

MACRO FORECASTS BY USE OF A NONPARAMETRIC MONETARY AGGREGATE

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Since neither the conventional simple sum aggregates which do not consider the user cost or the interest rate elasticity, nor the theoretically preferred approach, the Divisia index, which considers the optimizing behavior of economic agents, performs as well as expected in econometric studies, estimation, and forecasting, I propose an alternative aggregate, HM, by applying a nonparametric estimation. This paper shows HM performs better than the other two monetary aggregates in forecasting major economic variables such as real GDP growth rate, inflation, and changes in interest rate.

JEL Classification: C14, E47

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I. INTRODUCTION

This paper looks at possible improvement in forecasts of certain macro variables (real GDP growth rate, inflation, and changes in interest rate) from using a nonparametric transform of monetary aggregates based on a money demand model. Conventional aggregation of the money stock is the simple sum of various components. In general there is little explanation provided as to why we should use this aggregation method. In fact, this method of aggregation is only plausible when the components of the money stock are perfect substitutes, an assumption that appears highly unlikely to be true. Nonetheless, many studies on money still rely upon these simple aggregates. Two commonly used simple sum aggregates are :

M1 (currency + total checkable deposit + travelers check), and

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M2 (M1 + total savings deposit + total small time deposit + money market mutual funds).

Because simple sum aggregation does not consider the optimizing behavior of economic agents, one line of research attempts to incorporate microeconomic foundations into monetary aggregation. Most well-known is so-called the Divisia indices proposed by William Barnett (1980, 1982). These are weighted measures reflecting the user costs of the various monetary assets. According to the Divisia index, the money stock can be aggregated as a weighted sum proportional to the user cost defined below:

$$S_{it} = \frac{\pi_{it} M_{it}}{\sum_{k=1}^K \pi_{kt} M_{kt}}, \quad \pi_{it} = P_t^* \left(\frac{R_t - r_{it}}{1 + R_t} \right). \quad (1)$$

where π_{it} is the opportunity cost of a component monetary asset or current-period nominal user cost, R_t is the return on a benchmark asset, and r_{it} is the own rate of return on the i -th asset.¹ The Divisia index is in principle very attractive from the view of monetary theory since it reflects all costs and benefits of each monetary component in order for its user to optimize consumption. Unfortunately, however, in practice it has not been uniformly successful.

In this paper a new approach is proposed, based on nonparametric estimation. I directly estimate the unknown transformation function of monetary components in each category of the money stock. When we are interested in estimating the money demand function, the following equation is one of the most popular models in the literature (Laidler, 1993):

$$M/P = f(I, R) + U, \quad (2)$$

where I is a scale indicator (e.g., income), and R is an opportunity cost. We can rewrite this equation as a common linear parametric form:

$$M/P = C + \beta_1 I + \beta_2 R + U. \quad (3)$$

The problem is that this linear equation loses its validity when the true relationship is not linear. There are basically two ways to cope with this problem. First, we could set up a nonlinear parametric model by specifying a specific functional form. But we have few clues as to the correct functional

¹ The Federal Reserve Bank of St. Louis publishes the Divisia index under the name of monetary services indexes. Monthly, quarterly, and annual data are available on its website (<http://www.stls.frb.org>).

form. A second approach is to estimate the model nonparametrically, without assuming an explicit functional form.

There are pros and cons for each approach. One drawback of the nonparametric approach is that we cannot easily propose an explicit functional form of estimator since it does not assume any.

II. NONPARAMETRIC ESTIMATION OF MONETARY AGGREGATES

Let Y be the growth rate of real money stock (M/P), X_1 be the growth rate of real GDP, and X_2 be the growth rate of the interest rate.² We can write a simple money demand equation as follows:

$$Y = F(X_1, X_2) + U. \quad (4)$$

For the sake of estimation, I took logs and differenced the data. The model considered by Horowitz (1996)³ is :

$$H(Y) = X\beta + U, \quad (5)$$

where X is $(n \times k)$, β is $(k \times 1)$, $H(\cdot)$ is an unknown monotonic function. This model belongs to a group called the semiparametric single-index model. Given the estimate of beta by minimizing the sum of squared residuals (Ichimura, 1993), I estimate the unknown transformation $H(y)$. Following are the procedures to derive it.

First, note that by (5), Y depends on X only through the index $Z \equiv X\beta$. Let $G(\cdot | Z)$ be the CDF of Y conditional on $Z = z$, and $F(\cdot)$ for the CDF of U or $H(Y) - X\beta$. Now define that $h(y) \equiv dH(y)/dy$, $f(y) \equiv dF(y)/dy$, and $G_y(y | z) \equiv \partial G(y | z)/\partial y$, $G_z(y | z) \equiv \partial G(y | z)/\partial z$. Also notice that equation (5) implies that

$$G(y | z) = F[H(y) - z]. \quad (6)$$

Therefore, $G_y(y | z) \equiv h(y)f[H(y) - z]$, $G_z(y | z) \equiv -f[H(y) - z]$, $h(y) = -G_y(y | z)/G_z(y | z)$, where $G_z(y | z) \neq 0$.

Hence we can obtain the following equation:

$$H(y) = - \int_{y_0}^y [G_y(v | z)/G_z(v | z)] dv. \quad (7)$$

² 3-month U.S. Treasury Bill rate.

³ Horowitz (1996) mostly considered cross-sectional data, but did not investigate macro time series or monetary aggregation as in this paper.

I used a kernel estimator to get the nonparametric distribution function, and then took derivatives with respect to y and z , respectively.

Estimation

First, the probability density function of z is estimated by the following kernel estimation:

$$p(z) = \frac{1}{nh_z} \sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right). \quad (8)$$

Based on this pdf, the cdf of y conditional on z can be estimated as follows:

$$G(y | z) = \frac{\frac{1}{nh_z} \sum_{i=1}^n 1(Y_i \leq y) K_z\left(\frac{Z_i - z}{h_z}\right)}{\frac{1}{nh_z} \sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right)}. \quad (9)$$

Now I take the partial derivatives with respect to y and z , respectively:

$$\begin{aligned} G_y(y | z) &= \frac{\frac{1}{h_y} \frac{1}{nh_z} \sum_{i=1}^n K_y\left(\frac{Y_i - y}{h_y}\right) K_z\left(\frac{Z_i - z}{h_z}\right)}{\frac{1}{nh_z} \sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right)} \\ &= \frac{1}{h_y} \frac{\sum_{i=1}^n K_y\left(\frac{Y_i - y}{h_y}\right) K_z\left(\frac{Z_i - z}{h_z}\right)}{\sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right)}. \end{aligned} \quad (10)$$

