (2). As expected, (5) verifies that the Frisch elasticity of the labor supply is $1/\gamma$. Lansing (1998) studied in the case that the elasticity $1/\gamma$ of labor supply is infinite, i.e. $\gamma = 0$, by assuming indivisible labor supply as in Rogerson (1988) and Hansen (1985); and the case that $\gamma = 1.7$ by introducing home production in a business cycles model. Turnovsky (1990) assumed that the elasticity of labor supply, i.e. $1/\gamma = 0.5$, in the utility function with endogenous productive expenditure and endogenous labor.

2. The Problem of the Representative Firm

Firms choose private capital stock k and labor demand l, but they take public production services G as given.[16] That is to say, the flow of public services G (rather than the stock of public capital) increases the marginal product of capital and labor.[17] The production function $F(k,l;G)$ is increasing and twice continuously differentiable in (k,l,G). It also satisfies the Inada condition for (k,l,G). Public production services G are assumed to be nonrival and nonexclusive. That is, public spending G is considered a productive public good.

Following Barro (1990), Turnovsky (2000), Lansing (1989) and many others, the firm's production function is specified as

[16] Following Barro (1990) the output production function is a function of the private inputs and the public capital services in the public-finance endogenous growth models (Kneller, Bleaney, and Gemmell, 1999; Park and Philippopoulos, 2004). Moreover, the business cycles literature also adopts this production function to study a role of fiscal policies on economic fluctuations (Lansing, 1998; Raurich, 2000; Park, 2009).

[17] Alternatively, Glomm and Ravikumar (1997) used the stock of public capital, as did Baxter and King (1993) and Lansing (1998), in their calibrated real business cycle models. Because the public capital adds one more dimension to the dynamics, it is more likely to generate indeterminacy in models with public capital stocks (Raurich, 2000), which strengthens the indeterminacy result with public services in this study.

$$y = F(k,l;G) \equiv AG^{\phi}k^{\alpha}l^{\beta}\ , \tag{6}$$

where $A > 0$, $\phi > 0$, $0 < \alpha, \beta < 1$, $\alpha + \beta \leq 1$, and $\phi + \alpha \leq 1$. The condition $\alpha + \beta \leq 1$ is needed for a solution to the firm's problem in the competitive economy. Note that, ceteris paribus, an increase in G increases the marginal productivity of both inputs and that the aggregate production function exhibits overall increasing returns to scale in the three factors. Note also that the increasing returns in the aggregate technology is due to productive fiscal expenditures. As in Benhabib and Farmer (2004) the condition $\phi + \alpha = 1$ permits persistent capital accumulation in the long run and thereby persistent economic growth.[18] Similar to the present paper, Turnovsky (1990) specified the parameter values as $\phi = 0.08$, $\alpha = 0.82$, and $\beta = 0.08$,[19] while Lansing (1998) picked them with $\phi = 0.03$, $\alpha = 0.36$, and $\beta = 0.61$. With a logarithmic utility function of consumption in (2), the Cobb–Douglas production function is the only formulation of technology that is consistent with constant labor supply in a growing economy.

In detail for the productive public spending, first, Lansing's (1998) business cycles model with fiscal policies takes ϕ being equal to 0.03 which is based on the average profit ratio of in US industries in Basu and Fernald (1997). Second, Aschauer's (1989) estimates of ϕ for the productivity of public expenditure are around 0.34∼0.39 under the constant returns to scale technology with labor, private capital and public expenditure. Third, Turnovsky (1990) picked $\phi = 0.08$ in an AK-model with public capital services in an endogenously growing economy.

The representative firm acts competitively by taking prices and public services as given. It maximizes profits π given by

$$\pi = y - rk - wl\ . \tag{7}$$

The first-order conditions for k and l are, respectively,

$$r = \alpha AG^{\phi}k^{\alpha-1}l^{\beta} \tag{8}$$

and

$$w = \beta AG^{\phi}k^{\alpha}l^{\beta-1}\ . \tag{9}$$

Under the conditions on technology $F(k,l;G)$, a unique solution exists, and conditions (8) and (9) are necessary and sufficient for optimality in the firm's problem.

---

[18] Note that Benhabib and Farmer's (2004) value of α and β comprises the effects of capital and labor externalities, respectively, in the aggregate production function (Bennett and Farmer, 2000).

[19] A seemingly high value of the share of capital is because the capital stocks include human capital in an endogenous growth model.

3. The Government's Balanced Budget Constraint

Let us assume that the government balances its budget at each point in time.[20] We also assume that the total tax revenue is used only for productive public services; I examine the effect of consumptive public spending in a later discussion. Thus, using (8) and (9), the government's budget constraint is

$$G(\tau_k, \tau_l) = \tau_k rk + \tau_l wl = (\alpha\tau_k + \beta\tau_l)y \,. \tag{10}$$

In particular, I consider the case in which the income tax rate τ_k and τ_l are constant over time. Thus, public spending is not predetermined but endogenous under the balanced budget rules,[21] implying that government expenditure is procyclical. That is, $G(\tau_k, \tau_l)$ and y move together with no countercyclical tax rates.[22] Contrary to its specification in Schmitt-Grohe and Uribe (1997) and Guo and Harrison (2008), the balanced budget rule makes no expectation on income tax rate changes along with economic fluctuations.

According to the classification in IMF *Government Finance Statistics Yearbook*, the government expenditure G can include productive spending, general public spending, defense expenditure, education expenditure, health expenditure, housing expenditure, transport and communication expenditure (c.f., Devarajan, Swaroop, and Zou, 1996). Knellar, Bleaney, and Gemmell (1999) reported that an average productive spending is about 14.69% of the average GDP of 22 OECD countries with a standard deviation 4.57 during 1970-1995. They also showed that the average growth rate increases in 0.23∼0.43 as does productive government spending, thereby showing that government expenditure is not necessarily wasteful but conducive to economic growth in OECD countries. Turnovsky's (1990) calibration shows that a growth rate increases in about 0.24% when G/Y is in 0.08∼ 0.16. In sum, the above studies confirm that the productivity of public spending ϕ is neither negative nor zero as in the present paper.

This paper postulates that a specification of balanced budget rules plays an important role in affecting aggregate instability. There are two types of plausible

[20] The imposition of a period-by-period balanced budget constraint may not be very realistic, but one of the requirements for joining the European Monetary Union according to the Maastricht Treaty was that the deficit be kept below 3% of the gross national product, which required governments to meet a balanced budget at given a time limit. This imposition is not critical for the main results of my study as long as balanced budget rules are applied for some finite time intervals.

[21] Under the same condition on fiscal policies, except endogenous productive public spending and constant income taxation, Schmitt-Grohe and Uribe (1997, p. 977) claimed that endogenous business cycles cannot possibly exist; Utaka (2003) and Guo and Harrison (2004) verified their claim.

[22] Nourry, Seegmuller, and Venditti (2011) discussed in detail how procyclicality of public spending and contercyclicality of tax rates affect business cycles under balanced budget constraints.

specifications of balanced budget rules in business cycles models. The first type is implemented by adjusting tax rates given predetermined government expenditure. This induces the countercyclical tax policy under balanced budget rules (Schmitt-Grohe and Uribe, 1997; Giannitsarou, 2006). On the other hand, the second type is implemented by adjusting endogenously government expenditure along with a predetermined schedule of exogenous tax rates. This can be viewed as the procyclical expenditure policy under balanced budget rules (Guo and Harrison, 2004; Chang, Guo, Shieh, and Wang, 2014; Goken, 2006; Raurich, 2001). Under the formal specification with countercyclical tax policy, Schmitt-Grohe and Uribe (1997) showed that indeterminacy and aggregate instability appears, whereas under the later specification with procyclical expenditure policy indeterminacy disappears and thus an aggregate equilibrium path is unique and stable as in Guo and Harrison (2004) when government expenditure is lump-sum in a competitive market economy.

As in the specification of balanced budget rules in this paper, the usual analysis in real business cycles models is based on the constant tax and endogenous expenditure under balanced budget constraints. Lansing (1998, p. 354) claimed that this procyclical nature of public expenditure is not a stabilizer unlike Keynesian notion. In the U.S. data the tax rate is weakly correlated with output, while the public expenditure is highly correlated with output. Lansing's simulation illustrates that the correlation between GDP and public expenditure is 0.92. Furthermore, Ambler and Paquet's (1996) calibration also finds that the correlation is about 0.90 under exogenous tax rates.

4. Dynamics of Competitive Equilibrium Allocations

I now characterize the competitive equilibrium with any feasible fiscal policies. Under balanced budget rules with endogenous public spending and exogenous income tax rates, the fiscal policy can be fully summarized by a capital income tax rate τ_k and a labor income tax rate τ_l. First, plugging (10) into (6), the economy-wide output in a competitive equilibrium is

$$y = F(k,l;G(\tau_k,\tau_l)) = \Delta(\tau_k,\tau_l)[k^{\alpha}l^{\beta}]^{\frac{1}{1-\phi}}, \tag{11}$$

where $\Delta(\tau_k,\tau_l) \equiv \left[A(\alpha\tau_k+\beta\tau_l)^{\phi}\right]^{\frac{1}{1-\phi}} > 0$.

