

Inflation Risk, Settlement Cycles, and Monetary Policy*

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A monetary model is constructed to explore the risk-sharing role of multiple installment payments for credit settlement against inflation risk when the choice of cash and credit is endogenous. Economic individuals acquire liquidity for cash consumption and credit settlement. In equilibrium, the choice of a debt rollover may adjust the demand for liquidity in credit settlement to dampen consumption loss. Welfare benefits result from inflation. The optimal money growth is positive with a zero nominal interest rate and a one-period settlement cycle.

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I. Introduction

This study explores the risk-sharing role of multiple installments of credit-settlement payments against inflation risk when the choice of cash and credit is endogenous. This exploration aims to answer the three following questions: (1) What is the role of a credit debt rollover against inflation risk? (2) How do multiple settlement processes of credit payments affect the demand for liquidity and the choice of multiple payment means? (3) What is the optimal monetary policy in the presence of multiple credit installment payments?

The use of credit cards has become widespread recently. The settlement-process period of credit card payments varies from one economic individual to another.

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Some individuals pay credit-card debt by cash within a month on a designated date. Others pay the debt over the subsequent months by making minimum monthly payments or by paying nominal interest. Economic individuals may smooth out their demand for cash across periods through a debt repayment plan, such as multiple credit installment payments. Multiple credit installment payments may dampen consumption loss against inflation risk by relaxing the liquidity constraint, which may also affect the degree of intratemporal and intertemporal substitution between cash and credit.

Although a concern to economic individuals with regard to smoothing consumption, the presence of debt rollover is a determinant of the demand for money and nominal reserves. The settlement cycle, which is the period taken to pay off credit payments, may represent the degree of fluctuations in liquidity across periods in a payment system. The determinants of money demand are important to implement effective monetary policy, so monetary authorities should understand the characteristics of the settlement cycle. However, the implications of credit-settlement cycles in the choice of payment instruments, demand for liquidity, and monetary policy have yet to be extensively studied.

Previous studies on multiple means of payments, including Ireland (1994), Lacker and Schreft (1996), Aiyagari et al. (1998), Freeman and Kydland (2000), He et al. (2005, 2008), Alvarez and Lippi (2009), Sanches and Williamson (2010), Choi (2011, 2015), and Choi and Lee (2016), pay attention to the choice of cash and alternative payment means given the cost and welfare implications without multiple installment payments in credit payments. For example, Ireland (1994), Lacker and Schreft (1996), Aiyagari et al. (1998), and Choi and Lee (2016) study a cash-credit economy in which current credit balances are settled only within a period. On the contrary, Freeman and Kydland (2000) and Choi (2015) investigate a cash-credit economy in which current credit balances are settled only in the next period. When an endogenous debt rollover in credit transactions is absent as in these models, economic individuals may not adjust the demand for liquidity across periods and cannot insure themselves effectively against inflation risk.

Another relevant body of literature is the design of the payment system, including Freeman (1996), Zhou (2000), Kahn and Roberds (2001), Temzelides and Williamson (2001), Kahn et al. (2003), Williamson (2003), and Williamson (2009).¹ Most of these studies discuss the optimal payment system with its relevant risks and policies in intraday overdrafts and reserves. Models studied by Williamson (2009) and Choi (2011, 2015) are particularly relevant in that they analyze the role of credit in short-run monetary policy implications when money is non-neutral because of financial market segmentation. The optimal money growth rate can be positive for

¹ See Nosal and Rocheteau (2006), Chiu and Lai (2007), and Kahn and Roberds (2009) for a detailed literature review.

the redistribution of wealth. However, in their models, credit is exogenously settled on a net basis once a period, and little discussion is available on the role of multiple settlements.

The key factor of this model is that households' financial activities, including cash transfers and the choice of multiple credit installment payments, provide insurance for consumption loss against inflation risk. Credit settlement can occur multiple times, early in the current period and late in the next period. Early settlement requires a fixed installment fee that does not need to bear nominal interest. By contrast, late settlement, which allows a debt rollover, is an interest-bearing process with a proportional interchange fee. In practice, late settlement may represent the settling of multi-period installment credit that bears the nominal interest rate and interchange and fixed fees as the installment commission. In equilibrium, an increase in the size of a debt rollover can fully insure consumption loss against inflation. The positive effect of inflation dominates. The cost of borrowing decreases, and consumption increases. The optimal money growth is positive, which drives the nominal interest rate to zero—in line with the Friedman rule—and the settlement cycle to one.