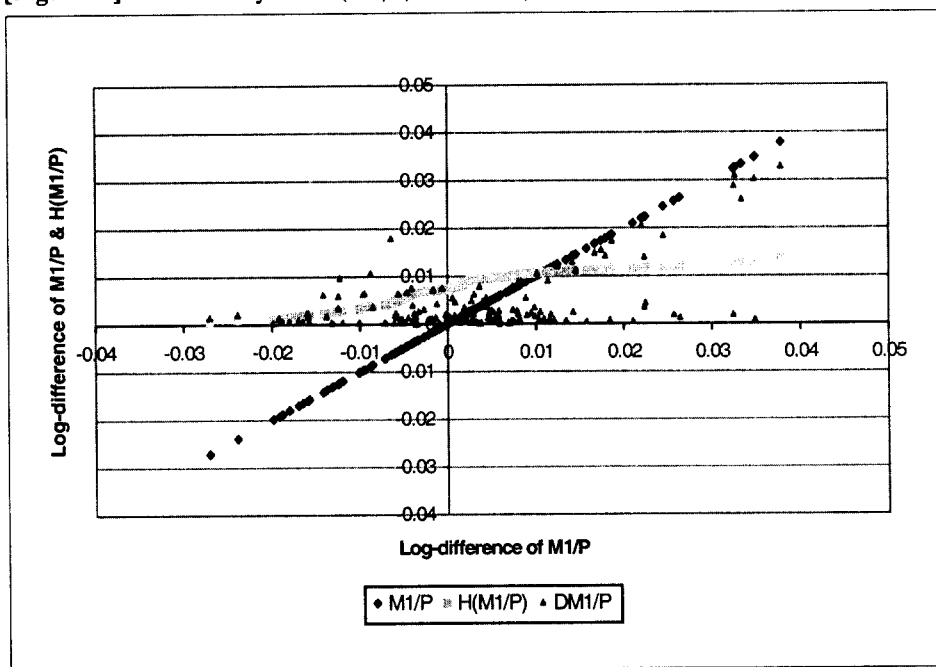
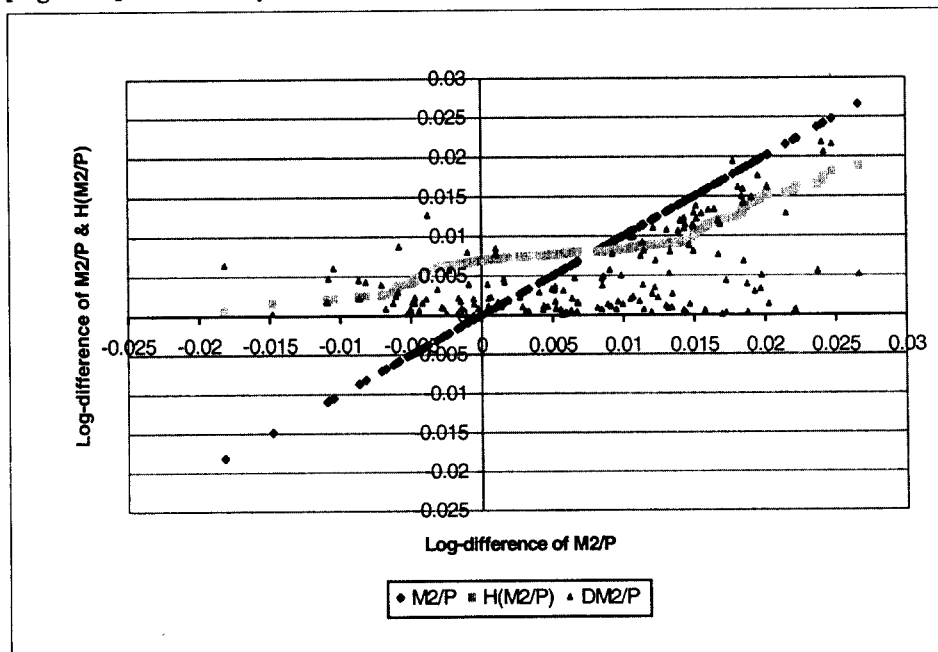
The derivation of $G_z(y | z)$ is shown in the Appendix.

To prevent the denominator becoming zero, I introduce a weight function $w(z)$ such that

$$\int_{s_w} w(z) dz = 1. \quad (11)$$

Therefore,

$$H(y) = - \int_{y_0}^y \int_{s_w} w(z) [G_y(v | z) / G_z(v | z)] dv dz. \quad (12)$$

[Figure 1] Nonlinearity of $H(M1/P)$ over $M1/P$.[Figure 2] Nonlinearity of $H(M2/P)$ over $M2/P$.

Since equation (5) holds if $H(y)$ and U are replaced by $H(y) + c$ and $U + c$ where c is a constant, and if $H(y)$, β , and U are replaced by $cH(y)$, $c\beta$, and cU , respectively for any $c > 0$,⁴ I set $H(y_0) = 0$ for some finite y_0 for location normalization, and for scale normalization I followed the literature on estimation of β in semiparametric single-index model. That is, arrange the components of X so that the first component of β satisfies $|\beta| = 1$. Figure 1 and figure 2 illustrate how the nonparametric estimate $H(Y)$ looks like compared to the simple sum aggregate or the Divisia index, where Y is the simple sum aggregate divided by price (M1/P or M2/P), and DM1 or DM2 is the Divisia index. These are scattered diagrams sorted by the simple sum aggregates. As we can see, $H(Y)$ shows significant nonlinearity against the simple sum aggregates whereas the Divisia indices are scattered irregularly around them. Thus, we expect that the nonlinearity of $H(Y)$ could give us an improvement in estimation and forecasting.

III. COMPARISON OF MONETARY AGGREGATES

There are many ways to compare different monetary aggregates. Here I compare the forecasting ability of different aggregates in forecasting the growth rate of real GDP, the price level, and the interest rate. Two more tests are introduced to compare the forecast ability, which are the forecast encompassing test (Chong and Hendry, 1986) and the equality test of forecast accuracy (Diebold and Mariano, 1995).

3.1. Model for Forecasting Ability

To do this I estimate the model over the first half of the data, and based on this estimation, I predict the subsequent value, and iterate these steps until I obtain the last value for prediction. I then predict the relevant variables by applying the same procedure and compare the results between the three monetary aggregates. A VAR (vector autoregressions) model forms my benchmark model for these three variables⁵. The lags were determined by Schwartz Information Criterion (SIC). That is, the combination of lags of each variable was chosen to minimize the following (Enders, 1995):

$$SIC = T \ln(\text{residual sum of squares}) + n \ln(T), \quad (13)$$

⁴ Horowitz (1996) proved that this location and scale normalization do not affect the result of the proposed estimation.

⁵ Since both Engle-Granger (1987) and Johansen cointegration test reject the null hypothesis of no cointegration, a VEC (vector error correction) model could perform better. But I choose a simple VAR model for benchmark because this paper mainly focuses on the comparison of forecasts based on different monetary aggregates rather than the forecast accuracy itself.

where n is the number of parameters estimated ($p+q$ + possible constant term), and T is the number of usable observations. Data are seasonally adjusted real GDP, the GDP deflator, the federal funds rate, and three different sets of monetary aggregates (simple sum, Divisia, and HM for both M1 and M2). The data range is 1960:1- 2001:4 and all data are log-differenced.

