

## ON OPTIMAL FOREIGN BORROWING

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*This paper derives an optimal borrowing rule from an intertemporal optimizing model in which uncertainty and risk premium are explicitly incorporated. The optimal borrowing rule is that the risk-adjusted marginal product of capital, net of depreciation, must be equal to the risk-adjusted, country-specific risk premium-adjusted interest rate of borrowing. The key difference between the optimal borrowing rule in earlier studies and the revised rule is that optimality criteria in foreign indebtedness should be associated with the agent's risk aversion and the size of the stochastic disturbances in profit flows. The revised rule demonstrates that uncertainties may play an important role in deciding the relationship between capital stock and foreign debt stock and between domestic saving and foreign saving in heavily indebted economies.*

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### 1. INTRODUCTION

There has been a hot debate in the literature as to the causes of the Asian currency and financial crisis of 1997-98. In one view (Corsetti, Pesenti, and Roubini, 1998; Krugman, 1998a, b), the origin of the crisis was a serious deterioration of macroeconomic fundamentals combined with poor economic policies and structures in the Asian countries. An alternative view (Radelet and Sachs, 1998a, b) is that the key source was sudden shifts in market expectations

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and confidence, i.e., financial panic.

Although a consensus on this issue has not been reached yet, the stylized fact shared by the affected economies is that high economic growth rates coincided with very large and persistent current account deficits before the crisis. The financing of these deficits took mostly the form of short-term borrowing in foreign currency, leading to a large accumulation of foreign liabilities. In contrast to many Latin American debt crises of the 1980s, where borrowing abroad took place to sustain high rates of consumption, foreign borrowing sustained high rates of investment at home. The problem is, however, that excessive investment, concentrated in risky, unprofitable projects, led to overborrowing.

[Table 1] Real interest rates of borrowing

(percent)

	1990	1991	1992	1993	1994	1995	1996	1997
Real effective interest rates								
Indonesia	-2.9	-3.6	-2.4	-3.8	-3.6	-4.4	-2.9	-1.7
Korea	-3.6	-4.5	-1.7	-0.1	-2.4	1.4	0.7	0.4
Malaysia	4.4	3.7	1.6	1.6	1.0	-0.6	1.8	3.4
Philippines	15.0	-13.7	-3.3	-0.9	-3.1	-2.6	-3.6	-1.1
Thailand	2.8	1.3	2.4	0.8	-0.9	-0.8	-0.1	0.3
Average real interest rates of new commitments								
Indonesia	-2.6	-3.2	-1.7	-4.0	-3.4	-3.7	-2.6	-1.1
Korea	-0.6	-1.6	0.8	0.9	-1.1	2.3	1.7	2.1
Malaysia	1.6	4.8	1.2	1.6	2.9	0.5	3.0	3.3
Philippines	15.4	-13.6	-3.3	-1.4	-4.4	-3.2	-2.8	-1.5
Thailand	0.4	-0.7	2.5	1.4	-0.2	-0.4	-1.2	0.3

Source: the World Bank, Global Development Finance 2001;

IMF, World Economic Outlook, various issues.

Note: Real effective interest rates = nominal effective interest rates minus inflation (CPI); Nominal effective interest rates = the ratio of interest payments to outstanding debt at the end of preceding year.

Table 1 shows the two measures of the *ex post* real interest cost of borrowing for the five Asian economies over the years of 1990-97. During the 1990s, the real cost of borrowing has been negative only for Indonesia and the Philippines; it is a mix of negative and positive for the other countries. Up to 1994, no trend of the real cost of borrowing is observed among the economies. However, the cost begins to rise in 1995 and accelerates during 1996-97 for all of these economies. This rise reflects the fact that the risk premium the Asian economies paid rose as the currency crisis approached.

**[Table 2]** Rate of return on capital and real interest rates in Korean manufacturing sector (percent)

	Rate of return <sup>a</sup> on capital(A)	Depreciation (B)	C <sup>b</sup> =A-B	Real interest rates(D)	E <sup>c</sup> =C-D
1990	14.57	15.32	-0.75	-0.6	-0.15
1991	14.29	14.74	-0.45	-1.6	1.15
1992	13.85	13.71	0.14	0.8	-0.66
1993	14.85	13.48	1.37	0.9	0.44
1994	14.76	13.37	1.39	-1.1	2.51
1995	16.18	15.14	1.04	2.3	-1.27
1996	13.92	14.36	-0.44	1.7	-2.14
1997	12.31	13.57	-1.26	2.1	-3.38

Source: the World Bank, Global Development Finance 2001;  
IMF, World Economic Outlook, various issues;  
The Bank of Korea, Financial Statement Analysis, various issues.