The remainder of this paper is organized as follows. Section II describes the baseline environment of the model. Section III discusses the role of a debit rollover in credit settlement and its monetary policy implications. Section IV concludes the paper.

II. Model

The model is built on Freeman and Kydland (2000) with a few elements of Choi (2015). Time is discrete and indexed by $t = 0, 1, 2, \dots$. A continuum of infinitely lived households with unit mass and a continuum of consumption goods indexed by $i \in [0, 1]$ exist. Each household consists of a shopper and a worker. A shopper purchases consumption goods in a goods market, and a worker sells them. The representative household has preferences given by²

$$U(\{c_i(i)\}_{i=0}^{\infty}) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln \left[\min_{i \in [0, 1]} \left(\frac{c_i(i)}{2i} \right) \right] \quad (1)$$

where \mathbb{E}_0 is the expectation operator conditional on information in period 0, β

² Log preferences do not lose any key results of this paper and simplify the equilibrium analysis. Moreover, no disutility arises from aggregate credit-transaction costs unlike Choi (2015). Credit costs will be introduced as resource costs which turn out to be important for the equilibrium analysis later.

is the discount factor, and $c_i(i)$ is perishable consumption goods purchased at market i .

At the beginning of each period t , the household starts with M_t amount of money, where $M_0 > 0$ is given, B_t represents units of one-period government nominal bonds, and $\tau_t L_t$ represents credit balances issued in period $t-1$, where $\tau_t \in (0,1)$ is the share of outstanding credit balances (L_t) settled in period t . The household receives constant endowments y and cannot consume its own endowments.

At the opening of the financial market, the household learns the money growth rate (μ_t) for the current period, which is independent and identically distributed over time with the time invariant cdf $F(\mu)$ and $\mu_t \in (\underline{\mu}, \bar{\mu})$. The realization of μ_t is the only source of uncertainty.³ All shoppers from households participate in the financial market and perform three financial transactions.⁴ The first is cash transfer (X_t) for subsequent cash trades in a goods market. The second is settlement of late credit payments from the previous period ($\tau_t L_t$). The third is determining the current share of late settlement ($\tau_{t+1} \in (0,1)$) from a new line of credit issued. Cash serves as a medium of credit settlement as well as a medium of exchange in the goods market.

First, shoppers transfer cash by exchanging between one-period government nominal bonds (B_t) and money. Each bond of B_{t+1} sells for $q_t \leq 1$ units of money in period t and is a claim to one unit of money in period $t+1$. The government bond cannot be liquidated for cash before its maturity, and as a book-entry bond, it cannot be circulated as a medium of exchange. Next, shoppers settle $\tau_t L_t$ issued in period $t-1$ by using cash acquired through exchanging B_t and money with others on a gross basis. Finally, after settling $\tau_t L_t$, shoppers can determine a new line of credit ($q_t L_{t+1}$) and the share of late settlement (τ_{t+1}) by comparing the costs of early and late settlement. In period t , each credit of L_{t+1} is issued with a price of q_t , and it can be paid off early at the end of period t or late in the following period on a gross basis.⁵ If it is paid off late to sellers with one unit of money, then it is called late-settlement credit. Late-settlement credit bears the nominal interest rate and the proportional interchange fee of γ units of consumption goods paid in the financial market in advance, which provides adequate guarantees to the household's counterparty. By contrast, if credit is paid

³ This model is constructed to allow monetary policy to induce inflation risk to address the insurance role of multiple installment payments in a tractable manner. However, monetary policy may have a risk-sharing role against other sources of uncertainty including income shocks in various models. See Levine (1991), Molico (2006), and Choi (2011).

⁴ In this model, shoppers play the role of bankers or financial intermediaries. Each household does not have a separate banker to keep the model simple and its analysis tractable.

⁵ The price of credit equals that of bonds (q_t) because no (financial) market frictions exist to cause a positive liquidity premium on the price of credit.

off early to sellers using q_t units of money, then it is early-settlement credit. Early-settlement credit bears fixed installment charge γ , which covers the household's forgone interest and management cost from early settlement. Unlike late settlement, early credit payments are funded using incoming cash, such as revenue from the sale of consumption goods, at the end of period t .

