THE KOREAN ECONOMIC REVIEW
Volume 17, Number 1, Summer 2001

ENDOGENOUS GROWTH WITHOUT SCALE EFFECTS IN A PROCESS INNOVATION MODEL

YONG-SANG SHYN*

One of the key issues related to the endogenous growth models based on R&D is whether the long-run economic growth rate is endogenously or semi-endogenously determined. This paper extends this debate to another type of endogenous growth model, a process innovation model with learning-by-doing in the production of new technology. The model supports the semi-endogeneity of long-run economic growth and the effectiveness of public policy on economic welfare. The driving forces of these results are the negative externality of past successful innovation in the production of new technology and the public good property of new technology.

JEL Classification: O41, O30
Keywords: knowledge, learning-by-doing, process innovation, scale effects, semi-endogenous growth

I. INTRODUCTION

One of the main issues related to the endogenous growth models is whether the long-run economic growth rate is endogenously or semi-endogenously determined. This debate has mainly resided in R&D-based endogenous growth literature. However, there are diverse types of innovative activities in a real world.

Thus, as an alternative, we propose another type of endogenous growth model with a learning-by-doing process as a mechanism of technology production in this paper. In R&D-based model, innovators create differentiated designs for producer durables and sell the patents for these designs to producers in the intermediate good sector, acting as local monopolists for the differentiated intermediate goods. However, in this paper, we replace the product innovation model

Received for publication: Jan. 29, 2001. *Revision accepted:* April. 6, 2001.

* Reserach Fellow, Korea Institute of Industrial Technology Evaluation and Planning, 395-67 Shindaebang-Dong Dongjak-Ku Seoul 156-010, South Korea.
Phone: 02-8298-771, Fax: 02-8298-801, E-Mail: ysshin@mail.itep.re.kr

based on R&D with a model of process innovation through a learning-by-doing process, in which there is a single intermediate good and new innovation make this intermediate good more productive. This model supposes the following two basic assumptions: knowledge production through a learning-by-doing process without scale effects and a perfectly competitive intermediate good market due to the pure public good property of technology.[1] That is, in this paper, producers in the intermediate good sector obtain new knowledge through a learning-by-doing process in which the creation of the next generation technology becomes more and more difficult as technology advances. Perfect competition in the intermediate good market originates from the assumption that knowledge is a pure public good in the sense that no producer in the intermediate good sector can prevent knowledge from flowing freely. With this process innovation model based on learning-by-doing, we examine what the main endogenous factors of the long-run economic growth are and whether economic growth is endogenous or semi-endogenous.

This model supports the semi-endogeneity of long-run economic growth. That is, the long-run economic growth rate is completely determined by exogenously given parameters: the growth rate of the labor force, innovation productivity, and capital-intensiveness. This result brings us back to the conclusions of Segerstorm's (1998) R&D-based semi-endogenous growth model, in which long-run economic growth does not depend on structural characteristics of the economy, so does not depend on government policy such as subsidies for new innovation.

In contrast to the Romer-type R&D-based endogenous growth models, there are no scale effects in this model, i.e., level changes in resources employed for the production of new knowledge do not affect the long-run economic growth rate. In this model, the long-run economic growth rate is proportional to the growth rate of the labor force but not its level, as in Jones (1995a, 1995b), Kortum (1997) and Segerstrom (1998). On the other hand, in this model, a change in the share of labor force devoted to the intermediate good sector does affect the long-run level of output (income) along the balanced growth path. A change in the level of population affects the level of output but not its long-run growth rate in this model. Thus, if the population is fixed, exponential growth of per capita output dose not arise in this model.

According to Segerstrom (1998), in general, endogenous growth has two definitions. The first definition is that both the economic growth rate and knowledge growth rate are endogenously determined by the optimal decisions of economic agents in the model. The second definition is a model in which government's public policies, such as subsidies for innovation, can influence the

---

[1] According to Brezis, Krugman and Tsiddon (1993), in general, normal technological change is likely to advance largely through learning-by-doing processes, while huge technological breakthroughs, which make nations start fresh and take the form of fundamental changes in methods of production, appear occasionally. Thus, in our model, we take learning-by-doing as the process of technological advance.

steady state economic growth rate. In contrast, semi-endogenous growth means that the economic growth rate is never affected by government policies, while it is endogenously determined in the model.[2] Thus, as noted by Li (2000), whether long-run economic growth is endogenous or semi-endogenous has important policy implications in the sense that if it is endogenous, the government has power affecting long-run economic growth. In contrast, if growth is semi-endogenous, government policy is powerless in inducing a permanent change in economic growth.

The endogenous versus semi-endogenous growth debate comes from the prediction of scale effects: the growth rate of the economy should be positively related to the aggregate innovative effort undertaken in the economy. Romer (1990) argues that knowledge is basically non-rivalrous in the sense that once any technology is developed, there is no need to reproduce it for use in other simultaneous activities. Non-rivalry causes intertemporal technology spillovers in innovative activities, which means that technological information produced by past innovative efforts contributes to improve the productivity of future innovative activities. He finds that in a system with positive externalities in the production process of technology, economic growth depends on the growth of new knowledge. Total profit that can be obtained by successful innovations depends on the scale (size) of market for new intermediate goods embodying new technologies, which in turn depends on the growth rate of income and consumption opportunities. In addition, in turn, this increased return to successful innovation leads to an increase in innovative activities. Consequently, this increase in innovative activities by the expansion of market size should lead to faster economic growth. This logic induces the expectation that all Romer-type R&D models such as Romer (1990), Grossman and Helpman (1991a, 1991b) and Aghion and Howitt (1992) should exhibit scale effects.[3]

However, despite an enormous increase in resources devoted to innovative activities in the major industrial countries since World War II, the economic growth rates of these countries have remained constant or even declined. As noted by Jones (1995a, 1995b), these industrial countries have not experienced, over the last 40 years, the scale effects that are strongly predicted by the standard knowledge-based endogenous growth models. On the basis of this historical fact, Jones (1995a, 1995b), Kortum (1997) and Segerstrom (1998) have developed alterna-

[2] The models of Jones (1995a, 1995b), Kortum (1997) and Segerstrom (1998) satisfy the first definition but not the second one. However, Jones (1995a) calls his model a semi-endogenous growth model in the sense that it has the first characteristic even though it dose not have the second one. Although our modified model in this paper does not satisfy the second definition, we call it a semi-endogenous growth model since it satisfies the first one.

