THE KOREAN ECONOMIC REVIEW
Volume 19, Number 1, Summer 2003

GOVERNMENT SIZE, TRADE AND GROWTH*

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We examine the effect of public sector size in an overlapping generations model in which human capital formation is endogenous and economic growth depends crucially on the stock of human capital in the economy. First, we investigate how factor composition and growth rate are affected by the different levels of fiscal policy in autarky. Second, we show that the trade pattern and growth rate can be determined by the extent of fiscal policy in an open economy.

JEL Classification: F1, H2, H3
Keywords: Government Size, Human Capital, Trade, Growth

Ⅰ. INTRODUCTION

The objective of this paper is to incorporate fiscal policy within an overlapping generations growth model with two production sectors and then to explain how the different levels of fiscal policy can alter not only growth rate but also comparative advantage by changing factor composition.

Some of the research on productivity growth has focused on the role of the government. Jones and Manuelli (1990), Rebelo (1991) etc. suggest a class of models in which cross-country differences in economic policy can generate heterogeneity in growth experiences. Most of these works study the effect of distortionary taxation on capital accumulation in a closed economy. Furthermore, they focus on the effect of international capital mobility caused by different level of capital income tax in an open economy under exogenous or endogenous growth framework.[1] On the other hand, Barro (1990) and Barro & Sala-i-Martin

Received for publication: Dec. 18, 2002. *Revision accepted:* April 9, 2003.
* The author is grateful to R. Findlay, R. Perotti, D. Foley, two referees and workshop participants at Columbia University for many helpful comments. This study was supported by the Research Fund of Hankuk University of Foreign Studies of 2003.
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(1992) try to set up a model to include tax-financed government services that affect production or utility. Empirically, cross-country comparisons reveal the existence of a strong negative correlation between the growth rate of an economy and the relative share of government spending in GDP, thus imparting support to the opinion that an economy with smaller government sectors is associated with faster growth rate.[2]

While there is a large body of literature on tax policy issues in neoclassical and endogenous growth models that high income tax rates translate into lower economic growth, these models do not give satisfactory explanation for the alteration of both national factor composition and trade pattern.[3] Exceptionally, Baxter (1992) applies tax and expenditure policy to the neoclassical trade theory with endogenous physical capital accumulation. However, it does not enable us to explain endogenous growth as well as the determination of comparative advantage through the alteration of human capital level.

On the other hand, Clarida and Findlay (1992) show that trade patterns can be altered by public choice between public capital —infrastructure— and public consumption good based on a Ricardo-Viner model.[4] Their analysis of the effect of government intervention under the static case of fixed factor endowment emphasizes the composition of government expenditure but does not deal with the private incentive effects that result from tax imposition. Wilson (1987) examines a system of governments that finance public expenditures with taxes on mobile capital (e.g., a property tax) under fixed factors. He provides an example in which the property tax is solely responsible for the existence of commodity trade between regions.

In this paper, the approach is different from that of the existing literature. In most of growth works, fiscal policy is treated within neoclassical exogenous and recent endogenous growth models with factor accumulation. These models have some difficulty in explaining how both trade pattern and growth rate are

[1] Jones, Manuelli & Rossi (1997) and Turnovsky (1996) provide a good survey for the taxation effect on growth. One stream of literature discusses the settings of income taxes in order to maximize the utility of an infinitely lived representative agent with perfect foresight subject to the competitive equilibrium behavior and the need to fund a fixed stream of government expenditures. Another stream of researches (ex. Buiter and Kletzer (1995)) investigates tax changes and capital mobility in an overlapping generations setting with exogenous or endogenous growth.

[2] The literature related is Grier and Tullock (1989), Landau (1986), Barth and Bradley (1987), Barro (1990 & 1991), Barro and Lee (1994) etc..

[3] Koester and Kormendi (1988) showed using cross sectional regressions that the marginal tax rate has an independent, negative effect on the output growth rates. Dowrick (1992) also found a strong negative effect of personal income taxation, but no impact of corporate taxes on output growth in a sample of OECD countries between 1960 and 1985. Chamley (1981) have demonstrated that in a nonconcave growth model, tax policy can have a potentially deep impact on the long-term growth rates.

[4] Banerji (1996) extended the analysis of Clarida and Findlay (1992) within the static Heckscher-Ohlin framework.

transformed through fiscal policy. Here, the engine of economic growth is the accumulation of knowledge capital through learning-by-doing in high tech sector whose production process is performed in a heavily human capital intensive fashion. Human capital formation is accomplished endogenously by each homogeneous agent's lifetime optimal career decision.

It is argued that public policy can exert a powerful influence on the average economic growth rates, since policies influence private incentives for savings and human capital formation. Investigating how an economy is modified through the tax policy —proportional wage income tax— in autarky, we show that higher income tax imposition depresses human capital formation and then reduces economic growth rate. This means that a small government is better for economic growth. Even if the initial conditions such as endowment and technology are equal, different distortionary tax rates bring about not only diverse factor composition but also various growth rates.

Elsewhere, it is demonstrated that even if both economies have the same initial endowment, the trade pattern can be altered by the different factor composition through the levels of fiscal policy. Hence, government size can influence growth rate as well as comparative advantage. Through the analysis, it is revealed that the incentive effects of fiscal policy can influence economic activity: the extent of income taxation can lead to a different path of economic growth and the alteration of comparative advantage through various levels of factor composition.

This paper is organized as follows. Section Ⅱ presents the general model in detail. Section Ⅲ studies the effects of fiscal intervention on economic growth rate and factor composition in autarky and how the tax rate is determined. In section Ⅳ, we examine the effects of permanent alteration in fiscal policy on factor composition, trade pattern and growth rates in a small open economy. Section Ⅴ studies how fiscal policy determines comparative advantage and growth rates under the general equilibrium of a two-country world. Concluding remarks with a brief summary follow in section Ⅵ.

Ⅱ. MODEL

The model is a two production sector dynamic general equilibrium model with intertemporal optimization and endogenous human capital formation. Consumer's side is characterized by overlapping generations with agents who live for two periods and without bequest motivation. High and low tech consumption goods are manufactured in two production sectors different in their factor intensity. A high tech good is produced in a more skill intensive fashion than a low one at all factor prices. Two production sectors are operated under a perfectly competitive market structure. Firms do not face dynamic problems and hire production factors in spot markets in every period.

The factor supplies are endogenously determined through the lifetime career paths of agents. In each period t, a new generation of agents enters the economy with identical ability. A young agent born at period t must make a decision about whether to participate in education as a student or to take part in production as an unskilled worker. Current period's students become the stock of human capital who will be employed either in the education institution as a teacher or in the production of high tech good as a skilled labor in the following period. Unskilled labors who work in period t retire in period $t+1$. They are employed in either high or low tech sector.

Unskilled labors who participate in economic activity in the first period consume part of their wage income and save the remainder to support their consumption for the retirement period. Agents who choose education in the first period as students do not receive any income during the schooling, and borrow against their second period income as skilled labors in order to finance consumption and education. The credit market regulates the savings of unskilled labors and the borrowings of students at an endogenous interest rate "γ_t" in every period.

Endogenous growth is performed through the accumulation of knowledge capital caused by external learning by doing.[5] It is assumed that the production experiences in high tech sector which utilizes human capital intensively generate knowledge capital through the learning and that the productivity improvement is accomplished throughout the economy owing to inter-industry spillover and diffusion effects.