Overall, by symmetry, the financial constraint of each household is given by

$$X_t + \tau_t L_t + P_t \tau_{t+1} \gamma = B_t - q_t B_{t+1}. \quad (2)$$

On the left-hand side of (2), $\tau_{t+1} \gamma$ is the interchange fee cost for late credit payments issued in period t ($\tau_{t+1} L_{t+1}$) in advance of the settlement in period $t+1$ to prevent a settlement failure. On the right-hand side, $B_t - q_t B_{t+1}$ is the amount of net liquidity transferred to the household. Assume no default in credit settlement because the household's government nominal bonds and income can act as collateral. In (2), every household participates in financial market activities, unlike in Choi (2011, 2015).

Finally, in the financial market, the government controls the money supply through open market operations and its budget constraint is

$$B_t^s - q_t B_{t+1}^s = M_{t+1}^s - M_t^s = \mu_t M_t^s, \quad (3)$$

where B_t^s is the balance of government bonds issued in period $t-1$, B_{t+1}^s is the balance of new issuances in period t , M_t^s is the money supply at the beginning of period t , M_{t+1}^s is the money supply at the closing of the financial market, and $\mu_t > -1$ is the net growth rate of money. After all financial transactions are complete, the worker and the shopper from each household go to the goods market.

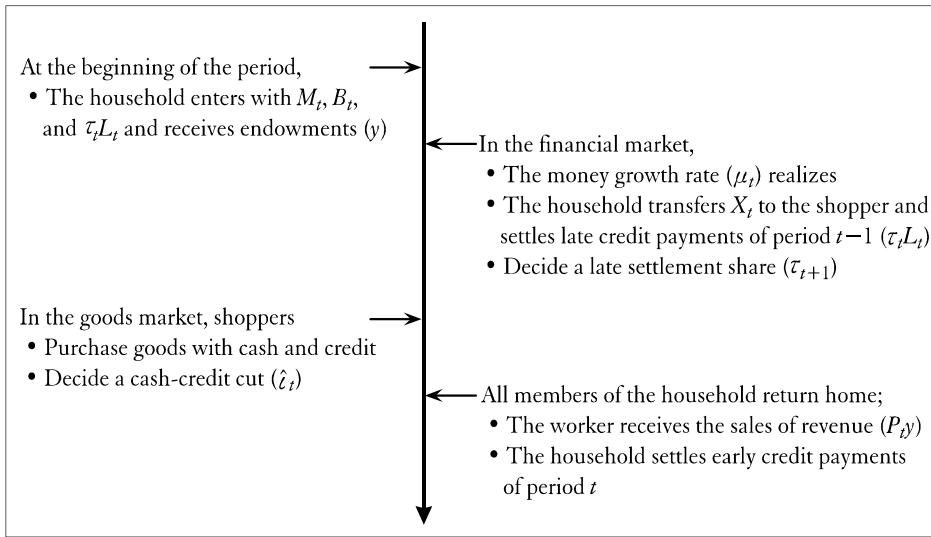
At the goods market, workers sell consumption goods, and shoppers acquire them using cash and credit. First, shoppers choose either cash or costly credit in each market i by comparing the opportunity costs of holding money and using credit. Market thresholds $[\hat{t}_i \in (0,1)]$ exist for cash and credit purchases. Cash purchases do not incur transactions costs, but credit purchases incur three types of costs in market i . One cost is the borrowing cost for late settlement, which equals the nominal interest of government bonds to avoid an arbitrage opportunity.⁶ Another cost is the interchange fee proportional to the late-settlement share ($\tau_{t+1} \gamma$). The third cost is the fixed installment fee (γ) for early credit settlement. Both

⁶ The presence of credit market frictions, for example, a default risk or limited commitment, may result in an endogenous spread ($\kappa_t > 0$) between the deposit rate (the return on bonds, r_t) and the lending rate (the borrowing cost of credit, $\tilde{r}_t = r_t + \kappa_t$). It can change the level of equilibrium outcomes but not the effects of monetary policy on equilibrium outcomes qualitatively, which are our focus, because every household participates in the financial market.

interchange and installment fees are resource costs.

At the end of each period, all agents return home. Workers deposit revenues from sales ($P_t y$) at their households. The household uses cash to settle early credit payments issued in period t . No further trade or barter is allowed. The household's budget constraint is presented in the next section. Figure 1 summarizes the itinerary of events of the representative household in period t .

