The Korean Economic Review
Volume 37, Number 1, Winter 2021, 65-84.
DOI : 10.22841/kerdoi.2021.37.1.003

Payments Systems, Liquidity, Collateral, and Central Banking*

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A monetary model is constructed to explore the risk-sharing role of gross settlement as a determinant of money demand for consumption in a credit economy. Due to a deferred payment system, the costs of gross and net settlement are sensitive to the nominal interest rate. Gross settlement may dampen a consumption loss against interest-rate risk arising from inflation by acquiring additional cash from a financial market. Hence, it is optimal for the government to influence inflation and to drive net settlement out of a payment system. For payment policy, the optimal collateral requirement ratio is one whereas for monetary policy the optimal money growth rate is infinity. Payment policy can be a useful alternative to monetary policy.

JEL Classification: E42, E44, E50
Keywords: Payments Systems, Liquidity, Collateral, Central Banking

I. Introduction

The purpose of this paper is to explore the risk-sharing role of deferred gross settlement for money demand, consumption and central banking in a credit economy when gross settlement plays as a determinant of money demand.

In a payment system, gross settlement in which the settlement of funds transfers occurs individually among trading partners usually requires more cash to settle credit payments when compared to net settlement in which the settlement of funds

Received: Jan. 27, 2020. Revised: April 26, 2020. Accepted: July 27, 2020.

* This work was supported by the Ministry of Education of the Republic of Korea and the National Research Foundation of Korea (NRF-2017S1A5A2A01024058). I am very grateful to Steve Williamson, Gabriele Camera, Changjoon Kim, and seminar participants at Kyoto University for their helpful comments and discussions. I would also like to thank two anonymous reviewers and the editors for their insightful suggestions. A previous version of this paper was circulated under the title “Payments Systems, Interest, Distribution, and Monetary Policy.”

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transfers occurs by netting positions by trading partners. The nominal interest rate foregone in gross settlement is greater. However, gross settlement can provide a crude insurance to economic individuals against a uninsurable risk if it played as a determinant of the demand for liquidity by adjusting their asset portfolios in a financial market. For example, in the rise of an interest-rate risk, gross settlement enables economic individuals to liquidate cash from the financial market to dampen a consumption loss. The implications of this risk-sharing role of gross settlement on the payment system and central banking have not yet been studied much.

The contribution of this paper is to capture the transmission mechanism of deferred gross settlement that provides a crude insurance on consumption and social welfare in the credit economy. Given a deferred payment system, credit settlement occurs in the subsequent period, and the opportunity costs of deferred gross and net settlement are sensitive to the nominal interest rate, which disagrees with the study of Choi (2019). For example, a lower interest rate during inflation would reduce the value of money and favor gross settlement which entails a larger amount of money as credit payments liquidated from the bond market. Hence, gross settlement may dampen a consumption loss arising from inflation and may improve social welfare.

There is a set of literature on the design of the payment system including Freeman (1996), Kahn and Roberds (1998, 2001), Temzelides and Williamson (2001), Kahn et al. (2003), Koeppl et al. (2008), Kahn (2013), and Choi (2018, 2019).[1] In particular, Choi (2018), whose model is built on Freeman and Kydland (2000), studies the risk-sharing role of realtime and deferred gross credit-settlement in the choice of cash and costly credit against an inflation risk. During inflation, a debt rollover eases the demand for money in credit-settlement and dampens a consumption loss. Inflation can improve welfare. Hence, under the gross-settlement system, it is desirable for the government to favor deferred settlement to realtime settlement by encouraging inflation and keeping the money growth rate positive. On the other hand, Choi (2019), whose model is built on Ireland (1994), addresses the role of gross and net settlement with an operational risk in the choice of cash and a costly debit card. During inflation, gross settlement would demand a larger amount of money for debit-card settlement, which results in a consumption loss unlike Choi (2018). The choice of net settlement may reduce this loss from inflation. Hence, it is optimal for the government to have the hybrid system of gross and net settlement by discouraging inflation and keeping the money growth rate negative. However, he fails to capture the welfare-enhancing role of the payment system in the presence of the interest-rate risk by comparing the costs and benefits of deferred gross and net settlement.

[1] See Nosal and Rocheteau (2006), Chiu and Lai (2007), and Kahn and Roberds (2009) for a detailed literature survey.

This paper constructs a monetary model of credit settlement with elements from the studies by Williamson (2009). Credit is the sole medium of exchange in goods trades due to its smaller transactions costs compared to those of cash unlike Freeman and Kydland (2000) and Ireland (1994). Credit transactions are settled through cash payments in the financial market of the subsequent period. There are two types of credit settlements - gross settlement and net settlement. While the former requires government nominal bonds as collateral in a government account, the latter involves the payment of an idiosyncratic transaction cost. When settling credit payments the participants of the payment system can decide the share of gross and net settlement by comparing their costs.

In equilibrium, gross settlement may dampen a consumption loss against the interest-rate risk arising from inflation by acquiring additional cash from a financial market. There are two transmission channels of payment and monetary policy. One is an inflation-rate channel that operates through the aggregate demand for money and the other is an interest-rate channel that operates through the aggregate demand for government nominal bonds. The short-run liquidity effect may arise in that a higher inflation rate decreases the nominal interest rate. Inflation with a smaller return on savings reduces the value of money and favors gross settlement that requires cash to settle credit payments. The greater share of gross settlement can increase consumption and improve social welfare. Net settlement alone fails to improve welfare, unlike Temzelides and Williamson (2001) and Williamson (2003). Hence, the pure gross-settlement system is considered optimal. It would be optimal for the government to influence inflation to drive net settlement out of the payment system. Payment policy controls the collateral requirement ratio in gross settlement and the optimal ratio is one. However, monetary policy controls money supply through open market operations and the optimal money growth rate is infinity.

Another strand of the relevant literature is on the interaction between the payment system and monetary policy including Green (2002), and Williamson (2003, 2009). Particularly, in Williamson (2003), net settlement improves the efficiency of the economy with deterministic fluctuations in aggregate productivity in a spatial separation model. An efficient allocation is achieved with a zero overnight discount rate. Moreover, in Williamson (2009), various monetary policies under net settlement are redistributive in a market-segmentation framework. Monetary policy can mitigate the negative effects of payment-system shocks, but cannot achieve a Pareto optimum.

The model here captures a novel transmission channel of payment policy in the context of gross settlement as the determinant of money demand. Unlike Williamson (2003, 2009), payment policy not only stabilizes the payment system but also controls the money supply. The participants of the payment system who received the effects of payment policy can insure themselves by adjusting the share of gross settlement.

The remainder of the paper is organized as follows. In the second section, the baseline environment of the model is described. The third section studies a net-settlement only economy and monetary policy implications. The fourth section discusses a hybrid-settlement economy and monetary policy implications. The last section presents the conclusion.