Notice that due to productive public spending economy-wide technology exhibits increasing returns to scale as the degree of $1/(1-\phi)$ (i.e., the last expression in (11)). When ϕ equals $1-\alpha$ for an endogenously growing economy (Barro, 1990), the output is linear in aggregate capital k; the production is a variant of linear technology in capital, augmented with the endogenous labor market.[23] The income tax rates also affect the (induced) total factor productivity $\Delta(\tau_k, \tau_l)$ and thus partially determine growth rates over time.[24]

Second, plugging (10) into (8), the return to capital is

$$r = \alpha y/k = \alpha\Delta(\tau_k, \tau_l) k^{\frac{\phi-(1-\alpha)}{1-\phi}} l^{\frac{\beta}{1-\phi}}. \tag{12}$$

Equation (12) is the return that drives private consumption/saving decisions in the competitive economy. Similarly, (9) and (10) give the wage rate as

$$w = \beta y/l = \beta\Delta(\tau_k, \tau_l) k^{\frac{\alpha}{1-\phi}} l^{\frac{\phi-(1-\beta)}{1-\phi}}. \tag{13}$$

Here, the labor demand curve (13) in the aggregate economy shows that its elasticity is $(1-\phi)/[\phi-(1-\beta)]$, given the income tax rate τ_k and τ_l, and capital stock k. Note that the individual labor demand elasticity of the individual firm equals $-1/(1-\beta)$ from (9), whereas this aggregate labor demand elasticity in the economy is weighted by the productivity ϕ of public production services G. Even though the individual labor demand curve is always downward sloping (see (9)), the positive slope of the aggregate labor demand can arise in the presence of the productive public spending $G(\tau_k, \tau_l)$. The two elasticities are identical when public production service $G(\tau_k, \tau_l)$ is absent, that is, $\phi = 0$ in (6); this wedge is due to the productive government spending. More preciously, the aggregate labor demand schedule is upward sloping when the marginal productivity of $G(\tau_k, \tau_l)$ exceeds that the residual of the share of an individual labor input (i.e., $\phi > 1-\beta$). Hence, the relative size of the productivity of public spending in private production determines the slope of private labor supply. This labor market property plays a critical role in the indeterminacy results in this economy.

---

[23] Following from endogenous growth models (Romer, 1986; Barro, 1990), private production has constant returns to scale, whereas aggregate technology exhibits increasing returns to scale. This technology ensures a persistently growing competitive equilibrium, which is socially inefficient. Following the same framework, economy-wide production exhibits increasing returns as in (11). Therefore, this modified AK-technology augmented with endogenous labor supply violates the convexity condition, and thus no efficient allocation is supported in a social planning problem (Benhabib and Farmer, 1999). Hence, I unfortunately cannot characterize the efficient allocation corresponding to the growing competitive equilibrium.

[24] My model maintains a linear technology for a growing economy; government expenditure can be financed solely by capital income taxes (or by labor income tax). Moreover, when a capital income tax rate is the same as the labor income tax rate, this income tax system is equivalent to a single tax on the final good. This model feature indicates that the income tax structure is not regarded as an indispensable source of instability of an economy.

Third, the firm's realized profit is

$$\pi = (1-\alpha-\beta)y\,. \tag{14}$$

Because this model allows that $\alpha+\beta=1$, the profit π equals zero in the equilibrium allocation. Hence, unlike in monopolistic competition models (see Benhabib and Perli, 1994), indeterminacy, if it exists, in my model does not depend on the existence of this positive profit in the failure of a competitive market.

Fourth, when the production function has decreasing returns $\alpha+\beta<1$ in the growing competitive economy $\phi+\alpha=1$, the profit is positive: $\pi=(1-\alpha-\beta)y>0$ (see (14)). Suppose that we also impose a profit tax at a constant rate $0\le\tau_\pi\le 1$; the modified household budget constraints are $c+\dot{k}=(1-\tau_k)rk+(1-\tau_l)wl+(1-\tau_\pi)\pi$ (see (3)), and the modified government balanced budget constraints are $G(\tau_k,\tau_l,\tau_\pi)=\tau_k rk+\tau_l wl+\tau_\pi\pi=(\alpha\tau_k+\beta\tau_l+(1-\alpha-\beta)\tau_\pi)y$ (see (10)). Hence, the economy-wide output is $y=\Delta(\tau_k,\tau_l,\tau_\pi)[k^\alpha l^\beta]^{1/(1-\phi)}$, where $\Delta(\tau_k,\tau_l,\tau_\pi)\equiv\left[A(\alpha\tau_k+\beta\tau_l+(1-\alpha-\beta)\tau_\pi)^\phi\right]^{1/(1-\phi)}>0$ (see (11)). Therefore, most of the results of this paper including indeterminacy, if it exists, hold under the modified the government expenditure $G(\tau_k,\tau_l,\tau_\pi)$ and the induced total factor productivity $\Delta(\tau_k,\tau_l,\tau_\pi)$.

Finally, using (12), (13), and (14) in (3), (4), and (5), the system of dynamic equations for equilibrium allocation k, c, and l in the competitive economy is

$$\dot{k}=(1-\alpha\tau_k-\beta\tau_l)\Delta(\tau_k,\tau_l)k^{\frac{\alpha}{1-\phi}}l^{\frac{\beta}{1-\phi}}-c\,; \tag{15a}$$

$$\dot{c}=c\left[\alpha(1-\tau_k)\Delta(\tau_k,\tau_l)k^{\frac{\phi-(1-\alpha)}{1-\phi}}l^{\frac{\beta}{1-\phi}}-\rho\right]; \tag{15b}$$

$$l=\left[\frac{\beta}{B}(1-\tau_l)\Delta(\tau_k,\tau_l)\frac{1}{c}k^{\frac{\alpha}{1-\phi}}\right]^{\frac{1}{\xi}}; \tag{15c}$$

where $\xi\equiv\gamma-[\phi-(1-\beta)]/(1-\phi)$.

Therefore, under the balanced budget conditions, I solve for a competitive equilibrium for any feasible economic policy as summarized by the income tax rate τ_k and τ_l and productive spending $G(\tau_k,\tau_l)$. In this equilibrium, (a) household and firm decisions are privately optimal, (b) government balanced budget constraints are satisfied in each period, (c) consumption–capital and labor markets clear, and (d) the transversality condition,

$$\alpha(1-\tau_k)\Delta(\tau_k,\tau_l)k^{\frac{\phi-(1-\alpha)}{1-\phi}}l^{\frac{\beta}{1-\phi}}-\rho<\rho\,, \tag{15d}$$

is satisfied.[25] The competitive equilibrium paths are summarized by (15a), (15b), and (15c), including the transversality condition (15d). These conditions are also sufficient for the competitive equilibrium because each individual agent's decision problem is a convex problem.

Due to productive public spending (i.e., $0<\phi<1$), we observe the difference between the social wage rate, $w^*=[1/(1-\phi)]w$ from (11) and the perceived wage rate w in (13) by the representative agent. The productivity of the aggregate labor demand in (13) differs from the productivity of the individual labor demand in (9), because the latter does not internalize the effect of public spending. Hence, these differences lead to the wedge between the wage rate of the individual labor demand and the social wage rate of the aggregate labor demand. Similarly, the same observation illustrates the wedge between the social return to capital, $r^*=[1/(1-\phi)]r$ from (11) and the perceived net return to capital r in (12). In other words, these wedges exist between the social factor returns (w^*,r^*) that determine the feasible stream of income/output in (15a) and the perceived factor returns (w,r) that drive leisure and consumption/saving decisions in (15b). Of course, (w,r) and (w^*,r^*) coincide without productive public spending (i.e., $\phi=0$) in the model.

III. Long-Run Competitive Equilibrium

An examination of the properties of a competitive equilibrium in the long run is a useful preface for the remainder of this study. Because public production services are expected to generate long-term growth, I focus on a growing balanced growth path in this section, that is, a solution in which (a) consumption and capital can grow at the same rate and (b) labor supply and demand is constant over time. Hence, I impose the condition that $\phi=1-\alpha$ and $\alpha+\beta=1$ for a balanced growth path. Then, the conditions for nonnegative stationary balanced growth can be derived in the assumptions of a logarithmic utility and a linear aggregate technology in k.[26]

Following the usual practice for long-run growing economies, I define $z\equiv c/k$.

[25] Boundedness of lifetime utility also satisfies the transversality condition. I later show that convergence of equilibrium paths to a steady state ensures the transversaility condition. Hence, in absolutely stable (or indeterminate) steady state equilibria, the transversality condition is redundant.

[26] Also see Benhabib and Farmer (1994, p. 37), who also imposed the same condition for existence of positive balanced growth equilibrium.