[Figure 1] Itinerary of Representative Household in Period t



III. Equilibrium Dynamics

In equilibrium, given $\hat{c}_i = c_i(i) / 2i$ from (1), the cash-credit cutoff value (\hat{i}_t) is determined by

$$\frac{1}{2\hat{i}_t \hat{c}_t} \left(\frac{\gamma}{1 - \hat{i}_t} \right) \left(1 + \frac{\tau_{t+1}}{q_t} \right) = \frac{1}{q_t}, \quad (4)$$

where τ_{t+1} is the share for late settlement. The left-hand side of (4) captures the opportunity costs of a unit credit transaction at market \hat{i}_t given average per-market credit-settlement costs $[\gamma(1 + \tau_{t+1} / q_t) / (1 - \hat{i}_t)]$. The right-hand side is the cost of a unit cash transaction, which is constant over i . In (4), cash is used for it $i_t < \hat{i}_t$, and credit is used for it $i_t > \hat{i}_t$. $c_i(i)$ increases with i , so a cash trade is for smaller purchases, and a credit trade is used for larger purchases, in line with Freeman and Kydland (2000) and Choi (2015).

In (2) and (4), the cash-in-advance and late-settlement credit constraints are given by

$$P_t \hat{l}_t^2 \hat{c}_t \leq M_t + X_t, \quad (5)$$

$$P_t \tau_{t+1} (1 - \hat{l}_t^2) \hat{c}_t = q_t \tau_{t+1} L_{t+1}, \quad (6)$$

where $\hat{l}_t^2 \hat{c}_t$ and $\tau_{t+1} (1 - \hat{l}_t^2) \hat{c}_t$ are consumption using cash and late-settlement credit, and $P_t \tau_{t+1} \gamma$ represents the late-settlement interchange fees. Then, the budget constraint is given by

$$\begin{aligned} P_t \hat{c}_t + P_t \gamma + M_{t+1} + (1 - \tau_{t+1}) q_t L_{t+1} \\ = M_t + X_t + q_t L_{t+1} + P_t y - q_t \bar{L}_{t+1} + (1 - \bar{\tau}_{t+1}) q_t \bar{L}_{t+1} + \bar{\tau}_t \bar{L}_t, \end{aligned} \quad (7)$$

where $P_t \gamma$ is early-settlement costs, $(1 - \tau_{t+1}) q_t L_{t+1}$ is early credit payments, $q_t L_{t+1}$ is total new credit balances issued by the household, $q_t \bar{L}_{t+1}$ is total new credit balances issued by other households, $P_t y - q_t \bar{L}_{t+1}$ is the household income without the revenue from credit sales, and $(1 - \bar{\tau}_{t+1}) q_t \bar{L}_{t+1}$ and $\bar{\tau}_t \bar{L}_t$ are early and late payments collected from other households, respectively.

Definition: For given $\{\mu_t\}_{t=0}^\infty$, a symmetric competitive equilibrium is the sequence $\{\hat{c}_t, \hat{l}_t, \tau_{t+1}, X_t, M_{t+1}, B_{t+1}, L_{t+1}, M_t^s, P_t, q_t\}_{t=0}^\infty$ such that

1. The household solves the optimization problem subject to (5)–(7) given (2) with the nonnegativity constraints $L_{t+1} \geq 0$ and $M_{t+1} \geq 0$.
2. The government budget constraint is satisfied with (3).
3. Markets clear in every period: (a) Goods Market: $\hat{c}_t + (1 + \tau_{t+1}) \gamma = y$, (b) Money Market: $M_{t+1}^s = M_{t+1} = M_t + X_t$, (c) Bonds Market: $B_t = B_t^s$ and $B_t - q_t B_{t+1} = B_t^s - q_t B_{t+1}^s = \mu_t M_t$, and (d) Credit Market: $\tau_{t+1} = \bar{\tau}_{t+1}$ and $L_{t+1} = \bar{L}_{t+1}$.

Suppose λ_t , σ_t , and η_t are the Lagrange multipliers for (5)–(7), respectively, given (2). The choice for $[\hat{l}_t^2 \hat{c}_t, (1 - \hat{l}_t^2) \hat{c}_t, \tau_{t+1} (1 - \hat{l}_t^2) \hat{c}_t, M_{t+1}, B_{t+1}, L_{t+1}]$ is as follows:

$$\frac{1}{P_t \hat{c}_t} = \hat{l}_t^2 (\lambda_t + \eta_t) \quad \text{if purchased with cash;} \quad (8)$$

$$= (1 - \hat{l}_t^2) \eta_t \quad \text{if purchased with credit;} \quad (9)$$

$$= \tau_{t+1} (1 - \hat{l}_t^2) (\sigma_t + \eta_t) \quad \text{if purchased with credit settled late;} \quad (10)$$

$$\eta_t = \beta \mathbb{E}_t [\lambda_{t+1} + \eta_{t+1} \mid \mu_t]; \quad (11)$$

$$q_t(\lambda_t + \eta_t) = \beta \mathbb{E}_t[\lambda_{t+1} + \eta_{t+1} \mid \mu_t]; \quad (12)$$

$$q_t \tau_{t+1}(\sigma_t + \eta_t) = \beta \mathbb{E}_t[\tau_{t+1}(\lambda_{t+1} + \eta_{t+1}) \mid \mu_t]. \quad (13)$$