<sup>a</sup> the capital share in value added divided by total capital.

<sup>b</sup> Rate of return on capital net of depreciation.

<sup>c</sup> the capital rate of return over the new loan real interest rates.

Table 2 presents the *ex post* rate of return on capital and depreciation for the Korean manufacturing sector for the period of 1990-97.<sup>1</sup> As the table shows, the rate hovered in the low to mid-teen before it began to fall in 1995. Concurrent with the rate of return on capital, net of depreciation, going negative, the real cost of new loans rose from -1% in 1994 to about 2% by 1997. We claim that the negative values in the last column (the extent of the *ex post* capital rate of return over the new loan real interest rates) measure the degree of overborrowing. The numbers show little sign of overborrowing in the Korean manufacturing sector before the end of 1994, but the symptom of overborrowing appeared in 1995 and its extent grew through 1997. Notice that in contrast with the other Asian economies, the real cost of borrowing in Korea changed little in 1995-97, suggesting the deterioration of the rate of return on capital, which culminated just before the crisis, as the main cause of overborrowing.

The purpose of this paper is to reconstruct the *ex post* optimal rule for foreign borrowing discussed in the literature. Bardhan (1967) and Hamada (1969) made the first attempts to find the optimal level of foreign debt. In their work, the emphasis was on deriving optimality criteria in the context of intertemporal optimizing models. They maximize an intertemporal utility function in the framework of a one-sector neoclassical growth model under the assumption of a closed economy. An important innovation in Bardhan's model is the implicit incorporation of the country-specific risk premium through the specification of an upward-sloping foreign-debt supply function. Bhandari, Haque and Turnovsky

<sup>1</sup> Those for the other Asian economies are not shown in the table due to lack of data.

(1990) employ this upward-sloping supply function and construct an optimizing growth model for a highly indebted, small open economy. This specification is also used by McCabe and Sibley (1976) who do a theoretical analysis for optimal foreign borrowing in the presence of export-revenue uncertainty, a situation that generates national-income instability.<sup>2</sup>

The optimal borrowing rule obtained from all these models is that the marginal cost of borrowing is equal to the marginal product of capital. This conventional rule, however, ignores the uncertainty (associated with productivity, demand or borrowing cost) of future profit flows from borrowing abroad. The main objective of this paper is to construct an optimal borrowing rule that takes into account uncertainty in borrowing opportunities and the country's debt servicing capacities. The paper is based on a real, one-sector intertemporal optimizing, macroeconomic model employed by earlier studies on optimality criteria for borrowing. This model differs from earlier studies in two ways. First, this model explicitly incorporates uncertainty, which follows a specific stochastic process. Second, the country-specific risk premium is associated not only with the level of debt stock, but also with debt servicing capacities. Contrary to the conventional rule, the new optimal borrowing rule depends upon the agent's degree of risk aversion, the size of the stochastic disturbances, and the country's debt servicing capacities.

Using the optimal condition for borrowing, we next explore the relationship between capital stock and foreign debt stock and the relationship between domestic saving and foreign saving, both of which have long been studied in the literature. Previous theoretical and empirical studies ignored the role of uncertainties in evaluating the proposition that foreign debt stock deters investment and growth, and that foreign capital resources lower domestic saving. The new optimal borrowing rule demonstrates that the proposition is acceptable to the extent that the future profit flows from borrowing opportunities are less certain for heavily indebted economies.

The rest of the paper is organized as follows: section 2 presents a continuous-time, stochastic macroeconomic model<sup>3</sup> based on the optimizing behavior of a representative agent in a small, open economy. In the first subsection, uncertainty is exemplified by a stochastic productivity that follows a geometric Brownian motion. The country-specific risk premium is defined as an increasing function of the level of foreign debt stock relative to capital stock. The cost of borrowing is also uncertain in the second subsection. The interest rate prevailing internationally follows a geometric mean reversion. As a corollary of the optimality condition, in section 3, we evaluate how foreign debt, investment and saving are correlated with each other, especially for heavily

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<sup>2</sup> Other intertemporal optimizing models are Dornbusch (1983), Martin and Selowsky (1984) and Sachs (1981).

