Multiple Means of Payment, Excess Reserves, and Monetary Policy*

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The effects of choosing between cash and credit as a means of payment on bank's excess reserves are explored in the proposed model. The model incorporates the widespread recent features of payment patterns and financial services. Results suggest that credit increases excess reserves and generates leeway for banks to invest in interest-bearing assets. Given the growth rate of money, credit transactions increase, but welfare decreases. This phenomenon implies the optimality of the Friedman rule.

I. Introduction

Recently, many financial institutions have been offering sweep accounts to maximize customers' interest earnings by minimizing reserve holdings against cash withdrawals. Also, thanks to the widespread use of credit cards, consumers do not tend to demand cash as much as before. That is, compared with those in a money-oriented economy, they are less willing to hold cash as credit cards become widely available. This might bring about more idle balances in banks, which allow them to invest more in interest-bearing assets. However, the implications of these recent changes in payment patterns and financial services on banks' excess reserves and

monetary policy have not yet been studied much.

There has been a set of literature on money, banking, and alternative payment instruments including Champ et al. (1996), Smith (2002), He Huang Wright (2005, 2008), Berentsen et al. (2007), Williamson (2009), Williamson (2012) , and Huang and Lee (2015). Among them, particularly, Champ et al. (1996) and Smith (2002) considered bank's reserves explicitly. Champ et al. (1996) construct an overlapping generations model in which as in the U.S. National Banking System in the 19th century, a bank can issue banknotes. They show that bank panics can be effectively prevented if the bank is allowed to issue banknotes with no restriction. Smith (2002) adopts a similar framework of Champ et al. (1996) and explores the relationship between bank panics and the nominal interest rate as a proxy of the inflation rate. A bank allocates its portfolio between non-interest-bearing cash and interest-bearing investment. He shows that the probability of bank panics declines as the nominal interest rate decreases because the bank is willing to hold more cash as reserves by curtailing investment. However, this strand of literature mainly focuses on the reserves in connection with bank panics.

Another strand of the relevant literature is on the credit card debt puzzle.[1] Particularly, Telyukova and Wright (2008) and Telyukova (2013) provide a theoretical framework and quantitative evaluations on the demand for credit and liquidity assets to explain the puzzle. However, they leave out the interaction between the endogenous choice of cash and credit and a bank's portfolio choice, which can provide another explanation on the substitution between credit and liquid assets.

This paper attempts to explore the channel through which the endogenous choice of means of payment between cash and credit gives an effect on banks' reserves and then monetary policy. Our model is built on Ireland (1994), Bencivenga and Camera (2011), and Champ et al. (1996). There are a unit mass of households, a bank, and the government. Each household has a transactions account, from which cash can be withdrawn with no cost, and an interest-bearing one-period savings account, which is considered as a sweep account in that the balances in the account can be transferred into the transaction account. The bank offers a deposit contract in the savings account maximizing an expected return of the household subject to a bank's no-default constraint. In the beginning of a period, households observe the technology shock to the cost of credit transaction but the bank cannot. In the financial market, the bank chooses its portfolio between interest-bearing bonds and non-interest bearing reserves. In the goods market, households choose how much to purchase consumption goods in cash and costly

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[1] According to the 2010 Survey of Consumer Finance, 39.4% of families carried credit card balances and 94.0% of them owned some financial assets. In particular, 92.5% of them owned some liquid transactions accounts, for example, checking, savings, and money market deposit accounts.

credit. The credit service is provided by the bank.

The key ingredient is that there are two opposite channels capturing the effect of the cash-credit choice on the bank's excess reserves and portfolios. The presence of credit increases excess reserves because of the uncertainty on the usages of credit by households. This will reduce investment in interest-bearing assets. However, it decreases money demand and increases bank's investment. On net, the former negative effect of credit on the investment in interest-bearing assets is dominated by the latter positive effect.

In an inflationary economy, the government money injection increases the frequency of credit transactions, but deteriorates welfare. However, as cash transactions decline, the bank holds less reserves and invests more. However, this positive effect on welfare is dominated by the negative effect stemming from an increase in the cost of credit transactions.

The remainder of the paper is organized as follows. Section 2 describes the background environment and Section 3 discusses equilibrium dynamics. In Section 4, the implications of an endogenous choice of means of payment between cash and credit on monetary policy are discussed. Section 5 summarizes the paper with a few concluding remarks.

II. Model

The background environment is a standard cash-in-advance model introduced in Ireland (1994), Bencivenga and Camera (2011), and Champ et al. (1996), among others.

