

## Coordination on Use of Non-deferred Electronic Payment Instruments\*

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*In the use of electronic payment technology, there is strategic complementarity and hence room for self-fulfilling multiple equilibria. But existing relevant literature is silent about how agents' expectations become coordinated. This paper resolves the coordination problem in the use of a non-deferred electronic means of payment, which can be represented by a debit card. We focus on that because it is almost a perfect substitute for cash. The presence of exogenous shocks that have a fundamental impact on the cost of the technology makes agents coordinate their expectations in a particular way. We also show that a high inflation and a distortionary financing scheme for debit-card transactions cost disturb coordination in the use of debit cards.*

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### I. Introduction

Recently, electronic payment transactions have increased dramatically. In particular, the growing trend of debit-card transactions is remarkable. Borzekowski, Kiser and Ahmed (2008) report that since 1996, debit-card transactions have grown at an average rate of more than 20 percent per year. As a consequence, debit cards have become one of the primary electronic means of payment for in-store purchases (for example, see Schuh and Stavins 2012).

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Such a rapid increase in the use of debit cards has drawn the attention of economists on the choice of means of payment, particularly between cash and debit cards. (for instance, see He, Huang and Wright 2008, Klee 2008, Kim and Lee 2010, Li 2011, Schuh and Stavins 2012, and Lotz and Vasseli 2013.) However, these works are silent on the economy-of-scale effect in the use of debit cards. An economy of scale is indeed one of the key features of an electronic payment technology. Relative to cash, debit cards in practice are a technology with a substantial fixed cost (setting up clearing and verification systems, issuing cards, etc.) but very low per-transaction marginal cost. This then implies that the benefits from debit-card transactions will be enhanced with agents' willingness to use a debit card by lowering its per-transaction cost.

By taking into account this economy-of-scale effect in the use of debit-card payment, Huang, Kim and Lee (2014) examine an optimal allocation scheme for its cost in a search-theoretic environment. However, they do not discuss much about the coordination issue. As mentioned, there are strategic complementarities with the use of debit cards in the sense that the payoff for each debit-card user relies on the actions of other agents. Therefore, as in typical models with strategic complementarities, this model essentially exhibits multiple equilibria: an agent's belief will not be uniquely determined according to the state of the economy and self-fulfilling prophecies could lead to either an equilibrium with all cash transactions or one with all debit-card transactions. We then should be able to say how agents' expectations become coordinated and which equilibrium will be played.

In this paper, we try to provide an answer for this coordination problem by introducing aggregate shock on the state of the economy as in Frankel and Pauzner (2000), Burdzy, Frankel and Pauzner (2001), and Araujo and Guimaraes (2014), for instance. Our model is similar to that of Huang, Kim and Lee (2014) and focuses on a debit card because as a representative non-deferred electronic means of payment (i.e., the point of delivering the good and the point of clearing the trade are coincident), it is an almost perfect substitute for cash. But in their model the cost structure of debit-card transactions is different in a way that it is constant over time, while it changes randomly in our model. Specifically, we introduce a shock to the state of the economy that has a fundamental impact on the cost of the technology such as the development of information technology. We then explore the interactions between the state of the economy and agents' willingness to coordinate in the use of debit cards. In doing so, we consider the following two cases: one with friction in switching means of payment between cash and debit cards, and the other with small friction in switching the means of payment (a limiting case of the former).

We first show that in an economy with friction, there is a unique equilibrium in which agents are always willing to coordinate in the use of debit cards if the currently realized shock on the state of the economy is greater than a threshold level

that is characterized as a function of the fraction of existing debit-card users. That is, agents' actions are uniquely predicted according to the state of the economy, which captures the key components of debit-card cost, and any other variables that are irrelevant to agents' payoffs such as sunspots do not play any role.

We then show that the uniqueness result is preserved in the limiting economy where agents are free to switch the means of payment. But it has a different feature in the sense that the threshold level does not depend on the fraction of existing debit-card users. This implies that even though the friction on switching the means of payment is small enough, the presence of an exogenous shock on the primitives of electronic payment technology can resolve an indeterminacy problem.

We also show that agents are less willing to coordinate in the use of debit cards in an inflationary economy. This is in stark contrast to Monnet and Roberds (2008) where inflation can stimulate the use of credit cards. This difference is mainly attributed to the fact that a credit card is a deferred payment instrument, whereas a debit card is a non-deferred payment instrument. In our model, a debit-card trader is willing to make a relatively large transaction compared with a cash trader, and hence a debit-card trader demands more money. This suggests that a debit-card trader is more susceptible to inflation and consequently, coordination in the use of debit cards is discouraged as the inflation rate increases. In a similar vein, it turns out that distortionary financing schemes for debit-card transaction cost disturb coordination in the use of debit cards.

## II. Model

The background environment is a version of Lagos and Wright (2005) with uncertainty about the user cost of electronic means of payment which, like a debit card, is a non-deferred one and does not have any credit function. Time is continuous and goes on forever, but trading opportunities occur in a sequence of periods at times  $t = 0, \tau, \dots$ . Some trades take place in a centralized market and others in a decentralized market with bilateral random matchings. There are many varieties of special goods and one general good. The general good can be produced and traded in the centralized market, whereas the special goods can be produced and traded in the decentralized market. All goods are divisible, perishable, and have to be consumed right after production. There is another object, called money which is perfectly divisible and durable, and evolution of its stock is controlled by the government.

The economy is populated with a  $[0, 1]$  continuum of infinitely lived agents, each of whom can consume a subset of specialized goods. Each agent can transform labor into one of the special goods, which cannot be consumed by himself. The utility from  $q$  units of special-good consumption is given by  $u(q)$  where

$u'' < 0 < u'$ ,  $u'(0) = \infty$ , and  $u'(\infty) = 0$ . The disutility from producing  $q$  units of a special good is given by  $c(q)$  where  $c' > 0$ ,  $c'' \geq 0$ , and  $c'(0) = 0$ . All agents can consume and produce the general good. The utility obtained from the net consumption  $x$  of the general good is simply  $x\tau$ , which implies that producing the general good for oneself is worthless. Agents discount future utility at the same rate  $e^{-\rho\tau}$  with  $\rho > 0$ . There is no discounting within a period  $\tau$ .

In the decentralized market, each agent receives an individual trading shock such that she will become either a buyer or a seller with an equal probability  $\alpha\tau \in (0, 1)$ , and each buyer is then randomly matched with a seller who produces the relevant consumption good. In a bilateral meeting, trading histories are private and agents cannot commit to future actions, which rules out any possibility of credit trades and hence a medium of exchange is essential. (See, for example, Kocherlakota 1998, Wallace 2001, Corbae, Temzelideset and Wright 2003, and Aliprantis, Camera and Puzzello 2007.) That is, transfer of money in exchange for goods produced should be made either in the form of cash or by use of a debit card.