With real GDP as the dependent variable, real GDP and the interest rate have four lagged variables while price and the money stock each have only one lagged variable.⁶

$$y_t = C_y + \begin{pmatrix} \beta_y \\ \beta_p \\ \beta_m \\ \beta_r \end{pmatrix}_{t-1} \begin{pmatrix} y \\ p \\ m \\ r \end{pmatrix}_{t-1} + \begin{pmatrix} \beta_y \\ \beta_r \end{pmatrix}_{t-2} \begin{pmatrix} y \\ r \end{pmatrix}_{t-2} + \begin{pmatrix} \beta_y \\ \beta_r \end{pmatrix}_{t-3} \begin{pmatrix} y \\ r \end{pmatrix}_{t-3} + \begin{pmatrix} \beta_y \\ \beta_r \end{pmatrix}_{t-4} \begin{pmatrix} y \\ r \end{pmatrix}_{t-4} + u_y \quad (14)$$

For price as the dependent variable, each variable has only one lagged value.

$$p_t = C_p + \begin{pmatrix} \beta_y \\ \beta_p \\ \beta_m \\ \beta_r \end{pmatrix}_{t-1} \begin{pmatrix} y \\ p \\ m \\ r \end{pmatrix}_{t-1} + u_p \quad (15)$$

And with the interest rate as the dependent variable, real GDP has three lags while other variables have four lags.

$$r_t = C_r + \begin{pmatrix} \beta_y \\ \beta_p \\ \beta_m \\ \beta_r \end{pmatrix}_{t-2} \begin{pmatrix} y \\ p \\ m \\ r \end{pmatrix}_{t-2} + \begin{pmatrix} \beta_y \\ \beta_p \\ \beta_m \\ \beta_r \end{pmatrix}_{t-1} \begin{pmatrix} y \\ p \\ m \\ r \end{pmatrix}_{t-1} + \begin{pmatrix} \beta_y \\ \beta_p \\ \beta_m \\ \beta_r \end{pmatrix}_{t-3} \begin{pmatrix} y \\ p \\ m \\ r \end{pmatrix}_{t-3} + \begin{pmatrix} \beta_p \\ \beta_m \\ \beta_r \end{pmatrix}_{t-4} \begin{pmatrix} p \\ m \\ r \end{pmatrix}_{t-4} + u_r \quad (16)$$

3.2. Four-Quarter-Ahead Forecasting of Different Monetary Aggregates

Based on the model specification determined by the Schwartz Information Criterion (SIC), four-quarter-ahead out-of-sample forecasts upon the growth rate

⁶ I estimated all possible combinations up to a maximum of four lags, except I did not allow holes in the lag structure.

of real GDP, the price level, and the interest rate were conducted respectively. The superiority of HM over the simple sum or the Divisia index becomes clear in these forecasts in terms of forecast errors.

Forecasting Real GDP

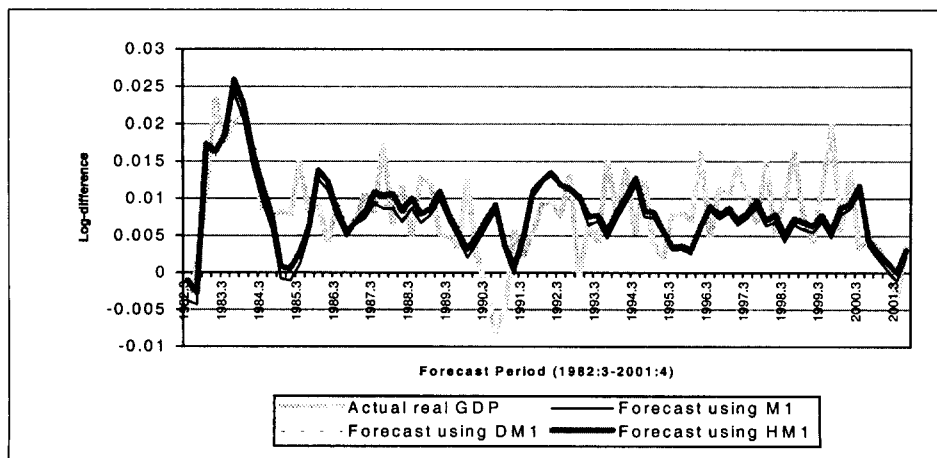
Real GDP was forecasted for 1982:3–2001:4 based on the data from 1960:1 to 1982:2 (Table 1). The superiority of HM is more evident in M1 group in terms of forecast errors (0.005586 for RMSE and 0.00003120 for APMSE). Notice that the forecast errors with HM are significantly smaller than those with the other two monetary aggregates. HM even performs better than the Divisia in forecasting real GDP while the forecast with the Divisia performs slightly better than that with the simple sum in both M1 and M2 group. The following two graphs (Figures 3 and 4) illustrate these differences easily.

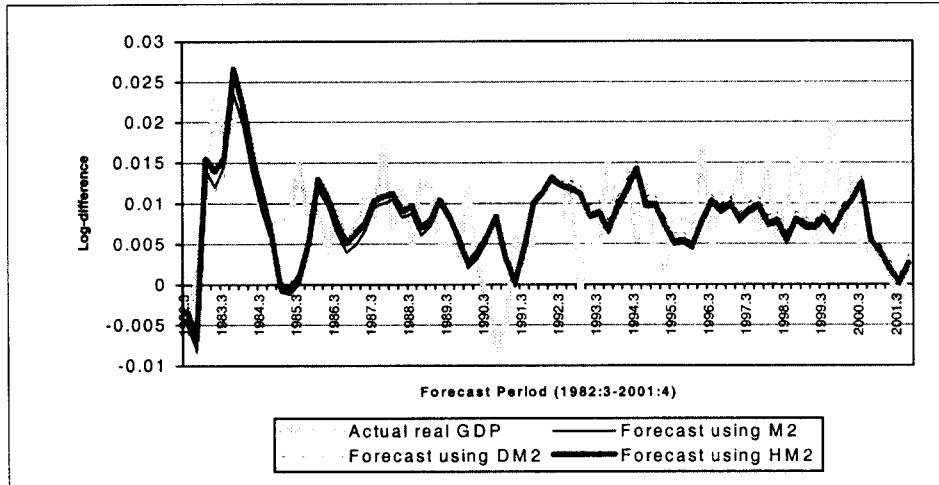
[Table 1] 4-Quarter-Ahead Forecast Errors of Different Monetary Aggregates on Real GDP

Money	Lag(y,p,m,r)	RMSE	APMSE
M1	(4,1,1,4)	0.00571900	0.00003271
DM1	(4,1,1,4)	0.00570448	0.00003254
HM1	(4,1,1,4)	0.00558594	0.00003120
M2	(4,1,1,4)	0.00569609	0.00003245
DM2	(4,1,1,4)	0.00565694	0.00003200
HM2	(4,1,1,4)	0.00559159	0.00003127

Notes: M is simple sum, DM is Divisia, HM is the nonparametric estimate.