Assume that $\beta \mathbb{E}_t[(\hat{c}_t / \hat{c}_{t+1})(\hat{l}_t^2 / \hat{l}_{t+1}^2)(P_t / P_{t+1})] < 1$ for a binding cash-in-advance constraint. In equilibrium, the budget constraint in (5)–(7) is first simplified to $M_{t+1} + P_t \gamma + q_t L_{t+1} - \tau_t L_t = P_t y$, and the real demand for money and credit-settlement net flow are determined by

$$\frac{M_{t+1}}{P_t} = \phi_t y, \quad (14)$$

where $\phi_t \in (0, 1)$ is the share of y for the demand for money. From (14), the inflation rate is given by

$$\frac{P_t}{P_{t-1}} = (1 + \mu_t) \frac{\phi_{t-1}}{\phi_t}. \quad (15)$$

Now, suppose $\delta_t < 1$ is a proportion of the money injection for late-settlement from (2) and (3). Then, $(X_t / P_t, \tau_t L_t / P_t)$ is then given by

$$\frac{X_t}{P_t} = \frac{(1 - \delta_t) \mu_t M_t}{P_t} - \tau_{t+1} \gamma, \quad (16)$$

$$\frac{\tau_t L_t}{P_t} = \frac{\delta_t \mu_t M_t}{P_t}, \quad (17)$$

where $\delta_t \mu_t > 0$ for any μ_t .⁷ From (5), (6), (14), and (15)–(17), consumption using cash ($\hat{l}_t^2 \hat{c}_t$) is given by⁸

$$\hat{l}_t^2 \hat{c}_t + \tau_{t+1} \gamma = \left(1 - \frac{\Lambda}{1 + \mu_t} \right) \phi_t y, \quad (18)$$

where $\Lambda = \mathbb{E}_t[\delta_{t+1} \mu_{t+1}] \in (0, 1)$ is the expected share of the net money growth rate for late settlement. In (16) and (17), the government money injection goes to not

⁷ Suppose the government injects money ($\mu_t > 0$). The household then transfers cash and pays off credit payments, $[X_t / P_t > 0, \delta_t \in (0, 1)]$. However, if $\mu_t < 0$, then nominal bonds are acquired through cash and credit transactions, $(X_t / P_t < 0, \delta_t < 0)$.

⁸ See appendix A for the derivation.

only $\tau_t L_t$ but also X_t . That is, τ_t affects the government money injection share for X_t and consumption in (18). Given late credit settlement, non-neutralities of money arise through consumption using cash ($\hat{i}_t^2 \hat{c}_t$) and using credit $[(1 - \hat{i}_t^2) \hat{c}_t]$ without financial market segmentation, unlike in Williamson (2009) and Choi (2011, 2015).

From (8)–(13), the dynamics of money, bonds, and credit demand are given by

$$1 = \beta \mathbb{E}_t \left[\left(\frac{\hat{c}_t}{\hat{c}_{t+1}} \right) \frac{1 - \hat{i}_t^2}{\hat{i}_{t+1}^2} \left(\frac{1}{1 + \mu_{t+1}} \right) \frac{\phi_{t+1}}{\phi_t} \right] = \frac{\beta \Psi (1 - \hat{i}_t^2) \hat{c}_t}{\phi_t}, \quad (19)$$

$$q_t = \beta \mathbb{E}_t \left[\left(\frac{\hat{c}_t}{\hat{c}_{t+1}} \right) \frac{\hat{i}_t^2}{\hat{i}_{t+1}^2} \left(\frac{1}{1 + \mu_{t+1}} \right) \frac{\phi_{t+1}}{\phi_t} \right] = \frac{\beta \Psi \hat{i}_t^2 \hat{c}_t}{\phi_t}, \quad (20)$$