In a bilateral meeting, a buyer makes a take-it-or-leave-it offer  $(q, p)$  to a seller where  $q$  denotes quantity of goods produced by a seller for a buyer and  $p$  denotes the amount of money transferred by a buyer to a seller. A buyer can pay  $p$  in cash (hereinafter C-payment) at the disutility cost of  $\eta\phi p$  where  $\phi$  denotes the value of money in terms of the general good. That is, an agent carrying cash incurs disutility  $\eta$  per each unit of real cash balance. This disutility cost can be interpreted as capturing the inconvenience of carrying cash around, the risk of loss or theft, and the foregone interest (see, for instance, Baumol 1952, Tobin 1956, Humphrey 2004, He, Huang and Wright 2008, and Monnet and Roberds 2008).

A buyer can also transfer  $p$  via a debit card (hereinafter E-payment): i.e., money is transferred from a buyer's account to a seller's account directly. As a payment-service provider, the government processes a record-keeping technology for checking accounts, but not for agents' trading histories. As typical in real-world practice, it is also assumed that checking-account balances cannot be negative.<sup>1</sup> Transactions via an E-payment system incur the government (service provider) a fixed cost  $\Omega(z)\tau$  per each period  $\tau$  in terms of the general good and its users pay the relevant cost in the subsequent centralized market by producing the general good.<sup>2</sup> We assume  $\Omega'(z) < 0$  where  $z$  captures the state of the economy, which has a fundamental impact on the E-payment cost such as the development of information technology. As in Burdzy, Frankel and Pauzner (2001),  $z$  follows a

<sup>1</sup> If the government can record agents' trading histories or if negative balances in checking accounts are permitted, then credit trades can be available. This would rule out a monetary equilibrium because buyers who do not hold any money can make trades in the decentralized market. We are indebted to a referee for suggesting this point.

<sup>2</sup> As in Hayashi and Keeton (2012) and Huang, Kim and Lee (2014),  $\Omega$  can be interpreted as the "social cost" of using an electronic payment system.

random walk such that either  $z_{t+\tau} = z_t + \nu\tau + \sigma\sqrt{\tau}$  or  $z_{t+\tau} = z_t + \nu\tau - \sigma\sqrt{\tau}$  with equal probability. Notice that the dynamics of  $z_t$  can be characterized by a variance  $\sigma^2$  and a trend  $\nu$ . The variance  $\sigma^2$  measures the size of the random component, whereas the trend  $\nu$  captures the deterministic part of  $z$ . We assume that C-payment dominates E-payment for  $z < -\bar{z}$ , whereas E-payment dominates C-payment for  $z > \bar{z}$ . Throughout, we assume  $\bar{z}$  is sufficiently large so that the dominant regions are remote from each other.<sup>3</sup> It is worthwhile to note here that if  $z$  is constant or follows an i.i.d. process over time, multiple equilibria exist. For instance, either an all E-payment equilibrium or an all C-payment equilibrium could arise due to sunspots which lead agents to believe that others will use E-payment (or C-payment).

The cost  $\Omega(z)$  is financed by imposing fee  $\omega$  on each E-payment where  $\omega$  should satisfy

$$\Omega(z) = S[\theta\omega + (1-\theta)\omega]. \quad (1)$$

Here,  $S$  denotes the instantaneous measure of E-payment transactions,  $\theta \in [0,1]$  is the share of cost allocated to a buyer, and  $(1-\theta) \equiv \bar{\theta} \in [0,1]$  is that allocated to a seller. It is worthwhile to note that as shown in Huang, Kim and Lee (2014),  $\theta$  can be interpreted as taxation on a buyer's labor and has a lump-sum feature in the sense that it does not affect quantity consumed in exchange for money transferred via a debit card. Meanwhile,  $\bar{\theta}$  can be interpreted as taxation on a buyer's consumption and is distortionary in the sense that it affects quantity consumed in the debit-card transaction. For the case  $S < \gamma$ , we assume that  $\omega$  is given by  $\omega = \Omega(z)/\gamma$  where  $\gamma$  is assumed to be very small so that the economy of scale ( $\omega$  declines as  $S$  increases) is valid almost everywhere. Here,  $\gamma$  is simply a technical device ensuring a finite  $\omega$  and can be regarded as the government subsidy that is given to the E-payment system if there are too few E-payment transactions ( $S < \gamma$ ). Notice that imposing a fee on no-traders or cash traders is not feasible because such transactions are not recorded in the system and the cost is collected not on the spot but in the centralized market.

The economy has friction such that in each period a measure  $k\tau \in (0,1)$  of agents is randomly selected and they are given opportunities to change means of

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<sup>3</sup> As we will see, the assumption that there are dominance regions in which a particular action is strictly dominant is critical for a uniqueness result. From the real-world perspective, our interpretation for the dominance regions is as follows. Notice first that advanced information technology can correspond to a higher  $z$  in our model because it can reduce E-payment cost by saving information processing cost. Then it is strictly dominant to choose E-payment in an economy having sufficiently advanced information technology so that  $\Omega$  is negligible, while it is strictly dominated to choose E-payment in an economy having poor information technology so that  $\Omega$  is enormous. We owe this point to an anonymous referee.

payment for subsequent bilateral trades. Putting it differently, it is costless to switch means of payment for some fraction of agents, whereas it is very costly for the remainder. The realization of this opportunity is independent across time and agents. In section 4, we will also consider the case in which friction becomes small where a larger  $k$  implies a smaller friction in the sense that  $1/k$  is the average duration of locking into a particular payment instrument.

Finally, the sequence of events in each  $\tau$  is as follows. First, the state of the economy ( $z$ ) and preference shock (either a buyer or a seller in the decentralized market) are realized. A buyer then moves to a seller's location with either cash or a debit card depending on her choice of the means of payment.<sup>4</sup> Trade occurs if agreement is reached between a buyer and a seller. After bilateral trades but before entering into the centralized market, the opportunity to switch the means of payment between cash and debit card for the next period is realized. Upon arriving in the centralized market, each agent receives a lump-sum transfer of money from the government and then trades the general good, but unlike in the decentralized market, E-payment is not available.

### III. Equilibrium

We now formulate an equilibrium in a recursive manner and work backward from the centralized market to the decentralized market in a generic period  $\tau$ .