Accordingly, $\dot{z}/z = \dot{c}/c - \dot{k}/k$. Let $\Gamma_c(z) \equiv \dot{c}/c$ and $\Gamma_k(z) \equiv \dot{k}/k$. Thus, $\dot{z}/z = \Gamma_c(z) - \Gamma_k(z)$. Now I rewrite (15a) and (15b), combined with (15c), in terms of z:

$$\Gamma_k(z) = (1-\alpha\tau_k - \beta\tau_l)\Delta(\tau_k,\tau_l)\left[\frac{\beta}{B}(1-\tau_l)\Delta(\tau_k,\tau_l)\frac{1}{z}\right]^{\frac{\beta}{\alpha\xi}} - z\,; \tag{16a}$$

$$\Gamma_c(z) = \alpha(1-\tau_k)\Delta(\tau_k,\tau_l)\left[\frac{\beta}{B}(1-\tau_l)\Delta(\tau_k,\tau_l)\frac{1}{z}\right]^{\frac{\beta}{\alpha\xi}} - \rho\,; \tag{16b}$$

$$l = \left[\frac{\beta}{B}(1-\tau_l)\Delta(\tau_k,\tau_l)\frac{1}{z}\right]^{\frac{1}{\xi}}; \tag{16c}$$

where $\xi \equiv \gamma - [\phi - (1-\beta)]/(1-\phi)$, $\phi = 1-\alpha$ and $\alpha + \beta = 1$. The dynamics are then governed by the variables z and l. Hence, the balanced growth conditions (i.e., consumption and capital can grow at the same rate, and labor supply and demand are constant over periods) imply $\dot{z} = 0$ and $\dot{l} = 0$; thus, the long-run allocation $\tilde{z}$ and $\tilde{l}$, if it exists, satisfies $\Gamma_k(\tilde{z}) = \Gamma_c(\tilde{z})$ in (16a), (16b), and (16c). In addition, once we solve for $\tilde{z}$ in (16a) and (16b), we can also solve for $\tilde{l}$ in (16c). This algebra yields

$$\tilde{z} = \rho + (1-\alpha-\beta\tau_l)\Delta(\tau_k,\tau_l)\tilde{l}^{\frac{\beta}{\alpha}}\,; \tag{17a}$$

$$\tilde{l} = \left[\frac{\beta}{B}(1-\tau_l)\Delta(\tau_k,\tau_l)\frac{1}{\tilde{z}}\right]^{\frac{1}{\xi}}. \tag{17b}$$

Note that $\tilde{z}$ and $\tilde{l}$ are defined implicitly in (17a) and (17b), so that I cannot explicitly solve for them. However, I analytically illustrate the existence of solutions. In the next two subsections, I consider two separate cases that examine the existence and uniqueness of a long-run equilibrium, according to whether ξ is positive or negative.

1. Global Uniqueness of Balanced Growth Path

Suppose that $\xi \equiv \gamma - [\phi - (1-\beta)]/(1-\phi) > 0$ where $\phi = 1-\alpha$, and $\alpha + \beta = 1$. I show that a unique balance growth path exists. First, note that from (16a) and (16b), $\Gamma_k(z) \to \infty$ and $\Gamma_c(z) \to \infty$, respectively, as $z \to 0$, and $\Gamma_k(z) \to 0$ and $\Gamma_c(z) \to 0$, respectively, as $z \to \infty$. Second, the functions $\Gamma_k(z)$ and $\Gamma_c(z)$ are twice continuously differentiable in z. In fact, the first derivatives of $\Gamma_k(z)$ and

$\Gamma_c(z)$ are, respectively,

$$\Gamma_k'(z) = -\frac{\beta}{\alpha\xi}\frac{\Gamma_k(z)+z}{z} - 1; \tag{18a}$$

$$\Gamma_c'(z) = -\frac{\beta}{\alpha\xi}\frac{\Gamma_c(z)+\rho}{z}. \tag{18b}$$

Obviously, $\Gamma_k'(z) < 0$ and $\Gamma_c'(z) < 0$ because $\xi > 0$, $\Gamma_k(z) > 0$ and $\Gamma_c(z) > 0$. That is, $\Gamma_k(z)$ and $\Gamma_c(z)$ are decreasing in z. Third, the second derivatives of $\Gamma_k(z)$ and $\Gamma_c(z)$ are, respectively,

$$\Gamma_k''(z) = \frac{\beta(1+\gamma)}{\alpha\xi^2}\frac{\Gamma_k(z)+z}{z^2}; \tag{19a}$$

$$\Gamma_c''(z) = \frac{\beta(1+\gamma)}{\alpha\xi^2}\frac{\Gamma_c(z)+\rho}{z^2}. \tag{19b}$$

Clearly, $\Gamma_k''(z) > 0$ and $\Gamma_c''(z) > 0$. Thus, $\Gamma_k(z)$ and $\Gamma_c(z)$ are convex in z. In addition, because the coefficient of $\Gamma_k(z)$, $(1-\alpha\tau_k-\beta\tau_l) > 0$ in (16a), is larger than the coefficient of $\Gamma_c(z)$, $\alpha(1-\tau_k) > 0$ in (16b), clearly $|\Gamma_k''(z)| > |\Gamma_c''(z)|$ for each z; therefore, $\Gamma_k(z)$ is more convex than $\Gamma_c(z)$. Fourth, combining with these prosperities for $\Gamma_k(z)$ and $\Gamma_c(z)$, the unique interior solution exists, say $\tilde{z}_1$, for $\Gamma_k(\tilde{z}_1) = \Gamma_c(\tilde{z}_1)$. Figure 1 illustrates this steady state solution $\tilde{z}_1$. Finally, by using the solution $\tilde{z}_1$, equation (17b) yields the unique solution $\tilde{l}_1$. Hence, the unique balanced growth path $(\tilde{z}_1, \tilde{l}_1)$ exists whenever $\xi > 0$.

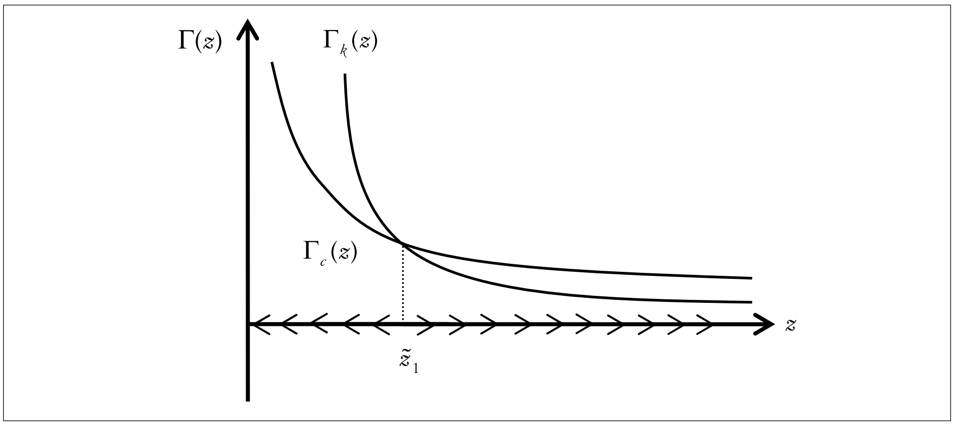
I summarize the previous argument as follows.

Proposition 1: *In a competitive economy with government fiscal policies with productive public spending $G(\tau_k, \tau_l)$, a fixed capital income tax rate τ_k, and a fixed labor income tax rate τ_l, the unique balanced growth path (c, k, l) exists under the balanced budget rules $G(\tau_k, \tau_l) = (\alpha\tau_k + \beta\tau_l)y$ when $\xi \equiv \gamma - [\phi - (1-\beta)]/(1-\phi) > 0$ with $\phi = 1-\alpha$ and $\alpha + \beta = 1$.*

The implication of the parameter values for the uniqueness of balanced growth is self-evident in the case when $\xi > 0$: Referring to (5), the slope of the labor supply curve is always positive because γ is nonnegative, whereas (13) shows that the sign of $\phi - (1-\beta)$ determines whether the labor demand curve has a positive or negative slope. When $\phi - (1-\beta) < 0$, the labor demand has a negative slope, which also implies that $\xi > 0$. Therefore, the labor market condition leads to the conclusion that a positive value of ξ is guaranteed when the slope of the labor demand curve is negative, because the labor supply curve is positively sloped.

However, note that even with a positive slope of the labor demand (i.e., $\phi-(1-\beta)>0$), the balanced growth path is unique as long as $1/(1-\phi)<(1+\gamma)/\beta$. Therefore, when the productivity $1/(1-\phi)$ of public spending is small, the labor market condition determines the uniqueness of a balanced growth path.[27] The aggregate technology with no productive public spending (i.e., $1/(1-\phi)=1$) ensures the negative slope of the labor demand and thereby a unique steady state as in a neoclassical growth model.

[Figure 1] $\xi>0$



2. Global Indeterminacy of Balanced Growth Paths

Suppose that $\xi\equiv\gamma-[\phi-(1-\beta)]/(1-\phi)<0$ where $\phi=1-\alpha$, and $\alpha+\beta=1$. In this case, I establish that either a balanced growth path does not exist or, more likely, two such paths exist. Once again, using the previous calculations as in (18a), (18b), (19a), and (19b), we can characterize the equations of $\Gamma_k(z)$ and $\Gamma_c(z)$. First, note that from (16a), $\Gamma_k(z)\to 0$ as $z\to 0$ and $\Gamma_k(z)\to\infty$ as $z\to\infty$. Also, from (18a), $\Gamma_k(z)$ decreases and then increases when z increases.