$$q_t = \beta \mathbb{E}_t \left[\left(\frac{\hat{c}_t}{\hat{c}_{t+1}} \right) \frac{\tau_{t+1} (1 - \hat{i}_t^2)}{\hat{i}_{t+1}^2} \left(\frac{1}{1 + \mu_{t+1}} \right) \frac{\phi_{t+1}}{\phi_t} \right] = \frac{\beta \Psi \tau_{t+1} (1 - \hat{i}_t^2) \hat{c}_t}{\phi_t}, \quad (21)$$

where $\Psi = \mathbb{E}_t [\phi_{t+1} / [\hat{i}_{t+1}^2 \hat{c}_{t+1} (1 + \mu_{t+1})]]$. The price of credit (q_t) depends on the intertemporal marginal rate of substitution of consumption, the late-settlement share (τ_{t+1}), the cash–credit choice (\hat{i}_t), and the inflation rate. From (19)–(21), q_t equals the ratio of cash purchases to credit purchases in terms of \hat{i}_t as well as τ_{t+1} , as follows:

$$q_t = \tau_{t+1} = \frac{\hat{i}_t^2}{1 - \hat{i}_t^2} \in (0, 1), \quad (22)$$

where $\hat{i}_t^2 \in (0, 1/2)$ holds. In (22), dynamic interactions exist among q_t , τ_{t+1} , and \hat{i}_t . As the borrowing cost ($1/q_t$) decreases, the household becomes late in settling a greater share of credit payments, or τ_{t+1} increases. In addition, the household spends less on credit, or \hat{i}_t increases because the return on savings decreases. Finally, from (22), late-settlement credit purchases equal cash purchases, $[\tau_{t+1} (1 - \hat{i}_t^2) \hat{c}_t = \hat{i}_t^2 \hat{c}_t]$.

The settlement cycle for per-unit credit in period t [$h_t \in (0, 1)$], which is a measure for the degree of interest-bearing credit settlement against inflation risk as a combination of τ_{t+1} and q_t , can be defined by

$$h_t = \tau_{t+1} + (1 - \tau_{t+1}) q_t. \quad (23)$$

A comparison of the burden of the nominal interest rate in late settlement and the

cost of inflation risk in early settlement shows that the settlement cycle lies between zero and one.⁹ To avoid the inflation risk, shoppers settle every payment late ($\tau_{t+1}=1$) by bearing the nominal interest rate ($1/q_t$). The settlement period becomes one ($h_t=1$). By contrast, to reduce the interest rate burden, shoppers settle every credit payment early ($\tau_{t+1}=0$) by facing inflation risk. The settlement period is given by $h_t=q_t \leq 1$. In (22) and (23), if the borrowing cost ($1/q_t$) decreases, then τ_{t+1} and h_t increase.¹⁰ That is, an increase is observed for τ_{t+1} and h_t .

IV. Monetary Policy Implications

This section discusses the short-run effects of monetary policy on equilibrium outcomes and optimal monetary policy. From (4) and (18)–(22), the choice for $(\phi_t, \hat{t}_t, q_t, \tau_t, \hat{c}_t)$ in terms of μ_t is determined by¹¹

$$\phi_t = \left[\frac{1 + \mu_t}{1 + (1 + \Lambda)\mu_t} \right] \left(\frac{y - \gamma}{y} \right), \quad (24)$$

$$\hat{t}_t^2 = \frac{q_t}{1 + q_t} = \frac{\tau_{t+1}}{1 + \tau_{t+1}} = \left[\frac{1 - \Lambda + \mu_t}{1 + (1 + \Lambda)\mu_t} \right] \left(\frac{y - \gamma}{y} \right), \quad (25)$$

$$\hat{c}_t = \hat{t}_t y = \left[\frac{1 - \Lambda + \mu_t}{1 + (1 + \Lambda)\mu_t} \right]^{\frac{1}{2}} [(y - \gamma)y]^{\frac{1}{2}}. \quad (26)$$

From (25) and (26), the nonnegativity conditions of the cash-credit choice and consumption imply that μ_t is bounded: for $y > 2\gamma / (1 - \Lambda)$,

$$1 + \mu_t \in \left(\Lambda, \frac{\Lambda(y - 2\gamma)}{y(1 - \Lambda) - 2\gamma} \right], \quad (27)$$

which also ensures a positive return on government nominal bonds that binds the cash-in-advance constraint.

⁹ This study focuses on $h_t \in (0, 1)$ for the sake of tractability. h_t can be extended to two periods or longer; however, doing so complicates the model without adding much in terms of results.

¹⁰ As in footnote 6, in (22) and (23), the quadratic function $h_t = -q_t^2 + 2q_t$ increases with $q_t \in (0, 1)$.

¹¹ See appendix B for the derivation.