[3] This result clearly distinguishes the Romer-type R&D-based endogenous growth model from endogenous growth models based on the accumulation of rivalrous physical or human capital such as Lucas (1988) and Rebelo (1991), in which the market size does not affect economic growth rate. In general, on the other hand, learning-by-doing models with non-rivalrous knowledge such as Young (1991, 1993) also exhibit scale effects.

tive R&D-based endogenous growth models without scale effects in order to explain why economic growth has held no upward trend in spite of the substantial increases in resources devoted to the production of new knowledge. Jones (1995a, 1995b) explains the absence of intertemporal technology spillovers experienced in major industrial countries by assuming the existence of decreasing returns to scale in the process of technology production. Thus, in his model, to hold a given rate of economic growth, more and more resources must be employed in innovative activities as technology advances. Kortum (1997) considers a search-theoretic model of technological change combined with Grossman and Helpman's (1991a) quality ladder model in order to explain why inventions per unit of research effort decline over time and why total factor productivity growth has not increased with the level of research. In his theoretic model, past research efforts form a technological frontier representing the most advanced technology in the economy, new technology is considered as a blueprint that advances the technological frontier, and technological breakthroughs become harder and harder to find as the technological frontier advances. Consequently, the inventions per research effort decline over time as technological breakthroughs become increasingly difficult to come by. Segerstrom (1998) suggests an endogenous growth model without scale effects, starting with the fact that patent statistics have been roughly constant even though the resources devoted to innovative activities has risen enormously in the main advanced countries over the last 40 years. In his paper, to explain this historical fact and to exclude scale effects, he assumes that R&D difficulty grows progressively in creating new technology over time. That is, he assumes that the clearest technology is developed first, making it more difficult to discover the next generation of technology.

Although the above-mentioned three alternative R&D models without scale effects are very different in detail and contain far more insight, the common characteristic of them is that the long-run economic growth rate is completely determined by exogenously given parameters, like the growth rate of the population. Thus, most variables like education level, openness by trade liberalization, market size and aggregate effort for inventive activities, which are suggested as the main endogenous factors of economic growth in the standard endogenous growth models, have only level effects on output and consumer's welfare without any change in long-run economic growth rates.

The latest line of endogenous growth models is the two-R&D-sector model such as the models of Young (1998) and Aghion and Howitt (1998, Ch. 12), which set up technological progress in the dual form of variety innovation (or horizontal innovation) and quality innovation (vertical innovation). That is, research can create the variety of differentiated products or it can improve the quality of a specific product. In this line of endogenous growth model, economic growth is endogenously determined and does not depend on the scale of the economy. Also, an exogenous growth of economy scale (i.e., growth of population) does not affect the growth in research effort for quality innovation but only the one in the variety of differentiated products. These two-R&D-sector model are impressive in the sense that the long-run economic growth is affected by public policy and, at the same time, there are no scale effects in the process of knowledge production. However, these interesting results from the two-R&D-sector model completely depend on the assumption that there is no positive externality between the quality innovation activity and the variety innovation activity, and on the assumption that the variety of products is proportional to the size of the economy.[4] Jones (1999) and Li (2000) prove that if even one of these two assumptions is relaxed, semi-endogenous growth becomes a general case, while endogenous growth is reduced to a knife-edge case even in the two-R&D-sector model.

As noted above, this debate has mainly resided in the R&D-based endogenous growth model. This paper extends this debate to another type of endogenous growth model, a process innovation model with learning-by-doing process in the production of new technology.

The rest of this paper is organized as the follows. In the next section, we present a simple process innovation model through learning-by-doing process without scale effects. In section III, we provide the competitive analysis in a decentralized economy. In section IV, we examine the welfare implications of the model. Section V concludes.

II. MODEL SPECIFICATION

The economy considered in this paper has two types of manufacturing activities: production of the final good Y and production of the intermediate input X. Thus, we consider the profit maximization problem of a manufacturer in each of two manufacturing sectors. In this model, we assume that each individual has normalized human capital and can freely allocate his human capital between the final goods sector and the intermediate good sector, denoted by h_y and h_x, respectively $(h_y + h_x = 1)$. Denoting total human capital in the economy as H, we get the following relation: $H = H_x + H_y$. The population N grows at rate n and the total human capital H is equal to the population N, with labor supplied inelastically.

2.1. Final goods

We assume that the final good is produced with the following constant return to scale production function[5]:

---

[4] The two-R&D-sector endogenous growth model such as Young (1998) and Aghion and Howitt (1998) assume that an increase in the scale of the economy proportionally increases the variety of products without any increase in quality-upgrading innovation of particular product. However this assumption is inconsistent with the historical fact. According to Segerstrom (1998), the quality-improving R&D activity of particular product has steadily increased over time.