II. Model

Time is discrete and indexed by $t = 0,1,2,\dots$. There is a continuum of infinitely-lived households with unit mass indexed by $j \in [0,1]$ and a continuum of consumption goods indexed by $i \in [0,1]$. Each household consists of a worker and a shopper. A worker sells consumption goods and a shopper purchases them in a goods market. The household j has preferences given by

$$U(\{c_{j,t}(i)\}_{t=0}^{\infty}) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 u[c_{j,t}(i)]di , \tag{1}$$

where $\mathbb{E}_0$ is the expectation operator conditional on information in the period 0, β is the discount factor, $c_{j,t}(i)$ denotes perishable consumption goods at market i for the household j. Assume that $u(c)$ is twice continuously differentiable and strictly concave in c, where $u'(0) = \infty$, $\lim_{c\to 0} cu(c) = 0$, and $\sigma = -cu''(c) / u'(c) \in (0,1)$.[2] Each household j receives constant endowments y. Assume that a household cannot consume its own endowments.

At the beginning of each period t, the household j starts with $M_{j,t}$ amount of money, where $M_0 > 0$ is given, and $B_{j,t}$ units of one-period nominal bonds and $L_{j,t}$ outstanding credit balances are issued in the period $t-1$.

On opening the financial market, each household j receives an idiosyncratic net-settlement-cost shock $(\gamma_{j,t})$ that is independent and identically distributed over households with the time-invariant cumulative distribution function $F(\gamma)$, where $\gamma_{j,t} \in (\underline{\gamma}, \overline{\gamma})$, and $0 < \underline{\gamma} < \overline{\gamma}$. The shock $\gamma_{j,t}$ is the only source of uncertainty that would capture the cost eliminating a counterparty credit risk and finalizing settlement on time: for example, a system-participation cost or a monitoring/ enforcement cost. In aggregation, the cost is given by $\int_{\underline{\gamma}}^{\overline{\gamma}} \gamma_{j,t} dF(\gamma) = \gamma$, where $\gamma = y > 1$.[3] For some households, the net-settlement-cost shock $(\gamma_{j,t} > \gamma)$ is greater than y, while for others $\gamma_{j,t} < \gamma$ is smaller than y.

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[2] Given $\sigma \in (0,1)$, for a greater increase in the rate of return on assets, the demand for assets would increase. In other words, the substitution effect dominates the income effect.

[3] Here, $\gamma = y > 1$ is assumed to keep the equilibrium analysis tractable. As long as the aggregate net-settlement cost is assumed to be smaller than y, the model will not lose any key results.

The financial market consists of two submarkets – a bond market in which one-period government nominal bonds are traded with money and a payment-system market in which credit balances are issued and settled on a gross and net arrangement. The household j has an access to both markets and performs the following two financial transactions. One is the settlement of credit balances issued from the previous period $t-1$. The other is to open a line of new credit and to decide its gross-settlement share for subsequent goods trades.[4]

First, the household j settles credit payments $L_{j,t}$ issued in the period $t-1$ simultaneously with others in a centralized payment-system platform managed by the government. A fraction $\hat{\theta}_{j,t}$ of $L_{j,t}$ is paid off by cash through gross settlement and the rest $1-\hat{\theta}_{j,t}$ is done through net settlement.[5] The household j can acquire money for the gross settlement of $\hat{\theta}_{j,t}L_{j,t}$ by liquidating matured one-period government bonds ($B_{j,t}$). Each bond of $B_{j,t+1}$ sells for $q_t \leq 1$ units of money in the period t and is a claim to one unit of money in the period $t+1$. The government bond cannot be liquidated in cash before its maturity and it cannot be circulated as a medium of exchange. Once payments are made, the centralized payment system collects and redistributes them across households to finalize the settlement.

Second, the household j opens a line of new credit ($q_t L_{j,t+1}$) in the period t, that is, a one-period IOU, up to a household's income and pays the credit at the next period's financial market. Each credit of $L_{j,t+1}$ is issued with a price of q_t in the period t and it is a claim to one unit of money in the period $t+1$. The price of credit is assumed to be the same as the price of bonds (q_t) to avoid any arbitrage opportunities.[6]

The household j then decides a gross-settlement share ($\hat{\theta}_{j,t+1}$) of $L_{j,t+1}$ by comparing the nominal opportunity costs of per-unit gross and net settlement. For deferred gross settlement, the household j is required to deposit $\alpha\hat{B}_{j,t+1}$ amount of nominal bonds in a government account as collateral prior to the settlement of $\hat{\theta}_{j,t+1}L_{j,t+1}$, where $\alpha \in [0,1]$ represents the collateral requirement ratio set by the government.[7,8] Once settlement is completed, old collateral ($\alpha\hat{B}_{j,t}$) deposited at the

[4] In order to keep the equilibrium analysis simple, this model assumes that the household can make financial transactions instead of introducing a financial intermediary.

[5] The payment system operates gross settlement without a gridlock problem because every household can get access to the financial market to acquire sufficient liquidity in a timely manner, unlike what is indicated in Freeman (1996) and Williamson (2009).

[6] When financial market frictions arise from asymmetric information and limited commitment, the price of credit and bonds may be different. The lending rate (the borrowing cost of credit, $\tilde{r}_t$) would consist of the interest rate on savings ($r_t = 1/q_t$) and a nonnegative spread (κ_t); i.e., $\tilde{r}_t = r_t + \kappa_t$. However, for the sake of the tractability, the financial market is assumed to be perfect.

[7] Here, α is not greater than one because the government does not need collateral deposits exceeding the amount to settle the payment. Credit default would not arise in case of $\alpha \in [0,1]$ because the government would confiscate the household's nominal assets and income. Besides, $\alpha > 1$

period $t-1$ returns to the household before the current financial market closes. The household j does not receive the nominal interest rate $(1/q_t)$ on collateral deposits because it is used to maintain the payment system by the government.[9] Hence, the per-unit opportunity cost of gross settlement is given by $\alpha\hat{B}_{j,t+1}/q_t\hat{\theta}_{j,t+1}L_{j,t+1}$.

For deferred net settlement, the household j bears a stochastic lump-sum resource cost $\gamma_{j,t}$ for each unit of net settlement prior to one period to prevent the credit risk.[10] The resource cost in net settlement for the household j is $(1-\hat{\theta}_{j,t+1})\gamma_{j,t}$. Because the resource cost is paid a period earlier, there is also a foregone interest and the per-unit opportunity cost of net settlement is given by $(1/q_t)[P_t(1-\hat{\theta}_{j,t+1})\gamma_{j,t}/(1-\hat{\theta}_{j,t+1})L_{j,t+1}]$, where P_t is the average price level.

In sum, the household j's collateral-in-advance constraint of the financial market is given by

$$
\begin{aligned}
&P_t\int_0^1 c_{j,t}(i)di+\hat{\theta}_{j,t}L_{j,t}+P_t(1-\hat{\theta}_{j,t+1})\gamma_{j,t}\\
&=M_{j,t}+B_{j,t}+q_tB_{j,t+1}+\alpha\hat{B}_{j,t}-\alpha\hat{B}_{j,t+1}+q_tL_{j,t+1}, \qquad (2)
\end{aligned}
$$

where $P_t\int_0^1 c_{j,t}(i)di=q_tL_{j,t+1}$ is the credit constraint for consumption. The left-hand side of (2) denotes the demand for consumption and money for credit settlement and the right-hand side does the amount of new credit balances for consumption and an incoming cash transfer for credit settlement throughout the transactions of the bond market. Assume that credit payments cannot be paid off with government bonds and that the household j does not get access to incoming credit payments $\bar{\theta}_{j,t}\bar{L}_{j,t}$ from others before the current financial market closes and $\bar{\theta}_{j,t}\bar{L}_{j,t}$ does not show up in (2).