The government is supposed to finance its expenditure through a proportional wage income tax, "τ_t", levied on skilled and unskilled labors working in the economy. The interest income tax is ignored. The government provides public consumption goods which consist of high and low tech goods in every period.[6] These public consumption goods improve individual's welfare and are depreciated fully after one period. For simplicity, they are the pure public goods with both non-excludability and non-rivalry.

Production

Firms regard themselves as too small to affect equilibrium prices and pursue profit maximization subject to the technological constraints. The outputs of the two sectors are perishable goods respectively. Both of them will be purchased by consumers to enhance private utility and by the government to provide public

[5] The examples of the literature in which economic growth is done by learning by doing are Arrow (1962), Krugman (1987), Lucas (1988), and Young (1991) and so forth.

[6] Barro (1990 & 1991) shows that growth is inversely related to the share of government consumption in GDP, but insignificantly to the share of public investment for 98 countries in the period 1960-1985. Especially, Holtz-Eakin (1994) & Garcia-Mila and McGuire (1992) find a negligible role for public capital. However, Aschauer (1989) Munnell (1990) and so on argue that public infrastructure contributes to economic growth. Cost-benefit analysis is well provided by Morrison and Schwartz (1996).

consumption goods. High tech sector Y in production utilizes skilled labor much more intensively than unskilled at all factor prices. Low tech sector X uses only unskilled labor for the production. We can set up the following production functions

$$Y_t = A_t (H_t^Y)^{\alpha} (L_t^Y)^{1-\alpha}$$

$$where \ \frac{1}{2} < \alpha < 1 \quad (1)$$

$$X_t = A_t \ L_t^X \quad (2)$$

Here is $\Delta A_t \equiv A_{t+1} - A_t = \delta Y_t, \delta > 0.$ [7] (3)

Both sectors are operated under constant returns to scale from the viewpoint of each firm. The productivity in both sectors, " A_t", which represents production knowledge capital as of time t, is predetermined but endogenous. Knowledge capital is accumulated as a by-product of high tech good production experience. These learning-by-doing effects are purely external to the individual firms. With complete spillovers within intra and inter-sectors, each manufacturing firm treats productivity " A_t" as given when making production and employment decisions.[8] There seems to be substantial spillover effects in the development of knowledge capital across industries with technical innovations originating in particular industry finding important applications as well as promoting further technical change in another industry.[9] Besides, ranging from Bardhan's(1970) analysis of the optimal subsidy for an infant industry in a small open economy to Lucas's(1988) exposition of a global learning-by-doing equilibrium, learning-by-doing has long been used in the formal analysis of trade effects on endogenous growth. As in Bardhan's (1970) framework, it is assumed that high tech sector experiences more rapid learning-by-doing than low one which might

[7] Matsuyama (1991) set up two production sectors which are manufacturing and agricultural sectors. The productivity of a manufacturing sector is increased by learning by doing in the production process. However, in his model an exogenous fixed factor is given and human capital factor is not taken into account. In economic growth, the innovation of agricultural sector is emphasized.

[8] The concrete explanations are provided in Arrow (1962), Succar (1987), Jaffe (1986) and so forth.

[9] The concept of spillover effects in learning-by-doing has previously been incorporated in models by Boldrin and Sheinkman (1988) and Stokey (1988) and so on. Especially, Boldrin and Sheinkman (1988) and Grossman and Helpman (1990) have investigated the implications of inter sectoral learning-by-doing, where the various industries do not contribute equally to the creation of knowledge. Caballero and Lyons (1990) showed the empirical evidence in support of country specific, inter-industry or inter-sectoral, and aggregate externalities: specific industry shows higher total factor productivity when other industries within that country produce a higher level of output.

experience no learning within the sector so as to focus the role of human capital on economic growth.

Consumption

Individuals optimize their lifetime utility subject to their budget constraints and government policy rules. Each individual shares the identical taste. The preferences of those born at time t can be represented by the following total utility function.[10]

$$\begin{aligned} Max V &= (1-\lambda)\ln\{(C_t^X)^\eta(C_t^Y)\}+\lambda\ln(PC_t)+\beta[(1-\lambda)\ln\{(C_{t+1}^X)^\eta(C_{t+1}^Y)\} \\ &+\lambda\ln(PC_{t+1})]=(1-\lambda)[\ln\{(C_t^X)^\eta(C_t^Y)\}+\beta\ln\{(C_{t+1}^X)^\eta(C_{t+1}^Y)\}] \\ &+\lambda[\ln(PC_t)+\beta\ln(PC_{t+1})]=(1-\lambda)\cdot U + \lambda\cdot G \\ &\textit{where } 0<\lambda<1 \text{ and } 0<\beta<1. \end{aligned}$$

Here, "β" describes a subjective time discount rate and "η" is the ratio of expenditure on low tech good X relative to that on high one Y in consumption. In "$C_:^:$", superscript is a class of goods and subscript is period in consumption. "PC_t" describes the public consumption good composed of high tech good Y and low one X in period t. "U" is the sub-utility function determined only by private consumption and "G" is one decided by government policy. The total utility of an individual depends on the quantities of low and high tech goods which are purchased and consumed privately, and the amount of public consumption goods provided by the government. Note that the private good utility and the public good utility are additively separable.

Given fiscal policy rules −government tax and expenditure− we can separate the following which is determined only by the private intertemporal optimal decision from the total utility function

$$Max \quad U=\ln[(C_t^X)^\eta(C_t^Y)]+\beta\ln[(C_{t+1}^X)^\eta(C_{t+1}^Y)] \tag{4}$$

$$\begin{aligned} s.t \quad & P_t\cdot {}_LC_t^X+{}_LC_t^Y=w_t^L(1-\tau_t)-S_t,\ P_{t+1}\cdot {}_LC_{t+1}^X+{}_LC_{t+1}^Y=(1+\gamma_t)S_t \\ & P_t\cdot {}_HC_t^X+{}_HC_t^Y=B_t-\Psi\cdot w_t^H,\ P_{t+1}\cdot {}_HC_{t+1}^X+{}_HC_{t+1}^Y=w_{t+1}^H(1-\tau_{t+1}) \\ & -(1+\gamma_t)B_t \end{aligned} \tag{5}$$

where $\{\tau_t\}_{t=0}^{\infty}$ is a proportional wage income tax rate, $\{S_t, B_t\}_{t=0}^{\infty}$ are saving

[10] This total utility function form is similar to that of Clarida and Findlay (1992) in which the static analysis is accomplished under fixed factor endowment. Optimal tax rate is investigated in the government and welfare part of section 3.

and borrowing respectively and $\{\gamma_t\}_{t=0}^{\infty}$ is an interest rate at the credit market. As each agent enters the economy with the identical condition, he or she has to enjoy the same lifetime utility regardless of the choice of career path. $\{H, L\}$ denote the individual's career paths as a skilled labor and an unskilled one respectively. The first line of budget constraints is those of agent who chooses the career path of the first period working as an unskilled labor and the second period retirement. The second line of budget constraints is those of agents who select the career path of the first period education participation and the second period skilled labor. " Ψw_t^H " is the education cost per student which a young agent has to charge when intending to take part in education so as to become a skilled labor in the second period. In every time period, " P_t" is the relative price of low tech good in terms of high one.