#### 3.1. Bellman Equations

We let  $W_t^o(m, z)$  be the value function pertaining to the beginning of the  $t$ -period centralized market where an agent holds  $m$  units of money and is stuck in payment system  $o \in \{E, C\}$  with the realized state  $z$ . We let  $V_t^o(m, z)$  be the value function pertaining to the beginning of the  $t$ -period decentralized market where an agent as a buyer holds  $m$  units of money and is stuck in payment system  $o \in \{E, C\}$  with the realized state  $z$ . Let  $\phi_t$  be the price of money in terms of the general good in the  $t$ -period centralized market and  $b_t$  be the lump-sum transfer of money, by which the government controls the inflation rate  $\mu_t = \frac{\phi_t - \phi_{t+\tau}}{\phi_{t+\tau}\tau}$ .

Noting that the opportunity for switching means of payment arrives before opening the centralized market,  $W_t^o$  and  $V_{t+\tau}^o$  share the same  $o$ -system. Then the

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<sup>4</sup> In our model, the portfolio choice between cash holdings and checking-account deposits is straight-forward: i.e., if a buyer decides to use a debit card, she will deposit all money into a checking account because the E-payment fee is irrelevant to the amount of the transaction. From the point of view that such checking-account balances can be used to purchase the consumption good only, we can simply regard E-payment in our model as armored car services that facilitate the transfer of money from a buyer to a seller at a fixed cost. We owe this point to an anonymous referee.

centralized-market problem for an agent entering with  $m$  is given by

$$W_t^o(m, z) = \max_{x, m', o'} \left\{ x\tau + e^{-\rho\tau} \mathbb{E}_z [\alpha\tau V_{t+\tau}^o(m', z') + (1-\alpha\tau)(k\tau W_{t+\tau}^{o'}(m', z') + (1-k\tau)W_{t+\tau}^o(m', z'))] \right\} \quad (2)$$

subject to  $x\tau = \phi_t(m + b_t) - \phi_t m'$  where the expectation  $\mathbb{E}_z$  is taken over the distribution of the following  $z'$  conditional on the current  $z$ . Notice that  $o'$  is contingent on the realization of the next period state  $z'$ , while  $m'$  has to be chosen before the realization of  $z'$ . Notice also that we here use the fact that under the buyer-take-all bargaining rule, the payoff of a seller in the decentralized market is equivalent to that of a no-trader. Substituting  $x\tau$  from the constraint, we have

$$\begin{aligned} W_t^o(m, z) &= \phi_t(m + b_t) \\ &\quad + e^{-\rho\tau} \max_{m'} \{ \alpha\tau [\mathbb{E}_z (V_{t+\tau}^o(m', z')) - \phi_{t+\tau} m'] + (\phi_{t+\tau} - e^{\rho\tau} \phi_t) m' \} \\ &\quad + e^{-\rho\tau} (1-\alpha\tau) k\tau \mathbb{E}_z \{ \max_{o'} [W_{t+\tau}^{o'}(0, z')] \} \\ &\quad + e^{-\rho\tau} (1-\alpha\tau) (1-k\tau) \mathbb{E}_z [W_{t+\tau}^o(0, z')]. \end{aligned} \quad (3)$$

As in Lagos and Wright (2005), (3) implies that  $W_t^o(m, z) = \phi_t m + W_t^o(0, z)$ : i.e., regard-less of  $o \in \{E, C\}$ ,  $W_t^o(m, z)$  is linear in money holdings ( $m$ ). Noting that the choice of a means of payment is contingent on the realization of  $z'$ , the expectation operator ( $\mathbb{E}_z$ ) comes before the maximization operator ( $\max_{o'}$ ). The sequence  $\{\phi_t\}$  is controlled by the government and thus it is independent of the realization  $z'$ .

Given the buyer-take-all trading protocol,  $V_t^o(m, z)$  should satisfy

$$\begin{aligned} V_t^o(m, z) &= \max_{q, p, o'} [u(q) - \xi^o + k\tau W_t^{o'}(m - p, z) + (1-k\tau)W_t^o(m - p, z)] \\ &= \max_{q, p, o} [u(q) - \phi_t p - \xi^o + \phi_t m + k\tau W_t^{o'}(0, z) + (1-k\tau)W_t^o(0, z)] \end{aligned} \quad (4)$$

subject to a seller's participation constraint [ $c(q) \leq \phi_t p$  for  $o = C$  and  $c(q) \leq \phi_t p - \bar{\theta}\omega$  for  $o = E$ ], and the transaction cost function  $\xi^o$  takes the form of

$$\xi^o = \begin{cases} \theta\omega & \text{if } o = E \\ \eta\phi_t p & \text{if } o = C. \end{cases}$$

In addition, the take-it-or-leave-it offer  $(q, p)$  for  $o = C$  should satisfy

$$\max_{(q, p)} [u(q) - (1+\eta)\phi_t p] \quad (5)$$

$$\text{s.t. } c(q) = \phi_i p \text{ and } p \leq m$$

and that for  $o = E$  should satisfy

$$\begin{aligned} & \max_{(q,p)} [u(q) - \phi_i p] - \theta \omega \\ & \text{s.t. } c(q) = \phi_i p - \bar{\theta} \omega \text{ and } p \leq m. \end{aligned} \quad (6)$$

The solutions for (5) are given by

$$\begin{aligned} q^C &= \begin{cases} \hat{q} & \text{if } \phi_i m \geq \hat{q} \\ c^{-1}(\phi_i m) & \text{otherwise} \end{cases} \\ p^C &= \begin{cases} \hat{q} / \phi_i & \text{if } \phi_i m \geq \hat{q} \\ m & \text{otherwise} \end{cases} \end{aligned}$$

where  $\hat{q}$  is such that  $u'(\hat{q}) = (1 + \eta)c'(\hat{q})$ . The solutions for (6) are given by

$$\begin{aligned} q^E &= \begin{cases} q^* & \text{if } \phi_i m \geq c(q^*) + \bar{\theta} \omega \\ c^{-1}(\phi_i m - \bar{\theta} \omega) & \text{otherwise} \end{cases} \\ p^E &= \begin{cases} [c(q^*) + \bar{\theta} \omega] \phi_i^{-1} & \text{if } \phi_i m \geq c(q^*) + \bar{\theta} \omega \\ m & \text{otherwise} \end{cases} \end{aligned}$$

where  $q^*$  satisfies  $u'(q^*) = c'(q^*)$ . Notice that C-payment cost  $\eta$  is distortionary in the sense that the first-best allocation  $(q^*)$  can never be produced with C-payment.