<sup>3</sup> See Turnovsky (1995) for a systematic presentation of continuous-time stochastic macroeconomic models and their application to policy evaluations.

indebted developing countries. The final section summarizes the main findings of the paper.

## 2. UNCERTAINTY AND RISK PREMIUM

### 2.1 The Analytical Framework

We consider a real economy that consists of a single, infinitely lived representative agent by consolidating the household and production sectors. The agent produces a single good whose output is generated from capital ( $K$ ) only, which, in turn, is accumulated through investment. The capital stock is adjusted costlessly. Domestic output is used in part for consumption ( $C$ ) and in part for investment ( $I$ ), with the rest being exported.

The production function for output is  $Y = AF(K)$ ,  $F' > 0$ ,  $F'' < 0$ , implying that production is strictly increasing in capital, but subject to the diminishing marginal productivity of capital. The stochastic productivity coefficient,  $A$ , follows geometric Brownian motion:

$$dA(s) = \mu_A A(s) ds + \sigma_A A(s) dz(s), \quad (1)$$

where  $s$  and  $\sigma_A^2$  denote time and the instantaneous variance of  $A$ , respectively, and  $dz$  is the increment of a Wiener process  $z$  with mean  $E[ dz ] = 0$  and variance  $Var[ dz ] = ds$ . The drift term,  $\mu_A$ , denotes the expected growth rate of productivity. For simplicity, we assume  $\mu_A = 1$ , which will not affect any of the main theoretical results in the model.

The objective of the agent is to maximize the expected value of lifetime utility as measured by the concave utility function. The problem can be solved by dynamic programming: Define the value function  $G(D(t), K(t), t)$  as

$$G(D(t), K(t), t) \equiv \max_{C(s), I(s)} E_t \int_t^\infty U(C(s)) e^{-\rho(s-t)} ds, \quad U'(C) > 0, U''(C) < 0 \quad (2)$$

subject to the instantaneous budget constraint,

$$dD(s) = [ C(s) + I(s) + r^d D(s) - A(s)F(K(s)) ] ds - A(s)F(K(s)) \sigma_A dz(s), \quad (3)$$

and the capital accumulation constraint,

$$dK(s) = (I(s) - \delta K(s)) ds, \quad (4)$$

with initial values of the state variable, foreign debt stock,  $D(t) = D_t$ , and capital stock,  $K(t) = K_t$ . Here  $\rho$  and  $\delta$  denote the rate of time preference,

and the rate of depreciation, respectively.

Equation (3) is the current account balance, but with two modifications. First, the trade balance is stochastic because the output has a stochastic process. Second, the interest rate ( $r^d$ ) on foreign debt is an increasing function of the level of foreign debt stock relative to capital stock,

$$r^d(D/K) = r + h(D/K) \quad h' > 0, h'' > 0, \quad (5)$$

where  $r$  is the given interest rate prevailing internationally and  $h(D/K)$  is the country-specific risk premium. The rationale for this specification lies in the fact that the borrowing or lending rate of interest is not given, but depends on the size of foreign debt and the debtor's debt-servicing capacity. Unlike developed countries, heavily indebted developing countries must pay the country-specific risk premium over the prevailing international interest rate. The risk premium increases with the absolute level of foreign debt, but also decreases with foreign exchange earning capacities such as capital, output or export. In (5), the risk premium depends on the level of debt relative to capital stock, and it rises at an increasing rate. It is also assumed that the economy does not encounter borrowing constraints from the creditors.

The value function  $G(D(t), K(t), t)$  in (2) denotes a present-value formulation of the maximization problem. To examine the current behavior of the agent's choice of consumption and investment, we introduce the new value function as

$$J(D(t), K(t), t) = e^{\rho t} G(D(t), K(t), t).$$

Thus, the stochastic Bellman equation for the agent's maximization problem takes the form

$$\rho J(D(t), K(t), t) dt = \max_{C(t), I(t)} U(C(t)) dt + E_t \{ dJ(D(t), K(t), t) \}$$

for all time  $t$ . Expanding  $E_t dJ$  using Ito's lemma (and omitting time index  $t$  and the argument of  $J(\cdot)$ ), the equation is rewritten as

$$\rho J = \max_{C, I} \left\{ U(C) + J_D [C + I + r^d D - AF(K)] + J_K (I - \delta K) + J_I + \frac{1}{2} F_{DD} A^2 F(K)^2 \sigma_A^2 \right\}, \quad (6)$$