Education

As for the education system in this economy, we follow Eicher (1996) and Bhagwati and Srinivasan (1977).[11] In order to become a skilled labor, each individual must participate in a costly schooling process when he or she is young. Students have to not only pay education costs but also invest time during the schooling.[12] The schooling process is undertaken by the institution which hires some human capital factors as teachers. It is assumed that teacher-student ratio in the institution, " Ψ ", is given as constant and less than one half

$$\frac{H_t^T}{ST_t} = \Psi \tag{6}$$

where H_t^T is the stock of skilled labor —human capital— allocated to the education institution and ST_t is that of students who participate in education at time period t. Notice that the lower is Ψ —the reciprocal of education system efficiency—, the higher is the productivity of skilled labor employed in the education institution.

Factor Constraints

Under the above education system, the factor supplies are endogenously determined through the occupational choice of individuals. The generation's growth rate is set to be zero. For simplicity, the size of each generation is normalized to unity across periods. The labor constraints in period t can be

[11] Eicher (1996) explains growth with the model in which technical progress is achieved through the by-product accomplished during human capital formation in the schooling process.

[12] According to Psacharopolous and Woodhall (1985), teachers's salaries are composed of at least 70% of the total education cost in many developing countries.

written such as

$$1 = ST_t + L_t \qquad L_t = L_t^Y + L_t^X \qquad H_t = H_t^Y + H_t^T \qquad ST_t = H_{t+1} \tag{7}$$

The supply of unskilled workers at time t is allocated between high tech good production, L_t^Y and low one, L_t^X. In each period t, the supply of skilled workers H_t will be divided between teachers employed in the education institution, H_t^T, and skilled workers hired in the production of high tech good Y, H_t^Y. The wage of skilled workers in production is equally applied to teachers in the education sector. Also, the total supply of skilled workers in some period is equal to the number of individuals who chose education in the previous period as students.

Government Side

In each time period t, the government collects revenue from a proportional wage income tax levied on skilled and unskilled labors working in the economy. It does not issue interest-bearing bonds. In every time period, the government budget keeps balanced. Its flow budget constraint is given by

$$\tau_t(H_t w_t^H + L_t w_t^L) = PC_t = P_t G_t^X + G_t^Y \tag{8}$$

The expenditure of government revenue will be used to provide public consumption goods, PC_t, which are composed of low and high tech goods in order to improve the society welfare.

Ⅲ. AUTARKY

In autarky, we examine a perfect foresight equilibrium composed of the sequences $\{H_t, A_t, PC_t\}_{t=0}^{\infty}$ satisfying the model and investigate policy implications on factor composition and economic growth.

Consumer Optimization

Since each individual enters the economy as an identical agent with the same condition, each agent must be indifferent between two career paths. In other words, given public policy $\{\tau_t, PC_t\}_{t=0}^{\infty}$, two agents who choose different life style have to enjoy the same level of welfare for his or her lifetime. We can obtain the following relationship[13]

$$U_L = U_H \tag{9}$$

[13] Even if this condition can be altered to reflect the asymmetric ability density function and the credit constraint, we are able to get the same implications about the model.

In this private education system, in order to isolate the effects of taxation from those of government expenditures, it is assumed that government revenue is used to finance the provision of public consumption goods which do not affect either the marginal utility of private consumption of both high and low tech goods, or the production possibilities of private sector in the economy. From (4) and (5), we obtain

$$P_t = \frac{\eta \cdot {}_jC_t^Y}{{}_jC_t^X}, \quad P_{t+1} = \frac{\eta \cdot {}_jC_{t+1}^Y}{{}_jC_{t+1}^X} \qquad j = H, L \tag{10}$$

These are the conditions which satisfy the demand for high and low tech goods by consumers in period t or $t+1$ in order to maximize each period's utility. From (9), (10) and consumer budget constraints, we can derive the saving and borrowing determined by each individual's lifetime optimization. In other words,

$$S_t = \theta w_t^L (1 - \tau_t)$$

$$B_t = \theta \Psi w_t^H + \frac{\theta w_{t+1}^H (1 - \tau_{t+1})}{\beta (1 + \gamma_t)}$$

$$= \Psi w_t^H + \frac{\theta}{\beta} w_t^L (1 - \tau_t)$$

where $\theta = \frac{\beta}{1+\beta}, \quad 0 < \theta < \frac{1}{2}.$

From the career arbirage condition,

$$\frac{w_{t+1}^H (1 - \tau_{t+1})}{1 + \gamma_t} = \Psi w_t^H + w_t^L (1 - \tau_t) \tag{11}$$

where "θ" can be interpreted as the marginal propensity rate of savings out of disposable income. Since skilled workers obtain a wage in the second period of their lives, their optimal borrowing reflects their future discounted value of labor income to be received as a skilled labor. Bigger "β", more patience, gives rise to less borrowing and more saving since more consumption is put off into the second period of life. Since higher education cost decreases the first period consumption possibility for students, it makes them transfer more income from the second period to the first through the increased borrowing. Both saving and borrowing are negatively influenced by a proportional wage income tax rate. It is due to the fact that higher proportional wage income tax rate results in the less disposable income for unskilled labors and in the diminished ability for students to repay borrowing in the future. The career arbitrage implies that two agents who choose different career paths must have the equivalent level of disposable wealth from working or borrowing at each period in order to enjoy the equal extent of expenditure. This is a necessary and sufficient condition to guarantee the identical lifetime utility for two career paths. It is inferred from the fact that agents have the same lifetime utility function and confront the same intertemporal transformation possibility in this economy.

Credit Market Equilibrium

As for the consumption credit market, total savings made by unskilled labor in working period for the retirement period consumption should be equal to total borrowings required by students who intend to become a skilled labor in the second period of their lives. This gives us the following conditions about the credit market clearing;

$$L_t \cdot S_t = ST_t \cdot B_t \tag{12}$$

We can obtain an endogenous interest rate through the substitution of per capita savings and borrowings (11) into the equation (12);

$$(1+\gamma_t) = \frac{w_{t+1}^H (1-\tau_{t+1})}{\beta\left(\frac{L_t}{ST_t} w_t^L (1-\tau_t) - \Psi w_t^H\right)} \tag{13}$$

It is shown that the credit market clearing interest rate is contingent on the number of students and that of unskilled labors, time discount rate, proportional wage income tax rate and education cost per student. From the equation (12) and factor constraints (7), the stock of students can be displayed by saving and borrowing. That is

$$H_{t+1} = ST_t = \frac{S_t}{S_t + B_t} \tag{14}$$

After plugging the conditions of (11) from consumer optimization and career arbitrage into the equation (14), we can derive the level of investment in human capital. The stock of students who intend to become skilled labor in the second period can be written such as

$$H_{t+1} = ST_t = \frac{\theta w_t^L (1-\tau_t)}{\Psi w_t^H + w_t^L (1-\tau_t)} = \frac{\theta}{\Psi \frac{w_t^H}{w_t^L (1-\tau_t)} + 1} \tag{15}$$

From the first equation, the stock of students is positively influenced by saving (numerator) but negatively by the education cost and the opportunity cost (denominator). From the second expression, it is clear that the stock of skilled labors is positively related with constant savings rate and negatively with a measure of the education cost. Higher savings rate means the more abundant resources for students' borrowing. In addition, this equation implies that the higher relative wage of skilled labor in the current period will discourage human capital formation owing to higher education cost which students should charge. It is also straightforward that a higher proportional wage income tax will have a negative impact on the formation of human capital available for the following period. It is obvious from the fact that as an unskilled labor has less disposable income on account of higher income tax, his or her saving will decrease. This reflects that the students' group has more difficulty to borrow the fund which can support their consumption and education cost for the education period. We need to pay attention to the notion that the relative price of low tech good in terms of high one is implicit in the relative wage of skilled labor in terms of unskilled one which transmits price effects to influence the incentives to build up human capital in the economy. It will be made evident in the analysis of producer optimization (especially from (18)). Now we can obtain the equilibrium interest rate in the credit market such as $(1+\gamma_t)=\frac{w_{t+1}^H(1-\tau_{t+1})}{\Psi w_t^H + w_t^L(1-\tau_t)}$ which means that the interest rate is the ratio of benefit and cost in the investment for human capital.