[5] Romer (1990) assumes that final goods are produced by using human capital, unskilled labor,

$$Y = AH_y^{1-\alpha}X^{\alpha}; 0 < \alpha < 1 \tag{1}$$

where A is a productivity parameter dependent on the country's institution, α denotes parameter of technology. This production function shows that the final good is produced using human capital H_y and intermediate good X. The output of final goods can be partly consumed and partly invested for production of intermediate good. Since all markets should be cleared in steady state, a market clearing condition for final goods is

$$Y = C + \dot{K} \tag{2}$$

where C is an aggregate consumption and $\dot{K}$ is investment for production of intermediate good. For simplicity, we assume that there is no physical depreciation of capital. Since the production function of final goods exhibits constant returns to scale, we can rewrite the above equations (1) and (2) in per capita terms with the assumption that the population grows at a rate n:

$$y = f(h_y, x) = Ah_y^{1-\alpha}x^{\alpha}, \tag{3}$$

$$y = c + \dot{k} + nk, \tag{4}$$

where $y = Y/H$, $c = C/H$, $h_y = H_y/H$, $x = X/H$ and $k = K/H$. We assume that this normalized production function is strictly concave.[6]

2.2. intermediate good

Intermediate good are also produced by the following increasing returns to scale production function:

$$X = ZH_x^{1-\beta}K^{\beta} \text{ or } x = Zh_x^{1-\beta}k^{\beta}, \quad 0 < \beta < 1, \tag{5}$$

where $h_x = H_x/H$. β is a parameter of technology and Z is the knowledge stock. This means that intermediate good are produced with human capital H_x, accumulated knowledge stock Z, and physical capital stock K. Note that there are constant returns to scale in human capital stock H_x and physical capital stock K, holding Z constant in the intermediate good sector. But there are increasing

and intermediate inputs, while in this setting, unskilled labor is excluded by normalizing, and human capital is defined as the measure of productivity embodied in each labor input.

[6] We also assume that this production function satisfies the following Inada conditions: $\lim_{h_y \to 0} f(\cdot) = 0$, $\lim_{x \to 0} f(\cdot) = 0$, $\lim_{h_y \to 0} \frac{\partial f}{\partial h_y} = \infty$, $\lim_{x \to 0} \frac{\partial f}{\partial x} = \infty$, $\lim_{h_y \to \infty} \frac{\partial f}{\partial h_y} = 0$, $\lim_{x \to \infty} \frac{\partial f}{\partial x} = 0$, which is a satisfactory condition but not a necessary condition. In addition, we assume that $h_0 > 0$ and $k_0 > 0$, which means that this economy starts with some human and physical capital.

returns to scale to these inputs and Z together. The presence of increasing returns reflects the fact that knowledge is a pure public good: once a technology is developed, it does not need to be reproduced to expand production. Thus, manufacturers in the intermediate good sector freely use the technology as a public good without any cost. By this assumption, we can switch the assumption of monopolistic competition in the intermediate good sector with the assumption of perfect competition. By introducing this modified setup in the intermediate good sector, we can derive the solution path of a perfectly competitive equilibrium and some different meanings.[7]

2.3. Learning-by-doing knowledge production process without scale effects

In contrast to the models in Romer (1990), Jones (1995a) and Segerstrom (1998), in this paper we assume that new technologies are created through a learning-by-doing process in the intermediate good sector. That is, manufacturers in the intermediate good sector generate additions to knowledge in the process of producing intermediate good (but not consumer goods) and they have no tools to prevent this knowledge from flowing freely into the public. This means that knowledge is a pure public good in the sense that all manufacturers in the intermediate good sector can make use of the accumulated knowledge Z embodied in the existing technologies (non-rivalry of technological information) and have no need to pay for additional use of new ideas (non-excludability of technological information). Therefore, the innovation in this paper does not coincide with the product innovation which creates differentiated designs in R&D models. Instead, it is process innovation which improves the productivity of inputs. Thus, in contrast to Romer-type model, in this paper, new knowledge does not expand the range of intermediate goods. To reflect the above conditions, following Segerstrom (1998), we model this knowledge production function which is affected by human capital devoted to intermediate good sector:

$$\dot{Z} \equiv \frac{BH_x}{D(Z)} \tag{6}$$

where $B > 0$ is a given technology parameter and $D(Z)$ is a learning-by-doing difficulty index which measures the degree of negative externality in the knowledge production process.[8] This innovation difficulty reflects the fact that the

---

[7] In contrast to the R&D-based endogenous growth models in which a solution path of a monopolistic competition is considered, like in this model, a model with a perfect competitive equilibrium path as a solution path, in which intermediate good are manufactured by past knowledge accumulation as an input factor, is considered by Takahashi and Sakagami (1998). We follow their learning-by-doing approach. However, a major difference between our model and theirs is that ours does not exhibit the scale effect in the process of knowledge production.

[8] Jones (1995a, 1995b) introduced the following knowledge production function:

$$\dot{Z} = \gamma H_z Z^{\phi},$$

clearest innovations are discovered first in a learning-by-doing process in the intermediate good sector so that the probability that someone discovers new innovations tends to decrease as the stock of knowledge increases.[9] Thus, we assume that this learning-by-doing difficulty proportionally grows as the knowledge stock increases through learning-by-doing process in intermediate good sector:

$$D(Z)=\omega Z^{x}, \quad x>0 \tag{7}$$

This difficulty term includes a fixed cost of innovation, ω, and an exogenously given learning-by-doing difficulty parameter, x. This assumption of a negative externality in the knowledge production process removes scale effects in the knowledge production process.

2.4. Representative individual behavior

Following the usual convention, we assume that the preferences are homogeneous across individuals. Then, the individual optimization problem can be reduced to a representative individual utility maximization problem. Now to make this model be an intertemporal maximization problem, we introduce a representative individual consumer who chooses his consumption and savings in the form of asset accumulation in order to maximize his discounted future utility function subject to a dynamic budget constraint. If we normalize the price of the final good as unity (i.e., the consumption good is the numeraire), the representative consumers intertemporal utility maximization problem is as follows:

$$\max_{c_t,\, a_t} \int_0^{\infty} e^{-\rho t} u(c_t)dt, \quad \text{s.t.} \quad \dot{a}=ra+w-c, \tag{8}$$

where $u(c_t)=\dfrac{c_t^{1-\sigma}}{1-\sigma}$ is a momentary utility function, a denotes per capita

---

where measures the degree of externality in the R&D process. Including difficulty index in equation (6) corresponds to the negative external returns case ($\Phi<0$) in Jones(1995a, 1995b).