2.1. Representative Household

Time is discrete and indexed by $t = 0,1,2,\dots$. There is a continuum of infinitely-lived households with unit mass. Each household consists of a shopper and a worker. There is a continuum of spatially separated good markets indexed by $i \in [0,1]$ in each period. Similar to in Ireland (1994), the household's preference is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1-i_t^*)\ln(c_t^m) + i_t^* \ln(c_t^c) - \theta_t \int_0^{i_t^*} \ln\left(\frac{1}{1-i}\right) di \right] \tag{1}$$

where $\mathbb{E}_0$ is the expectation operator conditional on information in period 0, β is the discount factor, $c^m (c^c)$ represents distinct and perishable consumption goods purchased with cash (credit), and i^* is the cutoff between cash and credit

purchases determined endogenously later. As in Camera and Li (2008) and Choi (2011), $\ln(\frac{1}{1-i})$ represents the transaction cost of credit in the form of effort as disutility. The cost increases with i exponentially as a shopper spends more on credit for a greater variety of consumption goods, and hence a shopper uses credit to acquire good i for $i < i_t^*$ and cash for $i < i_t^*$ (see for instance, Ireland 1994). Finally, θ is a technology shock to the cost of credit transaction which is independent and identically distributed over time with the time-invariant cdf $F(\theta)$ where $\theta_t \in [\underline{\theta}, \overline{\theta}]$ and $\overline{\theta} > \underline{\theta} > 0$.

In each period, the household receives constant endowments y and cannot consume its own endowments. At the beginning of each period t, the household enters the period with M_t amount of money in a transactions account and D_t amount of deposits in a one-period savings account. The household can withdraw cash from a transactions account at anytime with no cost.

When the financial market opens, θ for the current period is realized but the bank cannot observe it until shoppers come by it. In the financial market, one-period interest-bearing government bonds are traded between the bank and the government. After the financial market closes, the bank opens and a shopper can withdraw cash X_t from her savings account. The financial constraint for the household is

$$[D_{t+1}/(1+r_t)] + X_t = D_t \tag{2}$$

where r_t is the net nominal interest rate on a one-period savings account matured at $t+1$.

In the good market, workers sell their endowments and shoppers purchase them with either cash or credit. The cash-in-advance constraint is

$$P_t(1 - i_t^*)c_t^m \le M_t + X_t \tag{3}$$

where P_t is the average price level of consumption goods.

At the end of each period, all agents return home. Workers arrive at home with the revenue from sales $P_t y$ and the household pays off a credit balances to the bank. The budget constraint is

$$P_t c_t + M_{t+1} = M_t + X_t + P_t y \tag{4}$$

where $c_t = i_t^* c_t^c + (1 - i_t^*)c_t^m$ represents aggregate consumption with cash and credit.

2.2. Representative Bank

A representative bank takes deposits from the household, holds reserves and

government bonds, and issues credit. The bank offers a deposit contract that maximizes an expected return of the household subject to a bank's no-default constraint. This is in line with Diamond and Dybvig (1983), Bencivenga and Camera (2011), and Williamson (2012).

Given the household's deposit Dt, the bank can choose how much to invest in one-period government bonds (B_{t+1}) and how much to hold reserves (Z_{t+1}) in the financial market. A bond is sold at q_t units of money in period t and is a claim to one unit of money in period $t+1$. A government bond cannot be liquidated in cash before the maturity. In addition, it is a book-entry bond and hence cannot be circulated as a medium of exchange. The bank's balance sheet is given by

$$[D_{t+1}/(1+r_t)] = q_t B_{t+1} + Z_{t+1} - X_t. \tag{5}$$

Because of an unobservable preference shock to the cost of credit (θ_t) in the financial market, the bank cannot have an exact estimate of the current liquidity demand of the household (X_t). Therefore, under the no-default constraint, the bank should hold reserves (Z_{t+1}) in preparation for the largest cash withdrawals given the distribution of θ such that

$$Z_{t+1} = \bar{X}_t = \max X(\theta). \tag{6}$$

When the bank opens, the bank provides interest on matured savings deposits. In the good market, it can issue credit for shoppers, which will be settled at the end of the period. A zero profit condition then implies