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is not desirable for the government because it results in excess savings and would reduce the quantity of new credit and consumption.

[8] Collateral deposits in a form of one-period government nominal bonds instead of cash are crucial to keep key payment-policy results in Section 4. Cash collateral deposits would have the opposite effect on the nominal interest rate and the decision on the gross-settlement share. For example, an increase in α would increase the demand for nominal bonds for collateral deposits. Hence, it decreases the nominal interest rate and increases the demand for cash along with the gross-settlement share. On the contrary, given cash collateral deposits, an increase in α would increase the demand for cash for collateral deposits, but decrease the demand for nominal bond. Hence, it increases the nominal interest rate and decreases the gross-settlement share.

[9] Gross settlement involves its cost in a form of the nominal interest rate $(1/q_t)$ in order to emphasize the interest-rate channel of payment policy between the payment system and households' consumption. This channel would disappear if gross settlement entailed a resource cost instead, similar to net settlement.

[10] Since there will be no balance to pay off in a symmetric equilibrium, for the sake of simplicity, a collateral requirement is not imposed on net settlement.

The government controls the money supply at a constant rate through open market operations; the government budget constraint is given by

$$B_t^s - q_t B_{t+1}^s = M_{t+1}^s - M_t^s = \mu M_t^s \tag{3}$$

where B_{t+1}^s is the balances of newly issued nominal bonds, M_{t+1}^s is money supply on closing the financial market, and $\mu > -1$ is the net growth rate of money. After the financial market closes, the worker and the shopper from each household leave for the goods market.

At the goods market, workers sell consumption goods and shoppers acquire them with costly credit across the market i instead of money. For each market i, the cost of per-unit cash transaction is associated with the nominal interest rate $(1/q_t)$ while that of per-unit credit transaction comprises of the weighted average cost of the gross and net settlement in terms of $\hat{\theta}_{j,t+1}$. Technological advances in credit services may dramatically lower the costs associated with the credit transactions and settlement, which implies that the opportunity cost of holding money is always no smaller than that of using credit,[11]

$$\frac{1}{q_t} \geq (1-\hat{\theta}_{j,t+1})\left(\frac{1}{q_t}\right)\frac{P_t \gamma_{j,t}}{L_{j,t+1}} + \hat{\theta}_{j,t+1}\frac{\alpha}{q_t}. \tag{4}$$

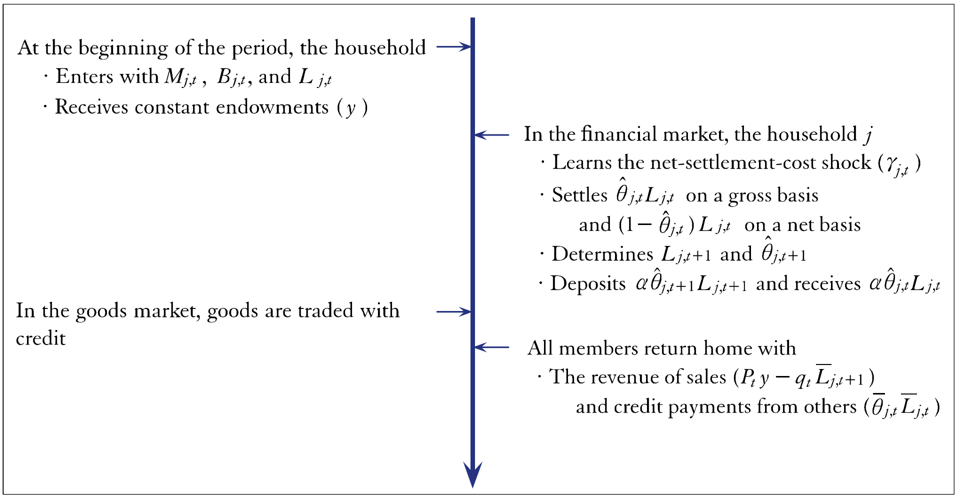
At the end of each period, all the agents return home. The household j receives the revenue from sales. Further trade or barter is not allowed. The budget constraint is given by

$$\begin{aligned} & P_t \int_0^1 c_{j,t}(i)di + \hat{\theta}_{j,t} L_{j,t} + P_t(1-\hat{\theta}_{j,t+1})\gamma_{j,t} + M_{j,t+1} \\ & \quad = M_{j,t} + B_{j,t} - q_t B_{j,t+1} + \alpha \hat{B}_{j,t} - \alpha \hat{B}_{j,t+1} + q_t L_{j,t+1} + P_t y - q_t \bar{L}_{j,t+1} + \bar{\theta}_{j,t} \bar{L}_{j,t}, \end{aligned} \tag{5}$$

where $M_{j,t+1}$ is the demand for cash at the end of the period t for the upcoming financial market transactions in the period $t+1$, $q_t \bar{L}_{j,t+1}$ are the new credit balances issued by other households, $P_t y - q_t \bar{L}_{j,t+1}$ is the revenue from sales, and $\bar{\theta}_{j,t} \bar{L}_{j,t}$ denote incoming payments from other households in gross settlement. Figure 1 summarizes the itinerary of events of the household j in the period t.

[11] Recent developments in payment-card technology would lower the opportunity cost of using a credit card further even though credit-card fraud is on the rise. On the contrary, there are other types of costs of holding money, missing in this model, including the risk of loss, counterfeiting, and the costs of replacing worn currency that could raise the opportunity cost.

[Figure 1] Itinerary of the Household j in the Period t



III. Net-Settlement Only System

This Section will explore a pure net-settlement economy and monetary policy implications to deliver a clear discussion on the risk-sharing role of gross settlement in the following section. Suppose the per-unit net settlement cost is always smaller than the per-unit gross settlement cost $(P_t\gamma_{j,t}/L_{j,t+1}<\alpha)$ for all j. Then, in equilibrium, every credit is settled on a net basis $(\hat{\theta}_{j,t+1}=0)$ bearing the total cost of $\gamma_{j,t}$ in the financial market, where $P_t\gamma_{j,t}/L_{j,t+1}<\alpha<1$ is satisfied with (4). In a pure credit economy without gross settlement, the household does not demand money in the financial market and, hence, does not make any exchange government nominal bonds for money, i.e. $B_{j,t}=q_tB_{j,t+1}$. The determination of a line of new credit remains the only transaction in the financial market. The credit and budget constraints in (2) and (5) are given by

$$P_t\int_0^1 c_{j,t}(i)di = q_tL_{j,t+1}\,, \tag{6}$$

$$P_t\int_0^1 c_{j,t}(i)di + P_t\gamma_{j,t} = q_tL_{j,t+1} + P_ty - q_t\overline{L}_{j,t+1}\,, \tag{7}$$