[9] As noted by Kortum (1997) and Segerstrom (1998), the evidence that productivity of technology creation activities has dropped comes from the patent statistics in the industrial countries, i.e., the number of patents relative to R&D efforts has consistently declined for the last 40 years. This decline in the number of patents per R&D resource is a worldwide trend for the relevant time period. Also, an increase in innovation difficulty in the knowledge production process is a common trend across overall industries: according to Kortum (1997), in the cases of textile industry, chemicals industry, and pharmaceutical industry, innovations are becoming more and more difficult to create. According to Segerstrom (1998), this assumption is motivated by the experiences of industries like the microprocessor industry. There, the effort to improve the quality of a product seems to grow exponentially with the complexity of the devices. As another example of this negative externality of the knowledge production process, Li (1999) considers the silicon chip industry. Silicon chips are produced by printing circuit patterns on wafers of silicon. As more and more transistors are condensed on a single chip, the move to the next generation chip become more and more difficult. This means that, in these industries, the innovation difficulty grows with increases in the knowledge stock.

wealth, $0<\sigma<1$ is the inverse of the intertemporal elasticity of substitution, r is the interest rate paid on assets, and $\rho>0$ is the subjective time preference rate as discount factor.

III. COMPETITIVE LONG RUN EQUILIBRIUM IN A DECENTRALIZED ECONOMY

Now we solve the model for the balanced growth path of a decentralized economy where all endogenous variables grow at a constant rate.[10] The manufacturer in the final goods sector maximizes his current profit by hiring human capital and intermediate good with a given wage w_y and a given price p_x of intermediate good.

$$\max_{H_y, X} \{AH_y^{1-\alpha}X^{\alpha} - w_y H_y - p_x X\}. \tag{9}$$

Solving this problem with respect to H_y and X, we obtain the following conditional demand functions from the FOCs of the competitive equilibrium:

$$w_y = (1-\alpha)Ah_y^{-\alpha}x^{\alpha}, \tag{10}$$

$$p_x = \alpha A h_y^{-\alpha}x^{\alpha-1}. \tag{11}$$

On the other hand, the representative manufacturer in the intermediate good sector solves the following profit maximization problem with respect to human capital H_x and physical capital K whose prices are given as wage w_x and interest rate r, respectively:

$$\max_{K, H_x} \{p_x Z H_x^{1-\beta}K^{\beta} - w_x H_x - rK\}. \tag{12}$$

From the competitive equilibrium FOCs of this problem, we obtain the following relations:

$$w_x = (1-\beta)p_x Z h_x^{-\beta}k^{\beta}, \tag{13}$$

$$r = \beta p_x Z h_x^{1-\beta}k^{\beta-1}. \tag{14}$$

Workers choose freely between the final goods sector and the intermediate good sector. Thus, by the arbitrage condition of human capital between two sectors, the wage in the final goods sector must be equal to the wage of human capital in

[10] However, the balanced growth path in steady state does not necessarily need the same growth rate for all endogenous variables.

the intermediate good sector, i.e., $w_y = w_x$ in the steady state. Substituting equation (5) into equations (10) and (11) and solving the arbitrage condition in the human capital market, we can determine the fraction of the human capital devoted to intermediate good along a balanced growth path:

$$h_x = \frac{\alpha - \alpha\beta}{1 - \alpha\beta}. \tag{15}$$

With an exogenously given learning-by-doing difficulty parameter $x > 0$, equations (6) and (7) mean that the stock of knowledge grows by the following formulation:

$$\gamma_z \equiv \frac{\dot{Z}}{Z} = \frac{Bh_x H}{\omega Z^{1+x}} \tag{16}$$

However, the stock of knowledge should grow at a constant rate in the steady state. Thus, in the steady state where workers employed in the intermediate sector grow at the same rate as that of total labor force, logarithmically differentiating equation (6) using equation (7) gives us the following steady state growth rate of the knowledge stock:

$$\gamma_z = \frac{n}{1+x}, \tag{17}$$

where n is the exogenously given grow rate of total labor force. Equation (17) shows that the growth rate of the knowledge stock depends on the following exogenous parameters: the growth rate of the total labor force n and the learning-by-doing difficulty parameter x describing the condition for knowledge accumulation through the learning-by-doing process in the intermediate good sector. Thus, the knowledge stock grows over time at the constant rate :

$$Z_t = Z_0 e^{\gamma_z \cdot t};\ Z_0 > 0. \tag{18}$$

Using equations (3), (4), (5), (15), (16) and (17), we can obtain the following per capita long-run output growth rate and the per capita level of output along the balanced growth path without scale effects.[11]

$$\gamma_y = \frac{\alpha}{1 - \alpha\beta} \cdot \gamma_z = \xi \cdot \frac{n}{1+x}, \tag{19}$$

[11] From equation (4), we can obtain the relation that $\gamma_y = \gamma_c$ because n is constant and $\dot{k} = 0$ in steady state. Consequently, the following relation is satisfied: $\gamma_y = \gamma_k = (\gamma_c)$.

$$y = \psi \cdot \left[\frac{B}{\omega} \cdot \frac{1+x}{n} \cdot h_x \cdot H\right]^{\alpha/(1+x)} k^{\alpha\beta}, \tag{20}$$

where $\xi \equiv \frac{\alpha}{1-\alpha\beta}$ which reflects the combined capital-intensiveness of both manufacturing sectors in the economy; $\psi \equiv A\left[\frac{\alpha(1-\beta)}{1-\alpha\beta}\right]^{\alpha(1-\beta)}\left[\frac{1-\alpha}{1-\alpha\beta}\right]^{1-\alpha}$ is a positive constant. Thus equation (19) shows that the long-run economic growth rate depends completely on the growth rate of the labor force n, the learning-by-doing difficulty parameter x and the combined capital-intensiveness parameter ξ. The higher the growth rate of the total labor force (the bigger is n), the lower the learning-by-doing difficulty (the smaller is x), and the more capital-intensive the final goods sector and the intermediate good sector (the bigger are α and β), the more striking steady state economic growth is. This permanent economic growth reflects the increasing returns to scale in intermediate good sector, which results directly from the assumption that knowledge is a pure public good. This result brings us back to the conclusions of the Solow-type neoclassical growth model, in which long-run growth does not depend on structural characteristics of the economy, so does not depend on government policy such as subsidies to intermediate good production, and capital accumulation is the main source of economic growth.