$$D_t = B_t + Z_t - X_{t-1} - \bar{L}_t + L_t \tag{7}$$

where $\bar{L}_t$ is the credit issued by the bank in the good markets and L_t is its repayment by the household to the bank at the end of the period.
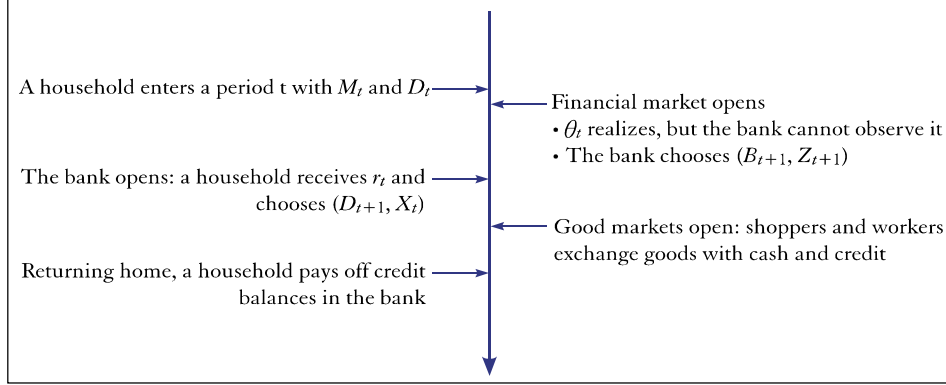
2.3. Government

The government controls the money supply through open market operations. The government budget constraint in period t is given by

$$B_t^s - q_t B_{t+1}^s = M_{t+1}^s - M_t^s = \mu_t M_t^s \tag{8}$$

where $\mu_t > -1$ is the growth rate of money with $\mu_t > 0$ denoting open market purchases by the government. Figure 1 summarizes the timing of events within a

representative period.

[Figure 1] Timing of events within a representative period

III. Equilibrium

A symmetric competitive equilibrium can now be defined as a set of $\{c_t^m, c_t^c, i_t^*,$
$X_t, D_{t+1}, Z_{t+1}, B_{t+1}, M_{t+1}\}_{t=0}^{\infty}$ and $\{P_t, q_t, r_t\}_{t=0}^{\infty}$ such that:
- The household solves the optimization problem subject to (2)-(4);
- The bank solves the optimization problem subject to (5)-(7);
- The government satisfies its budget constraint (8);
- Markets clear in every period;
    - Goods Market: $i_t^* c_t^c + (1 - i_t^*) c_t^m = y$
    - Money Market: $M_t^s = M_t$, $X_t = \mu_t M_t$, and $M_{t+1} = M_t + X_t$
    - Credit Market: $\bar{L}_t = L_t = P_t i_t^* c_t^c$
    - Bonds Market: $B_{t+1}^s = B_{t+1}$.

Let $\lambda_{1,t}$, $\lambda_{2,t}$, and $\lambda_{3,t}$ denote the Lagrange multipliers associated with the financial constraint in (2), the cash-in-advance constraint in (3), and the budget constraint in (4), respectively. Then, in equilibrium, $(c_t^m, c_t^c, X_t, D_{t+1}, M_{t+1})$ should satisfy

$$(c_t^m)^{-1} = (\lambda_{2,t} + \lambda_{3,t})P_t \tag{9}$$

$$(c_t^c)^{-1} = \lambda_{3,t}P_t \tag{10}$$

$$\lambda_{1,t} = \lambda_{2,t} + \lambda_{3,t} \tag{11}$$

$$\lambda_{1,t} = (1 + r_t)\beta\mathbb{E}_t(\lambda_{1,t+1}) \tag{12}$$

$$\lambda_{3,t} = \beta\mathbb{E}_t(\lambda_{2,t+1} + \lambda_{3,t+1}). \tag{13}$$

In addition, i_t^* is determined so that at the margin the benefit from a credit purchase is equal to that from a cash purchase,

$$\ln(c_t^c) - \theta_t \ln\left(\frac{1}{1-i_t^*}\right) - \lambda_{3,t} P_t c_t^c = \ln(c_t^m) - (\lambda_{2,t} + \lambda_{3,t}) P_t c_t^m. \tag{14}$$

3.1. Cash Withdrawal and Consumption

Hereinafter we focus on the case where the growth rate of money is constant, $\mu_t = \mu$ for all t. In order to express the nominal variable in real terms, we let $d_{t+1} = D_{t+1} / P_t$, $m_{t+1} = M_{t+1} / P_t$, $x_t = X_t / P_t$, and $z_{t+1} = Z_{t+1} / P_t$.