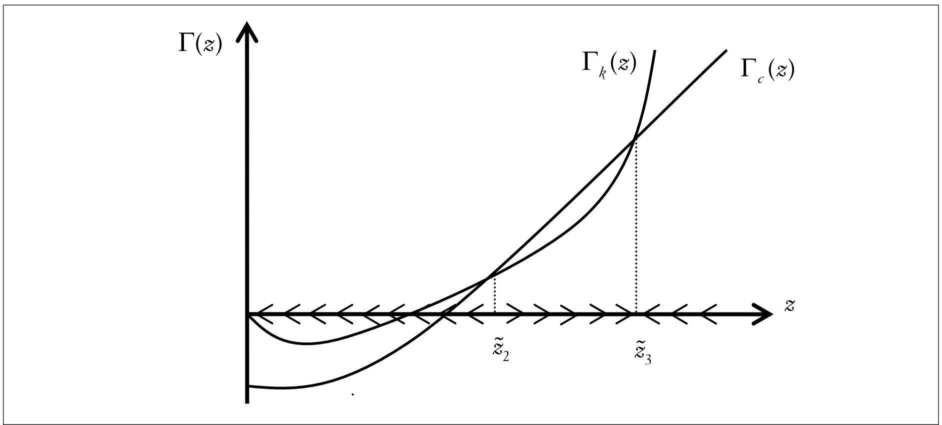
Second, from (16b), $\Gamma_c(z)\to-\rho$ as $z\to 0$ and $\Gamma_c(z)\to\infty$ as $z\to\infty$. Now, from (18b), $\Gamma_c(z)$ increases when z increases. Third, the second derivatives of $\Gamma_k(z)$ and $\Gamma_c(z)$ in (19a) and (19b), respectively, yield $\Gamma_k''(z)>0$ and $\Gamma_c''(z)>0$, respectively, for all z; that is, both functions are convex in z. Again, because $|\Gamma_k''(z)|>|\Gamma_c''(z)|$ for all $z\geq 0$, $\Gamma_k(z)$ is more convex than $\Gamma_c(z)$. Therefore, it follows that either no interior solution exists or two interior solutions exist, say $\tilde{z}_2$ and $\tilde{z}_3$.[28] Figure 2 illustrates this argument. Finally, by

[27] Refer to the aggregate production function in (11).

[28] Strictly speaking, when $\Gamma_c(z)$ is tangent to $\Gamma_k(z)$, a unique solution, say, $\tilde{z}_4$, exists to (16a) and (16b). In this case, $\tilde{z}_4=\beta\rho[\alpha(1+\gamma)]^{-1}$. Substituting $\tilde{z}_4$ into (17b) gives a unique level of employment: $\tilde{l}_4=\left[[\alpha(1+\gamma)/\beta\rho](1-\tau_l)\Delta(\tau_k,\tau_l)\right]^{1/\xi}$. In the next section I show that the bifurcation

using (17b), solutions $\tilde{z}_2$ and $\tilde{z}_3$ allow us to solve $\tilde{l}_2$ and $\tilde{l}_3$, respectively. Then, if any balanced growth paths exist, there are, at most, two: $(\tilde{z}_2,\tilde{l}_2)$ and $(\tilde{z}_3,\tilde{l}_3)$. Therefore, the economy in which $\xi<0$ is most likely to display global indeterminacy.

[Figure 2] $\xi<0$



Therefore, I establish the existence of multiple balanced growth paths as follows.

Proposition 2: *In a competitive economy with government fiscal policies with productive public spending* $G(\tau_k,\tau_l)$*, a fixed capital income tax rate* τ_k*, and a fixed labor income tax rate* τ_l*, no interior balanced growth path* (c,k,l) *exists or, at most, two balanced growth paths exist. More specifically, when* $\xi\equiv\gamma-[\phi-(1-\beta)]/(1-\phi)<0$*,* $\phi=1-\alpha$*, and* $\alpha+\beta=1$*, two balanced growth paths exist if there exists* $\bar{z}\geq 0$ *such that* $\Gamma_c(\bar{z})>\Gamma_k(\bar{z})$*; otherwise, no interior balanced growth path exists under the balanced budget rules* $G(\tau_k,\tau_l)=(\alpha\tau_k+\beta\tau_l)y$.

Global indeterminacy suggests that economic fundamentals cannot determine whether the economy will have a lower or higher balanced growth path. This model is different from the usual multiplicity of steady states for which the equilibrium is unique once the initial conditions are given. More specifically, the globally indeterminate balanced growth paths entail that, given any initial level of capital k, nothing places the economy on either of the two balanced growth paths, $(\tilde{z}_2,\tilde{l}_2)$ and $(\tilde{z}_3,\tilde{l}_3)$. In fact, the level of the initial capital stock is irrelevant; rather, critical to pinning down a realized balanced path is the choice of either the initial consumption or the initial leisure–labor. That is, the free initial consumption c and labor–leisure l must be chosen in such a way that puts the economy on the

emerges at this balanced growth path $(\tilde{z}_4,\tilde{l}_4)$.

stable manifold of either one of the two balanced growth paths, $(\tilde{z}_2, \tilde{l}_2)$ and $(\tilde{z}_3, \tilde{l}_3)$. The property of the associated stable manifold for each balanced growth path is studied in the next section.

As previously shown, the existence of two balanced growth paths is summarized by the labor market condition and productive public spending under the balanced budget rules. Specifically, when $\xi \equiv \gamma - [\phi - (1-\beta)]/(1-\phi) < 0$ where $\phi = 1-\alpha$, and $\alpha + \beta = 1$, the slope of labor demand should be positive (i.e., $[\phi - (1-\beta)]/(1-\phi) = (\beta - \alpha)/\alpha > 0$). Therefore, labor demand with a positive slope is a necessary (but, not a sufficient) condition for the global indeterminacy. In sum, the positive slope of the labor demand is possible only when public spending is productive (i.e., $\phi \neq 0$) in the aggregate economy. The labor market condition is also compatible with the set of parameter values taken by growth models in previous works. For example, as in Farmer and Guo (1996) and Caballero and Lyons (1992), the set of conventional parameter values $\gamma = 0.25$, $\alpha = 0.3$, $\beta = 0.7$, and $\phi = 0.7$ satisfies $\xi < 0$.

These parameter values are also consistent with the indeterminacy condition on the effect of the productivity of government expenditure $1/(1-\phi)$ in the aggregate technology in (11). For example, even though their increasing returns in the aggregate production come from external effects of input factors, Benhabib and Farmer's (1994, p. 30) indeterminacy also requires that the effect of their externalities exceeds 2.49, which also satisfies the condition that $1/(1-\phi) > 2.49$ in the set of the previous parameters including $\phi = 1-\alpha = 0.7$. Also, when the elastic labor supply is infinite (i.e., $\gamma = 0$; Hansen, 1985) and public spending is productive enough (i.e., $\phi > 0.5 > 1-\beta$; Caballero and Lyons, 1992),[29] the parameter values satisfy both my indeterminacy condition as well as the Benhabib and Farmer's indeterminacy condition. However, these parameter values yield unrealistically high interest rates and thus long-run growth rates. This feature is often recognized in endogenous growth models where a productive public good is an engine of growth in the aggregate AK-production function (e.g., Barro, 1990; Turnovsky, 2000; Raurich, 2000).[30]

Under a fiscal policy that government spending is endogenously financed by a predetermined constant tax rate, Schmitt-Gorhe and Uribe (1997) conjectured and Guo and Harrison (2004) showed that the economy displays saddle stability and uniqueness under balanced budget rules. Therefore, the policy recommendation of

---

[29] Baxter and King (1993) estimated that an elasticity of aggregate production in the United States is around 1.05, which is much smaller than one with the parameters of this study. But their estimates are based on the definition that the productive public spending is regarded as a capital stock, not a flow as in this study. Hence, their estimates are not directly applicable to mine.

[30] To improve numerical values of the economy, we may add some variables in the model (e.g., capital depreciation, population changes, weighted public services, etc.), but, as I mentioned previously, that is not my main purpose.

the balanced budget rules with countercyclical tax policy is to limit the government's ability to change tax rates to stabilize the economy against self-fulfilling business cycles fluctuations. On the contrary, the present paper demonstrates that balanced budget rules with the limiting tax changes and procyclical expenditure generate local and global indeterminacy, thereby causing aggregate instability in a perfect foresight competitive economy when the government spending is productive and the wage-hours in the aggregate labor market are positively related in equilibrium. Hence, unlike usual policy recommendation (Schmitt-Grohe and Uribe, 1997, p. 978), balanced budget rules with restricting the government's ability of changing tax rates does not rule out the emergence of endogenous fluctuations in perfect foresight general equilibrium models.

Previous studies have offered various interpretations for an upward sloping labor demand curve. For example, in Schmitt-Grohe and Uribe (1997), the labor demand curve is upward sloping because increasing aggregate employment is accompanied by decreasing income tax rates under balanced budget constraints. Benhabib and Farmer (1994) and Benhabib and Perli (1994) introduced externalities both in capital and labor markets, and their positively sloped labor demand curves are due mainly to the presence of sufficiently large externalities in the labor market. In my model, by contrast, equilibrium employment is upward sloping because the productive public spending increases the marginal productivity of the labor input in the aggregate production. In general, the allocative government policy invokes "fiscal increasing returns" with various income taxes under balanced budget constraints, thereby induces increasing the marginal productivity in the labor market and, thus, increasing returns to scales as in a growing economy.[31]

However, the nonstandard slopes of the labor demand curves may come under criticism. A few studies have demonstrated indeterminacy under a conventional labor market condition by introducing more than one sector in the economy. For example, in two-sector models with a standard (i.e., negative) slope of the labor demand, indeterminacy can arise when at least one sector in production processes exhibits increasing returns and externalities (see the survey in Benhabib and Farmer, 1999; Ortigueira, 2000). Despite this improvement over the nonstandard labor market condition in one-sector models, two-sector models often require a nonconvex aggregate production for indeterminacy, and, therefore, can collapse to a one-sector model with increasing returns technology.[32] Bennet and Farmer (2000)

[31] Also note that for explaining a role of fiscal policies on business cycles, Blanchard and Summer (1987) and Velasco (1996) introduced the notion of fiscal increasing returns in a labor market and in a capital market, respectively.