Producer Optimization

We choose the price of high tech good as the numeraire. It means that in every period, $P_t^Y=1$, $P_t=\frac{P_t^X}{P_t^Y}$. In this economy, the rewards of production factors employed in both high and low tech sectors are the value of marginal productivity out of constant returns to scale at the viewpoint of each firm. They can be written such as

$$w_t^H=\alpha A_t(H_t^Y)^{\alpha-1}(L_t^Y)^{1-\alpha},\quad w_t^L=(1-\alpha)A_t(H_t^Y)^{\alpha}(L_t^Y)^{-\alpha},\quad w_t^L=P_t\cdot A_t \tag{16}$$

From (16), perfectly competitive market and zero profit conditions, we can get

$$1=\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}A_t^{-1}(w_t^H)^{\alpha}(w_t^L)^{1-\alpha},\quad P_t=A_t^{-1}\cdot w_t^L \tag{17}$$

Utilizing the equations of (17), we are capable of obtaining the relationship between the relative price and the relative wage.

$$P_t=\frac{P_t^X}{P_t^Y}=\chi\left(\frac{w_t^H}{w_t^L}\right)^{-\alpha}\ where\ \chi=\alpha^{\alpha}(1-\alpha)^{(1-\alpha)} \tag{18}$$

It is shown that the relative price is negatively related with the relative wage. This is evident from factor intensity in both production sectors. In other words, it is inferred from the fact that high tech good is produced in a skilled labor intensive fashion, while low one is manufactured only by unskilled labor. Combining factor rewards (16) in high tech sector, we can also get the following about the relative wage and the employment ratio of unskilled and skilled labors in the production of high tech good.

$$\omega_t = \frac{w_t^H}{w_t^L} = (\frac{\alpha}{1-\alpha})(\frac{L_t^Y}{H_t^Y}) \tag{19}$$

From (18) & (19), we can derive the following expression about the relative price and the employment ratio of high tech sector;

$$P_t = (1-\alpha)(\frac{H_t^Y}{L_t^Y})^\alpha \tag{20}$$

This expression mirrors the factor market equilibrium condition under which the wage of unskilled labor should be equal in both high and low tech sectors from (16). Hence, from (7), (19) & (20), the relative wage and the relative price can be displayed such as

$$\omega_t = \frac{w_t^H}{w_t^L} = (\frac{\alpha}{1-\alpha})(\frac{L_t^Y}{ST_{t-1} - \Psi ST_t}), \quad P_t = (1-\alpha)(\frac{ST_{t-1} - \Psi ST_t}{L_t^Y})^\alpha \tag{21}$$

This shows that the relative wage and the relative price in period t are described by the previous and current period stock of students and the stock of unskilled labor which is employed in the production of high tech good.

The Growth Rate of Productivity

At the beginning, I assumed that the productivity of this economy is increased by the accumulation of knowledge capital through learning-by-doing in the production of high tech good which utilizes skilled labor intensively. We can get the following equation about the growth rate of productivity from (1) and (3);

$$\frac{\Delta A_t}{A_t} = \delta(H_t^Y)^\alpha (L_t^Y)^{1-\alpha} \tag{22}$$

This means that economic growth depends on the factor quantities of skilled and unskilled labor employed in the production of high tech good.[14] In the partial equilibrium, it is shown that productivity growth rate is positively related to the stock of skilled and unskilled labor hired in high tech sector, respectively. In the general equilibrium, however, it will be demonstrated that the stock of skilled labor —human capital— in the economy plays a crucial role for economic growth.[15] This will be made clear in the steady state analysis.

[14] Schultz (1975) surveyed a variety of indirect evidence that technological progress is positively related to average level of human capital in technologically advanced sectors.

Goods Market Equilibrium

In autarky, domestic supply should be equal to domestic demand in order to clear both high and low tech good markets. From the assumption that government tax revenue is used to finance the provision of public consumption goods which do not affect the marginal utility of private consumption or the production possibilities of both goods, we can be aware that the composition of public consumption good does not influence the relative share of high and low tech goods allocated to the private consumption. From the aggregation of per capita consumption over all individuals and the condition of goods market equilibrium, the relative price can be written as the ratio of consumption or production;

$$P_t = \frac{\eta C_t^Y}{C_t^X} = \frac{\eta G_t^Y}{G_t^X} = \frac{\eta(C_t^Y + G_t^Y)}{C_t^X + G_t^Y} = \frac{\eta Y_t}{X_t} \tag{23}$$

From production side, we get the following expression about the ratio of products and that of factors which are employed in the production of both high and low tech goods

$$\frac{Y_t}{X_t} = \left(\frac{H_t^Y}{L_t^Y}\right)^a \frac{L_t^Y}{L_t^X} \tag{24}$$

From (19), (23) & (24), the division of unskilled labor between high and low tech production sectors out of the market clearing condition can be derived such as

$$\frac{Y_t}{X_t} = \left(\frac{P_t}{1-a}\right)\left(\frac{L_t^Y}{L_t^X}\right) \qquad \frac{1-a}{\eta} = \frac{L_t^Y}{L_t^X} \tag{25}$$

This shows that the ratio of unskilled labors employed in both production sectors is determined by factor intensity in high tech good production and expenditure share in consumption. Higher "a" implies that high tech good should be produced in more skilled labor intensive manner. We can infer that as "a" is getting larger, the production of high tech requires less unskilled labor relatively. Higher "η" means that the share of private and government expenditure on low tech good is larger. It requires the more production of low

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[15] Empirically, average years of schooling (Benhabib and Spiegel (1994)), school enrollment rates (Mankiw, Romer and Weil (1990) and Barro (1991)), and the stock of scientists and engineers (Romer (1989)) have been shown to correlate positively with GDP growth. Theoretically, it is shown that the economy with a larger stock of human capital experiences faster growth in Romer (1990) and Grossman & Helpman (1991) which emphasize that human capital abundant economy can invest more human capital in R&D.

tech good in order to satisfy increased demand. From factor constraints (7) and (25), we can obtain the following results

$$1-ST_t=L_t^X+L_t^Y=(\frac{1+\eta-\alpha}{\eta})L_t^X=(\frac{1+\eta-\alpha}{1-\alpha})L_t^Y \tag{26}$$

This describes the relationship about the allocation share of unskilled labor between low and high tech good productions and the stock of students in time period t. From (20) & (26) and labor constraints, the relative price and the relative wage can be written as

$$\begin{aligned} P_t &= (1-\alpha)[\frac{ST_{t-1}-\Psi ST_t}{(\frac{1-\alpha}{1+\eta-\alpha})(1-ST_t)}]^{\alpha}, \\ \omega_t &= \frac{w_t^H}{w_t^L}=(\frac{\alpha}{1-\alpha})[\frac{(\frac{1-\alpha}{1+\eta-\alpha})(1-ST_t)}{ST_{t-1}-\Psi ST_t}] \end{aligned} \tag{27}$$

So, we are aware that the relative price and the relative wage in period t are determined by the stock of students in both period t and $t-1$. In this general equilibrium, we need to mention how the previous stock of students influences the relative wage and the relative price in the current period. The larger stock of students in the previous period brings forth more abundant stock of skilled labor in the current one. That brings about the downward pressure on the relative wage of skilled labor in terms of unskilled one in the current period. Naturally, the relative price of low tech good in terms of high one will be increased in the current period. On the other hand, the current stock of students has an ambiguous effect on the current relative wage. More students make unskilled labor scarce only to put downward pressure on the relative wage (numerator), but require the more skilled labor for education only to cause upward pressure on the relative wage in the production sector (denominator). The effect of a proportional wage income tax is transmitted in both the relative price and the relative wage through the alteration of factor composition. It will be made apparent in the below steady state analysis. From (15) and (27), we get the following difference equation of the stock of students between the previous and current periods

$$ST_t=\frac{\theta}{\frac{m(1-ST_t)}{(1-\tau_t)(ST_{t-1}-\Psi ST_t)}+1} \quad where\ m=\frac{\Psi\alpha}{1+\eta-\alpha}. \tag{28}$$

We can conjecture that the stock of students in period t is influenced crucially by a proportional wage income tax rate in period t.