As regards cash withdrawal, first notice that from (3) and (4), a fraction of revenue from sales is used to clear credit debt and the remainder is carried into the next period in the form of cash:

$$m_{t+1} + i_t^* c_t^c = y. \tag{15}$$

Let $\phi_t \in (0,1)$ be the ratio of real money demand (m_{t+1}) to income (y):

$$m_{t+1} = \phi_t y. \tag{16}$$

Then from (2), (5), (7), and (8), cash withdrawal from the bank is given by

$$x_t = \frac{\mu \phi_t y}{1+\mu} \tag{17}$$

where we use $(1+\mu) = (M_{t+1} / M_t) = (P_t \phi_t) / (P_{t+1} \phi_{t+1})$.

Lemma 1 *A real cash withdrawal in a money-credit economy (x_t) is smaller than that in a money-only economy.*

Proof. Notice that the amount of cash withdrawal from the savings account when the credit is not available is $[\mu / (1+\mu)]y$. The amount of cash withdrawal when the credit is available is given by (17) which can be rewritten as

$$x_t = \left(\frac{\mu}{1+\mu}\right)y - \left(\frac{\mu}{1+\mu}\right)(1-\phi_t)y. \tag{19}$$

This, together with $\phi_t \in (0,1)$, implies the claim.   ∎

Now, from (3) and (18) together with the money market clearing condition $(M_{t+1} = M_t + X_t)$, cash consumption is given by

$$(1 - i_t^*)c_t^m = \phi_t y \tag{19}$$

while from (15) and (16), credit consumption is given by

$$i_t^* c_t^c = (1 - \phi_t)y . \tag{20}$$

3.2. Real Money Demand and Cash-Credit Choice

From (9), (10), (11), and (14), the choice of cash and credit, i_t^*, is determined by the intertemporal marginal rate of substitution between consumption with credit and that with cash,

$$\left(\frac{1}{1 - i_t^*}\right)^{\theta_t} = \frac{c_t^c}{c_t^m} . \tag{21}$$

In addition, from (13), we can obtain

$$\frac{c_t^c}{\phi_t} = \frac{1 + \mu}{\beta \Psi} \tag{22}$$

where $\Psi = \mathbb{E}_t(\phi_{t+1} / c_{t+1}^m)$. From (9), (12), and (22), the return on deposit is equal to the marginal rate of substitution between cash consumption and credit consumption,

$$1 + r_t = \frac{c_t^c}{c_t^m} \tag{23}$$

and as in Ireland (1994), $c_t^c > c_t^m$ if $r_t > 0$: i.e., credit is used for larger purchases, while cash is used for smaller ones.

Now, from (19)-(22), i_t^* and ϕ_t are respectively characterized as a function of preference shock to the cost of credit transaction (θ_t):

$$i_t^* = 1 - \left(\frac{\beta \Psi y}{1 + \mu}\right)^{\frac{1}{1 + \theta_t}} \tag{24}$$

$$\phi_t = \frac{1}{1 + \frac{1+\mu}{\beta \Psi y} - \left(\frac{1+\mu}{\beta \Psi y}\right)^{\frac{\theta_t}{1+\theta_t}}} \tag{25}$$

where $[(\beta \Psi y)/(1+\mu)]^{\frac{1}{1+\theta_t}} \in (0,1)$.

3.3. Bank's Excess Reserves and Investment

Let z_t^{er} denote the bank's excess reserves, which are defined as $z_t^{er} = z_{t+1} - x_t$. Given unobservable θ_t, from (6), (17), and (18), the bank's excess reserves are then given by

$$z_t^{er} = z_{t+1} - x_t = \overline{x}_t - x_t = \left(\frac{\mu}{1+\mu}\right)[\overline{\phi}(\mu, \overline{\theta}_t) - \phi(\mu, \theta_t)]y \geq 0 \tag{26}$$

where $\overline{\theta}_t = \arg\max X(\theta_t)$. The following lemma manifests the effect of θ_t on the bank's reserves.

Lemma 2 *If preference shock (θ_t) is observable, the bank does not hold excess reserves, $z_t^{er} = z_{t+1} - x_t = 0$.*

Proof. The claim is implied by (26) because $\overline{\phi}_t = \phi_t$ if there is no uncertainty on θ_t. ■

It is worthwhile to note that the technology shock to the cost of credit transaction (θ_t) is irrelevant to cash transactions and hence as in Lemma 2, there is no excess reserve in a no-credit economy. Therefore (26) captures net effect of credit on the bank's excess reserves in a credit-money economy compared to a no-credit economy.

Now the following proposition suggests that in the presence of credit, uncertainty about the extent of its usages by the households induces excess reserves. But even in the case, the bank's investment in interest-bearing government bonds can increase further compared with one in a no-credit economy.