[32] See Benhabib and Perli (1994), who faced a difficulty to illustrate implications of indeterminacy in a multisector model and then reduced its dimensions to a corresponding one-sector model including endogenous labor supply. Nevertheless, the degree of increasing returns is less stringent for existence of

suggested a high cross elasticity between consumption and leisure in nonseparable felicity function but required the Frisch labor supply function to have a negative slope for indeterminacy. Thus, the existence of indeterminacy in this economy with fiscal policies offers an alternative justification for adopting the one-sector model with an endogenous labor supply.

3. Existence of Balanced Growth Paths

The existence analysis implicitly assumes that all balanced growth rates are strictly positive. A slight digression specifies the condition for positive long-run growth and makes welfare comparison across balanced growth paths in the long run. First, from (15b), nonnegative balanced growth, $\dot{k}/k = \dot{c}/c \geq 0$, holds if and only if the long-run growth rate $\Gamma = \alpha(1-\tau_k)\Delta(\tau_k, \tau_l)\tilde{l}^{\beta/\alpha} - \rho$ is nonnegative. By using both (17a) and (17b), the persistent growth condition follows immediately:[33]

$$\tilde{z} \geq \frac{\rho}{\alpha}\left[\frac{1-\alpha\tau_k - \beta\tau_l}{1-\tau_k}\right]. \tag{20}$$

Again in (15b), the long-run growth rate is positively related to the stationary labor allocation $\tilde{l}$, and when $\xi < 0$, $\tilde{l}$ is also positively related to $\tilde{z}$ in (17b). Combining these relations shows that the growth rate increases in $(\tilde{z}, \tilde{l})$.

I summarize the existence condition for positive long-run growth and welfare comparison across balanced growth paths in the following proposition:[34]

Proposition 3: *Under the conditions in Proposition 1 and 2, consumption c and capital k grow without a finite limit in the long run, so the long-run growth rate is nonnegative if and only if $\tilde{z} \geq \frac{\rho}{\alpha}[\frac{1-\alpha\tau_k-\beta\tau_l}{1-\tau_k}]$. Moreover, when $\xi \equiv \gamma - [\phi - (1-\beta)]/(1-\phi) < 0$ with $\phi = 1-\alpha$ and $\alpha + \beta = 1$, the growth rate of $(\tilde{z}_3, \tilde{l}_3)$ is higher than the growth rate of $(\tilde{z}_2, \tilde{l}_2)$ where $\tilde{z}_3 > \tilde{z}_2$ and $\tilde{l}_3 > \tilde{l}_2$.*

Proposition 3 reports that a higher consumption–capital ratio (i.e., $\tilde{z}_3 > \tilde{z}_2$) is positively associated with a higher level of employment (i.e., $\tilde{l}_3 > \tilde{l}_2$ in (17b)) in the long run and, therefore, with higher growth rates (i.e., (16a), (16b) and (16c)).[35]

---

indeterminacy in multisector models (see Benhabib and Farmer, 1999).

33 Alternatively, we can use (15a) for obtaining the same condition as in (20). We also notice that the tax rates are critical to ensure the positive long-run growth condition from $\Delta(\tau_k, \tau_l)$.

34 Although beyond the scope of this study, the ability for welfare comparison to alternative equilibria may allow us to design a select mechanism for the Pareto-superior equilibrium (see e.g., Palivos, Yip, and Zhang, 2003).

35 Unfortunately, I cannot compare the growth rate of $(\tilde{z}_1, \tilde{l}_1)$ in the case of $\xi > 0$ to the growth

The proposition makes intuitive sense when the (positive) slope of the labor demand curve exceeds the (positive) slope of the labor supply curve: Decreasing the stock of capital, which shifts the labor demand curve downward, tends to push the real wage rate higher and increase the level of employment in this case that $\xi<0$. Thus, under balanced budget rules with constant income tax rates, the higher income tax revenue increases productive public spending. As a consequence, the total factor productivity increases along with final output and, consequently, consumption. Hence, we have a positive relation between the consumption–capital ratio and the level of employment in the long run.

4. Uniqueness of a Steady State in a Nongrowing Economy

In this subsection, we briefly examine a nongrowing economy. Under $\phi+\alpha<1$, $0<\phi<1$, and $0<\alpha<1$, the competitive economy is not persistently growing because the market interest rate decreases in the long run (see (12)). As in Section 2, the system of dynamic equations for equilibrium allocation k, c, and l in the competitive economy satisfies (15a)$\sim$(15c). In the long run, a steady state path $\tilde{k}$, $\tilde{c}$, $\tilde{l}$ satisfies that $\dot{k}=\dot{c}=\dot{l}=0$. A simple observation shows that the balanced growth path is unique. That is, combining the steady state conditions $\tilde{c}=(1-\alpha\tau_k-\beta\tau_l)\Delta(\tau_k,\tau_l)\tilde{k}^{\alpha/(1-\phi)}\tilde{l}^{\beta/(1-\phi)}$ in (15a) and $\tilde{l}=[[\beta/B](1-\tau_l)\Delta(\tau_k,\tau_l)\tilde{c}^{-1}\tilde{k}^{\alpha/(1-\phi)}]^{1/\xi}$ in (15c) yields the unique $\tilde{l}$. Once the steady state labor $\tilde{l}$ is unique, we are also able to pin down the unique $\tilde{k}$ in (15b), $\alpha(1-\tau_k)\Delta(\tau_k,\tau_l)\tilde{k}^{(\phi-(1-\alpha))/(1-\phi)}\tilde{l}^{\beta/(1-\phi)}=\rho$. Again, by using (15a), we find that the steady state consumption $\tilde{c}$ is also unique. Therefore, the competitive steady state $\tilde{k}$, $\tilde{c}$, $\tilde{l}$ is unique; thereby, the nongrowing balanced growth path is globally determinate. This uniqueness property implies that a perpetually growing economy is necessary for global indeterminacy under productive public services and the balanced budget rules.

IV. Local Indeterminacy

In this section I investigate the possibility of local indeterminacy for competitive equilibrium paths with feasible fiscal policies. The dynamic system (16a) and (16b), combined with (16c), provides sufficient information on local indeterminacy by establishing the condition for the absolute stability property of the long-run equilibrium path. Recall that the consumption–capital ratio $z\equiv c/k$ is not a predeterminant variable. In this one-dimensional model, we can compute an

rate of either $(\tilde{z}_2,\tilde{l}_2)$ or $(\tilde{z}_3,\tilde{l}_3)$ in the case of $\xi<0$.

eigenvalue of the 1×1 Jacobian matrix of (16a)~(16c), evaluated at each balanced growth path of (17a) and (17b).

Hence, on the one hand, if the eigenvalue is positive in the dynamic system, the balanced growth path is locally saddle unstable; therefore, given the initial capital stock, we must choose a unique consumption and leisure level at which the equilibrium path leads to immediate convergence to the balanced growth path. In this case, the economy is on the unstable manifold of a balanced growth path. That is, the balanced growth path is determinate.

On the other hand, when the eigenvalue is negative, we can choose the set of infinitely many initial consumption and leisure levels at the beginning of time, all of which satisfy the optimality conditions for a competitive equilibrium. In this case, the economy is on the attracting or stable manifold of a balanced growth path. Therefore, a balanced growth path is absolutely stable and local indeterminacy emerges. The local indeterminacy for the growing economy means that many sets of self-fulfilling beliefs exist—each of which is consistent with a dynamic equilibrium path that converges to the same growth rate of consumption and capital but not to the same level of consumption, leisure, capital, and employment.

Formally, to determine the sign of the eigenvalue of the dynamic equations (16a) and (16b), combined with (16c), I linearize the $\dot{z}-$ equation around a balanced growth path $(\tilde{z},\tilde{l})$ in (17a) and (17b). Simple algebra yields

$$\Theta \equiv \frac{d\dot{z}(\tilde{z},\tilde{l})}{dz} = \tilde{z} + \frac{\beta}{\alpha\xi}\left[\tilde{z}-\rho\right]. \tag{21}$$

Therefore, $\Theta>0$ if and only if a balanced growth path is (locally) unstable and therefore unique; $\Theta<0$ if and only if a balanced growth path is (locally and absolutely) stable and therefore indeterminate. Moreover, because $\tilde{z}-\rho>0$ from (17a) the sign of Θ depends on the sign of ξ.