where  $J_i (i = D, K, t)$  are partial derivatives of the value function. Finally, the transversality condition can be written as

$$\lim_{t \rightarrow \infty} e^{-\rho t} J(D, K, t) = 0. \quad (7)$$

The optimality conditions for consumption and investment are obtained by taking the derivatives of (6) with respect to  $C$  and  $I$ ,

$$U'(C) = -J_D, \quad (8)$$

$$J_K = -J_D. \quad (9)$$

Equation (8) is the first-order optimality condition for consumption, equating the marginal utility of consumption to  $-J_D$ , the negative of the marginal disutility of debt, or the shadow value of wealth. In turn, investment optimality in (9) implies that the shadow value of wealth must be equal to the shadow value of installed capital. Rearranging terms in (9) leads to  $q (\equiv -J_K/J_D) = 1$ , which is the familiar optimal investment rule in which the market value of installed capital should equal the marginal cost of investing an additional unit of capital.

## 2.2 Optimal Borrowing

Using the first order conditions (8) and (9), Appendix A shows the optimal borrowing condition to be

$$AF'(K^*) \left[ 1 - \frac{J_{DD}}{J_D} AF(K^*) \sigma_A^2 \right] - \delta = r + h + \frac{D^*}{K^*} h' - \left( \frac{D^*}{K^*} \right)^2 h'. \quad (10)$$

where  $*$  denotes an optimal value. Put in words, the risk-adjusted marginal product of capital, net of depreciation, must be equal to the risk premium-adjusted interest cost of borrowing. In the bracket of the LHS,  $J_{DD}/J_D > 0$  measures the agent's absolute risk aversion. Since foreign debt is the negative of foreign asset, it is equivalent to the negative of the coefficient of absolute aversion to wealth risk.  $AF(K)\sigma_A^2$  denotes the size of the stochastic disturbances in output arising from a productivity shock.  $AF'(K)$  corresponds to the expected value of the marginal product of capital that must be adjusted downward for risk by subtracting the product of the risk-aversion coefficient and the stochastic disturbance term,  $(J_{DD}/J_D)AF(K)\sigma_A^2$ . If the agent is risk averse ( $J_{DD} < 0$ ), he/she will own less capital under uncertainty than with certainty since the rate of return on capital becomes smaller. This implies that the presence of the stochastic disturbances in the output leads to borrowing less for investment given the interest cost of borrowing.

The third and fourth terms in RHS indicate the change in the risk premium relevant to foreign debt and capital stock, respectively. This implies that the effective cost of borrowing becomes higher to the extent that foreign debt accumulates more compared to the debt servicing capacity of the economy.

If there is no uncertainty, the left-hand side of (10) becomes  $AF'(K) - \delta$ .

If the risk premium on borrowing is associated only with the amount of foreign debt stock, in addition, the optimal borrowing rule becomes

$$AF'(K^*) - \delta = r + h(D^*) + D^* h'(D^*). \quad (11)$$

This is the linear form of the optimal borrowing rule shown in Bardhan (1967), McCabe and Sibley (1976), and Bhandari, Haque and Turnovsky (1990), in which the marginal product of capital, net of depreciation, must be equal to the marginal cost of an additional unit of debt facing the agent.

The contribution of the conventional rule is that it postulates the cost of debt to increase with the absolute level of foreign debt stock. This is well described by Bhandari, Haque and Turnovsky (1990) as follows:

But the assumption that such [highly indebted small open] economies face a perfectly elastic supply of debt is clearly unrealistic. Experience with external borrowing in such economies has shown that debt repayments are not always made on time. Overborrowing, resulting from inadequate perceptions of domestic growth potential, has occurred on occasion. (p. 389).

One difference between (10) and (11) is that the former endogenizes the capacity to service debt in the cost of borrowing. That is, a lower risk premium is charged at each level of debt, allowing debtors to borrow more. However, the critical difference is that the latter does not take into account the uncertain rate of return to capital; thus the conventional rule leads to overborrowing given the interest cost of borrowing. Under this rule, a more risk-averse agent and a greater variance of the stochastic disturbances in the output will both increase the magnitude of overborrowing. The conventional rule survives only when the agent is risk neutral,  $J_{DD} = 0$ .