[Figure 3] 4-Quarter-Ahead Forecast of Different Monetary Aggregates on Real GDP (M1 Group).



[Figure 4] 4-Quarter-Ahead Forecast of Different Monetary Aggregates on Real GDP (M2 Group).

Forecasting Price

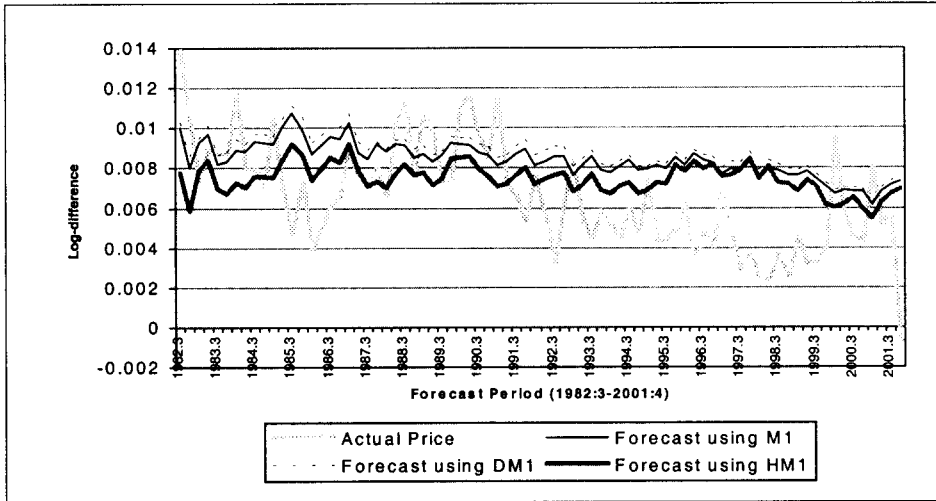
Price was forecasted for 1982:3–2001:4 based on the data from 1960:1 to 1982:2 (Table 2). Four-quarter-ahead forecasts on price reinforce the superiority of HM over the other two forecasts. RMSE and APMSE of the forecast with HM are significantly smaller than those of the other two (0.002874, 0.00000826 for M1 group, and 0.002406, 0.00000579 for M2 group, respectively). Notice that the RMSE and APMSE of the forecast with the Divisia (0.003303, 0.00001091 for M1 group, and 0.003104, 0.00000964 for M2 group, respectively) are much bigger than those with the simple sum (0.003088, 0.00000953 for M1 group, and 0.002509, 0.00000629 for M2 group, respectively). Thus, the forecast for price using HM performs best while the forecast using the Divisia index performs worst here. These differences can be easily identified in the following two graphs (Figures 5 and 6).

[Table 2] 4-Quarter-Ahead Forecast Errors of Different Monetary Aggregates on Price

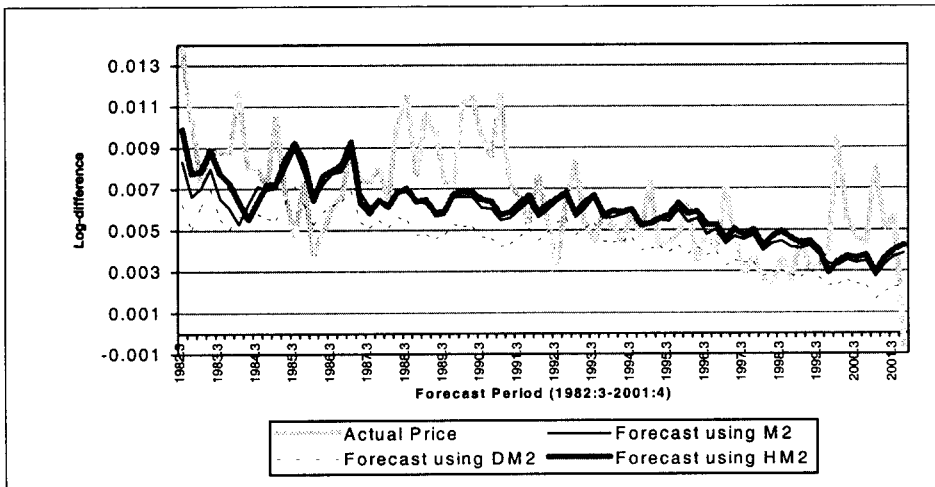
Money	Lag(y,p,m,r)	RMSE	APMSE
M1	(1,1,1,1)	0.00308765	0.00000953
DM1	(1,1,1,1)	0.00330307	0.00001091
HM1	(1,1,1,1)	0.00287364	0.00000826
M2	(1,1,1,1)	0.00250890	0.00000629
DM2	(1,1,1,1)	0.00310405	0.00000964
HM2	(1,1,1,1)	0.00240634	0.00000579

Notes: M is simple sum, DM is Divisia, HM is the nonparametric estimate.

[Figure 5] 4-Quarter-Ahead Forecast of Different Monetary Aggregates on Price (M1 Group).



[Figure 6] 4-Quarter-Ahead Forecast of Different Monetary Aggregates on Price (M2 Group).



Forecasting the Interest Rate

Interest rate was forecasted for 1982:3–2001:4 based on the data from 1960:1 to 1982:2 (Table 3). Overall the forecast errors are smaller in M2 group than those in M1 group. The superiority of HM is more evident in M1 group in terms of forecast errors (0.007370 for RMSE and 0.00005432 for APMSE), for these forecast errors are significantly smaller than those of the other two

forecasts. And the Divisia M1 performs slightly better than the simple sum in forecasting the interest rate (0.008266 for RMSE and 0.00006833 for APMSE).

For M2 group, the model with HM2 also forecasts best in RMSE and APMSE among the three forecasts (0.007438, 0.00005532, respectively). The Divisia M2 also performs better than the simple sum in forecasting the interest rate (0.007481 for RMSE and 0.00005596 for APMSE).

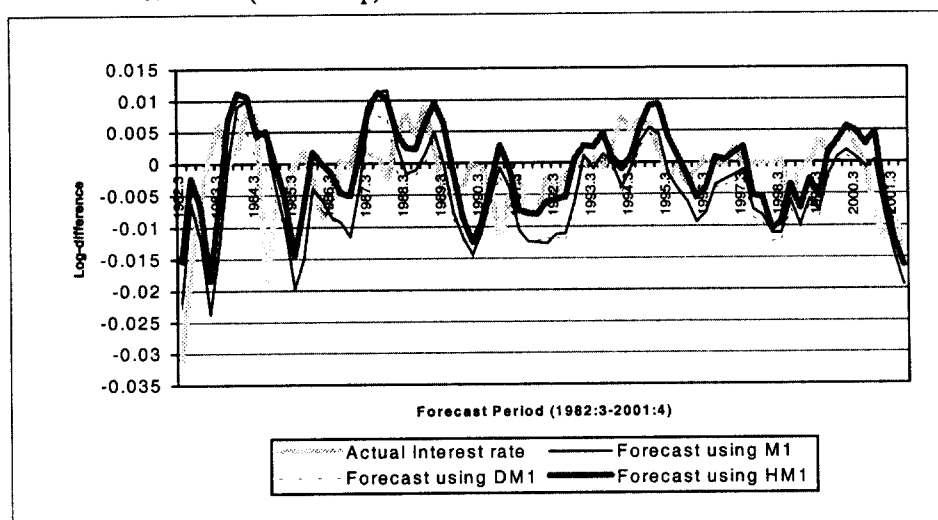
Thus both HM and the Divisia outperform the simple sum in forecasting the interest rate. Notice that the interest rate used here is not real but nominal interest rate (log-differenced federal funds rate). The following two graphs (Figures 7 and 8) illustrate these differences easily.