Policy Analysis at the Steady State

Now, it is assumed that a proportional wage income tax rate is always constant; " $\tau_t = \tau$". We define the steady state stock of students — human capital — as $H^* = ST^* = ST_t = ST_{t-1}$.[16] (see Appendix about the existence and uniqueness proof of steady state). From (28) and factor constraints, the stock of skilled and unskilled labor can be written such as

$$H^* = ST^* = \frac{\theta(1-\tau)-n}{(1-\tau)-n}, \quad L^* = 1 - ST^* = \frac{(1-\theta)(1-\tau)}{(1-\tau)-n} \qquad (29)$$

$$where\ n = \frac{m}{1-\Psi} = \frac{\Psi a}{(1-\Psi)(1+\eta-a)}.$$

We require the following condition " $\theta > n$" for the meaningful economy. In the general equilibrium, the stock of skilled labor — human capital — has a positive relationship with the marginal saving rate, θ. The higher marginal rate of saving will bring forth the larger stock of human capital since it causes the favorable credit market for human capital formation due to increased available resources for students. It is obvious from factor constraints that the stock of unskilled labor has a negative relationship with the marginal saving rate. Now, from (27), we can acquire the relationships about the relative price and the relative wage at the steady state such as

$$P^* = (1-a)\left[\frac{(1-\Psi)H^*}{\left(\frac{1-a}{1+\eta-a}\right)(1-H^*)}\right]^a,$$

$$\omega^* = \left(\frac{w^H}{w^L}\right)^* = \left(\frac{a}{1-a}\right)\left[\frac{\left(\frac{1-a}{1+\eta-a}\right)(1-H^*)}{(1-\Psi)H^*}\right] \qquad (30)$$

From (30), we can get the results that the relative price depends positively on the stock of skilled labor, while the relative wage depends negatively on the stock of human capital at the steady state in the general equilibrium;

$$\frac{dP^*}{dH^*} > 0, \quad \frac{d\omega^*}{dH^*} < 0 \qquad (31)$$

These reflect just Rybczynski effect. In other words, the increase of human capital stock makes unskilled labor scarce in the economy. As a result, the relative price of low tech good produced in an unskilled labor intensive way

[16] It can be easily shown that there exists a unique, stable stationary equilibrium from (28). The concrete proof is provided in Appendix.

gets higher and the relative wage of skilled labor becomes lower. Differentiating the stock of skilled and unskilled labor by a proportional wage income tax rate, we get

$$\frac{dP^*}{d\tau} = \frac{-(1-\theta)n}{(1-\tau-n)^2} < 0, \quad \frac{dL^*}{d\tau} = \frac{n}{(1-\tau-n)^2} > 0 \tag{32}$$

These imply that the stock of skilled labor depends negatively on a wage income tax rate due to less available resources, while that of unskilled labor depends positively on a wage income tax rate from the factor constraint condition. As in the partial equilibrium analysis, higher tax rate results in the less disposable income for savings and hence the less available resources for agents who want to choose education. From (31) & (32), we can get the following results

$$\frac{dP^*}{d\tau} = \frac{\partial P^*}{\partial H^*}\frac{\partial H^*}{\partial \tau} < 0, \quad \frac{d\omega^*}{d\tau} = \frac{\partial \omega^*}{\partial H^*}\frac{\partial H^*}{\partial \tau} > 0 \tag{33}$$

These display how the change of a proportional wage income tax rate affects the relative price negatively and the relative wage positively through the alteration of factor composition in the economy. From the equation (22), the growth rate of productivity at the steady state can be written such as

$$g = \frac{\Delta A_t}{A_t} = \delta[(1-\Psi)H^*]^{\alpha}[(\frac{1+\eta-\alpha}{1-\alpha})(1-H^*)]^{1-\alpha} \tag{34}$$

It is shown that the growth rate of productivity is influenced crucially by the stock of skilled labor. That is, in spite of different production and market structure from the other literature, it is shown that the stock of human capital is decisive to economic growth in this economy. If we differentiate productivity growth rate (g) with regard to human capital (H^*), we obtain the following relationship

$$\frac{dg}{dH^*} > 0 \tag{35}$$

It is demonstrated that economic growth rate depends positively on the stock of human capital. Notice that a lower ratio of teacher-student $-\Psi-$ brings a higher growth rate of productivity because it enables this economy not only to allow more skilled labor −human capital− for the production of high tech good Y but also to motivate more agents to participate in schooling by cheaper education cost per student. From (32) & (35), the relationship about growth rate and tax rate can be written such as

$$\frac{dg}{d\tau} = \frac{\partial g}{\partial H^*}\frac{\partial H^*}{\partial \tau} < 0 \tag{36}$$

It is obvious that productivity growth rate is negatively related by the size of a proportional wage income tax levied on unskilled and skilled labor who participate in economic activity. We can define the quantity of " τ " as government size. It will be made clear in the below government and welfare part analysis.

Proposition 1

In the above economy,

1. the stock of human capital and productivity growth rate have a negative relationship with a proportional wage income tax rate.

2. the economy with a small government sector has more stock of human capital than that with a large one.

3. the economy with a small government sector has the lower relative price of high tech good in terms of low one and the lower relative wage of skilled labor in terms of unskilled one than that with a large one.

Government and Welfare

In the above analysis, we revealed that higher wage income tax rate for more welfare policies results not only in shrinking private consumption but also in influencing the individual's saving-borrowing incentive and changing factor composition of the economy. In this economy, the current period unskilled labor's saving is crucial to finance students who will become skilled labors in the next period. It was shown that the national stock of skilled labor is the most decisive factor for economic growth.

In the previous part, endogenous growth and factor composition were derived under the assumption of given fiscal rules. Fiscal policy variables were not the target of choice. In this part, we relax this assumption and treat a proportional wage income tax rate " τ " as choice variable in order to answer what would be the best choice.