First, consider the case that $\xi>0$. Obviously, we always have $\Theta>0$ when $\xi>0$. In fact, this is the first case in the previous section in which the steady state $(\tilde{z}_1,\tilde{l}_1)$ is a globally unique balanced growth path (Proposition 1). Therefore, the unique positive balanced growth path $(\tilde{z}_1,\tilde{l}_1)$ is locally unstable, along with the positive growth condition $\tilde{z}\geq\rho\alpha^{-1}(1-\alpha\tau_k-\beta\tau_l)/(1-\tau_k)$ (see Proposition 3). The initial level of consumption and leisure should be chosen on the stable branch so that the associated equilibrium path converges to the balanced growth path $(\tilde{z}_1,\tilde{l}_1)$, but all other paths from other initial levels of consumption and leisure diverge and violate the transversality condition. This instability of the balanced growth path is depicted in terms of $\Gamma_k(z)$ and $\Gamma_c(z)$ in Figure 1.[36] Hence, a

[36] Clearly, when $\dot{z}/z=\Gamma_c(z)-\Gamma_k(z)>0$, z increases; otherwise, z decreases.

unique equilibrium path exists from a uniquely chosen level of consumption and leisure in the balanced growth path. That is, any transitional dynamic path is vanishing; therefore, there are no transitional dynamics. Hence, $(\tilde{z}_1, \tilde{l}_1)$ is locally determinate.[37]

Now consider the second case in which $\xi < 0$. Recall that in this case two balanced growth paths most likely exist (Proposition 2). Again, by observing (21), we find Θ is strictly positive if and only if $\tilde{z} < \rho\beta\left[\alpha\xi + \beta\right]^{-1}$, whereas Θ is strictly negative if and only if $\tilde{z} > \rho\beta\left[\alpha\xi + \beta\right]^{-1}$.[38] Therefore, along with the positive long-run growth condition $\tilde{z} \geq \rho\alpha^{-1}(1 - \alpha\tau_k - \beta\tau_l)/(1 - \tau_k)$ in Proposition 3, $\tilde{z} > \rho\beta\left[\alpha\xi + \beta\right]^{-1}$ guarantees the local indeterminacy of a balanced growth path. Otherwise, a balanced growth path is determinate.[39]

Next, I examine whether $(\tilde{z}_2, \tilde{l}_2)$ and $(\tilde{z}_3, \tilde{l}_3)$ satisfy the inequality condition $\tilde{z} > \rho\beta\left[\alpha\xi + \beta\right]^{-1}$ for indeterminacy. First, (18a) and (18b) clearly show that at the balance growth path $(\tilde{z}_2, \tilde{l}_2)$, the slope of $\Gamma_c(\tilde{z}_2)$ is steeper than the slope of $\Gamma_k(\tilde{z}_2)$, as also indicated in Figure 2. It is equivalent to the inequality condition $\tilde{z}_2 < \rho\beta\left[\alpha\xi + \beta\right]^{-1}$. This slope condition for $\tilde{z}_2$ coincides with the previous analysis on the condition for determinacy that Θ is strictly positive. It follows that $(\tilde{z}_2, \tilde{l}_2)$ is locally determinate: Given the initial level of capital, a unique dynamic equilibrium path exists from a uniquely chosen level of consumption and leisure in the neighborhood of the balanced growth path $(\tilde{z}_2, \tilde{l}_2)$.

On the other hand, at the balanced growth path $(\tilde{z}_3, \tilde{l}_3)$, (18a) and (18b) imply that the slope of $\Gamma_c(\tilde{z}_3)$ is less steep than the slope of $\Gamma_k(\tilde{z}_3)$. It is equivalent to the inequality condition $\tilde{z}_3 > \rho\beta\left[\alpha\xi + \beta\right]^{-1}$. This condition is also identical to the condition that Θ is strictly negative when $\xi < 0$. Hence, the balanced growth path $(\tilde{z}_3, \tilde{l}_3)$ is locally indeterminate. Thus, given the initial level of capital, a continuum of equilibrium paths exist, each of which—from each initial level of consumption and leisure in the neighborhood of the balanced growth path $(\tilde{z}_3, \tilde{l}_3)$—converges to $(\tilde{z}_3, \tilde{l}_3)$. That is, two balanced growth paths exist: one is locally determinate and the other is locally indeterminate. Absolute Stability of the two balanced growth paths is illustrated in terms of $\Gamma_k(z)$ and $\Gamma_c(z)$ in Figure 2.

I summarize the local indeterminacy properties as follows.

---

[37] Note that in the one-dimensional system of equations, the stable branch consists of the single point of the balanced growth path and the transitional path becomes trivial.

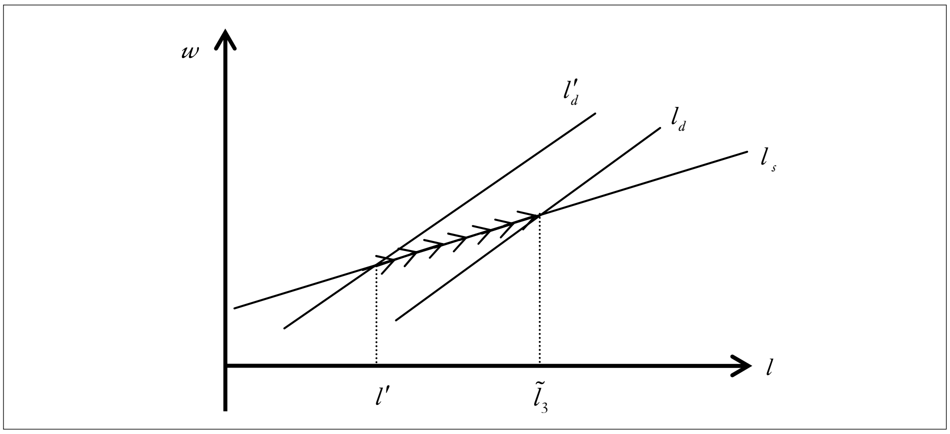
[38] Because $\tilde{z}$ is an endogenous variable, these conditions do not explicitly characterize indeterminacy. However, by using (17a, 17b), we can implicitly pin down a balanced growth path $(\tilde{z}, \tilde{l})$ for indeterminacy.

[39] In addition, when $\Theta = 0$, bifurcation emerges at, say, $\hat{z}$, $\hat{z} = \rho\beta\left[\alpha\xi + \beta\right]^{-1}$. Noticeably, when a balanced growth path is unique in the case that $\xi < 0$, the bifurcation point $\hat{z}$ emerges at the unique balanced growth path $(\tilde{z}_4, \tilde{l}_4)$, which, as previously noted, is solved as follows. Strictly speaking, when $\Gamma_c(z)$ is tangent to $\Gamma_k(z)$, a unique solution, say, $\tilde{z}_4$, exists to (16a) and (16b). In this case, $\tilde{z}_4 = \beta\rho[\alpha(1+\gamma)]^{-1}$. Substituting $\tilde{z}_4$ into (17b) gives a unique level of employment: $\tilde{l}_4 = \left[[\alpha(1+\gamma)/\beta\rho](1-\tau_l)\Delta(\tau_k, \tau_l)\right]^{1/\xi}$.

Proposition 4: *Under the conditions in Proposition 1, 2, and 3, the balanced growth path* $\tilde{z}_1$ *is locally determinate if and only if* $\xi \equiv \gamma - [\phi - (1-\beta)]/(1-\phi) > 0$ *with* $\phi = 1-\alpha$ *and* $\alpha + \beta = 1$. *On the other hand, when* $\xi \equiv \gamma - [\phi - (1-\beta)]/(1-\phi) < 0$ *with* $\phi = 1-\alpha$ *and* $\alpha + \beta = 1$, *two balance growth paths exist: one locally determinate and one locally indeterminate. Specifically, the lower growth balanced path* $(\tilde{z}_2, \tilde{l}_2)$ *is locally determinate if* $\tilde{z}_2 < \rho\beta\left[\alpha\xi + \beta)\right]^{-1}$, *whereas the higher balanced growth path* $(\tilde{z}_3, \tilde{l}_3)$ *is locally indeterminate if* $\tilde{z}_3 > \rho\beta\left[\alpha\xi + \beta\right]^{-1}$.

The mechanism of local indeterminacy is as follows: Consider z', $z' > \tilde{z}_3$. Then, $\Gamma_k(z') > \Gamma_c(z')$ in the neighbor of $\tilde{z}_3$. That is, the capital stock grows unevenly faster than consumption does in $\tilde{z}'$. Recall that, under the condition that $\xi < 0$, the labor demand function has the positive slope. The level of employment and the wage rate are lower in z' than in $\tilde{z}_3$ because the labor demand function shifts upward to l_d' from l_d by an increase in the capital stock. The fiscal increasing returns to productive public spending induce the positive equilibrium relation between employment and wage. Figure 3 depicts that the labor demand shift leads to the employment from $\tilde{l}_3'$ to l' in the labor market, corresponding to $\tilde{z}_3$ and z', respectively. As consequence, the consumption growth in z' tends to increase the real wage and employment. Along the equilibrium trajectory, the final good increases, and, eventually, the consumption–capital ratio also is accompanied by the growth of employment. Therefore, the increase in the ratio of consumption–capital reverses the process by the gradual labor demand shift back to l_d as in Figure 3. This process illustrates the absolute stability in $\tilde{z}_3$, and, thereby, local indeterminacy.

[Figure 3] Absolute Stability in the Labor Market



The conditions for global indeterminacy continue to play an important role in local indeterminacy. The balanced growth path is locally determinate, and the equilibrium path is unique in the economy with global determinacy, whereas local indeterminacy emerges for one of the two balanced growth paths in the economy with global indeterminacy.[40] The sensitivity analysis for local indeterminacy also involves the same parameter space with the rate of time preferences ρ as for the existence of global indeterminacy. The set of parameter values α, β, γ, and ϕ for local indeterminacy shows that the labor market condition and the productivity of public spending are crucial. The analysis in the previous section on the elasticity of labor supply $1/\gamma$ and the weighted elasticity $(1-\phi)/[\phi-(1-\beta)]$ of the labor demand affected by productive public spending in aggregate production also applies to the analysis of local indeterminacy.[41] The more elastic the labor supply and the less elastic the (positive-sloped) labor demand, the more likely that local indeterminacy will appear. In the context of equilibrium, the larger productivity of public spending $1/(1-\phi)$, the more elastic the labor supply, and the larger the labor input share β in aggregate production, the more likely that the indeterminate balanced growth path exists.[42] These parameters for indeterminacy indicate the tradeoff between the elasticity of labor supply and the elasticity of labor demand along with allocative fiscal policy. As in neoclassical growth models, the long-run equilibrium is unique when either the labor supply or the labor demand is inelastic (i.e., if $\gamma \to \infty$, or $\beta \to 0$, then $\xi > 0$) and public spending is not so productive for individual firms (i.e., $1/(1-\varphi) \to 1$).