[Table 3] 4-Quarter-Ahead Forecast Errors of Different Monetary Aggregates on Interest rate

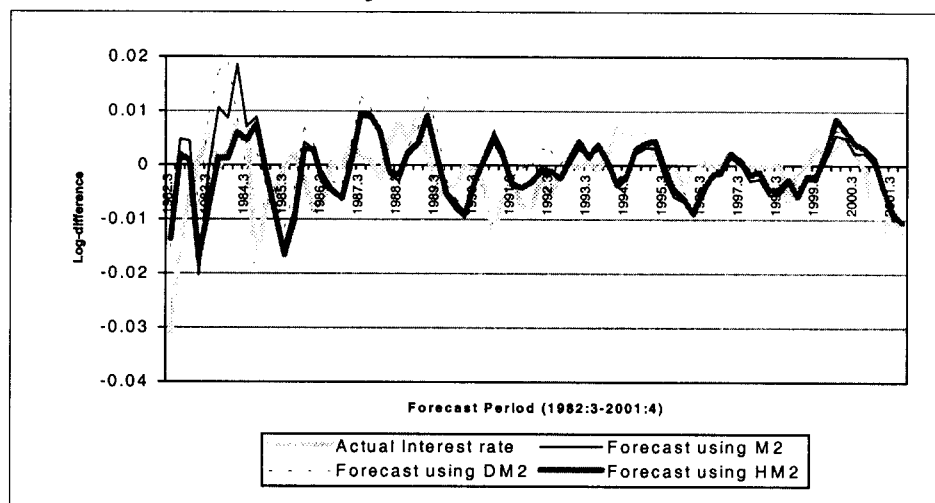
Money	Lag (y, p, m, r)	RMSE	APMSE
M1	(3,4,4,4)	0.00835459	0.00006980
DM1	(3,4,4,4)	0.00826600	0.00006833
HM1	(3,4,4,4)	0.00737046	0.00005432
M2	(3,4,4,4)	0.00793875	0.00006302
DM2	(3,4,4,4)	0.00748058	0.00005596
HM2	(3,4,4,4)	0.00743759	0.00005532

Notes: M is simple sum, DM is Divisia, HM is the nonparametric estimate.

[Figure 7] 4-Quarter-Ahead Forecast of Different Monetary Aggregates on Interest Rate (M1 Group).



[Figure 8] 4-Quarter-Ahead Forecast of Different Monetary Aggregates on Interest Rate (M2 Group).



3.3. Encompassing Tests for Four-Quarter-Ahead Forecast

The comparison of forecast ability involves many different approaches. To compare the relative size of mean square errors is one of the most common approaches in the literature. But the size of the MSE cannot tell us everything we want to know. Often we are interested in whether the information available in one forecast helps explain forecast errors in another forecast. Suppose there are two competing forecasts, forecast *A* and forecast *B*. If forecast *A* can help explain forecast errors in forecast *B* while forecast *B* cannot help explain forecast errors in forecast *A*, then forecast *A* contains more information than forecast *B*. This forecast encompassing test was considered by Chong and Hendry(1986).⁷

Consider the same forecasts, $\{\hat{y}_t^A\}_{t=1}^T$ and $\{\hat{y}_t^B\}_{t=1}^T$ of the time series $\{y_t\}_{t=1}^T$. We can write an artificial compound model:

$$y_t = \alpha \hat{y}_t^A + (1 - \alpha) \hat{y}_t^B, \quad (17)$$

where \hat{y}_t^L ($L=A, B$) is the forecasts. If α equals one, forecast *A* is sufficient to forecast *y*, and if α equals zero, forecast *B* is sufficient to forecast *y*. We can rearrange the above equation as

⁷ Chong and Hendry (1986), *Econometric Evaluation of Linear Macroeconomic Models*, *Review of Economic Studies* Vol.53, 676-679. Also see Jansen and Kishan (1996), *An Evaluation of Federal Reserve Forecasting*, *Journal of Macroeconomics* Vol.18, 105-107.

$$y_t - \hat{y}_t^A = (1 - \alpha)[(y_t - \hat{y}_t^A) - (y_t - \hat{y}_t^B)] + e_t, \quad (18)$$

and

$$y_t - \hat{y}_t^B = \alpha[(y_t - \hat{y}_t^B) - (y_t - \hat{y}_t^A)] + e_t. \quad (19)$$

Likewise, if α is significantly different from one in (18), forecast B can be said to encompass forecast A , and if α is significantly different from zero in (19), forecast A can be said to encompass forecast B . If α is not significantly different from one in (18) while α is significantly different from zero in (19), then forecast A is clearly superior to forecast B since the former contains information not available in the latter. We can easily rewrite the above two regressions as

$$e_t^A = (1 - \alpha)(e_t^A - e_t^B) + \varepsilon_t, \quad (20)$$

and

$$e_t^B = \alpha(e_t^B - e_t^A) + \varepsilon_t. \quad (21)$$

The following two tables (Tables 4 and 5) show the encompassing tests between the simple sum, the Divisia, and HM on three variables (real GDP, price, and the interest rate). The test compares three cases: simple sum vs. the Divisia, simple sum vs. HM, and the Divisia vs. HM.

First, I compare the test between the simple sum and the Divisia. The forecast for real GDP does not encompass each other because none of the test statistics is significant in both M1 and M2 group. However, the interest rate forecast with the Divisia encompasses that with the simple sum at the 1.4% for M1 group (t-statistic 2.52) and 0% significance level for M2 group (t-statistic 4.74), but the latter does not. Thus, the Divisia is superior to the simple sum in forecasting the interest rate. For price, the forecast with the simple sum encompasses that with the Divisia at the 0% significance level for both M1 group (t-statistic 3.94) and M2 group (t-statistic 4.25) while the latter encompasses the former at the 0.3% significance level for M1 group but not for M2 group (t-statistic -1.14, probability value 0.257). Thus, it can be said that the simple sum is superior to the Divisia in forecasting price at least for M2 group.