Proposition 1 From (24)–(27), the effects of monetary policy on $[\phi_t, \hat{t}_t, q_t, \tau_{t+1}, \hat{c}_t]$ are (1) $\partial \phi_t / \partial \mu_t < 0$, (2) $\partial \hat{t}_t / \partial \mu_t > 0$, (3) $\partial q_t / \partial \mu_t = \partial \tau_{t+1} / \partial \mu_t > 0$, and (4) $\partial \hat{c}_t / \partial \mu_t > 0$.

Corollary 1 The policy effects on $[\hat{t}_t^2 \hat{c}_t, \tau_{t+1}(1 - \hat{t}_t^2) \hat{c}_t, (1 - \tau_{t+1})(1 - \hat{t}_t^2) \hat{c}_t, (1 - \hat{t}_t^2) \hat{c}_t]$ are (1) $\partial \hat{t}_t^2 \hat{c}_t / \partial \mu_t = \partial \tau_{t+1}(1 - \hat{t}_t^2) \hat{c}_t / \partial \mu_t > 0$, (2) $\partial (1 - \tau_{t+1})(1 - \hat{t}_t^2) \hat{c}_t / \partial \mu_t = \partial (1 - \tau_{t+1}) \phi_t \Lambda y / \partial \mu_t < 0$, and (3) $\partial (1 - \hat{t}_t^2) \hat{c}_t / \partial \mu_t = \partial \phi_t \Lambda y / \partial \mu_t < 0$.

Corollary 2 From (23) and (25), the settlement cycle (h_{t+1}) increases with μ_t [$\partial h_t / \partial \mu_t = 2(1 - \tau_{t+1}) \partial \tau_{t+1} / \partial \mu_t > 0$].

When the government injects money, inflation arises. The value of cash decreases, whereas the real value of credit debt decreases. The negative effect of inflation can be insured by adjusting a cash transfer ($X_t > 0$) to goods trades and the share of multiple installment of payments. Cash transactions increase for a greater variety of markets, and consumption using cash ($\hat{t}_t^2 \hat{c}_t$) increases. The borrowing cost, that is, the nominal interest rate ($1/q_t$), decreases, and both the share for late settlement and the settlement cycle increase. Consumption using late-settlement credit increases, whereas that using early-settlement credit decreases. The net effect is a decrease in consumption using credit $[(1 - \hat{t}_t^2) \hat{c}_t]$. Given $X_t > 0$, the increase in $\hat{t}_t^2 \hat{c}_t$ dominates the decrease in $(1 - \hat{t}_t^2) \hat{c}_t$. Thus, a positive income effect exists. Aggregate consumption increases, and social welfare defined by $W(\mu_t) = \ln(\hat{c}(\mu_t))$ increases with the money growth rate, as follows:

$$\frac{\partial W_t}{\partial \mu_t} = \frac{1}{\hat{c}_t} \frac{\partial \hat{c}_t}{\partial \mu_t} > 0. \quad (28)$$

Proposition 2 From (25)–(28), (1) The optimal money growth rate is positive $\{\mu_t^* = \Lambda(y - 2\gamma) / [y(1 - \Lambda) - 2\gamma] - 1 > 0\}$ for $y > 2\gamma / (1 - \Lambda)$ and $\Lambda \in (1/2, 1)$, and the net nominal interest rate is zero ($q_t^* = 1$). (2) Given $\mu^* > 0$, the late-settlement share and the settlement cycle are equal to one ($\tau_{t+1}^* = 1 = h_t^*$).

Given the cash transfer in the goods market, the positive optimal money growth rate drives the nominal interest rate to zero, in line with the Friedman rule. The late-settlement share and the settlement cycle are extended to one because there is no borrowing cost.

V. Conclusion

This study investigates the risk-sharing role of multiple credit installment payments against inflation risk. Credit can be settled multiple times in a period. In the presence of multiple credit settlement, the settlement cycle increases with inflation. It may dampen consumption loss and improve welfare. The optimal money growth rate is positive, where the nominal interest rate reaches zero. Hence, the optimal settlement cycle is one.

To delve into various aspects of a debt rollover and its relevant policy implications, we can introduce a credit-market friction which requires collateral to settle credit payments in the model. The presence of collateral deposits may induce an endogenous interest spread between a deposit rate and a loan rate and allow the central banker to have multiple policy devices, that is, money supply and the collateral requirement ratio. Payment policy, which is a change in the collateral requirement ratio, directly affect the payment system. It has different transmission channels through collateral compared with monetary policy, that is, a change in the money growth rate.