Now from (4),  $V_m^C(m, z)$ , the marginal value of money for a buyer in the decentralized market with  $o = C$ , can be obtained as follows:

$$V_m^C(m, z) = \begin{cases} \phi_i & \text{if } \phi_i m \geq \hat{q} \\ \left[ \frac{u'}{c'} \right]_{\phi_i m} \phi_i - \eta \phi_i & \text{otherwise} \end{cases}$$

and that with  $o = E$ ,  $V_m^E(m, z)$ , is given by

$$V_m^E(m, z) = \begin{cases} \phi_i & \text{if } \phi_i m \geq c(q^*) + \bar{\theta} \omega \\ \left[ \frac{u'}{c'} \right]_{\phi_i m - \bar{\theta} \omega} \phi_i & \text{otherwise.} \end{cases}$$

Finally, (3) implies that the money demand for the next decentralized market  $(m')$

is irrelevant to the current money holdings ( $m$ ) and solves

$$\max_{m'} \{ \alpha \tau [ \mathbb{E}_z(V_t^o(m', z')) - \phi_t m' ] + [ (\phi_t - e^{\rho \tau} \phi_{t-\tau}) m' ] \}. \quad (7)$$

The first order condition for  $o = C$  is then

$$\alpha \tau \left( \frac{u'}{c'} \Big|_{c^{-1}(\phi_t m')} - 1 - \eta \right) = e^{\rho \tau} (1 + \mu_{t-\tau} \tau) - 1 \quad (8)$$

and that for  $o = E$  is

$$\alpha \tau \left( \mathbb{E}_z \left( \frac{u'}{c'} \Big|_{c^{-1}(\phi_t m' - \bar{\theta} \omega)} \right) - 1 \right) = e^{\rho \tau} (1 + \mu_{t-\tau} \tau) - 1. \quad (9)$$

The balance of money carried into the next period is determined by the trade-off between the inflation cost of holding money [the second bracket in (7)] and its expected benefit from a bilateral trade [the first bracket in (7)], which differs between C-payment and E-payment. Notice that  $\mu_{t-\tau}$  must be greater than zero so that a solution exists, and for a given  $\mu_{t-\tau} > 0$ , the productions in (8) and (9) are uniquely determined respectively.

### 3.2. Iterative Conditional Dominance

Let  $X_t$  be the fraction of agents who are locked into an E-payment system after the choices are revised just before the  $t$ -period centralized market. The public history at time  $t$  is the evolution of the environment until  $t - \tau$  such that  $\{X_v, z_v, \phi_v\}_{v=0, \tau, \dots, t-\tau}$  where the initial values of  $X_0$  and  $z_0$  are given. An agent's private history at time  $t$  consists of her actions and the details of her matches through period  $t$ . An agent's information set at time  $t = \tau, 2\tau, 3\tau, \dots$  is composed of both public history and her private history. Strategies are functions mapping from the set of all information sets to the set of mixtures over  $\{E, C\}$ , which indicate what an agent will do if she receives an opportunity to change the means of payment. Notice that the effect of money-holding distribution is reflected in the price ( $\phi_t$ ) and hence its evolution is not included in the public history.

When an agent receives an opportunity to change the means of payment, she chooses the payment instrument using the probability distribution over paths  $(z_{t+i\tau})_{i=0}^\infty$ , the deterministic path  $(\phi_{t+i\tau})_{i=0}^\infty$  which is controlled by the government, and her beliefs about path  $(X_{t+i\tau})_{i=0}^\infty$  which will result from any given realization of  $(z_{t+i\tau})_{i=0}^\infty$  and  $(\phi_{t+i\tau})_{i=0}^\infty$ . Notice that an agent readjusts her money holdings in

the centralized market for the upcoming bilateral trade regardless of whether she receives a switching opportunity. If she is locked into  $o_i \in \{E, C\}$ , her expected payoff with respect to an individual shock is

$$\begin{aligned} W_i^{o_i}(m_i^{o_i}, z_i) &= \phi_i(m_i^{o_i} + b_i) \\ &+ e^{-\rho\tau} \{ \alpha\tau \mathbb{E}_z [u(q_{i+\tau}^{o_i}) - \phi_{i+\tau} p_{i+\tau}^{o_i} - \xi_{i+\tau}^{o_i}] + (\phi_{i+\tau} - e^{\rho\tau} \phi_i) m_{i+\tau}^{o_i} \} \\ &+ e^{-\rho\tau} k\tau \mathbb{E}_z \max_{o_{i+\tau}} [W_{i+\tau}^{o_{i+\tau}}(0, z_{i+\tau})] + e^{-\rho\tau} (1 - k\tau) \mathbb{E}_z [W_{i+\tau}^{o_i}(0, z_{i+\tau})] \end{aligned} \quad (10)$$

where  $m_i^o$  and  $m_{i+\tau}^{o_i}$  are the money carried into the  $t$ -period centralized market and that carried into the decentralized market in the following period, respectively. [See Appendix for derivation of (10).] The term  $\mathbb{E}_z [u(q_{i+\tau}^{o_i}) - \phi_{i+\tau} p_{i+\tau}^{o_i} - \xi_{i+\tau}^{o_i}]$  is the benefit from a bilateral trade for a buyer, which will depend on  $z_{i+\tau}$  through its effect on the cost-sharing scheme (1) and the trade specified in (6) for  $o = E$ . The term  $(\phi_{i+\tau} - e^{\rho\tau} \phi_i) m_{i+\tau}^{o_i}$  is the change in the value of money holdings because of inflation.

Notice that if  $z_i$  is realized, (7) uniquely determines the following period's money holdings  $m_{i+\tau}$  which, together with the following period  $\omega_{i+\tau}$ , gives  $(q_{i+\tau}^E, p_{i+\tau}^E)$ . In (9),  $(q_{i+\tau}^E, p_{i+\tau}^E)$  is contingent on the realization of  $(z_i, z_{i+\tau})$ , even though the expectation operator is taken as if  $(q_{i+\tau}^E, p_{i+\tau}^E)$  depends on the whole sequence of  $z_{i+i\tau}$ . Then the relative payoff for choosing E-payment in the period  $t[\Delta(E, z_i) \equiv W_i^E(m_i^E, z_i) - W_i^C(m_i^C, z_i)]$  is given by<sup>5</sup>

$$\begin{aligned} \Delta(E, z_i) &= \sum_{i=1}^{\infty} e^{-\rho\tau i} (1 - k\tau)_z^{i-1} \mathbb{E}_z \left\{ \alpha\tau u(q_{i+i\tau}^E) + (1 - e^{\rho\tau i} \frac{\phi_{(i-1)\tau+i}}{\phi_{i+i\tau}} - \alpha\tau) c(q_{i+i\tau}^E) \right. \\ &\quad \left. + [(1 - e^{\rho\tau i} \frac{\phi_{(i-1)\tau+i}}{\phi_{i+i\tau}}) \bar{\theta} - \alpha\tau] \omega_{i+i\tau} \right\} \\ &\quad - \sum_{i=1}^{\infty} e^{-\rho\tau i} (1 - k\tau)^{i-1} \left\{ \alpha\tau [u(q_{i+i\tau}^C) - \phi_{i+i\tau} p_{i+i\tau}^C - \eta \phi_{i+i\tau} p_{i+i\tau}^C] + \right. \\ &\quad \left. [\phi_{i+i\tau} - e^{\rho\tau i} \phi_{(i-1)\tau+i}] m_{i+i\tau}^C \right\}. \end{aligned} \quad (11)$$