Second, I compare the test between the simple sum and HM. The superiority of HM over the simple sum is most evident here. For real GDP, the forecast with HM encompasses that with the simple sum at the 8.6% for M1 group (t-statistic 1.74) and 12.2% significance level for M2 group (t-statistic 1.56) while the latter does not encompass the former for both M1 and M2 group. For

price, the forecast with HM encompasses that with the simple sum at the 3.5% for M1 group (t-statistic 2.15) and 2.3% significance level for M2 group (t-statistic 2.32) while the latter does not encompass the former for both M1 and M2 group. For the interest rate, the forecast with HM encompasses that with the simple sum at the 0% significance level for both M1 group (t-statistic 5.95) and M2 group (t-statistic 4.57) while the latter does not encompass the former for both M1 and M2 group. Thus, HM is superior to the simple sum in forecasting ability.

Third, I compare the test between the Divisia and HM. The superiority of HM over the Divisia is also most evident in forecasting price and the interest rate. For price, the forecast with HM encompasses that with the Divisia at the 0.4% for M1 group (t-statistic 2.97) and 0% significance level for M2 group (t-statistic 4.79) while the latter does not encompass the former for both M1 and M2 group. For the interest rate, the forecast with HM encompasses that

[Table 4] 4-Quarter-Ahead Forecast Encompassing Tests of Different Monetary Aggregates (M1 Group)

Forecasted Variable	Dependent Variable	Independent Variable	t-statistic (marginal significance)
4-Quarter-Ahead real GDP	e^{M1}	$(e^{M1} - e^{DM1})$	0.77 (0.443)
	e^{DM1}	$(e^{DM1} - e^{M1})$	-0.46 (0.646)
4-Quarter-Ahead Price	e^{M1}	$(e^{M1} - e^{DM1})$	-3.12 (0.003)
	e^{DM1}	$(e^{DM1} - e^{M1})$	3.94 (0.000)
4-Quarter-Ahead Interest rate	e^{M1}	$(e^{M1} - e^{DM1})$	2.52 (0.014)
	e^{DM1}	$(e^{DM1} - e^{M1})$	-1.35 (0.182)
4-Quarter-Ahead real GDP	e^{M1}	$(e^{M1} - e^{HM1})$	1.74 (0.086)
	e^{HM1}	$(e^{HM1} - e^{M1})$	-0.22 (0.825)
4-Quarter-Ahead Price	e^{M1}	$(e^{M1} - e^{HM1})$	2.15 (0.035)
	e^{HM1}	$(e^{HM1} - e^{M1})$	-0.07 (0.943)
4-Quarter-Ahead Interest rate	e^{M1}	$(e^{M1} - e^{HM1})$	5.95 (0.000)
	e^{HM1}	$(e^{HM1} - e^{M1})$	0.29 (0.771)
4-Quarter-Ahead real GDP	e^{DM1}	$(e^{DM1} - e^{HM1})$	1.61 (0.113)
	e^{HM1}	$(e^{HM1} - e^{DM1})$	-0.28 (0.782)
4-Quarter-Ahead Price	e^{DM1}	$(e^{DM1} - e^{HM1})$	2.97 (0.004)
	e^{HM1}	$(e^{HM1} - e^{DM1})$	-0.30 (0.764)
4-Quarter-Ahead Interest rate	e^{DM1}	$(e^{DM1} - e^{HM1})$	6.69 (0.000)
	e^{HM1}	$(e^{HM1} - e^{DM1})$	0.95 (0.345)

Estimated equation: $e_t^A = \alpha(e_t^A - e_t^B) + \varepsilon_t$, where ε_t follows a MA(3).

with the Divisia at the 0% for M1 group (t-statistic 6.69) and 0.6% significance level for M2 group (t-statistic 2.81) while the latter does not encompass the former for both M1 and M2 group. For real GDP, it is not so clear which one is superior in M2 group though the forecast with HM weakly encompasses that with the Divisia at the 11.3% significance level (t-statistic 1.61) in M1 group.

[Table 5] 4-Quarter-Ahead Forecast Encompassing Tests of Different Monetary Aggregates (M2 Group)

Forecasted Variable	Dependent Variable	Independent Variable	t-statistic (marginal significance)
4-Quarter-Ahead real GDP	e^{M2}	$(e^{M2} - e^{DM2})$	1.29 (0.202)
	e^{DM2}	$(e^{DM2} - e^{M2})$	0.37 (0.713)
4-Quarter-Ahead Price	e^{M2}	$(e^{M2} - e^{DM2})$	-1.14 (0.257)
	e^{DM2}	$(e^{DM2} - e^{M2})$	4.25 (0.000)
4-Quarter-Ahead Interest rate	e^{M2}	$(e^{M2} - e^{DM2})$	4.74 (0.000)
	e^{DM2}	$(e^{DM2} - e^{M2})$	-0.72 (0.474)
4-Quarter-Ahead real GDP	e^{M2}	$(e^{M2} - e^{HM2})$	1.56 (0.122)
	e^{HM2}	$(e^{HM2} - e^{M2})$	-0.27 (0.787)
4-Quarter-Ahead Price	e^{M2}	$(e^{M2} - e^{HM2})$	2.32 (0.023)
	e^{HM2}	$(e^{HM2} - e^{M2})$	-0.94 (0.349)
4-Quarter-Ahead Interest rate	e^{M2}	$(e^{M2} - e^{HM2})$	4.57 (0.000)
	e^{HM2}	$(e^{HM2} - e^{M2})$	-1.29 (0.202)
4-Quarter-Ahead real GDP	e^{DM2}	$(e^{DM2} - e^{HM2})$	0.93 (0.358)
	e^{HM2}	$(e^{HM2} - e^{DM2})$	0.46 (0.647)
4-Quarter-Ahead Price	e^{DM2}	$(e^{DM2} - e^{HM2})$	4.79 (0.000)
	e^{HM2}	$(e^{HM2} - e^{DM2})$	-0.81 (0.423)
4-Quarter-Ahead Interest rate	e^{DM2}	$(e^{DM2} - e^{HM2})$	2.81 (0.006)
	e^{HM2}	$(e^{HM2} - e^{DM2})$	1.71 (0.091)

Estimated equation: $e_t^A = \alpha(e_t^A - e_t^B) + \varepsilon_t$, where ε_t follows a MA(3).

3.4. Equality Tests for Four-Quarter-Ahead Forecast Accuracy

There are many ways to compare the predictive accuracy of time series econometric models in the literature. Among them, testing equality of forecast accuracy is one of the most common and convenient approaches. Here I introduce an equality test for forecast accuracy studied by Diebold and Mariano (1995).

Consider two forecasts, $\{\hat{y}_{1,t}\}_{t=1}^T$ and $\{\hat{y}_{2,t}\}_{t=1}^T$, of the time series $\{y_t\}_{t=1}^T$.