One interesting implication for the optimal rule may be associated with the functional forms of production. We assumed the neoclassical production function, which satisfies the Inada condition with diminishing marginal returns to capital. However, production technology may be characterized with constant or increasing marginal returns to capital as suggested by endogenous growth models, which have flourished over the last decade.

Let us illustrate an extreme case. Suppose that there is no risk premium and the interest cost is exogenously given  $r$ . The utility function of the agent has the form  $U(C) = e^{-\eta C} / -\eta$  where  $\eta$  is the coefficient of absolute risk aversion. The production technology is linear  $Y = AK$  such as the production function of Rebelo (1991). In this case, the optimal rules shown above can be transformed into

$$A - \delta = r(1 + \eta A^2 K^* \sigma_A^2), \quad (10a)$$

$$A - \delta = r. \quad (11a)$$

The first implication of this example is that the optimal levels of capital and foreign debt stock become indeterminate in the case of conventional optimal rule (11a), which consists of all constants. If  $A - \delta > r$ ,  $K$  and  $D$  approach  $\infty$ , which violates the transversality condition. If  $A - \delta < r$ , they become 0, which is impossible. However, finite  $K^*$  and thus finite  $D^*$  can be determined in (10a) where risk in the productivity of capital is explicitly taken into account. If the risk premium in borrowing cost is added, both rules produce finite solutions, but the conventional rule still entails overborrowing.

Second, it should be noticed that the two equations can be rearranged such that they have the same cost of borrowing, but the different marginal product of capital, net of depreciation. Thus we can reinterpret the risk-related rates of return to capital as the additional cost of borrowing which leads to less borrowing under uncertainty.

### 2.3 Borrowing Cost Uncertainty

In the real world, the cost of borrowing is subject to uncertainty. The interest rate prevailing internationally is stochastic, not given; the literature reports that they follow mean-reverting stochastic processes.<sup>4</sup>

Suppose the borrowing rate in the world financial market,  $r$ , follows a geometric mean reversion (GMR), called the geometric Ornstein-Uhlenbeck process,

$$dr(s) = \theta(\bar{r} - r(s))r(s)ds + \sigma_r r(s)dv(s), \quad (12)$$

shown in Merton (1975). Here,  $\theta$  is the speed of reversion and  $\bar{r}$  is the constant 'mean' or 'normal' level of  $r$  (the level to which  $r$  tends to revert).  $v$  is a Wiener process and  $\sigma_r^2$  is the instantaneous variance of  $r$ . Equation (12) implies that if shocks to interest rates ( $dv$ ) push  $r$  above (below)  $\bar{r}$ , the effective trend is pushed down (up), thereby driving interest rates back toward their mean rates. If  $\theta = 0$ , the interest rate follows a geometric Brownian motion (GBM) without drift. If  $\theta \rightarrow \infty$ ,  $r$  can never deviate from  $\bar{r}$ , which means that  $r$  is constant as assumed in the previous section.

It is assumed here that the two sources of uncertainty are not correlated with each other,  $E(dz, dv) = 0$ . Therefore, the optimal borrowing condition must be revised as such:

<sup>4</sup> See Table 5 of Ingersoll and Ross (1992) for various stochastic processes of interest rates.

$$\begin{aligned}
 & AF'(K^*) \left[ 1 - \frac{J_{DD}}{J_D} AF(K^*) \sigma_A^2 \right] - \delta \\
 & = \theta(\bar{r} - r)r + \frac{J_{DD}}{J_D} r^2 D^* \sigma_r^2 + h + \frac{D^*}{K^*} h' - \left( \frac{D^*}{K^*} \right)^2 h'. \tag{13}
 \end{aligned}$$

Compared with (10), where the world interest rate is given, the interest cost of borrowing is not only risk-adjusted but country-specific risk premium-adjusted. If the interest rate follows GBM, the borrowing cost rises absolutely by the stochastic disturbance term,  $r^2 D \sigma_r^2$ , but the extent of its effect depends on the agent's risk aversion,  $J_{DD}/J_D$ . If it follows GMR, however, the effect of interest-rate uncertainty on borrowing cost also depends on whether the current interest rate is below or above the normal level: higher in the case of  $r < \bar{r}$  and lower in the case of  $r > \bar{r}$ . This additional effect on the cost would be smaller with a greater speed of reversion,  $\theta$ .