In equilibrium, from (6) and (7), the real cost of net settlement and new credit balances are given by $\phi_{j,t}y=\gamma_{j,t}$ and $c_{j,t}=q_tL_{j,t+1}/P_t=(1-\phi_{j,t})y$, where $\phi_{j,t}\in(0,1)$ is the household j's share of y. Hence, given $\gamma=y>1$, the choice of $(\phi_{j,t},c_{j,t})$ is as follows:

$$\phi_{j,t} = \begin{cases} \gamma_{j,t} / y & \text{if } \gamma_{j,t} < y, \\ 1 & \text{if } \gamma_{j,t} \geq y, \end{cases}$$

$$c_{j,t} = \begin{cases} y - \gamma_{j,t} & \text{if } \gamma_{j,t} < y, \\ 0 & \text{if } \gamma_{j,t} \geq y. \end{cases}$$

In the absence of money transfer in gross settlement, the household cannot share its net-settlement risk with others in the financial market. All households would consume a positive amount of goods if $\gamma_{j,t} < y$ and nothing if $\gamma_{j,t} \geq y$. Hence, net settlement alone in the payment system cannot provide any insurance to dampen a consumption loss and would fail to achieve efficiency, unlike Temzelides and Williamson (2001) and Williamson (2003). Further, monetary policy does not have any role in redistribution of wealth at all in the economy.

IV. Hybrid-Settlement Economy

This Section will introduce gross settlement in the payment system, in which the household j will make a choice between gross and net arrangements $[\hat{\theta}_{j,t+1} \in (0,1)]$, and will examine its risk-sharing role for consumption, social welfare, and central banking. Credit still remains as the sole medium of exchange in goods trades which is satisfied with (4) and money is used to settle credit payments in the payment system similar to those in Williamson (2009).

4.1. Equilibrium

Definition: A symmetric competitive equilibrium is given by the sequence $\{c_{j,t}, \hat{\theta}_{j,t+1}, M_{j,t+1}, B_{j,t+1}, L_{j,t+1}, M_t^s, P_t, q_t\}_{t=0}^{\infty}$ for $j \in [0,1]$ such that:

1. The household j maximizes (1) subject to (2) and (5) and the nonnegativity constraints, $L_{j,t+1} \geq 0$ and $M_{j,t+1} \geq 0$, given (4).
2. The government budget constraint is satisfied with (3).
3. The markets clear in each of the following periods: (a) goods market: $c_{j,t}(i) = c_{j,t}$ for every market i to avoid arbitrage opportunities and $\int_{\underline{\gamma}}^{\overline{\gamma}} [c_{j,t} + (1 - \hat{\theta}_{j,t+1})\gamma_{j,t}] dF(\gamma) = y$, (b) bonds Market: $\int_{\underline{\gamma}}^{\overline{\gamma}} (B_{j,t} - q_t B_{j,t+1} + \alpha \hat{B}_{j,t} - \alpha \hat{B}_{j,t+1}) dF(\gamma) = B_t^s - q_t B_{t+1}^s = \mu M_t$, (c) money Market: $M_{t+1} = (1+\mu) M_t = \int_{\underline{\gamma}}^{\overline{\gamma}} M_{j,t+1} dF(\gamma) = M_{t+1}^s$, and (d) credit Market: $\hat{\theta}_{j,t+1} = \overline{\theta}_{j,t+1}, L_{j,t+1} = \overline{L}_{j,t+1}$, and $\hat{B}_{j,t+1} = \hat{\theta}_{j,t+1} L_{j,t+1}$.

Suppose $\lambda_{j,t}$ and $\eta_{j,t}$ are the Lagrange multipliers associated with (2) and (5). Then, the choice for $(c_{j,t}, B_{j,t+1})$ is determined by

$$\frac{u'(c_{j,t})}{P_t} = \lambda_{j,t} + \eta_{j,t} \tag{8}$$

$$q_t(\lambda_{j,t} + \eta_{j,t}) = \beta \mathbb{E}_t[\lambda_{j,t+1} + \eta_{j,t+1}]. \tag{9}$$

In equilibrium, when $\hat{B}_{j,t+1} = \hat{\theta}_{j,t+1} L_{j,t+1}$ in terms of gross-settlement payments, the gross-settlement share $(\hat{\theta}_{j,t+1})$ is determined by

$$\frac{1}{q_t}\left(\frac{P_t \gamma_{j,t}}{L_{j,t+1}}\right) = \frac{\alpha}{q_t}, \tag{10}$$

where the nominal opportunity cost of per-unit net settlement is equal to that of per-unit gross settlement. In aggregation, from (10), net settlement needs $(1-\hat{\theta}_{j,t+1})\gamma_{j,t}$ amount of cash to pay off the transaction cost in the financial market. On the contrary, gross settlement requires a total $\alpha \hat{B}_{j,t+1}$ amount of government nominal bonds as collateral.

Further, from (4) and (10), $\alpha \leq 1$ holds. Credit is usually more advantageous to money as medium of exchange in the goods market. When $\alpha = 1$ it is indifferent for shoppers to use credit or cash.

Second, since every household can have an access to the financial market, the household j shares the same amount of money balances $M_{t+1} = M_{j,t} + B_{j,t} - q_t B_{j,t+1} + \alpha \hat{\theta}_{j,t} L_{j,t} - \alpha \hat{\theta}_{j,t+1} L_{j,t+1}$ against $\gamma_{j,t}$ with other households by exchanging money for one-period nominal government bonds. Hence, from (2) and (3), the collateral-in-advance constraint is given $P_t(1-\hat{\theta}_{j,t+1})\gamma_{j,t} + \hat{\theta}_{j,t} L_{j,t} = M_{t+1}$ and, from (10), it can be expressed in terms of the intertemporal real demand for credit balances,

$$\frac{\alpha(1-\hat{\theta}_{j,t+1})L_{j,t+1}}{P_t} + \frac{\hat{\theta}_{j,t} L_{j,t}}{P_t} = \frac{M_{t+1}}{P_t}. \tag{11}$$

The decision on $\hat{\theta}_{j,t+1}$ allows the household j to share its risk $(\gamma_{j,t})$ with others by adjusting the share of M_{t+1}/P_t for $L_{j,t+1}/P_t$. Money is nonneutral and central banking may have real effects on the contrary to what is seen in Section 3. However, if credit payments are entirely settled on a gross basis $(\hat{\theta}_{j,t+1} = 1)$ in (11), then each household demands the different amount of money $\hat{\theta}_{j,t} L_{j,t} / P_t$ and gross settlement alone cannot play the role of an insurance device anymore similar to the result from Section 3.