Proposition 1 *When preference shock is unobservable, (i) $z_t^{er} = \overline{x}_t - x_t > 0$ unless $\theta_t = \overline{\theta}_t$ and (ii) the presence of credit boosts the bank's investment in government bonds.*

Proof. The first claim is an obvious consequence from (26). As regards the second claim, from (18) and (26), the bank's net investment in government bonds compared to a no-credit economy is given by

$$\left(\frac{\mu}{1+\mu}\right)(1-\phi_t)y - \left(\frac{\mu}{1+\mu}\right)(\bar{\phi}_t - \phi_t)y = \left(\frac{\mu}{1+\mu}\right)(1-\bar{\phi}_t)y \tag{27}$$

which is positive because $\bar{\phi}_t \leq 1$. ■

Notice that the first term on the left-hand side of (27) is the effect of credit on cash withdrawal from a savings account and the second term is the effect of credit on the bank's excess reserves. Proposition 1 shows that the former positive effect of credit on the bank's capacity for investment dominates the latter negative effect of credit on the bank's capacity.

IV. Monetary Policy Implications

In this section, we investigate the effect of the growth rate of money on the credit transactions and the excess reserves of the bank.

Proposition 2 *In (24), i_t^* increases with μ:*

$$\frac{\partial i_t^*}{\partial \mu} = \frac{(\beta \Psi y)^{\frac{1}{1+\theta_t}}}{1+\theta_t}\left(\frac{1}{1+\mu}\right)^{\frac{2+\theta_t}{1+\theta_t}} > 0. \tag{28}t$$

This result is intuitively straightforward in the sense that shoppers are willing to use credit for a greater variety of consumption goods in a high-inflation economy. As a consequence, as shown below, the fraction of cash consumption declines and the volume of credit consumption increases.

Corollary 1 *In (25), ϕ_t decreases with μ:*

$$\frac{\partial \phi_t}{\partial \mu} = \left(\frac{\phi_t^2}{\beta \Psi y}\right)\left[\left(\frac{\beta \Psi y}{1+\mu}\right)^{\frac{1}{1+\theta_t}}\left(\frac{\theta_t}{1+\theta_t}\right) - 1\right] < 0. \tag{29}$$

Notice that as mentioned already, $\phi_t = 1$ in a no-credit economy and the bank's reserves are equal to cash withdrawals, $z_t = \bar{x}_t = x_t = \mu y/(1+\mu)$. There are no excess reserves, regardless of the growth rate of money. In the presence of credit, however, there arise excess reserves and the effect of the money growth rate on the excess reserves is not obvious because it affects the portfolio of the bank as well as the means-of-payment choice of the household. The following proposition suggests

that excess reserves and investment in government bonds increase with the growth rate of money at least in an inflationary economy.

Proposition 3 *Suppose* $\mu \geq 0$. *Then, as the growth rate of money increases, (i) the bank's excess reserves* (z_t^{er}) *increase and (ii) the investment in government bonds also increases.*

Proof. As regards the first claim, from (17) and (26),

$$\frac{\partial z_t^{er}}{\partial \mu} = \frac{y}{1+\mu}\left[\underbrace{\frac{\overline{\phi}_t - \phi_t}{1+\mu}}_{\text{net inflation effect}} + \underbrace{\mu\left(\frac{\partial \overline{\phi}_t}{\partial \mu} - \frac{\partial \phi_t}{\partial \mu}\right)}_{\text{portfolio adjustment effect}}\right].$$

Since, from (17) and (25),

$$\frac{\partial \overline{\phi}_t}{\partial \mu} - \frac{\partial \phi_t}{\partial \varpi} = \left(\frac{\overline{\phi}_t^2}{\beta \Psi y}\right)\left[\left(\frac{\beta \Psi y}{1+\mu}\right)^{\frac{1}{1+\overline{\theta}_t}}\left(\frac{\overline{\theta}_t}{1+\overline{\theta}_t}\right) - 1\right] - \left(\frac{\phi_t^2}{\beta \Psi y}\right)\left[\left(\frac{\beta \Psi y}{1+\mu}\right)^{\frac{1}{1+\theta_t}}\left(\frac{\theta_t}{1+\theta_t}\right) - 1\right]$$

$$= \begin{cases} > 0 \text{ if } 1+\mu = \beta \Psi y \\ \approx 0 \text{ if } 1+\mu \text{ is very large,} \end{cases}$$

we have $\partial z_t^{er} / \partial \mu > 0$ if $\mu \geq 0$. As regards the second claim, the derivative of (27) with respect to μ is given by

$$\frac{y}{(1+\mu)^2}\left[(1-\overline{\phi}_t) - \mu\frac{\partial \overline{\phi}_t}{\partial \mu}\right]. \tag{30}$$