With probability  $(1 - k\tau)^{i-1}$ , she has no switching opportunities between  $t$  and  $t+i\tau$ , and an agent chooses E-payment if the relative payoff in (11) is positive  $[\Delta(E, z_i) > 0]$  and cash if it is negative  $[\Delta(E, z_i) < 0]$ . Notice that inflation cost is positive so that the money holdings are finite  $(1 - e^{\rho\tau i} \frac{\phi_{(i-1)\tau+i}}{\phi_{i+i\tau}} < 0)$ . Notice also that a higher fraction of agents using E-payment implies a lower  $\omega_{i+i\tau}$ , a smaller restriction on production defined in (6) and hence a higher relative payoff  $\Delta(E, z_i)$ . That is, in our model, strategic complementarities come from the economy of scale from the use of an electronic payment system, which implies the following result.

<sup>5</sup> See Appendix for derivation of (11).

**Lemma 1 (Strategic complementarity)** *Consider the subsets of the space of states  $(X, z)$ ,  $S_A$  and  $S_B$ , and two associated strategies  $A$  and  $B$ , respectively. In strategy  $A$ , agents receiving an opportunity to switch the means of payment choose E-payment only in states  $(X, z) \in S_A$ . In strategy  $B$ , agents choose E-payment only in states  $(X, z) \in S_B$ . If  $S_A \subset S_B$ , then the relative payoff for choosing E-payment in case  $B$  is at least as large as in case  $A$ .*

As in Frankel and Pauzner (2000), we now solve the model using a solution concept of the iterative elimination of conditionally dominated strategies. The process makes use of strategic complementarity implied in (1). Notice that due to our assumption, E-payment is the dominant choice if  $z$  is sufficiently high, whereas C-payment is the dominant choice if  $z$  is sufficiently low. Although agents put most of the weight on payoffs that they receive in the near future if the switching opportunity arrives frequently, as we will see below, the presence of these dominance regions still remains important because of backwards induction.

Let  $Z_0(X)$  be the boundary of the region where an agent will choose E-payment regardless of the choices of other agents: i.e., even in the case in which she expects all other agents choosing after her to select C-payment, she will choose E-payment. Since an agent knows that other agents receiving an opportunity to switch the means of payment must choose E-payment to the right of  $Z_0$ , an agent actually wants to choose E-payment slightly to the left of  $Z_0$  as well because of strategic complementarity. Therefore, there is a new boundary  $Z_1$  to the left of  $Z_0$ . This process can be repeated infinitely, which gives a sequence of cut-off functions  $Z_2, Z_3, \dots$ . Let  $Z_\infty$  be the limiting cut-off function of this sequence. We then know that agents will choose E-payment when the current state is to the right of  $Z_\infty$ .

We now let  $Z'_0$  be the boundary of the region where an agent will choose C-payment regardless of the choices of other agents: i.e., even in the case in which she expects all other agents choosing after her to select E-payment, she will choose C-payment. Exactly the same iteration above can be applied and we can obtain another limiting cut-off function  $Z'_\infty$ . These two limiting cut-off functions ( $Z_\infty$  and  $Z'_\infty$ ) coincide with each other as shown in Theorem 1 of Frankel and Pauzner (2000). This argument gives the following uniqueness result.<sup>6</sup>

**Proposition 1** *There exists a unique equilibrium in the model such that agents choose E-payment if and only if  $z \geq Z_\infty(X)$ .*

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<sup>6</sup> For more formal proof in an analogous environment, see Frankel and Pauzner (2000).

#### IV. Small Friction Economy

In the previous section, we introduced the friction of switching the means of payment. We will now explore a case in which such friction shrinks to zero by letting  $\tau \rightarrow 0$  and then  $k \rightarrow \infty$ .<sup>7</sup> As shown in Appendix, we can first obtain the relative payoff for choosing E-payment when  $\tau \rightarrow 0$  by taking the limit to  $\Delta(E, z_t)$  in (11):

$$\lim_{\tau \rightarrow 0} \Delta(E, z_t) = \mathbb{E}_z \left\{ \int_0^\infty e^{-(k+\rho)v} \left\{ \alpha[u(q_{t+v}^E) - c(q_{t+v}^E) - \omega_{t+v}] - \mu_{t+v} c(q_{t+v}^E) \right\} dv \right. \\ \left. - \int_0^\infty e^{-(k+\rho)v} \left\{ \alpha[u(q_{t+v}^C) - \phi_{t+v} p_{t+v}^C - \eta \phi_{t+v} p_{t+v}^C] - \phi_{t+v} \mu_{t+v} m_{t+v}^C \right\} dv \right\}. \quad (12)$$

Notice that as shown in Frankel and Pauzner (2000), the dynamics of  $X$  as  $k \rightarrow \infty$  bifurcates either upward (all E-payment trades) or downward (all C-payment trades). Starting with a particular state  $(X^*, z^*)$ , the chance of bifurcating up to  $X = 1$  converges to  $1 - X^*$ , while the chance of bifurcating down to  $X = 0$  goes to  $X^*$ . The dynamics of  $X$  can be approximated as  $X_{t+v}^\uparrow = 1 - (1 - X^*)e^{-kv}$  with probability  $(1 - X^*)$  and  $X_{t+v}^\downarrow = X^*e^{-kv}$  with probability  $X^*$ . In addition, the movement in  $X$  is fast relative to the movement in  $z$ : in other words,  $z$  can be considered constant relative to the movement of  $X$ . Then if we consider a constant inflation rate  $\mu$ , the agent's relative payoff for choosing E-payment can be approximated as follows:

$$\bar{\Delta}(E, z_t) = (1 - X^*) \left\{ \int_0^\infty e^{-(k+\rho)v} \left\{ \alpha[u(q_*^E) - c(q_*^E) - \omega_{t+v}^\uparrow] - \mu c(q_*^E) - \mu \bar{\theta} \omega_{t+v}^\uparrow \right\} dv \right\} \\ + X^* \left\{ \int_0^\infty e^{-(k+\rho)v} \left\{ \alpha[u(q_*^E) - c(q_*^E) - \omega_{t+v}^\downarrow] - \mu c(q_*^E) - \mu \bar{\theta} \omega_{t+v}^\downarrow \right\} dv \right\} \\ - \int_0^\infty e^{-(k+\rho)v} \left\{ \alpha[u(q_*^C) - c(q_*^C) - \eta c(q_*^C)] - \mu c(q_*^C) \right\} dv \quad (13)$$

where  $\omega_{t+v}^\uparrow$  ( $\omega_{t+v}^\downarrow$ ) is the path associated with  $X_{t+v}^\uparrow$  ( $X_{t+v}^\downarrow$ ), and  $q_*^C$  and  $q_*^E$  satisfy respectively,

$$\alpha\left(\frac{u'}{c'} \Big|_{q_*^C} - 1 - \eta\right) = \mu \quad (14)$$

<sup>7</sup> By doing so, we can ensure that the effect of  $k\tau$  group's choice is surpassed by the effect of the random change in  $z$ . See, for instance, Burdzy, Frankel and Pauzner (2001) for details.

$$\alpha \left( \frac{u'}{c'} \Big|_{q_*^E} - 1 \right) = \mu. \quad (15)$$

That is, noting  $z_{t+\tau} \rightarrow z_t$  as  $\tau \rightarrow 0$ , (8) and (9) with  $\tau \rightarrow 0$  and constant  $\mu$  imply that the production levels characterized in (8) and (9) approach  $q_*^C$  and  $q_*^E$  in (14) and (15), respectively. Now, as shown in Appendix, (13) can be simplified as

$$\begin{aligned} \bar{\Delta}(E, z_t) = & \left( \frac{1}{\rho + k} \right) \{ \alpha [u(q_*^E) - c(q_*^E)] - \mu c(q_*^E) \} \\ & - \left( \frac{1}{\rho + k} \right) \{ \alpha [u(q_*^C) - c(q_*^C) - \eta c(q_*^C)] - \mu c(q_*^C) \} \\ & - \left( \frac{\alpha + \mu \tilde{\theta}}{k} \right) \left[ \int_{X^*}^1 \left( \frac{1-X}{1-X^*} \right)^{\frac{\rho}{k}} \omega(z_t, X) dX + \int_0^{X^*} \left( \frac{X}{X^*} \right)^{\frac{\rho}{k}} \omega(z_t, X) dX \right] \end{aligned} \quad (16)$$

where per-transaction cost  $\omega(\cdot)$  is given by (1). Now the cut-off  $z^*$  can be defined as  $\lim_{k \rightarrow \infty} \bar{\Delta}(E, z_t) = 0$ .

**Proposition 2** Suppose  $\tau \rightarrow 0$  and then  $k \rightarrow \infty$ . The division line  $Z_\infty$  is horizontal at  $z^*$  where  $z^*$  is defined by

$$\begin{aligned} & \{ \alpha [u(q_*^E) - c(q_*^E)] - \mu c(q_*^E) \} - \{ \alpha [u(q_*^C) - c(q_*^C) - \eta c(q_*^C)] - \mu c(q_*^C) \} \\ & = (\alpha + \mu \tilde{\theta}) \left[ \int_0^1 \omega(z^*, X) dX \right]. \end{aligned} \quad (17)$$

The result above implies that even though the friction on switching the means of payment is sufficiently small, the presence of an exogenous shock on the primitive of electronic payment technology can lead to a unique outcome. But  $Z_\infty$  is now constant and does not depend on  $X$  because when agents are free to switch, observing the current payment pattern conveys no information about the future payment pattern.

We now examine the effect of inflation on the willingness to coordinate in the use of E-payment. Notice that the derivative of the left-hand side of (17) with respect to  $\mu$  is  $[c(q_*^C) - c(q_*^E)]$ , which is strictly negative because  $q_*^E > q_*^C > 0$  from (14) and (15). Meanwhile, the right-hand side of (17) is strictly increasing in  $\mu$ . Therefore, a higher inflation rate  $\mu$  implies a higher threshold  $z^*$  because  $\omega$  declines as  $z$  increases from (1) and  $\Omega'(z) < 0$ . Put differently, a higher inflation rate disturbs coordination in using debit cards due to the real-balance

effect of inflation. That is,  $q_*^E > q_*^C$  from (14) and (15) implies  $m^E > m^C$ . In addition, since a buyer incurs  $\eta$  for each unit of real cash balance, C-payment cost is indeed irrelevant to inflation. Hence, compared with cash traders, debit-card traders are more susceptible to inflation and the relative benefit of E-payment over C-payment decreases as the inflation rate increases.<sup>8</sup> It is worthwhile to note that this result is in stark contrast to Monnet and Roberds (2008) where inflation can stimulate the use of credit cards. This difference is mainly attributed to the fact that a credit card is a deferred payment instrument, whereas a debit card is a non-deferred payment one.

Finally, as mentioned,  $\theta$  is non-distortionary taxation on a buyer's labor, whereas  $\tilde{\theta}$  is distortionary taxation on a buyer's consumption. Then, (17) implies that non-distortionary taxation to finance the E-payment cost encourages coordination in using E-payment: i.e., a higher  $\theta$  implies a lower threshold  $z^*$ . The background channel for this is very similar to that for  $\mu$ . That is,  $q_*^E$  is constant from (15) and then as  $\tilde{\theta}$  increases,  $m^E$  should increase to compensate a seller for bearing the greater cost of E-payment. Since money holdings are subject to inflation cost, a higher  $\tilde{\theta}$  (a lower  $\theta$ ) renders agents less willing to coordinate in the use of E-payment.

## V. Concluding Remarks

One of the crucial features of electronic payment technology is that it requires a substantial fixed cost but its operation cost is relatively small. Hence, as more people use the technology, per-transaction cost declines. This feature of an economy of scale implies that how agents end up the coordination in using an electronic payment would be a critical ingredient in explaining its adoption. However, existing literature does not say much about how to coordinate in the use of electronic payment instruments.

This paper attempted to solve this coordination problem in the context of global game reasoning. We show that the presence of exogenous shocks which have a fundamental impact on the cost of the electronic payment technology can resolve an indeterminacy problem related to agents' beliefs. Furthermore, it turns out that higher inflation as well as distortionary financing schemes for E-payment cost tend to disturb coordination in using debit cards.