Subsequently, in (2) and (5), $M_{j,t+1} + q_t L_{j,t+1} - \hat{\theta}_{j,t} L_{j,t} = P_t y$ and the real demand for money and the inflation rate are given by

$$\frac{M_{j,t+1}}{P_t} = \phi_{j,t} y , \tag{12}$$

$$\frac{P_t}{P_{t-1}} = (1+\mu)\frac{\Phi_{t-1}}{\Phi_t} , \tag{13}$$

where $\phi_{j,t} y$ is the household j's share of y for money demand and $\Phi_t = \int_{\underline{\gamma}}^{\overline{\gamma}} \phi_{j,t} dF(\gamma)$ is aggregate real money demand. Due to gross settlement, Φ_t may fluctuate over periods. Hence, from (8)-(13), the borrowing cost (q_t) depends on the share of gross settlement through Φ_t and Φ_{t+1} and is determined by

$$q_t = \beta \mathbb{E}_t \left[\frac{u'(c_{j,t+1})}{u'(c_{j,t})} \left(\frac{1}{1+\mu} \right) \frac{\Phi_{t+1}}{\Phi_t} \right] = \frac{\beta \Psi_j}{u'(c_{j,t})(1+\mu)\Phi_t} , \tag{14}$$

where $\Psi_j = \mathbb{E}_t [u'(c_{j,t+1})\Phi_{t+1}] > 0$.

Let us drop the household subscript j to avoid confusion because every household makes an identical decision through financial market transactions. From (2), (10), and (11)-(14), assuming $\delta_{j,t} \in (0,1)$ is a share of M_{t+1} for $\hat{\theta}_{j,t} L_{j,t}$, the choice for $(c_t, L_{t+1}/P_t, \hat{\theta}_{t+1} c_t)$ is determined by[12]

$$c_t = \frac{q_t L_{t+1}}{P_t} = \left(\frac{y}{\alpha} \right) q_t , \tag{15}$$

$$\hat{\theta}_{t+1} c_t = q_t (1+\mu) \Pi \Phi_t y , \tag{16}$$

where $y = \gamma = \int_{\underline{\gamma}}^{\gamma} \gamma_{j,t} dF(\gamma)$ and $\Pi = \mathbb{E}_t[\delta_{t+1}] \in (0,1)$ as the expected share of the money demand for gross settlement. Therefore, from (10)-(16), the resource constraint $[c_t + (1-\hat{\theta}_{t+1})\gamma = y,$ where $c_t = \int_{\underline{\gamma}}^{\overline{\gamma}} c_{j,t} dF(\gamma)]$, and $y = \gamma > 1$, the solution for $(\Phi_t, \hat{\theta}_{t+1}, c_t, q_t)$ is given by[13]

$$\frac{1}{\Phi_t} = 1 - \Pi + \alpha \Pi (1+\mu) , \tag{17}$$

$$\hat{\theta}_{t+1} = \alpha \Pi (1+\mu) \Phi_t , \tag{18}$$

$$c_t = [1 - (1-\Pi)\Phi_t] y , \tag{19}$$

$$q_t = \frac{\beta \Psi}{u'(c_t)(1+\mu)\Phi_t} , \tag{20}$$

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[12] See appendix A for the derivation of (16).

[13] See appendix B for the derivation.

where $\Psi = \mathbb{E}_t[u'(c_{t+1})\Phi_{t+1}] > 0$. In (17) and (18), Φ_t relies on the ratio of the expected net-settlement share to the current net-settlement share,

$$\frac{1}{\Phi_t} = \frac{1-\Pi}{1-\hat{\theta}_{t+1}}. \tag{21}$$

From (17) and (21), if $\Pi = 0$, then $\hat{\theta}_{t+1} = 0$ and it becomes a pure net-settlement economy with $\Phi_t = 1$, as seen in Section 3. If $\Pi = 1$, then $\hat{\theta}_{t+1} = 1$ and it is a pure gross-settlement economy with $\Phi_t \in (0,1)$.

Proposition 1 *In the coexistence of gross and net settlement with* $\Pi \in (0,1)$, c_t *in (19) is smaller than* y, *consumption under the pure gross-settlement system, but greater than* 0, *consumption under the pure net-settlement system,*

$$c_t = y - \underbrace{(1-\Pi)\Phi_t y}_{\text{Net-settlement loss}} \in (0, y). \tag{22}$$

Proof. From (17) and (21), if $\Pi = 0$, then $\hat{\theta}_{t+1} = 0$, $\Phi_t = 1$, and $c_t = 0$. If $\Pi = 1$, then $\hat{\theta}_{t+1} = 1$ and $c_t = y$. For $\Pi \in (0,1)$, $\Pi\Phi_t y \in (0, y)$ and $c_t = y - (1-\Pi)\Phi_t y$ $\in (0, y)$ in (17) holds. ■

Gross settlement dampens a consumption loss arising from net settlement. In (22), since consumption increases with Π, consumption in the pure gross-settlement system $(\Pi = 1)$ is greater than any other consumption in the hybrid system $[\Pi \in (0,1)]$.

4.2. Payment Policy Implications

This Section will discuss the short-run effects of payment policy on equilibrium outcomes and the optimal collateral requirement ratio. Payment policy plays a dual role in this economy. First, it works as an alternative policy instrument to control the money supply in addition to open market operations. Subsequently, the collateral requirement ratio would have a direct effect on the cost structure of the payment system.

Suppose the government permanently increases the collateral requirement ratio (α) of gross settlement. Then, given $\mu > -1$, the household j will demand more government nominal bonds as collateral for unit gross settlement, which will lead to a decline the stock of money in circulation.

Proposition 2 *From (17), (18), and (20), the effects of a payment policy on* $(\Phi_t, \hat{\theta}_{t+1},$

q_t) are determined by the following:(1) $\partial\Phi_t / \partial\alpha < 0$, (2) $\partial\hat{\theta}_{t+1} / \partial\alpha > 0$, and (3) $\partial q_t / \partial\alpha > 0$, where the payment policy elasticity of Φ_t is inelastic from (α / Φ_t) $(\partial\Phi_t / \partial\alpha) = -\hat{\theta}_{t+1} \in (-1,0)$.

Proof. See appendix C. ■

Corollary 1 *The effects of a payment policy on $[c_t, \hat{\theta}_{t+1}c_t, (1-\hat{\theta}_{t+1})c_t]$ are determined by the following:(1) $\partial c_t / \partial\alpha > 0$, (2) $\partial\hat{\theta}_{t+1}c_t / \partial\alpha > 0$, and (3) $\partial(1-\hat{\theta}_{t+1})c_t / \partial\alpha < 0$ for $1/\Phi_t > 2(1-\Pi)$.*

Proof. See appendix C. ■

There are two transmission effects of an increase in α. One is the negative effect on aggregate money demand (Φ_t). A greater demand for nominal bonds from an increase in α decreases the demand for money and pushes the inflation rate higher from (13) and (18). The other is the negative effect on the nominal interest rate $(1/q_t)$. The short-run liquidity effect arises because a higher inflation rate from an increase in α decreases the nominal interest rate. Even if the household j bears a greater collateral burden, inflation with a smaller return on savings reduces the value of money and favors gross settlement that requires cash to settle credit payments. The gross-settlement share $(\hat{\theta}_{t+1})$ increases in order to save the transactions cost of net settlement.