Then (29) together with $\mu \geq 0$ implies that (30) is strictly positive. ∎

The growth rate of money affects excess reserves via two channels: one is the non-negative net-inflation effect on the household and the bank; the other is the portfolio-adjustment effect, which is comprised of (i) the effect on the choice of a means of payment between money and credit, and hence on x_t, and (ii) the effect on the bank's portfolio between government bonds and reserves, and hence on z_t. These portfolio-adjustment effects depend on μ and they are non-negative if $\mu \geq 0$. Therefore, in an inflationary economy ($\mu \geq 0$), both inflation and portfolio effects are positive and z^{er} increases with μ. This positive relationship between excess reserves and inflation is somewhat interesting because the conventional

isdom seems to imply the negative relationship between them. In a deflationary economy ($\mu < 0$), however, it is not clear. Meanwhile, the bank's investment in government bonds increases with the growth rate of money in an inflationary economy because as we can see from (29), the demand for cash declines.

Finally, we examine whether the optimal growth rate of money is negative as in a typical cash-in-advance model. In order to discuss the optimality, we first define the welfare as instantaneous utility of a representative household:

$$W_t = (1 - i_t^*)\ln(c_t^m) + i_t^* \ln(c_t^c) - \theta_t \int_0^{i_t^*} \ln\left(\frac{1}{1-i}\right) di .$$

Since $\theta_t \ln(1/(1 - i_t^*)) = \ln(c_t^c / c_t^m)$ from (21), W_t can be represented simply in terms of c_t^c and i_t^* as follows:

$$
\begin{aligned}
W_t &= (1 - i_t^*)\ln(c_t^m) + i_t^* \ln(c_t^c) - \theta_t \int_0^{i_t^*} \ln\left(\frac{1}{1-i_t}\right) di \\
&= \ln(c_t^c) - \theta_t (1 - i_t^*) \ln\left(\frac{1}{1-i_t^*}\right) - \theta_t \int_0^{i_t^*} \ln\left(\frac{1}{1-i_t}\right) di \\
&= \ln(c_t^c) + \theta_t (1 - i_t^*) \ln(1 - i_t^*) - \theta_t (1 - i_t^*) \ln(1 - i_t^*) + \theta_t (1 - i_t^*) \\
&= \ln(c_t^c) + \theta_t (1 - i_t^*) .
\end{aligned}
\tag{31}
$$

Now the following proposition shows that inflation deteriorates welfare through the channel of a means-of-payment choice.

Proposition 4 *As the growth rate of money increases, welfare decreases,* $\partial W_t / \partial \mu < 0$.

Proof. See Appendix. ■

As shown in Proposition 2, households are more willing to use credit in the good markets as the growth rate of money increases. This causes positive as well as negative effect on welfare: the positive one is due to increases in the interest rate and investment in government bonds from (21), (23) and Proposition 3; the negative one is due to an increase in the cost of credit transactions. Proposition 4 suggests that the negative effect dominates the positive effect and hence the Friedman rule is essentially optimal. The optimal growth rate of money (μ^*) is given by $\mu^* = \beta \Psi y - 1$ at which there is no opportunity cost of cash holdings and hence, as we can see in (24) and (25), all transactions are made in cash.

V. Concluding Remarks

We explore the effect of credit on excess reserves and its implications for monetary policy in an environment where widespread recent features of payment patterns and financial services are incorporated. The presence of credit tends to induce excess reserves, but investment in interest-bearing assets still increases because of its effect on the portfolios of both the bank and households. With the growth rate of money, credit transactions increase in terms of both frequency and volume. Also, in an inflationary economy, it turns out that there is a positive relationship between excess reserves and inflation.

In order to delve into various aspects of unconventional balance-sheet monetary policy, we could consider variations of the model in which interest on excess reserves is explicitly incorporated. We could also add production in the model, which will eventually allow us to explore the role of a bank as an insurance provider for households against idiosyncratic liquidity shocks and a relationship with economic growth. In the current model, it is an endowment economy and the positive effect of inflation via an increase in banks investment on welfare is dominated by its negative effect via an increase in credit-transaction cost. However, it can exacerbate welfare loss even greater in the economy with production depending on the degree of the intertemporal rate of substitution. When households are not able to smooth out production and consumption against the shock, the bank can play a risk-sharing role by adjusting its portfolio and the supply of credit.