In contrast to the Romer-type R&D-based models and Young-type learning-by-doing models, this result shows that there are no scale effects: the levels of factors employed for the productions of goods and of new knowledge do not affect the long-run growth rate. In the situation that population is growing at a certain rate, the presence of scale effects brings about explosive economic growth in the long-run. This is how the presence of scale effects brings about a problem. In this model, the long-run growth rate is proportional to the population growth rate but not its level.

Equation (20) shows that changes in the share of human capital devoted to the intermediate good sector affects the long-run level of output (income) along the balanced growth path. So, changes in the size of the population affect the level of output but not its long-run growth rate in this model. Thus, if population is fixed, the exponential growth of per capita output is zero in this model.

Proposition 1. The long-run economic growth rate: *With the learning-by-doing endogenous growth mechanism without scale effects, the long-run economic growth rate is completely determined by the growth rate of the labor force(n), the innovation difficulty parameter(x) and the combined capital-intensiveness parameter(ξ). Thus long-run growth does not depend on structural characteristics of the economy, so does not depend on government policy such as subsidies for the production of intermediate good embodying more advanced technology.*

Ⅳ. SOCIAL OPTIMALITY AND TRANSITIONAL EFFECTS

4.1. Social optimality and transitional dynamics

Let us consider the following social planner problem of this growth model to study the social optimal growth rate and the transitional dynamics:

$$\max_{c_t,\, h_x} \quad \int_0^{\infty} e^{-\rho t} \frac{c_t^{1-\sigma}}{1-\sigma} dt \tag{21}$$

$$\text{s.t.} \quad y = A h_y^{1-\alpha} x^{\alpha}, \tag{22}$$

$$x = Z h_x^{1-\beta} k^{\beta}, \tag{23}$$

$$\dot{Z} = \frac{BH_x}{\omega Z^{x}}, \tag{24}$$

$$h_y + h_x = 1, \tag{25}$$

$$\dot{k} = y - c - nk \tag{26}$$

and k_0 and Z_0 are given positive initial values.

For simplifying the above constraints into one equation, substituting the intermediate good production function into the final goods production function, we obtain

$$y = AZ^{\alpha}(1-h_x)^{1-\alpha} h_x^{\alpha(1-\beta)} k^{\alpha\beta}. \tag{27}$$

For satisfying the market clearing condition of human capital market, we solve the following simple maximization problem. That is, differentiating equation (27) with respect to the per capita human capital devoted to the intermediate good sector gives us the following solution:

$$h_x = \frac{\alpha - \alpha\beta}{1-\alpha\beta}, \tag{28}$$

This is the same solution as the one in competitive equilibrium of human capital market, i.e., in the social planner formulation, the share of human capital devoted to the intermediate good sector along the balanced growth path is not changed in spite of the introduction of a knowledge accumulation process by learning-by-doing in intermediate good sector.

Substituting equation (28) into equation (27), we obtain the following reduced form of per capita production function of final goods:

$$y = \phi Z^{\alpha} k^{\alpha\beta}, \tag{29}$$

where $\phi \equiv A\left[\frac{\alpha(1-\beta)}{1-\alpha\beta}\right]^{\alpha(1-\beta)}\left[\frac{1-\alpha}{1-\alpha\beta}\right]^{1-\alpha}$ is a constant. The above reduced form of the per capita production function exhibits increasing returns to scale with respect to all input factors. Now substituting equation (18) into equation (29) and rearranging, we obtain the following form of per capita output production function:

$$y = \Psi \cdot \left(e^{\frac{\alpha}{1-\alpha\beta}\gamma_z \cdot t}\right)^{1-\alpha\beta} k^{\alpha\beta}, \tag{30}$$

where $\Psi \equiv Z_0^{\alpha}\phi$ is a constant. Let $\gamma \equiv \frac{\alpha}{1-\alpha\beta} \cdot \gamma_z$ and $\nu \equiv \alpha\beta$. Then $\gamma = \gamma_y = \frac{\alpha}{1-\alpha\beta} \cdot \frac{n}{1+x}$ in this autarkic case. If we let $\eta \equiv e^{\gamma \cdot t}$, then we get the following expression from equation (30):

$$y = \Psi \eta^{1-\nu} k^{\nu}. \tag{31}$$

Since the initial knowledge stock Z_0 is fixed, y and k grow at the rate of γ. Thus, γ can be regarded as the steady state growth rate of per capita output, consumption and the physical capital stock in the social planner problem (optimal growth model). This growth rate is the same as the one in equation (19), which is the long-run growth rate of competitive equilibrium in a decentralized economy. Since the sum of the exponents of η and k is unity, using the property of constant returns to scale, we can transform equation (31) to the following reduced form:

$$\hat{y} = \Psi \hat{k}^{\nu}, \tag{32}$$

where $\hat{y} = y/\eta$ and $\hat{k} = k/\eta$. In order to rewrite the social planner problem in terms of $\hat{y}$, $\hat{k}$ and $\hat{c}$, we change the physical capital accumulation equation to the following form:

$$\dot{\hat{k}} = \hat{y} - \hat{c} - (\gamma + n)\hat{k} \tag{33}$$

Since $\hat{c} = c/\eta$, we can have the following relation:

$$e^{-\rho t}\frac{c_t^{1-\sigma}}{1-\sigma} = e^{-[\rho-\gamma(1-\sigma)]t}\frac{\hat{c}_t^{1-\sigma}}{1-\sigma}. \tag{34}$$