And the forecast errors are given by $\{e_{1,t}\}_{t=1}^T$ and $\{e_{2,t}\}_{t=1}^T$. Define the loss differential equation by $d_t \equiv g(e_{1,t}) - g(e_{2,t}) = e_{1,t}^4 - e_{2,t}^4$. Then the null hypothesis of equal forecast accuracy for two forecasts is $E[g(e_{1,t})] = E[g(e_{2,t})]$, or $E[d_t] = 0$. Thus, the test statistic is:

$$\frac{\bar{d}_t - 0}{\sigma_d} = \frac{\frac{1}{T} \sum_{t=1}^T d_t}{\sqrt{\text{est. var}(d_t)/T}} \sim t\text{-distribution under the null hypothesis.}$$

If the first and the second forecast had exactly the same forecast accuracy, the null hypothesis could not be rejected. If the second forecast performed better than the first one, the loss differential equation would not be all zero. Hence we could reject the null hypothesis of equal forecast accuracy, concluding the second forecast is superior to the first forecast at the given significance level.

The test (D-M test) compares three cases for each forecast: simple sum vs.

[Table 6] 4-Quarter-Ahead D-M Tests of Different Monetary Aggregates (M1 Group)

Forecasted Variable	Null Hypothesis	D-M statistic (marginal significance)
4-Quarter-Ahead Real GDP	$E[d] \equiv [g(e^{M1}) - g(e^{DM1})] = 0$	-0.27 (0.605)
	$E[d] \equiv [g(e^{DM1}) - g(e^{M1})] = 0$	0.27 (0.395)
4-Quarter-Ahead Price	$E[d] \equiv [g(e^{M1}) - g(e^{DM1})] = 0$	-4.49 (1.000)
	$E[d] \equiv [g(e^{DM1}) - g(e^{M1})] = 0$	4.49 (0.000)
4-Quarter-Ahead Interest rate	$E[d] \equiv [g(e^{M1}) - g(e^{DM1})] = 0$	1.18 (0.120)
	$E[d] \equiv [g(e^{DM1}) - g(e^{M1})] = 0$	-1.18 (0.880)
4-Quarter-Ahead Real GDP	$E[d] \equiv [g(e^{M1}) - g(e^{HMI})] = 0$	0.95 (0.173)
	$E[d] \equiv [g(e^{HMI}) - g(e^{M1})] = 0$	-0.95 (0.827)
4-Quarter-Ahead Price	$E[d] \equiv [g(e^{M1}) - g(e^{HMI})] = 0$	1.25 (0.108)
	$E[d] \equiv [g(e^{HMI}) - g(e^{M1})] = 0$	-1.25 (0.892)
4-Quarter-Ahead Interest rate	$E[d] \equiv [g(e^{M1}) - g(e^{HMI})] = 0$	1.50 (0.069)
	$E[d] \equiv [g(e^{HMI}) - g(e^{M1})] = 0$	-1.50 (0.931)
4-Quarter-Ahead Real GDP	$E[d] \equiv [g(e^{DM1}) - g(e^{HMI})] = 0$	1.31 (0.097)
	$E[d] \equiv [g(e^{HMI}) - g(e^{DM1})] = 0$	-1.31 (0.903)
4-Quarter-Ahead Price	$E[d] \equiv [g(e^{DM1}) - g(e^{HMI})] = 0$	2.56 (0.006)
	$E[d] \equiv [g(e^{HMI}) - g(e^{DM1})] = 0$	-2.56 (0.994)
4-Quarter-Ahead Interest rate	$E[d] \equiv [g(e^{DM1}) - g(e^{HMI})] = 0$	0.62 (0.268)
	$E[d] \equiv [g(e^{HMI}) - g(e^{DM1})] = 0$	-0.62 (0.732)

Loss function: $g(e_t) = e_t^4$, D-M statistic: $\frac{\bar{d}}{\sigma_d}$.

[Table 7] 4-Quarter-Ahead D-M Tests of Different Monetary Aggregates (M2 Group)

Forecasted Variable	Null Hypothesis	D-M statistic (marginal significance)
4-Quarter-Ahead Real GDP	$E[d] = [g(e^{M2}) - g(e^{DM2})] = 0$	0.26 (0.397)
	$E[d] = [g(e^{DM2}) - g(e^{M2})] = 0$	-0.26 (0.603)
4-Quarter-Ahead Price	$E[d] = [g(e^{M2}) - g(e^{DM2})] = 0$	-3.33 (0.999)
	$E[d] = [g(e^{DM2}) - g(e^{M2})] = 0$	3.33 (0.001)
4-Quarter-Ahead Interest rate	$E[d] = [g(e^{M2}) - g(e^{DM2})] = 0$	1.31 (0.096)
	$E[d] = [g(e^{DM2}) - g(e^{M2})] = 0$	-1.31 (0.904)
4-Quarter-Ahead Real GDP	$E[d] = [g(e^{HM2}) - g(e^{DM2})] = 0$	1.12 (0.132)
	$E[d] = [g(e^{DM2}) - g(e^{HM2})] = 0$	-1.12 (0.868)
4-Quarter-Ahead Price	$E[d] = [g(e^{M2}) - g(e^{HM2})] = 0$	1.94 (0.028)
	$E[d] = [g(e^{HM2}) - g(e^{M2})] = 0$	-1.94 (0.972)
4-Quarter-Ahead Interest rate	$E[d] = [g(e^{M2}) - g(e^{HM2})] = 0$	1.91 (0.030)
	$E[d] = [g(e^{HM2}) - g(e^{M2})] = 0$	-1.91 (0.970)
4-Quarter-Ahead Real GDP	$E[d] = [g(e^{DM2}) - g(e^{HM2})] = 0$	0.77 (0.220)
	$E[d] = [g(e^{HM2}) - g(e^{DM2})] = 0$	-0.77 (0.780)
4-Quarter-Ahead Price	$E[d] = [g(e^{DM2}) - g(e^{HM2})] = 0$	3.32 (0.001)
	$E[d] = [g(e^{HM2}) - g(e^{DM2})] = 0$	-3.32 (0.999)
4-Quarter-Ahead Interest rate	$E[d] = [g(e^{DM2}) - g(e^{HM2})] = 0$	-0.82 (0.793)
	$E[d] = [g(e^{HM2}) - g(e^{DM2})] = 0$	0.82 (0.207)

Loss function: $g(e_t) = e_t^4$, D-M statistic: $\frac{\bar{d}}{\sigma_d}$.

the Divisia, simple sum vs. HM, and the Divisia vs. HM. Tables 6 and 7 report each case with one-sided p-values.

First, I compare the test between the simple sum and the Divisia. For real GDP, the test statistic is not significant so we cannot tell which one is better than the other for both M1 and M2 group. For price, D-M statistic of the forecast with the simple sum against the Divisia shows 4.49 with 0% p-value for M1 group and 3.33 with 0.1% p-value for M2 group. This indicates that the simple sum outperforms the Divisia in forecasting price, which is consistent with the previous encompassing test. For the interest rate, the test statistic shows a weak superiority of the Divisia over the simple sum at the 12% significance level for M1 group. But it shows a sufficient superiority of the Divisia over the simple sum at the 9.6% significance level for M2 group, which is also consistent with the previous result of the encompassing test.