It is worth noting that various alterations to the cost structures can be considered

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<sup>8</sup> Notice that C-payment cost is proportional to the real amount of the transaction in our model. But if C-payment cost is assumed to be proportional to the nominal amount of the transaction, the cost increases with inflation and thus in an inflationary economy, agents might be more willing to use E-payment rather than C-payment. We owe this point to an anonymous referee.

in our framework. We here assume that C-payment cost is borne by a buyer and is proportional, whereas E-payment cost is fixed. As an alternative, one might delve into a case in which sellers also bear some cash-handling cost or a case in which C-payment cost is not proportional, but fixed.

Finally, in our model, buyers eventually determine whether to use a debit card and we abstract from the sellers' problem of whether to accept a debit card. In the real world, buyers' adoption of a debit card would depend on its acceptance by sellers, and vice versa. In this context, it would also be worthwhile to explore the coordination problem between buyers and sellers.

## Appendix

**Derivation of (10):** Notice that if a buyer is locked in  $o \in \{E, C\}$ , her expected payoff with respect to an individual shock can be expressed as

$$W_t^{o_i}(m_t^{o_i}, z_t) = \phi_t(m_t^{o_i}, b_t) + e^{-\rho\tau} \left\{ \alpha\tau [\mathbb{E}_z(V_{t+\tau}^{o_i}(m_{t+\tau}^{o_i}, z_{t+\tau})) - \phi_{t+\tau} m_{t+\tau}^{o_i}] \right. \\ \left. + (\phi_{t+\tau} - e^{\rho\tau} \phi_t) m_{t+\tau}^{o_i} \right\} \\ + e^{-\rho\tau} (1 - \alpha\tau) \left\{ k\tau \mathbb{E}_z \max_{o_{t+\tau}} [W_{t+\tau}^{o_{t+\tau}}(0, z_{t+\tau})] \right. \\ \left. + (1 - k\tau) \mathbb{E}_z [W_{t+\tau}^{o_i}(0, z_{t+\tau})] \right\}. \quad (18)$$

Now substituting (4) into (18), we can obtain (10):

$$W_t^{o_i}(m_t^{o_i}, z_t) = \phi_t(m_t^{o_i}, b_t) + e^{-\rho\tau} \left\{ \alpha\tau \mathbb{E}_z [u(q_{t+\tau}^{o_i}) - \phi_{t+\tau} p_{t+\tau}^{o_i} - \xi_{t+\tau}^{o_i}] \right. \\ \left. + (\phi_{t+\tau} - e^{\rho\tau} \phi_t) m_{t+\tau}^{o_i} \right\} \\ + e^{-\rho\tau} \left\{ k\tau \mathbb{E}_z \max_{o_{t+\tau}} [W_{t+\tau}^{o_{t+\tau}}(0, z_{t+\tau})] \right. \\ \left. + (1 - k\tau) \mathbb{E}_z [W_{t+\tau}^{o_i}(0, z_{t+\tau})] \right\}.$$

**Derivation of (11):** The relative payoff for choosing E-payment,  $W_t^E(m_t^E, z_t) - W_t^C(m_t^C, z_t)$ , can be expressed as follows:

$$W_t^E(m_t^E, z_t) - W_t^C(m_t^C, z_t) = e^{-\rho\tau} \left\{ \alpha\tau \mathbb{E}_z [u(q_{t+\tau}^E) - \phi_{t+\tau} p_{t+\tau}^E - \theta \omega_{t+\tau}] \right. \\ \left. + (\phi_{t+\tau} - e^{\rho\tau} \phi_t) m_{t+\tau}^E \right\} \\ - e^{-\rho\tau} \left\{ \alpha\tau [u(q_{t+\tau}^C) - \phi_{t+\tau} p_{t+\tau}^C - \eta \phi_{t+\tau} p_{t+\tau}^C] \right. \\ \left. + (\phi_{t+\tau} - e^{\rho\tau} \phi_t) m_{t+\tau}^C \right\} \\ + e^{-\rho\tau} (1 - k\tau) \mathbb{E}_z [W_{t+\tau}^E(0, z_{t+\tau}) - W_{t+\tau}^C(0, z_{t+\tau})].$$

Now, by substituting successively, the right-hand side can be rearranged as

$$\sum_{i=1}^{\infty} e^{-\rho\tau i} (1 - k\tau)^{i-1} \mathbb{E}_z \left\{ \alpha\tau [u(q_{t+i\tau}^E) - \phi_{t+i\tau} p_{t+i\tau}^E - \theta \omega_{t+i\tau}] \right. \\ \left. + (\phi_{t+i\tau} - e^{\rho\tau} \phi_{(i-1)\tau+t}) m_{t+i\tau}^E \right\} \\ - \sum_{i=1}^{\infty} e^{-\rho\tau i} (1 - k\tau)^{i-1} \left\{ \alpha\tau [u(q_{t+i\tau}^C) - \phi_{t+i\tau} p_{t+i\tau}^C - \eta \phi_{t+i\tau} p_{t+i\tau}^C] \right. \\ \left. + (\phi_{t+i\tau} - e^{\rho\tau} \phi_{(i-1)\tau+t}) m_{t+i\tau}^C \right\}.$$

Since  $c(q_{t+i\tau}^E) + \tilde{\theta} \mathbb{E}_z \omega_{t+i\tau} = \phi_{t+i\tau} p_{t+i\tau}^E$ , we finally have (11):

$$\begin{aligned}
 W_t^E(m_t^E, z_t) - W_t^C(m_t^C, z_t) = & \\
 & \sum_{i=1}^{\infty} e^{-\rho \tau i} (1 - k\tau)^{i-1} \mathbb{E}_z \left\{ \alpha \tau u(q_{t+i\tau}^E) + (1 - e^{\rho \tau} \frac{\phi_{(i-1)\tau+t}}{\phi_{t+i\tau}} - \alpha \tau) c(q_{t+i\tau}^E) \right. \\
 & \quad \left. + [(1 - e^{\rho \tau} \frac{\phi_{(i-1)\tau+t}}{\phi_{t+i\tau}}) \tilde{\theta} - \alpha \tau] \omega_{t+i\tau} \right\} \\
 & - \sum_{i=1}^{\infty} e^{-\rho \tau i} (1 - k\tau)^{i-1} \left\{ \alpha \tau [u(q_{t+i\tau}^C) - \phi_{t+i\tau} p_{t+i\tau}^C - \eta \phi_{t+i\tau} p_{t+i\tau}^C] \right. \\
 & \quad \left. + (\phi_{t+i\tau} - e^{\rho \tau} \phi_{(i-1)\tau+t}) m_{t+i\tau}^C \right\}.
 \end{aligned}$$