From the above transformation, we can rewrite the above social planner problem with equations (21)-(26) to the following reduced forms:

$$\max_{\hat{c}_t,\ \hat{k}_t} \int_0^{\infty} e^{-[\rho-\gamma(1-\sigma)]t} \frac{\hat{c}_t^{\,1-\sigma}}{1-\sigma}, \tag{35}$$

$$\text{s. t. } \dot{\hat{k}} = \Psi \hat{k}^{\nu} - \hat{c} - (\gamma+n)\hat{k}, \tag{36}$$

$\hat{c}_0$ and $\hat{k}_0$ are also given positive values.

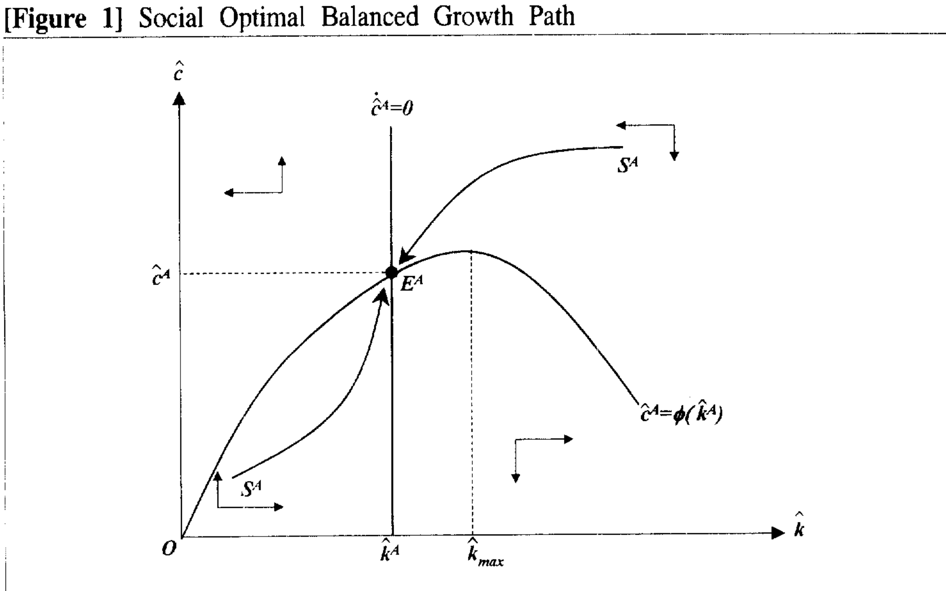
To ensure the existence of an steady state balanced growth path in this model, we assume that $\gamma < \frac{\rho}{1-\sigma}$ which means that maximal economic growth rate cannot exceed $\frac{\rho}{1-\sigma}$. Solving the above dynamic optimization problem, we obtain the following two differential equations:[12]

$$\gamma_{\hat{c}} = \frac{1}{\sigma}[\nu\Psi \hat{k}^{\nu-1} - (\rho+n+\sigma\gamma)], \tag{37}$$

$$\dot{\hat{k}} = \Psi \hat{k}^{\nu} - \hat{c} - (n+\gamma)\hat{k}. \tag{38}$$

Using the above two dynamic equations, let us analyze the phase diagrams of this system. Since $0<\nu=\alpha\beta<1$, there is always an intersection between the $\dot{\hat{c}}=0$ locus and the $\dot{\hat{k}}=0$ locus in the first quadrant. Now we define two functions as the $\dot{\hat{c}}=0$ locus and the $\dot{\hat{k}}=0$ locus:

[Figure 1] Social Optimal Balanced Growth Path



[12] The sufficient condition for a path to be optimal is $\lim_{t\to\infty}[e^{-[\rho-\gamma(1-\sigma)]t}\hat{k}_t \cdot u(\hat{c}_t)]=0$ which is called as the transversality condition.

$$\hat{k}=\left[\frac{\nu\Psi}{\rho+n+\sigma\gamma}\right]^{1/(1-\nu)}, \tag{39}$$

$$\phi(\hat{k})=\Psi\,\hat{k}^{\nu}-(n+\gamma)\,\hat{k}. \tag{40}$$

The locus starts from the origin, reaches a maximum at $\hat{k}_{\max}=\left(\frac{\nu\Psi}{n+\gamma}\right)^{1/(1+\nu)}$ at which $\phi'(\hat{k})=0$ and intersect with the horizontal axis at $\hat{k}'=\left(\frac{\Psi}{n+\gamma}\right)^{1/(1-\nu)}$. The $\hat{c}=0$ locus is vertical at $\hat{k}^{A}=\left[\frac{\nu\Psi}{\rho+n+\sigma\gamma}\right]^{1/(1-\nu)}$. By the assumption of $\gamma<\frac{\rho}{1-\sigma}$, we find that $0<\hat{k}^{A}<\hat{k}_{\max}<\hat{k}'$. With this fact and equations (39) and (40), we can draw the phase diagram as <Figure 1>. Anywhere above the locus $\dot{\hat{k}}=0$, $\hat{k}$ is decreasing, while $\hat{k}$ is increasing below the $\dot{\hat{k}}=0$ locus. Similarly, $\hat{c}$ is increasing to the left of the $\dot{\hat{c}}=0$ locus and decreasing to the right of this locus. The trajectory $S^{A}S^{A}$ is the stable balanced growth path that converges to the unique fixed saddle point $E^{A}(\hat{c}^{A},\hat{k}^{A})$. The social optimal solution for the system of equations (21)-(26) is completely summarized by the stable balanced growth path $S^{A}S^{A}$ in autarky. Therefore, the pre-normalized path $e^{\gamma\cdot t}(\hat{c},\hat{k})$ converges to the pre-normalized balanced growth path $e^{\gamma\cdot t}(\hat{c}^{A},\hat{k}^{A})$.