Hence, in corollary 1, consumption increases with α. In particular, consumption for gross settlement increases, while consumption for net settlement decreases because the negative effect on the net-settlement share dominates the positive effect on consumption. There exist welfare benefits of a collateral requirement, where social welfare is defined by $W_t = \int_{\underline{\gamma}}^{\overline{\gamma}} u(c_t)dF(\gamma)$,

$$\frac{\partial W_t}{\partial\alpha} = u'(c_t)\frac{\partial c_t}{\partial\alpha} > 0 .$$

Proposition 3 *Given $\mu > -1$ and $u'(y) = \beta\Psi$, the optimal collateral requirement rate is one $(\alpha^* = 1)$. From (18), the optimal gross-settlement share is one $(\hat{\theta}^*_{t+1} = 1)$. Further, the net nominal interest rate is zero $(q^*_t = 1)$.*

Proof. *In (20), if $\alpha = 1$, then q_t converges to one, given $u'((1+\mu)\Pi y / (1+\Pi\mu)) = \beta\Psi\ (1+\Pi\mu)/(1+\mu)$. In (15), $c_t = y$ so $\Pi = 1$, given $u'(y) = \beta\Psi$. Thus, in (17) and (18), $\hat{\theta}_{t+1} = 1$ and $1/\Phi_t = 1+\mu > 0$.* ■

Since a permanent increase in α results in a positive income effect on households, optimal payment policy involves operating a pure gross-settlement system with a full collateral ratio to lower the net nominal interest rate to zero in (20). This welfare-improving role of gross settlement may justify the recent replacement of net settlement with gross settlement in the large value payment system, as discussed in Martin (2005) and Bech et al. (2008).

4.3. Monetary Policy Implications

This Section will discuss the short-run effects of open market operations on equilibrium outcomes and the optimal money growth rate. Unlike payment policy, open market operations control the money supply but do not have a direct control on the payment system. Suppose the government permanently increases the money growth rate (μ) through open market purchases. Then, given $\alpha \in (0,1)$, money supply increases along with inflation rate, from (13).

Proposition 4 *From (17), (18), and (20), the effects of monetary policy on* $(\Phi_t, \hat{\theta}_{t+1}, q_t)$ *are determined by the following: (1)* $\partial\Phi_t / \partial\mu < 0$*, (2)* $\partial\hat{\theta}_{t+1} / \partial\mu > 0$*, and (3)* $\partial q_t / \partial\mu > 0$ *for* $\sigma > y / \alpha$*, where* $\sigma = -c_t u'(c_t) / u''(c_t) \in (0,1)$ *and the monetary policy elasticity of* Φ_t *is inelastic from* $[(1+\mu)/\Phi_t](\partial\Phi_t / \partial\mu) = -\hat{\theta}_{t+1} \in (-1,0)$.

Proof. See appendix D. ■

Corollary 2 *The effects of monetary policy on* $[c_t, \hat{\theta}_{t+1}c_t, (1-\hat{\theta}_{t+1})c_t]$ *are determined by the following:(1)* $\partial c_t / \partial\mu > 0$*, (2)* $\partial\hat{\theta}_{t+1}c_t / \partial\mu > 0$*, and (3)* $\partial(1-\hat{\theta}_{t+1})c_t / \partial\mu < 0$ *for* $1/\Phi_t > 2(1-\Pi)$.

Proof. See appendix D. ■

A permanent increase in μ results in inflation, and decreases both the aggregate money demand and the nominal interest rate, similar to that of payment policy. The short-run liquidity effect also arises. The higher inflation rate with a smaller return on savings reduces the value of money and favors gross settlement. The demand for cash in the credit settlement increases as the gross-settlement share $(\hat{\theta}_{t+1})$ increases in order to save the transactions cost of net settlement.

Hence, in corollary 2, consumption increases with μ due the smaller borrowing cost. In particular, consumption for gross settlement increases, while consumption for net settlement decreases because the negative effect on the net-settlement share dominates the positive effect on consumption, similar to that of payment policy. There exist welfare benefits of an inflation, where social welfare is defined by $W_t = \int_{\underline{\gamma}}^{\overline{\gamma}} u(c_t)dF(\gamma)$,

$$\frac{\partial W_t}{\partial \mu} = u'(c_t)\frac{\partial c_t}{\partial \mu} > 0 . \tag{23}$$

Proposition 5 *The optimal money growth rate is infinity* $(\mu^{**} = \infty)$. *From (17) and (18), the optimal gross-settlement share is one* $(\hat{\theta}^*_{t+1} = 1)$.

Since the permanent increase in μ results in a positive income effect on households, the optimal money growth rate should be infinity. The gross-settlement share converges to one in (18) whereas the aggregate real demand for money converges to zero in (18). Hence, monetary policy can achieve efficiency only with a very large money injection in the economy, while payment policy can achieve efficiency given money stock in circulation. Payment policy would be a useful policy device for monetary authorities who are not in favor of creating higher inflation in the economy.

V. Conclusion

In this study, a credit-settlement model is constructed to explore the effects of a deferred gross settlement in the credit economy and its central-bank-policy implications. It addresses the potential importance of payment policy. In the payment system, net settlement alone cannot achieve efficiency; however, a greater share of gross settlement dampens a consumption loss against the interest-rate risk arising from inflation and improves social welfare. Payment policy controls the collateral requirement ratio in gross settlement, and the optimal collateral ratio is one. On the other hand, monetary policy controls the money supply through open market operations, and the optimal money growth rate is infinity. Payment policy may be more desirable than monetary policy as the former maintains relatively stable money supply.

In order to delve into various aspects of payment and monetary policy, several variant models could be considered, in which money and credit are in circulation as medium of exchange or in which multiple types of collateral is required for gross settlement. Besides, in this paper, the credit risk in net settlement does not play a crucial role in the choice of payments systems since there is no balance to settle on a net basis in a symmetric equilibrium. It would be potentially another interesting extension of the model to introduce a financial market friction arising from asymmetric information or limited commitment to analyze its effects on the demand for money, the choice of payments system, and central banking.

Appendix

A. Derivation of (16)

First, in (11), suppose $\delta_{j,t} \in (0,1)$ is a share of M_{t+1} for $\hat{\theta}_{j,t} L_{j,t}$. Subsequently, from (2) and (11), $c_{j,i} = q_t L_{j,t+1} / P_t$ and $\hat{\theta}_{j,t} L_{j,t} = \delta_{j,t} M_{t+1}$ imply that

$$\hat{\theta}_{j,t} c_{j,t+1} = \frac{q_{t-1} \hat{\theta}_{j,t} L_{j,t}}{P_{t-1}} = \frac{q_{t-1} \delta_{j,t} M_{t+1}}{P_{t-1}},$$

and, given (13),

$$\begin{aligned} \hat{\theta}_{j,t+1} c_{j,t} &= q_t \mathbb{E}_t \left[\frac{\delta_{j,t+1} M_{t+2}}{P_t} \right] = q_t \mathbb{E}_t \left[\delta_{j,t+1} \Phi_{t+1} y (1+\mu) \frac{\Phi_t}{\Phi_{t+1}} \right] \\ &= q_t (1+\mu) \Pi \Phi_t y, \end{aligned} \tag{24}$$

$$\frac{L_{j,t+1}}{P_t} = \frac{(1+\mu)\Pi\Phi_t y}{\hat{\theta}_{j,t+1}}, \tag{25}$$

where $\Pi = \Pi_j = \mathbb{E}_t[\delta_{j,t+1}]$ for all j.