Another interesting future research direction is to empirically explore the relationship between the settlement cycle and monetary policy. In our theoretical model, many factors relevant in deciding on the settlement cycle are missing. First, in our model, the household can rollover its credit payments for only one period at most. In practice, the rollover may persist for many months or even more than a year. Next, credit market frictions are absent, for example, a default risk or limited commitment. In practice, the presence of credit market frictions induces an endogenous interest spread between a deposit rate and a loan rate that can influence the settlement cycle. Finally, certain factors affecting the amount of credit card debt are simplified, for example, a flow of income and a household portfolio. To test our theoretical results on the relationship between the settlement cycle and monetary policy empirically, we must control these factors in the model, which can be another interesting future project.

Appendix

A. Derivation of (18)

In (17), $\tau_t L_t / P_t = \delta_t \mu_t M_t / P_t$, and the late-settlement credit constraint in period $t-1$ in (6) is $\tau_t P_{t+1} (1 - \hat{l}_t^2) \hat{c}_t = q_{t-1} \tau_t L_t = q_{t-1} \delta_t \mu_t M_t$, and consumption with late-settlement credit in period t is given by

$$\begin{aligned} \tau_{t+1} (1 - \hat{l}_t^2) \hat{c}_t &= q_t \mathbb{E}_t \left[\frac{\delta_{t+1} \mu_{t+1} M_{t+1}}{P_t} \right] = q_t \mathbb{E}_t [\delta_{t+1} \mu_{t+1} \phi_t y] = q_t \phi_t \Lambda y, \\ \frac{L_{t+1}}{P_t} &= \frac{\phi_t \Lambda y}{\tau_{t+1}}, \end{aligned} \quad (29)$$

where $\Lambda = \mathbb{E}_t [\delta_{t+1} \mu_{t+1}] \in (0, 1)$. From (14), (17), and (29), $\delta_t \mu_t M_t / P_{t-1} = \tau_t L_t / P_{t-1} = \phi_{t-1} \Lambda y$, which implies that $\delta_t \mu_t = \Lambda$ and $\hat{l}_t^2 \hat{c}_t$ is given by (5) and (14)–(16), assuming $1 + \mu_t > \Lambda$,

$$\hat{l}_t^2 \hat{c}_t = \frac{M_t}{P_t} + \frac{X_t}{P_t} = \frac{M_t}{P_t} + \frac{(1 - \delta_t) \mu_t M_t}{P_t} - \tau_{t+1} \gamma = \left(1 - \frac{\Lambda}{1 + \mu_t} \right) \phi_t y - \tau_{t+1} \gamma. \quad (30)$$

B. Derivation of (24)–(27)

First, from (14), the following holds:

$$\gamma + q_t \frac{L_{t+1}}{P_t} - \frac{\tau_t L_t}{P_t} = (1 - \phi_t) y, \quad (31)$$

and from (15), (29), and (31), $\gamma + \phi_t \Lambda y - \phi_t \Lambda y / (1 + \mu_t) = (1 - \phi_t) y$ holds, which determines $\phi_t \in (0, 1)$ in (24). Next, from (19)–(22), $\beta \Psi \Lambda y = 1$, and $\hat{c}_t = \hat{l}_t^2 \hat{c}_t + (1 - \hat{l}_t^2) \hat{c}_t = \phi_t \Lambda y (1 + \tau_{t+1})$ hold, which implies that the resource constraint is given by

$$(\phi_t \Lambda y + \gamma) (1 + \tau_{t+1}) = y. \quad (32)$$

In (4), (18), (20), and (22),

$$\hat{l}_t \hat{c}_t = \hat{l}_t^2 \hat{c}_t + \tau_{t+1} \gamma = q_t (\phi_t \Lambda y + \gamma) \quad (33)$$

and from (22), (24), (32), and (33),

$$\hat{l}_t \hat{c}_t = \frac{q_t y}{1 + q_t} = \frac{\tau_{t+1} y}{1 + \tau_{t+1}} = \hat{l}_t^2 y = \left(1 - \frac{\Lambda}{1 + \mu_t}\right) \phi_t y, \quad (34)$$

which determines (25) and (26) from $\hat{c}_t = \hat{l}_t y$. Finally, from (25), $q_t \leq 1$ implies that $\hat{l}_t^2 \in (0, 1/2)$, and the range of μ_t is satisfied with (27) for $y > 2\gamma / (1 - \Lambda)$.

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