Note also that we are not considering the so-called command equilibrium in which one would choose fiscal policy variable and consumption-saving's decision rules all simultaneously from the beginning. Here, individuals who regard policy variables as given make consumption-saving and consumption-borrowing decisions to maximize the lifetime well-being. On the other hand, the government chooses fiscal policy with taking the response functions of individuals as given. In this overlapping generations economy without the bequest motivation, there is no justification that the government puts heavier discounts on the far future generations. For the entire set of social welfare functions, the discounting of time by a social planner causes the philosophical problem. While the infinitely lived agents could conceivably have an incentive to discount the present and future utility, two-period lived agents do not have any right to discount any other generations. However, in this simple overlapping generations model, we can think of the sub-optimal tax decision of the government which maximizes just one period utility. Before we analyze that, we consider the indirect utility function. That is,

$$Indirect\ V = (1-\lambda)\ln\frac{f(\tau_t, GDP_t, P_t;\mu)}{2} + \lambda \ln h(\tau_t, GDP_t, P_t;\mu).$$

We divide "f" by 2 —total population size—, since two generations co-exist in some time period. "f" implies private expenditure on high and low tech goods and "h" means public expenditure on both goods to provide public consumption good. Therefore, "f" is a function of disposable income and "h" is that of government revenue. Here, Gross Domestic Product (GDP), Disposable Income (DI) and Government Revenue (GR) in time period t can be expressed such as

$$GDP_t = w_t^H \cdot H_t + w_t^L \cdot L_t$$

$$DI_t = (1-\tau_t)(w_t^H \cdot H_t + w_t^L \cdot L_t)$$

$$GR_t = \tau_t\,(w_t^H \cdot H_t + w_t^L \cdot L_t).$$

We assumed that the expenditure share of the government on high and low tech goods is the same as the private one on both goods. Through the derivation of a proportional wage income tax rate to maximize the indirect social welfare function subject to the wages of skilled and unskilled labor and the relative price, we can obtain the result that "$\tau_t = \lambda$" from the first order condition of the indirect welfare function. This is due to the characteristic of log utility function. We know that a wage income tax rate is determined by how much weight the government puts on a public consumption good which is considered to be used in supporting diverse welfare programs. So we can define higher "λ" country as the welfare-oriented one and lower "λ" country as the development-oriented one. From the expressions of GDP and GR, it is obvious that we can regard the size of tax rate as government size.

IV. SMALL OPEN ECONOMY

We take into account the effects of fiscal policy in an open economy. The preceding model of a closed economy may be reinterpreted in a small open economy in which the relative price of low tech good in terms of high one is taken as being exogenously determined in the world goods market. It implies that both high and low tech goods have the perfectly elastic demand on the world goods market. It is assumed that only goods are internationally mobile but production factors such as skilled and unskilled labors are immobile across borders. In the credit market, domestic borrowings are financed only from domestic savings.[17] In order to focus on public finance issues, we abstract from the international diffusion of knowledge capital.[18] While there is a possibility of complete specialization in the production of high tech good, an incomplete specialization is assumed throughout the entire analysis.

Given the world relative price P^{world} of low tech good in terms of high one, the relative wage of skilled labor in terms of unskilled one can be obtained by substituting P^{world} in the equation (18);

$$P^{world} = \chi(\omega^{world})^{-\alpha} = \chi(\frac{w^H}{w^L})^{-\alpha} \tag{37}$$

Consequently, we can be aware that the relative price given in the world goods market fixes the relative wage in domestic factor markets. That is, since P^{world} determines the relative wage with a given factor intensity parameter of high tech α, the relative wage of a small open economy should be equalized to that of the world in the trading equilibrium.

From (19) and (20), we know that the ratio of unskilled and skilled labor employed in the production of high tech good is fixed. That is, we can get the following

$$\omega^{world} = (\frac{\alpha}{1-\alpha})(\frac{L^Y}{H^Y}), \qquad P^{world} = (1-\alpha)(\frac{H^Y}{L^Y})^{\alpha} \tag{38}$$

From these expressions, we can conjecture that the composition of two production factors in the economy will determine the product quantities of high and low tech goods with productivity knowledge capital. In a small open economy, the domestic supplies of low and high tech goods do not have to be equal to the domestic demands of both goods. The domestic goods market equilibrium condition in autarky does not hold in the trading world equilibrium any longer. Note that the individuals in the world share the same preference and their subutility function is homothetic in the consumption of both high and low tech goods.

In a small open economy, domestic credit market clearing, consumer

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[17] Using data for the OECD economies during the period 1960-1974, Feldstein and Horioka (1980) showed that net capital flows are small; there exists an almost perfect correlation between domestic savings and investment. This result is robust to changes in the sample size and period as well as the estimation procedure (Obstfeld (1986) & Frankel (1991)).

[18] The importance of the experiences of domestic producers rather than producers located abroad – national spillovers – is discussed and emphasized in Bardhan (1970), Krugman (1987), Lucas (1988) and Caballero & Lyons (1990).

optimization and career arbitrage give us the stock of students —next period human capital.

$$H_{t+1} = ST_t = \frac{\theta}{\Psi \frac{\left(\frac{\chi}{P^{world}}\right)^{\frac{1}{\alpha}}}{(1-\tau_t)} + 1} = \frac{\theta}{\Psi \frac{\omega^{world}}{(1-\tau_t)} + 1} \tag{39}$$

Given the relative price in the world markets, the stock of students depends negatively on the size of a proportional wage income tax levied on skilled and the unskilled labor who take part in economic activity. Under the domestic credit market constraint, saving and borrowing which are crucially influenced by a proportional wage income tax determine the stock of production factors.[19]

It is assumed that " $\tau_t = \tau$" at every period. In this small open economy, the steady state level of skilled labors will be written such as

$$H^* = ST^* = \frac{\theta}{\Psi \frac{\omega^{world}}{(1-\tau)} + 1} \quad \text{where} \quad P^{world} = \chi(\omega^{world})^{-\alpha} \tag{40}$$

We are aware that the stock of skilled labor has a positive relationship with the relative price given in the world goods markets through the effect transmitted by the relative wage. This implies that since the higher relative world price of low tech good in terms of high one causes the relative wage of skilled labor in terms of unskilled one to be lower —Stolper-Samuelson effect—, the education cost is getting cheaper. Individuals have far more incentive to take part in the schooling owing to cheaper education cost.[20] On the contrary, since the credit market is constrained domestically, higher tax rate causes the less saving of unskilled labor by the diminished disposable income. That brings about more difficulty for students to borrow funds to support consumption and education cost during the schooling period. As a result, the stock of skilled labor —human capital— in the economy gets decreased. That is, we get the following

$$\frac{dH^*}{dP^{world}} > 0, \qquad \frac{dH^*}{d\tau} < 0 \tag{41}$$

We can infer that the economy with smaller tax rate has more abundant skilled labor and exports high tech goods produced in a skilled labor intensive way at

[19] Barro, Mankiw and Sala-i-Martin (1995) show that their open-economy model conforms with the empirical evidence, only if human capital must be financed by domestic savings.