Second, I compare the test between the simple sum and HM. Overall the test shows a superiority of HM over the simple sum. For real GDP, the test shows

that the forecast with HM is superior to that with the simple sum at the 17.3% significance level for M1 group (D-M statistic 0.95) and 13.2% significance level for M2 group (D-M statistic 1.12), though they are not so significant. For price, HM is superior to the simple sum in forecasting at the 10.8% significance level for M1 group (D-M statistic 1.25) and 2.8% significance level for M2 group (D-M statistic 1.94). For the interest rate, HM is superior to the simple sum in forecasting at the 6.9% significance level for M1 group (D-M statistic 1.50) and 3.0% significance level for M2 group (D-M statistic 1.91). This result also confirms the superiority of HM over the simple sum aggregates consistently with the encompassing test.

Third, I compare the test between the Divisia and HM. For real GDP, HM is superior to the Divisia in forecasting at the 9.7% significance level for M1 group (D-M statistic 1.31) but the test statistic is not significant for M2 group (D-M statistic 0.77). For price, it is most evident that HM is superior to the Divisia in forecasting at the 0.6% significance level for M1 group (D-M statistic 2.56) and 0.1% significance level for M2 group (D-M statistic 3.32). For the interest rate, the test result is rather ambiguous because it does not show a significant test statistic and the direction of superiority is not consistent, either.

IV. CONCLUSIONS

The question of what is money is not settled conclusively. There are so many things to consider before settling down on an answer to this question, such as user cost, elasticity, and aggregation as well as acceptability, profitability, and stability. The aggregation issue matters especially when we are interested in using a quantity variable in a scientific model, because we need to know the exact volume or magnitude in order to explain the interrelationship among the relevant variables.

I investigated three different aggregations of money stock to compare the forecasting ability in this paper. The Divisia index is still one of the most theoretically preferred aggregations. However, the simple sum monetary aggregates are still widely used in the literature not because it is more preferred but because the Divisia index does not perform better in many analyses.

HM, newly nonparametrically estimated, comes from a concise concept based on a simple money demand model. I tried to show whether HM performs better than the other two monetary aggregates in analyzing and forecasting macroeconomic variables such as real GDP growth rate, inflation, and changes in interest rate. If so, notwithstanding the hesitance to replace the simple sum aggregates with HM, I carefully propose to utilize it as a complementary index. I believe this paper shows its usefulness at least in forecasting those variables.

Further study could expand the model of unknown transformation of dependent variable to incorporate different coefficients to the components of monetary aggregate. Also the difficulty to recover HM to pre-differenced aggregate limits

its use as an alternative monetary aggregate. Nonetheless, I hope the nonparametric approach studied in this paper will shed light on the issue of monetary aggregation and macro forecasting, giving us an additional avenue worth exploring.

APPENDIX

Estimation of $H(y)$

First, the probability density function is estimated by the following kernel estimation:

$$p(z) = \frac{1}{nh_z} \sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right). \quad (22)$$

Based on this pdf, the cdf of y conditional on z can be estimated as follows:

$$G(y | z) = \frac{\frac{1}{nh_z} \sum_{i=1}^n 1(Y_i \leq y) K_z\left(\frac{Z_i - z}{h_z}\right)}{\frac{1}{nh_z} \sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right)}. \quad (23)$$

Now take the partial derivatives with respect to y and z , respectively:

$$\begin{aligned} G_y(y | z) &= \frac{\frac{1}{h_y} \frac{1}{nh_z} \sum_{i=1}^n K_y\left(\frac{Y_i - y}{h_y}\right) K_z\left(\frac{Z_i - z}{h_z}\right)}{\frac{1}{nh_z} \sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right)} \\ &= \frac{1}{h_y} \frac{\sum_{i=1}^n K_y\left(\frac{Y_i - y}{h_y}\right) K_z\left(\frac{Z_i - z}{h_z}\right)}{\sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right)}. \end{aligned} \quad (24)$$

Here I use a normal distribution for kernel estimator. That is,

$$\begin{aligned} K\left(\frac{Z_i - z}{h_z}\right) &= \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{Z_i - z}{h_z}\right)^2}, \\ \text{and } K'\left(\frac{Z_i - z}{h_z}\right) &= \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{Z_i - z}{h_z}\right)^2} \cdot -\left(\frac{Z_i - z}{h_z}\right) \cdot \frac{1}{h_z} \\ &= \frac{1}{h_z} \cdot \left(\frac{Z_i - z}{h_z}\right) \cdot K\left(\frac{Z_i - z}{h_z}\right). \end{aligned}$$

Hence,

$$G_z(y | z) = \frac{d}{dz} \left(\frac{\frac{1}{nh_z} \sum_{i=1}^n 1(Y_i \leq y) K_z\left(\frac{Z_i - z}{h_z}\right)}{\frac{1}{nh_z} \sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right)} \right) =$$

$$\begin{aligned}
& \frac{1}{h_z} \frac{\left[\sum_{i=1}^n 1(Y_i \leq y) \cdot K_z\left(\frac{Z_i - z}{h_z}\right) \cdot \left(\frac{Z_i - z}{h_z}\right) \right] \cdot \sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right)}{\left[\sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right) \right]^2} \\
& - \frac{1}{h_z} \frac{\left[\sum_{i=1}^n 1(Y_i \leq y) K_z\left(\frac{Z_i - z}{h_z}\right) \right] \left[\sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right) \cdot \left(\frac{Z_i - z}{h_z}\right) \right]}{\left[\sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right) \right]^2}.
\end{aligned} \quad (25)$$

Therefore,

$$\begin{aligned}
& \frac{G_y(y | z)}{G_z(y | z)} = \\
& \frac{h_z}{h_y} \frac{\left[\sum_{i=1}^n K_y\left(\frac{Y_i - y}{h_y}\right) \cdot K_z\left(\frac{Z_i - z}{h_z}\right) \right] \cdot \sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right)}{\left[\sum_{i=1}^n 1(Y_i \leq y) \cdot K_z\left(\frac{Z_i - z}{h_z}\right) \cdot \left(\frac{Z_i - z}{h_z}\right) \right] \cdot \sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right)} \\
& - \frac{\left[\sum_{i=1}^n 1(Y_i \leq y) \cdot K_z\left(\frac{Z_i - z}{h_z}\right) \right] \left[\sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right) \cdot \left(\frac{Z_i - z}{h_z}\right) \right]}{\left[\sum_{i=1}^n K_z\left(\frac{Z_i - z}{h_z}\right) \right]^2}.
\end{aligned} \quad (26)$$

To prevent that the denominator becomes zero, I introduce a weight function, $w(z)$ such that $\int_{S_w} w(z) dz = 1$ with compact support S_w .

Finally obtain the estimator of $H(Y)$:

$$H(y) = - \int_{y_0}^y \int_{S_w} w(z) [G_y(v | z) / G_z(v | z)] dv dz. \quad (27)$$

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