**Derivation of (12):** The relative payoff for choosing E-payment when  $\tau \rightarrow 0$  can be expressed as follows:

$$\begin{aligned}
 & \lim_{\tau \rightarrow 0} \sum_{i=1}^{\infty} e^{-\rho \tau i} (1 - k\tau)^{i-1} \mathbb{E}_z \left\{ \alpha \tau u(q_{t+i\tau}^E) + (1 - e^{\rho \tau} \frac{\phi_{(i-1)\tau+t}}{\phi_{t+i\tau}} - \alpha \tau) c(q_{t+i\tau}^E) \right. \\
 & \quad \left. + [(1 - e^{\rho \tau} \frac{\phi_{(i-1)\tau+t}}{\phi_{t+i\tau}}) \tilde{\theta} - \alpha \tau] \omega_{t+i\tau} \right\} \\
 & - \lim_{\tau \rightarrow 0} \sum_{i=1}^{\infty} e^{-\rho \tau i} (1 - k\tau)^{i-1} \left\{ \alpha \tau [u(q_{t+i\tau}^C) - \phi_{t+i\tau} p_{t+i\tau}^C - \eta \phi_{t+i\tau} p_{t+i\tau}^C] \right. \\
 & \quad \left. + (\phi_{t+i\tau} - e^{\rho \tau} \phi_{(i-1)\tau+t}) m_{t+i\tau}^C \right\}
 \end{aligned}$$

which can be rearranged as

$$\begin{aligned}
 & \mathbb{E}_z \left\{ \int_0^{\infty} e^{-(k+\rho)v} \{ \alpha [u(q_{t+v}^E) - c(q_{t+v}^E) - \omega_{t+v}] \right. \\
 & \quad \left. - \mu_{t+v} c(q_{t+v}^E) - \mu_{t+v} \tilde{\theta} \omega_{t+v} \} dv \right\} \\
 & - \int_0^{\infty} e^{-(k+\rho)v} \{ \alpha [u(q_{t+v}^C) - \phi_{t+v} p_{t+v}^C - \eta \phi_{t+v} p_{t+v}^C] - \phi_{t+v} \mu_{t+v} m_{t+v}^C \} dv.
 \end{aligned}$$

**Derivation of (16):** Notice that at  $Z_{\infty}$ , an agent is indifferent between two means of payment if she expects all other agents to select E-payment to the right and C-payment to the left. Since the movement in  $X$  always pulls the state away from  $Z_{\infty}$ , the dynamics of  $(X, z)$  are unstable. With  $k \rightarrow \infty$ , the movement in  $X$  is fast relative to the movement in  $z$ . The system very quickly bifurcates, either upward (all E-payment trades) or downward (all C-payment trades). By Theorem 2 and Corollary 1 in Krzysztof, Frankel and Pauzner (1998), as the friction shrinks to zero, the amount of time that passes before a bifurcation occurs goes to zero. Starting with a particular state  $(X^*, z^*)$ , the chance of bifurcating up converges to  $1 - X^*$ , while the chance of bifurcating down goes to  $X^*$ . Also, the reasoning behind Theorems 1 and Theorem 2 in Burdzy, Frankel and Pauzner (2001) imply that we can ignore what happens in the distant future after  $X$  approaches zero or one. In other words,  $z$  can be considered constant (relative to the movement of  $X$ ) and is equal to  $z^*$ . The dynamics of  $X$  can be considered approximately as  $X_{t+v}^{\uparrow} = 1 - (1 - X^*)e^{-kv}$  with probability  $(1 - X^*)$  and  $X_{t+v}^{\downarrow} = X^*e^{-kv}$  with

probability  $X^*$ . And we only need to consider the process of  $X$  reaching zero or one. Therefore, if we consider a constant inflation rate  $\mu$ , the agent's relative payoff for choosing E-payment can be approximated as follows:

$$\begin{aligned} & (1 - X^*) \left\{ \int_0^\infty e^{-(k+\rho)v} \{ \alpha[u(q_*^E) - c(q_*^E) - \omega_{t+v}^\uparrow] - \mu c(q_*^E) - \mu \bar{\theta} \omega_{t+v}^\uparrow \} dv \right\} \\ & + X^* \left\{ \int_0^\infty e^{-(k+\rho)v} \{ \alpha[u(q_*^E) - c(q_*^E) - \omega_{t+v}^\downarrow] - \mu c(q_*^E) - \mu \bar{\theta} \omega_{t+v}^\downarrow \} dv \right\} \\ & - \int_0^\infty e^{-(k+\rho)v} \{ \alpha[u(q_*^C) - c(q_*^C) - \eta c(q_*^C)] - \mu c(q_*^C) \} dv \end{aligned}$$

where  $\omega_{t+v}^\uparrow(\omega_{t+v}^\downarrow)$  is the path associated with  $X_{t+v}^\uparrow(X_{t+v}^\downarrow)$ . By rearranging the above, we can finally obtain (16):

$$\begin{aligned} & \int_0^\infty e^{-(k+\rho)v} \{ \alpha[u(q_*^E) - c(q_*^E)] - \mu c(q_*^E) \} dv \\ & - \int_0^\infty e^{-(k+\rho)v} \{ \alpha[u(q_*^C) - c(q_*^C) - \eta c(q_*^C)] - \mu c(q_*^C) \} dv \\ & - (1 - X^*) \left\{ \int_0^\infty e^{-(k+\rho)v} (\mu \bar{\theta} + \alpha) \omega_{t+v}^\uparrow dv \right\} - X^* \left\{ \int_0^\infty e^{-(k+\rho)v} (\mu \bar{\theta} + \alpha) \omega_{t+v}^\downarrow dv \right\} \\ & = \left( \frac{1}{\rho + k} \right) \{ \alpha[u(q_*^E) - c(q_*^E)] - \mu c(q_*^E) - \alpha[u(q_*^C) - c(q_*^C) - \eta c(q_*^C)] + \mu c(q_*^C) \} \\ & - \left( \frac{\alpha + \mu \bar{\theta}}{k} \right) \left[ \int_{X^*}^1 \left( \frac{1-X}{1-X^*} \right)^{\frac{\rho}{k}} \omega(z_t, X) dX + \int_0^{X^*} \left( \frac{X}{X^*} \right)^{\frac{\rho}{k}} \omega(z_t, X) dX \right]. \end{aligned}$$

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