[20] There is a literature which investigates the effect of international trade on the factor composition. Findlay and Kierzkowski (1983), and Findlay (1995) show that trade causes the divergence of factor composition in the open trade economy.

the relative price given in the world goods market.
In a small open economy, the growth rate of domestic productivity can be described by the following equation

$$g=\frac{\Delta A_t}{A_t} = \delta(H^Y)^\alpha(L^Y)^{1-\alpha} = \delta(\frac{H^Y}{L^Y})^\alpha(L^Y) \tag{42}$$

The growth rate of knowledge capital will be determined by the quantities of factors employed in the domestic production of high tech good. The empirical underpinning of this result is that inter and intra-sectoral learning-by-doing externalities exist and these gains are geographically confined in the sense that they are not easily traded across borders.[21] Using the factor constraints and (38) & (40), we can get

$$g=\frac{\Delta A_t}{A_t} = \delta(\frac{1-\alpha}{p^{world}})^{\frac{1-\alpha}{\alpha}}(1-\Psi)H^* \tag{43}$$

It is shown that growth rate depends positively on the stock of skilled labor which are supplied in the domestic economy. More efficient education system brings about higher growth rate by allowing more skilled labor to be allocated in production statically and by the participation of more students in the schooling due to cheaper education cost per student dynamically. That is, we can derive

$$\frac{dg}{dH^*} > 0, \quad \frac{dg}{d\Psi} < 0 \tag{44}$$

From chain rule, (41) & (44), the relationship between economic growth and tax rate through the stock of skilled labor can be derived such as

$$\frac{dg}{d\tau} = \frac{\partial g}{\partial H^*}\frac{\partial H^*}{\partial \tau} < 0 \tag{45}$$

As we have analyzed in government and welfare part, the size of a proportional wage income tax rate means government size in the economy. The determination

[21] Even if we allow the spillover of knowledge capital among countries such as $g= \frac{\Delta A_t}{A_t} = F(Y_t^{domestic}, Y_t^{the\ rest\ of\ world}, \delta)$, we can get the same implication about the trade pattern which is determined by the factor composition in each country. In reality, since it is impossible to have the perfect spillover, the growth rate of each country is influenced decisively by the domestic stock of human capital.

of tax rate in an open economy is the same as in the government and welfare part in autarky section Ⅲ.

Proposition 2

As in autarky, in a small open economy, the country with small government size has the larger stock of human capital and higher growth rate.

Proposition 3

Compared with the rest of the world, the country with small government size has abundant human capital and exports high tech goods produced in a skilled labor intensive way.

V. TWO LARGE COUNTRIES

Now, we investigate the equilibrium of a two-country world economy in which both countries are large enough to affect equilibrium prices in the world goods market. Each country has the economy described by the model of section Ⅱ. We establish how government size affects comparative advantage as well as growth rate in each country. In other words, we are ready to answer one of the most fundamental questions: what is the determinant of the international patterns of production, trade and growth? Let's consider two countries, A and B, which have the identical tastes and the same initial endowment except for government policy. The assumption of identical and homothetic preferences in the private consumption of high and low tech goods and constant returns to scale production from each firm's viewpoint makes the model similar to the Heckscher-Ohlin trade model. Thus, the country with larger stock of human capital will have comparative advantage in high tech good produced in a humancapital intensive way. It is also accompanied with higher growth rate which is crucially dependent upon the accumulation of knowledge capital brought about by the production process of high tech good. Under free trade between two countries, the relative wages will be equalized as long as two countries are producing both low and high tech goods, even though the absolute wages are different on account of the difference of productivities. The welfare oriented country which provides more public consumption goods will impose higher wage income tax to finance government expenditure than the development oriented country. As a result, the welfare oriented country will have a smaller stock of skilled labor and thus slow down productivity growth. As we have discussed in the government and welfare part of section Ⅲ, the welfare oriented country is the country which has higher "λ" on the whole utility function. Hence it puts more weight on the social welfare. Here, as in a small open economy, it is assumed that there is no spillover of knowledge capital for productivity growth between countries.[22]

In this section, domestic borrowing is also financed only from domestic saving owing to the reasons expounded in the footnotes of section Ⅳ. Then, it is clear that the determinant of comparative advantage and economic growth is government policy in each economy. Even though two countries are the same but for the extent of government policy in the provision of public consumption goods, they will have a different level of human capital. Hence they have the different patterns of comparative advantage as well as productivity growth rate.

Proposition 4

In the two country world,

1. the welfare oriented country has a smaller stock of human capital and thus slows down growth rate than the development oriented one.

2. the development oriented country has comparative advantage in high tech sector, while the welfare oriented one has comparative advantage in low tech sector.

Ⅵ. CONCLUSION

In this paper, we have developed a two production sector overlapping generations model of international trade with endogenous human capital formation and endogenous growth through learning-by-doing which accumulates public knowledge capital. We explored the relationships between government policy, comparative advantage and economic growth in endogenous growth framework.

We established that there are the significant implications for the role of the government in influencing a country's growth potential and trade pattern. Here we investigated the question of how fiscal policy determines the national factor endowment, growth rate and trade pattern. Even if each economy has the same initial conditions such as factor endowment, preferences and so forth, economic growth and comparative advantage can be different as a result of the alteration of factor composition through fiscal policy which affects saving and borrowing behavior.

Before finishing the paper, we should point out several limitations. First, the total welfare implication brought about by the different levels of government policy is ambiguous in this analysis. Second, under this overlapping generations framework, the savings of unskilled labor are crucial to support human capital formation. Throughout the paper, it is assumed that there is no bequest motivation between generations. Third, we focused only on a proportional wage income tax and did not take into consideration consumption and production taxes together. In a closed economy, the same tax rate of production or consumption in both high and low tech goods will not affect factor composition. However,

[22] Even if we allow the spillover of knowledge capital between the two large countries, we obtain the same implication about the trade pattern. However, the growth rate of productivity will be influenced. As mentioned in section Ⅳ, note that the domestic growth rate depends crucially on the domestic human capital.

the different tax rate between two goods will distort the relative price and hence influence the whole economy in diverse manners. All these limitations should be dealt with further in future research.

Ⅶ. APPENDIX

We provide the proof about the existence of steady state which is unique and stable. In this process, we assume that government policy is given ; a proportional wage income tax rate is " $\tau_t = \tau$". We exclude the case in which initial human capital is zero because that has not much meaning in the real world.

Proposition: There exists a unique positive level of stable stationary equilibrium " $H^* = ST^*$" such that $H^* = ST^* = ST_t = ST_{t-1}$, $0 < H < \theta$ with a necessary and sufficient condition " $\theta > n$".

Step i) $\frac{dST_t}{dST_{t-1}} > 0$.

From (29), we can get the following

$$f(ST_t) \equiv [m + (1-\tau)\Psi] \cdot (ST_t)^2 - [m + \theta(1-\tau)\Psi + (1-\tau)ST_{t-1}] \cdot ST_t + \theta(1-\tau) \cdot ST_{t-1} = 0 \quad \text{(A1)}$$

If we plug θ into ST_t, we can know that $\theta \neq ST_t$. That is,

$$f(\theta) \equiv [m + (1-\tau)\Psi] \cdot \theta^2 - [m + \theta(1-\tau)\Psi + (1-\tau)ST_{t-1}] \cdot \theta + \theta(1-\tau) \cdot ST_{t-1} = m\theta(\theta-1) < 0 \quad where\ 0 < \theta < 1 \quad \text{(A2)}$$

If we take the total derivative of (A1), we get the following

$$\frac{dST_t}{dST_{t-1}} = \frac{(1-\tau)(\theta - ST_t)}{[m + \theta(1-\tau)\Psi + (1-\tau)ST_{t-1}] - [m + (1-\tau)\Psi]2ST_t} \quad \text{(A3)}$$

From the equation (22), we know that the numerator is positive.
Utilizing (6) and (22), we can show that the denominator is also positive with some manipulation. Therefore, the proof of step I) is completed.

Step ii) *There exists a unique continuous mapping* $ST_t = \sigma(ST_{t-1})$ *satisfying*

$$ST_t \frac{\theta}{\dfrac{m(1-ST_t)}{(1-\tau)(ST_{t-1}-\Psi ST_t)}+1} \quad where \quad m=\frac{\Psi a}{1+\eta-a}.$$

Let's utilize the condition of meaningful economy $-0 < ST_t < \theta-$. (A1) has two solutions. From (A2) and $\theta(1-\tau)ST_{t-1}$, one solution lies in $(0, \theta)$ and the other lies in (θ, ∞). We can conclude that both mappings are continuous ones from the implicit function theorem and step i. Therefore, the proof of step ii) is completed.