4.2. Transitional effects of government policy on economic welfare

In this section, we examine the transitional effects of a subsidization(taxation) on the production of intermediate good on economic growth and welfare. In doing so, we observe a change in the share of human capital devoted to intermediate good sector. If we introduce a subsidy for the production of intermediate good and finance it by a tax to individual total income, the budget constraint of equation (26) is changed to the following form:

$$\dot{k}=(1-\tau_y)y-c-nk\ ;\quad 0\le\tau_y<1 \tag{41}$$

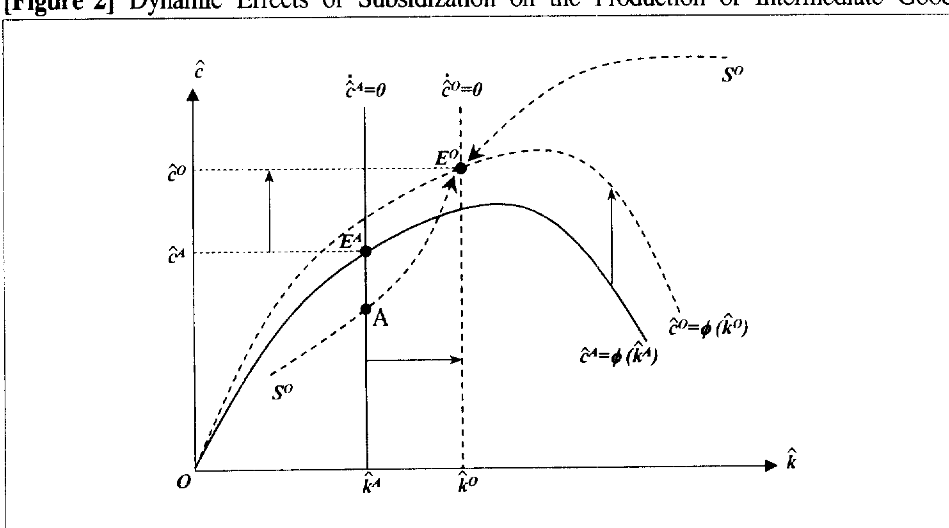
where τ_y is tax rate to individual total income.

Suppose that the subsidization rate on the production of intermediate good is s_x. With the assumption of balanced government budget (i.e., government spending $G=\tau_y Y=s_x X$), the share of human capital devoted to intermediate good sector after subsidy changes to $h_x=\frac{\alpha(1-\beta)(1+s_x)}{(1-\alpha\beta)+\alpha(1-\beta)s_x}$. To check how the subsidization on the production of intermediate good affects this share, differentiating h_x with respect to the subsidization rate s_x, we obtain $\frac{\partial h_x}{\partial s_x}=\frac{\alpha(1-\alpha)(1-\beta)}{[(1-\alpha\beta)+\alpha(1-\beta)s_x]^2}$. The subsidization (taxation) on the production of intermediate good increases the share of human capital for the production of intermediate good. The subsidization (taxation) also increases (decreases) the value of Ψ (i.e., $\frac{\partial\Psi}{\partial s_x}>0$). Thus, an increase in the subsidy rate s_x, shifts the $\dot{\hat{k}}=0$ locus upward and the $\dot{\hat{c}}=0$ locus to the right, while a decrease in the subsidy on production of intermediate good (negative subsidy, $-s_x$) shifts the $\dot{\hat{k}}=0$ locus downward and the $\dot{\hat{c}}=0$ locus to the left. Based on these facts, we examine the transitional dynamics of government policy for the production of intermediate good. In addition, the directions of these shifts is determined by whether the government policy is a subsidy or a tax.

Hereafter, we regard $E^A(\hat{c}^A, \hat{k}^A)$ as the pre-policy socially optimal saddle point. Then the $\dot{\hat{k}}=0$ locus and the $\dot{\hat{c}}=0$ locus without government policy are the same as those in <Figure 1>, and the trajectory S^AS^A is the stable balanced growth path without government policy, which converges to the pre-policy saddle point $E^A(\hat{c}^A, \hat{k}^A)$. Now suppose the government decides to subsidize the production of intermediate good and to finance this subsidy by a tax to individual total income. Then this government policy causes a rise in Ψ, thus the $\dot{\hat{k}}=0$ locus shifts upward and the $\dot{\hat{c}}=0$ locus to the right. Consequently, the balanced growth path shifts from the trajectory S^AS^A to the new trajectory S^OS^O which converges to the new optimal saddle point E^O where the levels of normalized consumption and capital stock, $\hat{c}$ and $\hat{k}$, increase.

In order to show the optimal transitional dynamics from the old saddle point E^A to the new saddle point E^O, we draw <Figure 2>. The figure is the same as <Figure 1>, except the increases in consumption and the capital stock normalized by $e^{\gamma \cdot t}$, accounting for the upward shift of the $\dot{\hat{k}}=0$ locus and the rightward shift of the $\dot{\hat{c}}=0$ locus after the subsidy on intermediate good production. At the time of subsidization, normalized consumption $\hat{c}$ jumps down so that the economy is at a point on the new transition path S^OS^O. In <Figure 2>, that is the point A. The trajectory S^OS^O is the optimal transition path since this path also satisfies the transversality condition. Thereafter $\hat{c}$ and $\hat{k}$ increase gradually toward their post-subsidy saddle point $E^O(\hat{c}^O, \hat{k}^O)$ along the new trajectory

[Figure 2] Dynamic Effects of Subsidization on the Production of Intermediate Good



$S^O S^O$. The new values of $\hat{c}$ and $\hat{k}$ are larger than their values on the pre-subsidy balanced growth path. The economic interpretation of these transitional dynamics caused by subsidization for production of intermediate good is as follows: under perfect foresight, all economic agents expect that the level of post-subsidy output is higher than the pre-subsidy level.