Another interesting potential work is to explore empirically the relationship between credit and excess reserves. In our theoretical model, many ingredients that give an effect on the choice of means of payment by consumers and the choice of portfolios by banks are missing. For instance, there is no intra-day borrowing from the government and the bank holds the maximum amount of reserves in preparation for the largest cash withdrawal under unobservable liquidity shocks. However, in practice, the availability of intra-day credit may distort reserve holdings because the bank can simply borrow from the government if necessary. In addition, as mentioned, our economy is an endowment one (no production economy) and credit is simply within-period IOUs issued by consumers through the bank. However, in practice, bank's reserve holdings would depend on credit loans to firms or economic activities as well. Finally, other than credit-transaction costs, there is no credit friction such as default risk. All in all, in order to test empirically our theoretical result on the relationship between credit and excess reserves, we should control these ingredients effectively.

Appendix: Proof of Proposition 4

From (22) and (31), the effect of monetary policy on welfare can be expressed as

$$
\begin{aligned}
\frac{\partial W_t}{\partial \mu} &= \frac{1}{c_t^c}\frac{\partial c_t^c}{\partial \mu} - \theta_t \frac{\partial i_t^*}{\partial \mu} = \frac{1}{c_t^c}\left[\frac{-y}{i_t^{*2}}\left\{i_t^*\frac{\partial \phi_t}{\partial \mu} + (1-\phi_t)\frac{\partial i_t^*}{\partial \mu}\right\}\right] - \theta_t\frac{\partial i_t^*}{\partial \mu} \\
&= -\frac{y}{i_t^* c_t^c}\frac{\partial \phi_t}{\partial \mu} - \frac{(1-\phi_t)y}{i_t^{*2} c_t^c}\frac{\partial i_t^*}{\partial \mu} - \theta_t\frac{\partial i_t^*}{\partial \mu} = -\left(\frac{1}{1-\phi_t}\right)\frac{\partial \phi_t}{\partial \mu} - \frac{1}{i_t^*}\frac{\partial i_t^*}{\partial \mu} - \theta_t\frac{\partial i_t^*}{\partial \mu} \\
&= -\left(\frac{1}{1-\phi_t}\right)\frac{\partial \phi_t}{\partial \mu} - \left(\theta_t + \frac{1}{i_t^*}\right)\frac{\partial i_t^*}{\partial \mu},
\end{aligned}
\tag{32}
$$

where $i_t^* c_t^c = (1-\phi_t)y$ from (20) and the effect of monetary policy on c_t^c is given by

$$
\frac{\partial c_t^c}{\partial \mu} = \frac{-y}{i_t^{*2}}\left[i_t^*\frac{\partial \phi_t}{\partial \mu} + (1-\phi_t)\frac{\partial i_t^*}{\partial \mu}\right].
$$

Next, in (24) and (25), $\frac{1}{\phi_t} = 1 + (\frac{1+\mu}{\beta\Psi y})i_t^*$ and the relationship between the effect of monetary policy on real money demand and that on the cash-credit choice is

$$
\frac{\partial \phi_t}{\partial \mu} = -\phi_t^2\left[\frac{i_t^*}{\beta\Psi y} + \left(\frac{1+\mu}{\beta\Psi y}\right)\frac{\partial i_t^*}{\partial \mu}\right].
\tag{33}
$$

Now from (32) and (33), the effect on welfare can be expressed in terms of the cash-credit choice:

$$
\begin{aligned}
\frac{\partial W_t}{\partial \mu} &= -\left(\frac{1}{1-\phi_t}\right)\frac{\partial \phi_t}{\partial \mu} - \left(\theta_t + \frac{1}{i_t^*}\right)\frac{\partial i_t^*}{\partial \mu} \\
&= \left(\frac{\phi_t^2}{1-\phi_t}\right)\frac{i_t^*}{\beta\Psi y} + \left[-\theta_t - \frac{1}{i_t^*} + \left(\frac{\phi_t^2}{1-\phi_t}\right)\left(\frac{1+\mu}{\beta\Psi y}\right)\right]\frac{\partial i_t^*}{\partial \mu}.
\end{aligned}
\tag{34}
$$