Step iii) *There exists a fixed point for mapping* σ *such that* $ST=\sigma(ST)$, *where* $0 < ST = H < \theta$.
Using (A1), we find one solution which satisfies the following

$$[m+(1-\tau)\Psi]\cdot ST^2-[m+(1-\tau)\cdot ST+\theta(1-\tau)\Psi]\cdot ST+\theta(1-\tau)\cdot ST=0 \qquad \text{(A4)}$$

The meaningful solution which satisfies the condition $0 < ST < \theta$ is obtained if and only if

$\theta > n$ *where* $n=\frac{m}{1-\Psi}=\frac{\Psi a}{(1-\Psi)(1+\eta-a)}$. The solution is $ST=\frac{\theta(1-\tau)-n}{(1-\tau)-n}$.

Step iv) *The solution is stable.*
In order to check where the solution is stable, we investigate the size of $\frac{dST_t}{dST_{t-1}}$ around the solution which is economically meaningful. Using (A3) and step iii), we can get the following

$$\frac{dST_t}{dST_{t-1}}\Big|_{ST_t=ST_{t-1}=ST}=\frac{(1-\tau)(\theta-ST)}{[m+\theta(1-\tau)\Psi+(1-\tau)ST]-[m+(1-\tau)\Psi]\cdot 2ST} \qquad \text{(A5)}$$

$$=\frac{\theta(1-\tau)-(1-\tau)ST}{\theta(1-\tau)-[m+(1-\tau)\Psi]ST}<1$$

This inequality is driven from $\theta > n$. That is, we use the fact that $\theta > n > m > 0$ and the parameter range condition implies $m+\Psi < 1$. In this process, we assume that tax rate is extremely small but it is valid at any rate.
Therefore, from steps i, ii, iii, and iv, with the initial period condition $0 < H < \theta$, we can obtain the result that $ST^*=H^*=\frac{\theta(1-\tau)-n}{(1-\tau)-n}$ is the unique stable stationary solution if and only if $\theta > n$ *where* $n=\frac{m}{1-\Psi}=\frac{\Psi a}{(1-\Psi)(1+\eta-a)}$.

REFERENCE

Arrow, K. J. (1962), "Economic Implications of Learning by Doing," *Review of Economic Studies* 29: 155-173.

Banerji, A. (1994), "The Effect of Public Capital Stock on Comparative Advantage and The Pattern of Trade," *Ph.D. dissertation*, Columbia University.

Bardhan, P. K. (1970), "Economic growth, Development, and Foreign Trade," New York: Wiley-Interscience.

Barro, R. J. (1990), "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy* 98: S103-S125.

Barro, R. J., Mankiw, G. N., and Sala-i-Martin, X. (1995), "Capital Mobility in Neoclassical Models of Growth," *American Economic Review* 85: 103-115.

Barro, R. J. and Xala-i-Martin, X. (1992), "Public Finance in Models of Economic Growth," *Review of Economic Studies.* 59: 645-661.

Barro, R. J. and Xala-i-Martin, X. (1996), "Economic Growth," New York, NY: McGraw Hill Company.

Barth, J. R. and Bradley, M. D. (1987), "The Impact of Government Spending on Economic Activity," *Manuscript*, George Washington University.

Baxter, M. (1992), "Fiscal Policy, Specialization, and Trade in the Two-Sector Model: The Return of Ricardo?" *Journal of Political Economy* 100: 713-744.

Benhabib, J. and Spiegel, M. (1992), "The Role of Human Capital in Economic Development: Evidence from Aggregate Cross-Country Data," *Journal of Monetary Economics* 34: 143-173.

Boldrin, M. and Sheinkman, J. A. (1988), "Learning By Doing, International Trade, and Growth: A Note," In Anderson, Philip W.; Arrow, Kenneth J.; and Pines, David, eds. *The Economy as an Evolving Complex System. Reading*, MA: Addison-Wesley Publishing Company.

Buiter, W. H. and Kletzer, K. M. (1995), "Capital Mobility, Fiscal Policy and Growth under Self-Financing of Human Capital Formation," *NBER working paper* # 5120.

Caballero, R. J. and Lyons R. K. (1990), "Internal versus External Economies in European Manufacturing," *European Economic Review.*

Chamley, C. (1981), "The Welfare Cost of Capital Taxation in a Growing Economy," *Journal of Political Economy* 89: 468-496.

Dowrick, S. (1992), "Estimating the Impact of Government Consumption on Growth: Growth Accounting and Optimizing Models," Australian National University, *memeo.*

Eicher, T. S. (1996), "Interaction Between Endogenous Technological Change and Endogenous Human Capital," *Review of Economic Studies* 63: 127-144.

Feldstein, M. and Horioka, C. (1980), "Domestic Saving and International Capital Flows," *Economic Journal* XC: 314-322.

Findlay, R. and Clarida, R. H. (1992), "Government, Trade, and Comparative

Advantage," *American Economic Review 82 (Papers and Proceedings)*: 122-127.

Findlay, R. and Kierzkowski, H. (1983), "International Trade and Human Capital: A Simple General Equilibrium Model," *Journal of Political Economy* 91: 957-978.

Frankel, J. (1991), "Quantifying International Capital Mobility in the 1980s," In D. Bernheim and J. Shoven, eds. *National Saving and Economic Performance*, Chicago, IL: University of Chicago Press.

Grier, K. B. and Tullock, G. (1989), "An Empirical Analysis of Cross-national Economic Growth, 1950-1980," *Journal of Monetary Economics* 24: 259-276.

Grossman, G. M. and Helpman, E. (1991), "Innovation and Growth in the Global Economy," Cambridge, MA: MIT Press.

Jones, L. E. and Manuelli, R. E. (1990), "A Convex Model of Equilibrium Growth: Theory and Policy Implications," *Journal of Political Economy* 98: 1008-1038.

Jones, L. E. Manuelli, R. E. and Rossi, P. E. (1997), "On the Optimal Taxation of Capital Income," *Journal of Economic Theory* 73: 93-117.

Koester, R. B. and Kormendi, R. C. (1988), "Taxation, Aggregate Activity and Economic Growth: Cross-Country Evidence on Some Supply-Side Hypotheses," *Economic Inquiry*.

Landau, D. L. (1986), "Government and Economic Growth in the Less Developed Countries: An Empirical Study for 1960-1980," *Economic Development and Cultural Change*: 35-75.

Lucas, R. E., Jr. (1988), "On the Mechanics of Economic Development," *Journal of Monetary Economics* 22:3-42.

Mankiw, G. N. Romer, D. and Weil, D. N.(1992), "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics* 107: 407-437.

Matsuyama, K. (1992), "Agricultural Productivity, Comparative Advantage, and Economic Growth," *Journal of Economic Theory* 58: 317-334.

Obstfeld, M. (1986), "Capital Mobility in the World Economy: Theory and Measurement," *Carnegie-Rochester Conference series on Public Policy*.

Psacharopoulos, G. and Woodhall, M. (1985), "Education for Development," Oxford University Press.

Rebelo, S.(1991), "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy* 99: 500-521.

Turnovsky, S. J. (1996), "Fiscal Policy, Growth, and Macroeconomic Performance in a Small Open Economy," *Journal of International Economics* 40: 41-66.

Young, A. (1991), "Learning by Doing and The Dynamic Effects of International Trade," *Quarterly Journal of Economics* 106: 369-405.