In sum, interest-rate uncertainty is more likely to raise borrowing cost, thus leading to borrow less than in the case of interest-rate certainty. Its extent relies not only on the size of the uncertainty but also on specific stochastic processes of interest rates. One implication of a mean-reverting interest rate is the timing of borrowing: for example, it is better to postpone borrowing when the interest rate is higher than its mean rate.

### 3. FOREIGN DEBT, INVESTMENT AND SAVING

The optimal conditions for borrowing can be employed to answer some questions that have long been explored in the literature. The first question is whether foreign debt stock crowds out domestic investment. The second is whether foreign saving crowds out domestic saving.

Empirical studies on the first question focus on testing the "debt overhang" argument (Krugman 1988; Sachs 1989): the stock of debt is in itself seen as a tax on the domestic economy and thus a deterrent to investment. However, empirical results are not conclusive; the negative relationship between foreign debt and investment is found only after the 1982 debt crisis in the countries that have been heavily indebted (Deshpande, 1997).

To answer the first question, totally differentiating (13) and (11) and rearranging terms gives

$$\tilde{F}dK^* = \left[ \tilde{D} + \frac{J_{DD}}{J_D} r^2 \sigma_r^2 \right] dD^* - \frac{D^*}{K^*} \tilde{D}dK^* \tag{14}$$

where  $\tilde{F} \equiv AF'' - \frac{J_{DD}}{J_D} A^2 (F'^2 + FF'') \sigma_A^2$ ,  $\tilde{D} \equiv \frac{1}{K^*} \left( 1 - \frac{D^*}{K^*} \right) \left( 2h' + \frac{D^*}{K^*} h'' \right)$  and

$$AF'' dK^* = (2h' + D^* h'') dD^*. \tag{15}$$

The latter equation is also implicitly shown in McCabe and Sibley (1976). Since  $F'' < 0$  and  $h'' > 0$ , equation (15), derived from the conventional rule, implies a negative correlation between capital stock and foreign debt stock. This is because the rising debt stock pushes the marginal borrowing cost up, requiring a higher marginal return to capital for optimality for which investment must decline. As an economy becomes more indebted, foreign debt crowds out more investment.

Empirical studies, however, found no clear relationship between capital stock and foreign debt stock even after the 1982 debt crisis (Cohen, 1993). Furthermore, the level of foreign debt was positively correlated with investment even among 13 severely indebted countries before the debt crisis (Deshpande, 1997). How can we explain these results? First, it depends upon where foreign capital goes. Equation (15) clearly shows the positive correlation between capital and debt stock in the case of increasing marginal returns to capital,  $F'' > 0$ . If funds are used for investment projects with scale economies such as constructing plants for nuclear energy and shipbuilding, the accumulation of foreign debt may accompany positive investment.

Next, their relationship is also associated with uncertainty in the cost of borrowing and in the returns to capital, which is accounted for in (14). To see the implication of (14), we can derive from (13) the effect of two sources of uncertainties on the stock of capital for a given foreign debt stock:

$$\frac{dK^*}{d\sigma_A^2} = \frac{(J_{DD}/J_D)A^2 F' F}{\tilde{F} + (D^*/K^*)\tilde{D}} \quad \text{and} \quad \frac{dK^*}{d\sigma_r^2} = \frac{(J_{DD}/J_D)r^2 D^*}{\tilde{F} + (D^*/K^*)\tilde{D}}.$$
 The effects depend critically upon the sign of  $\tilde{F}$  in a common denominator,  $\tilde{F}$  (technological factor)  $+ (D^*/K^*)\tilde{D}$  (debt servicing capacity). An extra term in  $\tilde{F}$ ,  $-(J_{DD}/J_D)A^2 (F'^2 + FF'')\sigma_A^2$ , which is likely to be negative,<sup>5</sup> denotes a term relevant to uncertain future returns to capital. Ignoring the term denoting debt servicing capacity, for  $F'' < 0$ , an increase in uncertainties in borrowing terms or/and productivities shall reduce investment, leading to a fall in the stock of capital. For  $F'' > 0$ , on the other hand, the relationship between capital stock and uncertainties becomes ambiguous: for the case where  $AF'' > (J_{DD}/J_D)A^2 (F'^2 + FF'')\sigma_A^2$ , that is,  $\tilde{F} > 0$ , uncertainties are positively correlated with capital stock. But, when uncertain factors dominate, and thus the inequality is reversed ( $\tilde{F} < 0$ ), their relationship becomes negative. What is evident here is that uncertainties negatively affect investment activities even though a positive relationship between uncertainty and capital stock is possible.