However, local indeterminacy is possible under a strong effect of productive public spending in a nongrowing economy: Again, by applying the usual practice to determine the sign of eigenvalues in the dynamic system, linearizing the dynamic equations of (15a), (15b), and (15c), and evaluating them at the steady state $\tilde{k}$, $\tilde{c}$, $\tilde{l}$ yields a 2×2 matrix as follows:

[40] The coexistence of global and local indeterminacy is also found in Lucas-type, two-sector growth models with labor and capital externalities (for details, see Benhabib and Perli, 1994; also, see a survey in Benhabib and Farmer, 1999).

[41] Under $\phi = 0.08$ along with $\alpha = 0.82$ and $\beta = 0.08$ Turnovsky (1990) yielded $\xi > 0$. Lansing's (1998) business cycles model with $\phi = 0.03$, $\alpha = 0.36$ and $\beta = 0.61$ also has that $\xi > 0$. In the both set of parameter values, the fiscal increasing returns ϕ with respect to the productivity α of private capital are too small to generate indeterminacy. However, the productivity ϕ of public spending $0.34 \sim 0.39$ in Aschauer (1989) and $0.23 \sim 0.43$ in Knellar, Bleaney, and Gemmell (1999) generates the positive wage-hours relation by fiscal increasing returns and thus allows a broad range of the private input shares α and β for indeterminate equilibra.

[42] More preciously, the local and global indeterminacy emerge when the competitively growing economy exhibits the fiscal increasing returns $1/(1-\phi)$ in the economy-wide production function in (11). As a result, we note that the tax distortion $\Delta(\tau_k, \tau_l)$ in the production function plays an essential role for long-run growth but no direct role in the existence of the local and global indeterminacy.

$$\begin{bmatrix} \dot{k} \\ \dot{c} \end{bmatrix} = \begin{bmatrix} \Psi\left[\dfrac{\alpha}{1-\varphi}\right]\left[\dfrac{\tilde{c}}{\tilde{k}}\right] & -\Psi \\ \left[\alpha\Psi+\varphi-1\right]\left[\dfrac{\rho}{1-\varphi}\right]\left[\dfrac{\tilde{c}}{\tilde{k}}\right] & \rho\left[1-\Psi\right] \end{bmatrix} \begin{bmatrix} k-\tilde{k} \\ c-\tilde{c} \end{bmatrix}, \tag{23}$$

where $\Psi \equiv 1+\beta/\xi(1-\phi)$ and $\xi \equiv \gamma-[\phi-(1-\beta)]/(1-\phi)$. The determinant of this matrix is $\Psi(\alpha-1+\phi)\left(\rho/(1-\phi)\right)\left[\tilde{c}/\tilde{k}\right]$. First, when $\xi>0$, we have $\Psi>0$. Hence, the determinant of the matrix is negative since $\phi+\alpha<1$; the one eigenvalue is positive, and the other is negative. The system of the equations is saddle stable, and thus the transitional path converges uniquely to the unique steady state $\tilde{k}$, $\tilde{c}$, $\tilde{l}$. Therefore, the steady state path is locally determinate in a nongrowing economy. However, when $\xi<0$, it is possible that $\Psi \equiv 1+\beta/\xi(1-\phi)<0$. Moreover, the determinant of the matrix is positive only if $(1+\gamma)/\beta<1/(1+\phi)$. By a simple computation, we show that the trace of this matrix is negative under the condition that $\tilde{c}/\tilde{k}>\rho\beta/\alpha(1+\gamma)$. Under these conditions, the two eigenvalues are negative. Therefore, when the strong effect of productive government spending, $1/(1+\phi)$, the high elasticity of leisure γ^{-1}, and the large productivity β of labor generate the locally indeterminate steady state in a nongrowing competitive economy. Roughly speaking, these conditions are consistent with those in the growing economy.

In a growing economy, the eigenvalue condition $\tilde{z}>\rho\beta\left[\alpha\xi+\beta\right]^{-1}$ for local indeterminacy in the neighbor of $\tilde{z}_3$ is difficult to interpret in terms of the parameter values because $\tilde{z}$ is also endogenously determined by the same parameter values in the model. Nonetheless, because the change in capital growth rates is larger than the change in consumption growth rates (i.e., $\Gamma'_k(z)>\Gamma'_c(z)$ around $\tilde{z}_3$), we know that the consumption–capital ratio stabilizes when z deviates from $\tilde{z}_3$. Hence, this relative speed of capital and consumption growth makes all equilibrium trajectories absolutely stable and induces convergence to the associated balanced growth path under these labor market conditions (see Figure 3).

The indeterminacy property in the economy with productive public spending is distinguished from other economies described in the related studies on balanced budget rules. First, local indeterminacy in this study arises regardless of the type of distortionary tax, for example, capital versus consumption tax (e.g., Giannitsarou, 2007; Nourry, Seegmuller, and Venditti, 2011), distortionary versus nondistortionary taxes (e.g., Utaka, 2003), or the composition of revenue from capital versus labor income tax (e.g., Schmitt-Grohe and Uribe, 1997). As a result, neither a type of tax nor a source of government revenue is necessarily seen as a primary factor of aggregate instability, although a different type of taxes affects unequally the rate of balanced growth (i.e., (20)) and the rate of changes in consumption and capital over time (i.e., (16a) and (16b)).

Second, the role of productive public spending for indeterminacy is important. Similarly to this study, Guo and Harrison (2008) obtained local indeterminacy with productive policies, but their economy is not growing in the long run and their indeterminacy does not coexist with global indeterminacy. Indeterminacy in a nongrowing economy exhibits no different level of consumption, capital, and labor in the long run. Raurich (2001) showed the existence of indeterminacy in a growth model with productive public spending. However, his results depends on productive public services congested by a number of agents and derived from public capital financed a fixed lump-sum tax in a Lucus-type two-sector growth model.

Third, Palivos, Yip, and Zhang (2003) also obtained indeterminacy in the endogenous growth model with fiscal policies; however, neglected the difference between the local and global indeterminacy conditions. Specifically, their analysis on local indeterminacy overlooks the effect of relative long-run growth rates between consumption and capital, the (positive) slope of demand for labor, the productivity of public spending, and the rate of time preferences. Furthermore, unlike aggregate business cycles in the present paper, their primary focus is to design a selection mechanism among two multiple balanced growth paths.

Fourth, predetermined public spending (e.g., Gokan, 2006; Pintus, 2004) is not required for indeterminacy as long as public spending affects the aggregate production process or consumption–leisure choices. The predetermined government expenditure or the countercyclical rate of taxation is not critical in this model. Unlike countercyclical tax rates under balanced budget rules in Schmitt-Grohe and Uribe (1997), and Nourry, Seegmuller, and Venditti (2011), the local indeterminacy result in my model depends on procyclical public spending with constant income taxation.

Finally, related with the previous point, indeterminacy holds true even with underlying endogenous government expenditures (see, e.g., Schmitt-Grohe and Uribe, 1997) in the present model with allocative fiscal policy. That is, when agents' optimistic expectation leads to higher investment and more working hours, the government is forced to increase its spending under the balanced budget constraints as final output rises. This productive spending policy can increase the realized productivities of capital and labor inputs and thus fulfills agents' initial optimistic expectation. It is because the endogenous productive spending generates increasing returns in the private input factors and fiscal increasing returns to scale in the labor market. Hence, multiple sunspots equilibria arise, and aggregate instability is driven by self-fulfilling belief.

In general, the previous works have established that indeterminacy is likely to emerge when an aggregate economy exhibits increasing returns to scale through either market imperfection or through presence of externalities found in an aggregate economy. In this study, the allocative fiscal policy exclusively derives fiscal increasing returns in aggregate technology. Noticeably, the economy-wide production function in my model exhibits increasing returns because productive public spending under a balanced budget constraint leads to a linear technology of capital with endogenous labor employment.

V. Concluding Remarks

I present a standard one-sector endogenous growth model to reexamine aggregate stability and business cycles arising from self-fulfilling beliefs under fiscal policies with a balanced budget condition. This model incorporates the additional feature of a fiscal policy in which public spending is productive and increases the total and individual factor productivity of aggregate production. In this economy, the parameter values for the weighted elasticity labor supply and demand by productive public spending in aggregate production determine the conditions for local and global indeterminacy. The analysis implies that if public spending is productive, local and global indeterminacy can exist regardless of the fiscal policies related to predetermined government expenditure, the source of government revenue, the type of distortionary taxes, and the procyclicality of government expenditures.