B. Derivation of (17)-(19)

First, since every household participates in the financial market, it shares identical new credit balances $(L_{j,t+1} / P_t)$ with other households. From (10) and (25), $\gamma / \alpha = (1+\mu)\Pi\Phi_t y / \hat{\theta}_{t+1}$. Given $y = \gamma$, (18) holds. Second, from (18), the aggregate net-settlement cost is given by

$$\int_{\underline{\gamma}}^{\overline{\gamma}} (1-\hat{\theta}_{j,t+1}) \gamma_{j,t} dF(\gamma) = \gamma - \int_{\underline{\gamma}}^{\overline{\gamma}} \hat{\theta}_{j,t+1} \gamma_{j,t} dF(\gamma) = \gamma - \alpha(1+\mu)\Pi\Phi_t y, \tag{26}$$

where $y = \gamma = \int_{\underline{\gamma}}^{\overline{\gamma}} \gamma_{j,t} dF(\gamma)$. Subsequently, from (2), (3), (12), and (16), $P_t(1-\hat{\theta}_{j,t+1})\gamma_{j,t} + \hat{\theta}_{j,t} L_{j,t} = M_{t+1}$ holds, and

$$\begin{aligned} \int_{\underline{\gamma}}^{\overline{\gamma}} (1-\hat{\theta}_{j,t+1}) \gamma_{j,t} dF(\gamma) &= \int_{\underline{\gamma}}^{\overline{\gamma}} \frac{M_{t+1}}{P_t} - \frac{\hat{\theta}_{j,t} L_{j,t}}{P_t} dF(\gamma) \\ &= (\Phi_t - \Pi\Phi_t) y = (1-\Pi)\Phi_t y, \end{aligned} \tag{27}$$

additionally, from (26) and (27), (17) holds. Third, (19) holds from (27), and the resource constraint can be expressed as

$$y=\int_{\underline{\gamma}}^{\overline{\gamma}} c_{j,t}+(1-\hat{\theta}_{j,t+1})\gamma_{j,t}dF(\gamma)=c_t+(1-\Pi)\Phi_t y\,.$$

Finally, (20) holds from (14), (17), and (19) with satisfying $c_t u'(c_t)[\alpha(1+\mu)]\Phi_t = \beta\Psi y$ in equilibrium.

C. Proof of Proposition 2 and Corollary 1

First, in (17), the effect of α on Φ_t is determined by

$$\frac{\partial\Phi_t}{\partial\alpha}=-\Pi(1+\mu)\Phi_t^2<0\,. \tag{28}$$

Subsequently, from (18) and (28), the effect on $\hat{\theta}_{t+1}$ is determined by

$$\frac{\partial\hat{\theta}_{t+1}}{\partial\alpha}=\Pi(1+\mu)\Phi_t\left(1+\frac{\alpha}{\Phi_t}\frac{\partial\Phi_t}{\partial\alpha}\right)>0\,, \tag{29}$$

where $\Pi<1$ and

$$\frac{\alpha}{\Phi_t}\frac{\partial\Phi_t}{\partial\alpha}=-\alpha\Pi(1+\mu)\Phi_t=-\hat{\theta}_{t+1}\in(-1,0)\,. \tag{30}$$

From (19), (20), and (28), the effects on c_t and q_t are determined by

$$\frac{\partial c_t}{\partial\varepsilon}=-(1-\Pi)y\frac{\partial\Phi_t}{\partial\alpha}>0\,, \tag{31}$$

$$\begin{aligned}\frac{\partial q_t}{\partial\alpha}&=\left(\frac{\beta\Psi}{1+\mu}\right)\left(\frac{1}{u'(c_t)\Phi_t}\right)^2\left[-u''(c_t)\Phi_t\frac{\partial c_t}{\partial\alpha}-u'(c_t)\frac{\partial\Phi_t}{\partial\alpha}\right]\\&=\left(\frac{\beta\Psi}{1+\mu}\right)\left(\frac{1}{u'(c_t)\Phi_t}\right)^2[u''(c_t)\Phi_t(1-\Pi)y-u'(c_t)]\frac{\partial\Phi_t}{\partial\alpha}>0\,,\end{aligned} \tag{32}$$

Second, it is straightforward to show the positive effect of α on $\hat{\theta}_{t+1}c_t$ from (29) and (31). Subsequently, in (18) and (29)-(31), the effect on $(1-\hat{\theta}_{t+1})c_t$ is negative from, for $1/\Phi_t>2(1-\Pi)$,

$$\frac{\partial(1-\hat{\theta}_{t+1})c_t}{\partial\alpha} = (1-\hat{\theta}_{t+1})\frac{\partial c_t}{\partial\alpha} - c_t\frac{\partial\hat{\theta}_{t+1}}{\partial\alpha}$$
$$= -(1-\hat{\theta}_{t+1})(1-\Pi)y\frac{\partial\Phi_t}{\partial\alpha} - c_t\Pi(1+\mu)\Phi_t\left(1+\frac{\alpha}{\Phi_t}\frac{\partial\Phi_t}{\partial\alpha}\right)$$
$$= -(1-\hat{\theta}_{t+1})(1-\Pi)y\frac{\partial\Phi_t}{\partial\alpha} - \frac{\hat{\theta}_{t+1}c_t}{\alpha}\left(1+\frac{\alpha}{\Phi_t}\frac{\partial\Phi_t}{\partial\alpha}\right)$$
$$= -(1-\hat{\theta}_{t+1})^2\left(\frac{y}{\alpha}\right)\frac{\alpha}{\Phi_t}\frac{\partial\Phi_t}{\partial\alpha} - \frac{\hat{\theta}_{t+1}c_t}{\alpha}\left(1+\frac{\alpha}{\Phi_t}\frac{\partial\Phi_t}{\partial\alpha}\right)$$
$$= -(1-\hat{\theta}_{t+1})^2\left(\frac{y}{\alpha}\right)\hat{\theta}_{t+1} - \frac{\hat{\theta}_{t+1}c_t}{\alpha}(1-\hat{\theta}_{t+1})$$
$$= \left(\frac{1-\hat{\theta}_{t+1}}{\alpha}\right)\hat{\theta}_{t+1}[(1-\hat{\theta}_{t+1})y - c_t]$$
$$= \left(\frac{1-\hat{\theta}_{t+1}}{\alpha}\right)\hat{\theta}_{t+1}[2(1-\Pi)\Phi_t y - y]$$
$$= \left(\frac{1-\hat{\theta}_{t+1}}{\alpha}\right)\hat{\theta}_{t+1}y\left[\frac{1-\Pi-\alpha\Pi(1+\mu)}{1-\Pi+\alpha\Pi(1+\mu)}\right] < 0\,.$$