In their empirical study, Aizenman and Marion (1999) test the relationship between uncertainty and investment for 46 developing countries with annual data from 1970 to 1992. They found that uncertainty is negatively related to private

<sup>5</sup> The necessary condition for  $F'' < 0$  to be negative is  $|FF''| < F'^2$ . If a functional form of the production technology is  $Y = AK^\alpha$ , the necessary condition implies  $\alpha < \frac{1}{2}$ .

investment, but positively related to public investment. One interpretation they gave for the latter case is that, if a benevolent planner determines public investment, it may increase in periods of heightened volatility to compensate for the reluctance of the private sector to invest. The correct reason, however, may be that public investment projects are largely tied to infrastructure with scale economies such as power plants, dams, airports, port facilities, etc., which require large indivisible capital.

Therefore, for the case of decreasing marginal returns to capital ( $F'' < 0$ ), greater uncertainty in borrowing terms and productivities leads to a greater likelihood that foreign debt stock will crowd out investment. For the case of increasing returns to capital ( $F'' > 0$ ), uncertainty mitigates the extent of the positive relationship between capital and debt stock and even leads them to be negatively correlated with each other. Thus, equation (14) suggests that the extent of uncertainty is a critical element in deciding the relationship between debt and capital stock of an economy.

Turning to the second question, earlier studies (Griffin and Enos, 1970; Weisskopf, 1972; Papanek, 1972, 1973; Gupta, 1975) identified a negative relationship between foreign saving (net foreign resource inflows) and domestic saving. However, the magnitude of this relationship differs among these studies. The basic reason is that if present consumption is a normal good, additional resource inflows should augment consumption and depress saving.

Though the majority of later studies<sup>6</sup> support the proposition that foreign saving crowds out domestic saving, the consensus is not reached yet, especially for individual countries. The optimal borrowing rules in this text imply that if funds are tied to investment projects, the relationship between foreign resource inflows and domestic saving is dependent on the returns to capital and the extent of uncertainty associated with them.

Since  $S \equiv Y - C = dK - dD$ , equation (14) yields

$$S^* = \frac{[1 - (D^*/K^*)]\tilde{D} + (J_{DD}/J_D)r^2\sigma_r^2 - \tilde{F}}{\tilde{F} + (D^*/K^*)\tilde{D}} dD^*, \quad (16)$$

where  $S$  and  $dD$  denote domestic saving and net foreign resource inflow, respectively. Likewise, the conventional rule generates

$$S^* = \frac{2h' + D^*h'' - AF''}{AF''} dD^*, \quad (17)$$

which is exactly the equation shown in McCabe and Sibley (1976).<sup>7</sup>

Equation (17) shows that domestic saving and foreign saving are negatively

<sup>6</sup> Among them are Gupta (1975), Edwards (1995), Schmidt-Hebbel and Servén (1995), Reinhardt and Talvi (1998), and Obstfeld (1999).

<sup>7</sup> McCabe and Sibley (1976) replace  $S^*$  by  $dS^*$ , which is conceptually wrong.

correlated along an optimal path since  $h' > 0$ ,  $h'' > 0$  and  $F'' < 0$ . However, the relationship may be completely reversed with increasing marginal returns to capital,  $F'' > 0$ , unless the risk premium in borrowing cost rises at a lower rate than the marginal returns to capital. The latter case implies that saving and investment may both be positively related to net foreign resource inflows. In contrast, Obstfeld (1999) asserts that foreign resource inflows are negatively related to national saving and positively related to domestic investment. Obstfeld's logic is based on income-account identity where saving and investment must move in opposite directions. On the other hand, saving and investment are endogenously determined here such that foreign resource inflows crowd in investment, which, in turn, crowds in saving.

Equation (16) accounts for uncertainty and debt servicing capacity. Uncertainty, in borrowing terms, negatively affects domestic saving for a given level of foreign saving. However, uncertainty in productivities has an ambiguous impact on savings. As Sandmo (1970) and Brock (1991) have noted, uncertainty of the type associated with stochastic shocks to output produces two types of risk: income risk and capital risk. Income risk raises the discount rate for the future consumption of the agent, thereby raising the agent's saving. Capital risk arises because investment expenditures generate an uncertain future return on capital. Thus capital risk causes the agent to save less by lowering wealth in the form of capital as discussed above. On the other hand, income risk causes the agent to increase wealth, in the form of internationally traded bonds, or to decrease debt to foreigners.