However, the income tax for financing subsidization on production of intermediate good reduces instantaneously the levels of its consumption and capital stock. Then the economy gradually increases the levels of consumption and capital stock along the transition path at the new steady state as level of output is growing.

Equations (16), (17), (19) and (20) support the above results. Starting from the balanced growth path, a permanently and unexpectedly subsidization on the production of intermediate good leads to an increase in human capital devoted to intermediate good sector, and thus rises in the growth rates of knowledge and output. However, these effect is only temporary. The increase in the stock of knowledge increases innovation difficulty faster. Consequently, this increased innovation difficulty makes the growth rates of knowledge and of output fall back to the steady state growth rates. However, equation (20) shows that an increase in h_x by the permanent subsidization for the production of intermediate good can increases the steady state per capita output level. In summary, one of the main implications in the analysis of transitional effects of government subsidization on production of intermediate good is that a permanent rise in the share of human capital in intermediate good sector dose not cause a permanent change in the economic growth rate. Whether or not the appropriate policy is a tax or a subsidy, such policy has an important level effect on output and economic welfare along a transition path to the new steady state.

Proposition 2. Welfare effects of government policy: *With the process innovation model through learning-by-doing process without scale effects, the subsidization (taxation) on the production of intermediate good clearly has positive(negative) level effects on output and economic welfare, without any change in the long-run economic growth rate.*

V. CONCLUSION

This paper develops a process innovation (productivity innovation) model through learning-by-doing process without scale effects to explain economic growth, technological process, and the role of public policy on economic welfare. In contrast to the Romer-type R&D-based endogenous growth models, there are no scale effects in this model, i.e., level changes of resources employed for the productions of products and technology do not affect the long-run economic growth rate. Thus a change in the size of the population affect the level of output but not its long-run growth rate. Thus, if population is fixed, the exponential growth of per capita output is disapperared in this model. According to this process innovation model through learning-by-doing, the long-run economic growth rate is completely determined by exogenously given parameters including the growth rate of population, similar to the semi-endogenous growth models developed by Jones (1995a), Kortum (1997), and Segerstrom (1998). Thus, the long-run economic growth is not affected by structural characteristics of the economy, and thus the government is powerless in determining the trend of economic growth. However, government policy has important level effects on output and consumer welfare. The changes in the share of human capital devoted to the intermediate good sector, which is brought about by a subsidy (or tax) on the production of intermediate good, affect the levels of knowledge production and output along the balanced growth path. This means that the share of resources employed in a specific manufacturing sector along the balanced growth path is not optimal.

REFERENCES

Aghion, P. and P. Howitt, (1992), "A model of growth through creative destruction," *Econometrica* 60, 223--247.

________________, (1998), *Endogenous Growth Theory,* Cambridge: MIT Press.

Barro, R. and X. Sala-i-Martin, (1995), *Economic Growth,* New York: McGrow-Hill.

Brezis, E., P. Krugman and D. Tsiddon, (1993), "Leapfrogging in international competition: A theory of cycles in national technological leadership," *American Economic Review* 83, 1211--1219.

Eaton, J. and S. Kortum, (1999), "International technology diffusion: Theory and measurement," *International Economic Review* 40, 537--570.

Griliches, Z., (1992), "The search for R&D spillovers," *Scandinavian Journal of Economics* 94, S29--S49.

Grossman, G. and E. Helpman, (1991a), "Quality ladders and product cycles," *The Quarterly Journal of Economics* 106, 557--586.

________________, (1991b), *Innovation and growth in the global economy*, Cambridge: MIT Press.

Jones, C., (1995a), "Time series tests of endogenous growth models," *The Quarterly Journal of Economics* 110, 495--525.

________, (1995b), "R&D-based models of economic growth," *Journal of Political Economy* 103, 759--784.

________, (1998a), "Population and ideas: A theory of endogenous growth," *Manuscript*, Stanford University.

________, (1998b), "Sources of U.S. economic growth in a world of idea," *Manuscript*. Stanford University.

________, (1999), "Growth: With or without scale effects?," *American Economic Review* 89, S139--S144.

Kortum, S., (1997) "Research, patenting, and technology change," *Econometrica* 65, 1389--1419.

Li, C., (2000), "Endogenous vs. semi-endogenous growth in a Two-R&D sector model," *Economic Journal*, forthcoming.

Mankiw, G., D. Romer and D. Weil, (1992), "A contribution to the empirics of economic Growth," *The Quarterly Journal of Economics* 107, 407--437.

Romer, D., (1990), "Endogenous technological change," *Journal of Political Economy* 98, S71--S102.

_________, (1996), *Advanced Macroeconomics,* New York: McGraw-Hill.

Segerstrom, P., (1998), "Endogenous growth without scale effects," *American Economic Review* 88, 1290--1310.

____________, T. Anant and E. Dinopoulos, (1990), "A schumpeterian model of the product life cycle," *American Economic Review* 80,1077--1092.

Takahashi, H. and T. Sakagami, (1998), "Transitional dynamics of economic integration and endogenous growth," *Journal of Economic Behavior & Organization* 33, 543--555.

Xie, D., (1998), "An endogenous growth model with expanding ranges of consumer goods and producer durables," *International Economic Review* 39, 439--460.

Young, A., (1991), "Learning by doing and the dynamic effects of international trade," *The Quarterly Journal of Economics* 106, 369--405.

________, (1993), "Invention and bounded learning by doing," *Journal of Political Economy* 101, 443--472.

________, (1998), "Growth without scale effects," *Journal of Political Economy* 106, 41--63.