D. Proof of Proposition 4 and Corollary 2

First, in (17), the effect of μ on Φ_t is determined by

$$\frac{\partial\Phi_t}{\partial\mu} = -\alpha\Pi\Phi_t^2 < 0\,. \tag{33}$$

From (18), (19), and (33), the effects on $\hat{\theta}_{t+1}$ and c_t are determined by

$$\frac{\partial\hat{\theta}_{t+1}}{\partial\mu} = \alpha\Pi\left[\Phi_t + (1+\mu)\frac{\partial\Phi_t}{\partial\mu}\right] = \alpha\Pi\Phi_t[1-\alpha\Pi(1+\mu)\Phi_t]$$
$$= \alpha\Pi\Phi_t(1-\hat{\theta}_{t+1}) > 0\,, \tag{34}$$
$$\frac{\partial c_t}{\partial\mu} = -(1-\Pi)y\frac{\partial\Phi_t}{\partial\mu} > 0\,. \tag{35}$$

Then, from (17) and (20),

$$u'(c_t)q_t = \frac{\beta\Psi}{(1+\mu)\Phi_t}\,, \tag{36}$$

and from (33) and (36), the effect on q_t is determined by

$$[u'(c_t)+q_t u''(c_t)]\frac{\partial q_t}{\partial \mu}=-\frac{\beta\Psi}{[(1+\mu)\Phi_t]^2}\left[\Phi_t+(1+\mu)\frac{\partial \Phi_t}{\partial \mu}\right], \tag{37}$$

where $\sigma=-c_t u'(c_t)/u''(c_t)\in(0,1)$. Subsequently, by rearranging (37) with (15), assuming $\sigma>y/\alpha$, the effect on q_t is positive from

$$\begin{aligned}\frac{\partial q_t}{\partial \mu}&=-\frac{\beta\Psi}{u'(c_t)(1-\frac{\sigma\alpha}{y})(1+\mu)^2\Phi_t}\left[1+\left(\frac{1+\mu}{\Phi_t}\right)\frac{\partial \Phi_t}{\partial \mu}\right]\\&=-\frac{\beta\Psi(1-\hat{\theta}_{t+1})}{u'(c_t)(1-\frac{\sigma\alpha}{y})(1+\mu)^2\Phi_t}>0,\end{aligned} \tag{38}$$

where in (18) and (33),

$$\left(\frac{1+\mu}{\Phi_t}\right)\frac{\partial \Phi_t}{\partial \mu}=-\alpha\Pi(1+\mu)\Phi_t=-\hat{\theta}_{t+1}\in(-1,0).$$

Second, it is straightforward to show the positive effect of α on $\hat{\theta}_{t+1}c_t$ from (34) and (35). Subsequently, in (17)-(19) and (33)-(35), the effect on $(1-\hat{\theta}_{t+1})c_t$ is negative from, for $1/\Phi_t>2(1-\Pi)$,

$$\begin{aligned}\frac{\partial(1-\hat{\theta}_{t+1})c_t}{\partial \mu}&=(1-\hat{\theta}_{t+1})\frac{\partial c_t}{\partial \mu}-c_t\frac{\partial \hat{\theta}_{t+1}}{\partial \mu}\\&=-(1-\hat{\theta}_{t+1})(1-\Pi)y\frac{\partial \Phi_t}{\partial \mu}-c_t\alpha\Pi\Phi_t(1-\hat{\theta}_{t+1})\\&=(1-\hat{\theta}_{t+1})\alpha\Pi\Phi_t[(1-\Pi)\Phi_t y-c_t]\\&=-(1-\hat{\theta}_{t+1})\alpha\Pi\Phi_t[1-2(1-\Pi)\Phi_t]y\\&=(1-\hat{\theta}_{t+1})\alpha\Pi\Phi_t y\left[\frac{1-\Pi-\alpha\Pi(1+\mu)}{1-\Pi+\alpha\Pi(1+\mu)}\right]<0.\end{aligned}$$

References

Bech, M., C. Preisig, and K. Soramäki (2008), "Global Trends In Large-Value Payments," *FRBNY Economic Policy Review*, Federal Reserve Bank of Ney York, September, 59–81.

Chiu, J., and A. Lai (2007), "Modelling Payments Systems: A Review of the Literature," Working Paper 2007-28, Bank of Canada.

Choi, H. (2018), "Inflation Risk, Settlement Cycles, and Monetary Policy," *Korean Economic Review*, Vol. 34, 267–282.

________ (2019), "Money, Debit Card, Gross-Settlement Risk, and Central Banking," *North American Journal of Economics and Finance*, Vol. 50, 100993.

Freeman, S. (1996), "The Payments System, Liquidity, and Rediscounting," *American Economic Review*, Vol. 86, 1126–1138.

Freeman, S., and F. Kydland (2000), "Monetary Aggregates and Output," *American Economic Review*, Vol. 90, 1125–1135.

Green, E. J. (2002), "Payment Arrangements and Inflation," *American Economic Review Papers and Proceedings*, Vol. 92, 51–57.

Ireland, P. N. (1994), "Money and Growth: An Alternative Approach," *American Economic Review*, Vol. 84, 47–65.

Kahn, C. M. (2013), "Private Payment Sytems, Collateral, and Interest Rates," *Annals of Finance*, Vol. 9, 83–114.

Kahn, C. M., J. McAndrews, and W. Roberds (2003), "Settlement Risk under Gross and Net Settlement," *Journal of Money, Credit and Banking*, Vol. 35, 591–608.

Kahn, C. M., and W. Roberds (1998), "Payment System Settlement and Bank Incentives," *Review of Financial Studies*, Vol. 11, 845–870.

________________________ (2001), "Real-Time Gross Settlement and the Cost of Immediacy," *Journal of Monetary Economics*, Vol. 47, 299–319.

________________________ (2009), "Why Pay? An Introduction to Payments Economics," *Journal of Financial Intermediation*, Vol. 18, 1–23.

Koeppl, T., C. Monnet, and T. Temzelides (2008), "A Dynamic Model of Settlement," *Journal of Economic Theory*, Vol. 142, 233–246.

Martin, A. (2005), "Recent Evolution of Large-Value Payment Systems: Balancing Liquidity and Risk," *Economic Review*, Federal Reserve Bank of Kansas City, First Quarter, 33–57.

Nosal, E., and G. Rocheteau (2006), "Economics of Payments," *Policy Discussion Paper*, No. 14, Federal Reserve Bank of Cleveland.

Temzelides, T., and S. D. Williamson (2001), "Payments Systems Design in Deterministic and Private Information Environments," *Journal of Economic Theory*, Vol. 99, 297–326.

Williamson, S. D. (2003), "Payments Systems and Monetary Policy," *Journal of Monetary Economics*, Vol. 50, 475–495.

_______________ (2009), "Transactions, Credit, and Central Banking in a Model of Segmented Markets," *Review of Economic Dynamics*, Vol. 12, 344–362.