In sum, the literature has explored the proposition that for heavily indebted developing countries, foreign debt stock tends to deter investment and growth, and foreign resources may not raise national saving. While empirical results for this proposition have not been conclusive, the conventional optimal rule for borrowing accepts the proposition but suggests that the result can be reversed if funds are used for investment projects with scale economies. In reality, however, uncertainties ignored by the conventional rule have been present in most of the sectors in these economies where investment projects are also very risky. The new rule demonstrates that the proposition is more acceptable to the extent that the future profit flows from borrowing opportunities are more uncertain for heavily indebted economies.

#### 4. CONCLUSION

Debtors' overborrowing has been the source of concern in the international financial market since the middle of the twentieth century. The extent of debtors' indebtedness may be exacerbated by the conventional optimal borrowing rule in the literature because, primarily, the old rule takes no account of uncertainty in borrowing decisions.

The revised optimal borrowing rule is that the risk-adjusted marginal product

of capital, net of depreciation, must be equal to the risk-adjusted, country-specific risk premium-adjusted interest rate of borrowing. The main improvement of this new rule is that optimality criteria in foreign indebtedness should be associated with the agent's degree of the risk aversion and with the size of the stochastic disturbances in profit flows.

The conventional rule supports a negative relationship between capital stock and foreign debt stock and between saving and net foreign resource inflows, but these relationships have not been confirmed in all empirical studies. However, the outcome depends critically upon marginal returns to capital. The revised rule suggests that their relationships are more likely to be negative to the extent that the future profit flows from borrowing opportunities are more uncertain and borrowers are heavily indebted relative to their debt servicing capacities.

The model can be extended to incorporate other sources of uncertainty and other stochastic processes with no change in the basic results here. The actual process of a stochastic variable must be determined from empirical studies, which remains to be done in the future work.

#### Appendix A: Derivation of optimal borrowing condition (10)

According to Ito's lemma, the stochastic differential of the shadow value of debt,  $J_D$ , is given by

$$dJ_D = J_{DD} dD + J_{DK} dK + J_{D} dt + \frac{1}{2} J_{DDD} dD^2. \quad (\text{A1})$$

Next, take the partial derivative of the Bellman equation (6) with respect to  $D$ , to obtain

$$0 = (U' + J_D)C_D + J_D[r^d + (D/K)h' - \rho] + J_{DD}[C + I + r^d D - AF(K)] \\ + J_{DK}(I - \delta K) + J_{D} + \frac{1}{2} J_{DDD} A^2 F(K)^2 \sigma_A^2. \quad (\text{A2})$$

Use optimality condition (8) and substitute  $J_{D}$  obtained from (A2) into (A1), resulting in the following stochastic differential equation for  $J_D$ ,

$$dJ_D = J_D[\rho - (r + h) - (D/K)h']dt - J_{DD}AF(K)\sigma_A dz. \quad (\text{A3})$$

This equation is a stochastic version of the optimal consumption rule: Taking expected values of this equation and employing (8) leads to

$$\rho - \frac{E dU(C)/dt}{U(C)} = (r + h) + (D/K)h'. \quad (\text{A4})$$

The rule implies that the expected marginal rate of return on consumption must be equal to the marginal cost of an additional unit of debt. This can also be interpreted as the continuous-time version of the Euler equation taking the risk premium into account.

For the optimal borrowing rule, taking the stochastic derivative of the market value of installed capital,  $q$ , yields

$$\frac{dq}{q} = \frac{1}{J_K} dJ_K - \frac{1}{J_D} dJ_D + \frac{1}{J_D^2} (dJ_D)^2 - \frac{1}{J_D J_K} dJ_K dJ_D. \quad (\text{A5})$$

The stochastic behavior of  $J_K$  is needed to solve this last equation. A process analogous to obtaining (A3) produces the stochastic differential equation for  $J_K$ :

$$dJ_K = J_D \left\{ AF'(K) + (D/K)^2 h' - \frac{J_{DD}}{J_D} A^2 F'(K) F(K) \sigma_A^2 \right\} dt + J_K (\rho + \delta) dt - J_{DK} AF(K) \sigma_A dz. \quad (\text{A6})$$

Using the fact that optimality condition (9) implies  $q=1$ , equations (A3), (A5) and (A6) together give rise to optimal borrowing condition (10) in the text.

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