Finally, from (20) and (22),

$$
\frac{1-\phi_t}{\phi_t i_t^*} = \frac{1+\mu}{\beta\Psi y}.
\tag{35}
$$

In (34) and (35), the effect on welfare in terms of the cash-credit choice is

$$\frac{\partial W_t}{\partial \mu} = \left(\frac{\phi_t i_t^*}{1-\phi_t}\right)\frac{\phi_t}{\beta\Psi y} + \left[-\theta_t - \frac{1}{i_t^*}\left\{1-\phi_t\left(\frac{\phi_t i_t^*}{1-\phi_t}\right)\left(\frac{1+\mu}{\beta\Psi y}\right)\right\}\right]\frac{\partial i_t^*}{\partial \mu}$$

$$= \frac{\phi_t}{1+\mu} + \left(-\theta_t - \frac{1-\phi_t}{i_t^*}\right)\frac{\partial i_t^*}{\partial \mu}$$

$$= \frac{\phi_t}{1+\mu} - \left\{\theta_t + \phi_t\left(\frac{1+\mu}{\beta\Psi y}\right)\right\}\frac{\partial i_t^*}{\partial \mu}. \tag{36}$$

Then, in (24), (25), (28), (29), and (36),

$$\frac{\partial W_t}{\partial \mu} = \frac{\phi_t}{1-\mu}\left[1-(1+\mu)\left(\frac{\theta_t}{\phi_t}+\frac{1+\mu}{\beta\Psi y}\right)\frac{\partial i_t^*}{\partial \mu}\right]$$

$$= \frac{\phi_t}{1+\mu}\left[1-\left(\frac{\theta_t}{\phi_t}+\frac{1+\mu}{\beta\Psi y}\right)\left(\frac{1}{1+\theta_t}\right)\left(\frac{\beta\Psi y}{1+\mu}\right)^{\frac{1}{1+\theta_t}}\right]$$

$$= \frac{\phi_t}{1+\mu}\left[1-\left\{\theta_t+\theta_t\left(\frac{1+\mu}{\beta\Psi y}\right)-\theta_t\left(\frac{1+\mu}{\beta\Psi y}\right)^{\frac{\theta_t}{1+\theta_t}}+\frac{1+\mu}{\beta\Psi y}\right\}\left(\frac{1}{1+\theta_t}\right)\left(\frac{\beta\Psi y}{1+\mu}\right)^{\frac{1}{1+\theta_t}}\right]$$

$$= \left(\frac{\phi_t}{1+\mu}\right)\frac{A^{\frac{-1}{1+\theta_t}}}{1+\theta_t}\left[(1+\theta_t)A^{\frac{1}{1+\theta_t}}-\theta_t-(1+\theta_t)A+\theta_t A^{\frac{\theta_t}{1+\theta_t}}\right]$$

$$= -\left(\frac{\phi_t}{1+\mu}\right)\frac{A^{\frac{-1}{1+\theta_t}}}{1+\theta_t}\left[(1+\theta_t)A-\theta_t A^{\frac{\theta_t}{1+\theta_t}}-(1+\theta_t)A^{\frac{1}{1+\theta_t}}+\theta_t\right]$$

$$= -\left(\frac{\phi_t}{1+\theta_t}\right)(\beta\Psi y)^{\frac{1}{1+\theta_t}}(1+\mu)^{\frac{-2-\theta_t}{1+\theta_t}}H(A;\theta_t), \tag{37}$$

where in (29), $\frac{1}{\phi_t} = 1+\frac{1+\mu}{\beta\Psi y}-(\frac{1+\mu}{\beta\Psi y})^{\frac{\theta_t}{1+\theta_t}}$, $A = \frac{1+\mu}{\beta\Psi y} > 1$, and

$$H(A;\theta_t) = (1+\theta_t)A-\theta_t A^{\frac{\theta_t}{1+\theta_t}}-(1+\theta_t)A^{\frac{1}{1+\theta_t}}+\theta_t. \tag{38}$$

In (38), H_t is a convex function of A minimized at $\overline{A} < 1$,

$$\overline{A} \in \left\{A\left|\frac{\partial H_t}{\partial A} = 1+\theta_t-\frac{\theta_t^2}{1+\theta_t}A^{\frac{-1}{1+\theta_t}}-A^{\frac{-\theta_t}{1+\theta_t}} = 0\right.\right\} \tag{39}$$

where

$$\left.\frac{\partial H_t}{\partial A}\right|_{A=1} = \frac{1}{1+\theta_t} > 0 \;. \tag{40}$$

Therefore, in (39) and (40) with $H(A=1;\theta_t)=0$ from (38), H_t is positive for all $A > 1$ and monotonically increasing.

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