This study concludes that indeterminacy emerges as a consequence of allocative fiscal policy under a balanced budget constraint that allows for feedback from private sectors to the future effects of productive public spending. That is, balanced budget rules facilitate the feedback effect of fiscal policies in equilibrium. This result suggests an additional reason why an economically productive fiscal policy under balanced budget rules can destabilize a competitive economy in the absence of persistent shocks to economic fundamentals. In addition, an endogenous growth model that combines the indeterminacy property with productive fiscal policy may explain endogenous propagation mechanisms, including empirically observed autocorrelations, business cycle frequencies, and impulse responses in business cycles models.

References

Ambler S. and A. Paquet (1996), "Fiscal Spending Shocks, Endogenous Government Spending, and Real Business Cycles," *Journal of Economic Dynamics and Control*, Vol. 20, pp. 237-256.

Arrow K. and M. Kurz (1970), *Public Investment, the Rate of Return and Optimal Fiscal Policy*, Jones Hopkins University Press.

Aschauer D. (1989), "Is Public Expenditure Productive?" *Journal of Monetary Economics*, Vol. 23, pp. 177-200.

Azariadis C. (1981), "Self-fulfilling Prophecies," *Journal of Economic Theory*, Vol. 25, pp. 380-396.

Barro R. (1979), "On the Determination of Public Debt," *Journal of Political Economy*, Vol. 87, pp. 940-971.

Barro R. (1990), "Government Spending in a Simple Model of Economic Growth," *Journal of Political Economy*, Vol. 98, pp. S103-S125.

Baxter M. and R. King (1993), "Fiscal Policy in General Equilibrium," *American Economic Review*, Vol. 83, pp. 315-334.

Benhabib J. and R. Farmer (1999), "Indeterminacy and Sunspots in Macroeconomics," *Handbook of Macroeconomics*, Vol. 1A, eds. Taylor J. and M. Woodford, North-Holland, Amsterdam, pp. 387-448.

Benhabib J. and R. Farmer (1994), "Indeterminacy and Increasing Returns," *Journal of Economic Theory*, Vol. 63, pp. 19-41.

Benhabib J. and R. Perli (1994), "Uniqueness and Indeterminacy: On the Dynamics of Endogenous Growth," *Journal of Economic Theory*, Vol. 63, pp. 113-142.

Bennett R and R. Farmer (2000), "Indeterminacy and Non-separable Utility," *Journal of Economic Theory*, Vol. 93, pp. 118-143.

Blanchard O. and L. Summers (1987), "Fiscal Increasing Returns, Hysteresis, Real Wages and Unemployment," *European Economic Review*, Vol. 31, pp. 543-559.

Caballero R. and R. Lyons (1992), "External Effects in US Procyclical Productivity," *Journal of Monetary Economics*, Vol. 29, pp. 209-226.

Cazzavillan G. (1996), "Public Spending, Endogenous Growth and Endogenous Fluctuations," *Journal of Economic Theory*, Vol. 71, pp. 394-415.

Chang J., J. Gou, J. Shieh, and W. Wang (2014), "Sectoral Composition of Government Spending and Macroeconomic (in)Stability," *Working Paper* in IEAS.

Cooly T. and G. Hansen (1992), "Tax Distortion and a Neoclassical Monetary Economy," *Journal of Economic Theory*, Vol. 58, pp. 290-316.

de Hek P. (1998), "An Aggregative Model of Capital Accumulation with Leisure-dependent Utility," *Journal of Economic Dynamics and Control*, Vol. 23, pp. 255-276.

Devarajan S., V. Swaroop and H. Zou (1996), "The Composition of Public Expenditure and Economic Growth," *Journal of Monetary Economics*, Vol. 37, pp. 313-344.

Farmer R. and J.-T. Guo (1996), "The Econometrics of Indeterminacy: An Applied Study," *Carnegie-Rochester Conference Series on Public Policy*, Vol. 43, pp. 225-271.

Gali J. (1994), "Government Size and Macroeconomic Stability," *European Economic Review*, Vol. 36, 117-132.

Giannitsarou C. (2006), "Balanced Budget Rules and Aggregate Instability: The Role of Consumption Taxes," *Economic Journal*, Vol. 117, pp. 1423-1435.

Glomm G. and B. Ravikumar (1997), "Productive Government Expenditures and Long-run Growth," *Journal of Economic Dynamics and Control*, Vol. 21, pp. 183-204.

Gokan Y. (2006), "Dynamic Effects of Government Expenditure in a Finance Constrained Economy," *Journal of Economic Theory*, Vol. 127, pp. 323-333.

Greenwood and Huffman (1991), "Tax Analysis in a Real-business-cycle Model: On Measuring Harberger Triangles and Okun Gaps," *Journal of Monetary Economics*, Vol. 27 pp. 167-190.

Guo J.-T and S. Harrison (2008), "Useful Government Spending and Macroeconomic (in)Stability under Balanced-budget Rules," *Journal of Public Economic Theory*, Vol. 10, pp. 383-397.

Guo J.-T and S. Harrison (2004), "Balanced-budget Rules and Macroeconomic (in)Stability," *Journal of Economic Theory*, Vol. 119, pp. 357-363.

Hansen G. (1985), "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, Vol. 16, pp. 309-325.

King, R., C. Plosser and S. Rebelo (1988), "Production, Growth and Business Cycles: II. New direction," *Journal of Monetary Economics*, Vol. 12, pp. 309-341.

Kneller R., M. Bleaney and N. Gemmell (1999), "Fiscal Policy and Growth: Evidence from OECD Countries," *Journal of Public Economics*, Vol. 74, pp. 171-190.

Ladron-de-Guevara A., S. Ortigueira and M. Santos (1999), "A Two-sector model of Endogenous Growth with Leisure," *Review of Economic Studies*, Vol. 66, pp. 609-631.

Lansing K. J. (1998), "Optimal Fiscal Policy in a Business Cycles Model with Public Capital," *Canadian Journal of Economics*, Vol. 31, pp. 337-364.

Lucas R. (1988), "On the Mechanics of Economic Development," *Journal of Monetary Economics*, Vol. 22, pp. 3-42.

Milesi-Ferretti G. and N. Roubini (1998), "On the Taxation of Human and Physical Capital in Models of Endogenous Growth," *Journal of Public Economics*, Vol. 70, pp. 237-254.

Mueller, D. (1989), *Public Choice III*, Cambridge University Press, Cambridge.

Nishimura K., C. Nourry, T. Seegmuller, and A. Venditti (2012), "Destabilizing Balanced Budget Consumption Taxes in Multi-sector Economies," *Working Paper* in the Universite de la Mediterranee Aix-Marseille II and GREQAM.

Nourry C., T. Seegmuller, and A. Venditti (2011), "Aggregate Instability under Balanced-budget Consumption Taxes: Re-examination," *Working Paper* in the Universite de la Mediterranee Aix-Marseille II and GREQAM.

Ortigueira S. (2000), "A Dynamic Analysis of an Endogenous Growth Model with Leisure," *Economic Theory*, Vol. 16, pp. 43-62.

Palivos T., C. Yip and J. Zhang (2003), "Transitional Dynamics and Indeterminacy of Equilibria in an Endogenous Growth Model with a Public Input," *Review of Development Economics*, Vol. 7, pp. 86-98.

Park H. (2009), "Ramsey Fiscal Policies in an Endogenous Growth Model," *Economic*

Theory, Vol. 39, pp. 377-398.

Park H. and A. Philippopoulos (2004), "Indeterminacy and Fiscal Policies in a Growing Economy, *Journal of Economic Dynamics and Control*, Vol. 28, pp. 645-660.

Pintus P. (2004), "Aggregate Instability in the Fixed-cost Approach to Public Spending," *Working Paper* in the Universite de la Mediterranee Aix-Marseille II and GREQAM.

Raurich X. (2001), "Indeterminacy and Government Spending in a Two-sector Model of Endogenous Growth," *Review of Economic Dynamics*, Vol. 4, pp. 210-229.

Raurich X. (2000), "Global Indeterminacy in an Endogenous-growth Model with Public Capital," *Journal of Economics*, Vol. 71, pp. 255-280.

Rogerson R. (1988), "Indivisible Labor, Lotteries and Equilibrium, *Journal of Monetary Economics*, Vol. 21, pp.3-16.

Romer P. (1986), "Increasing Returns and Long-run Growth," *Journal of Political Economy*, Vol. 94, pp. 1002-1037.

Schmitt-Grohe S. and M. Uribe (1997), "Balanced Budget Rules, Distortionary Taxes and Aggregate Instability," *Journal of Political Economy*, Vol. 105, pp. 976-1000.

Shigoka T. (1994), "A Note on Woodford's Conjecture: Constructing Stationary Sunspot Equilibria in a Continuous Time Model," *Journal of Economic Theory*, Vol. 64, pp. 531-540.

Shell K. (1967), "A Model of Incentive Activity and Capital Accumulation," in *Essays on the Theory of Optimal Economic Growth,* edited by K. Shell, The MIT Press.

Turnovsky S. (2000), "Fiscal Policy, Elastic Labor Supply and Endogenous Growth," *Journal of Monetary Economics*, Vol. 45, pp. 185-210.

Utaka A. (2003), "Income Tax and Endogenous Business Cycles," *Journal of Public Economic Theory*, Vol. 5, pp. 135-145.

Velasco A. (1996), "Animal Spirits, Investment and International Capital Movements," *Journal of International Money and Finance*, Vol. 15, pp. 221-237.

Woodford M. (1986), "Stationary Sunspot Equilibria in a Finance Constrained Economy," *Journal of Economic Theory*, Vol. 40, pp. 128-137.